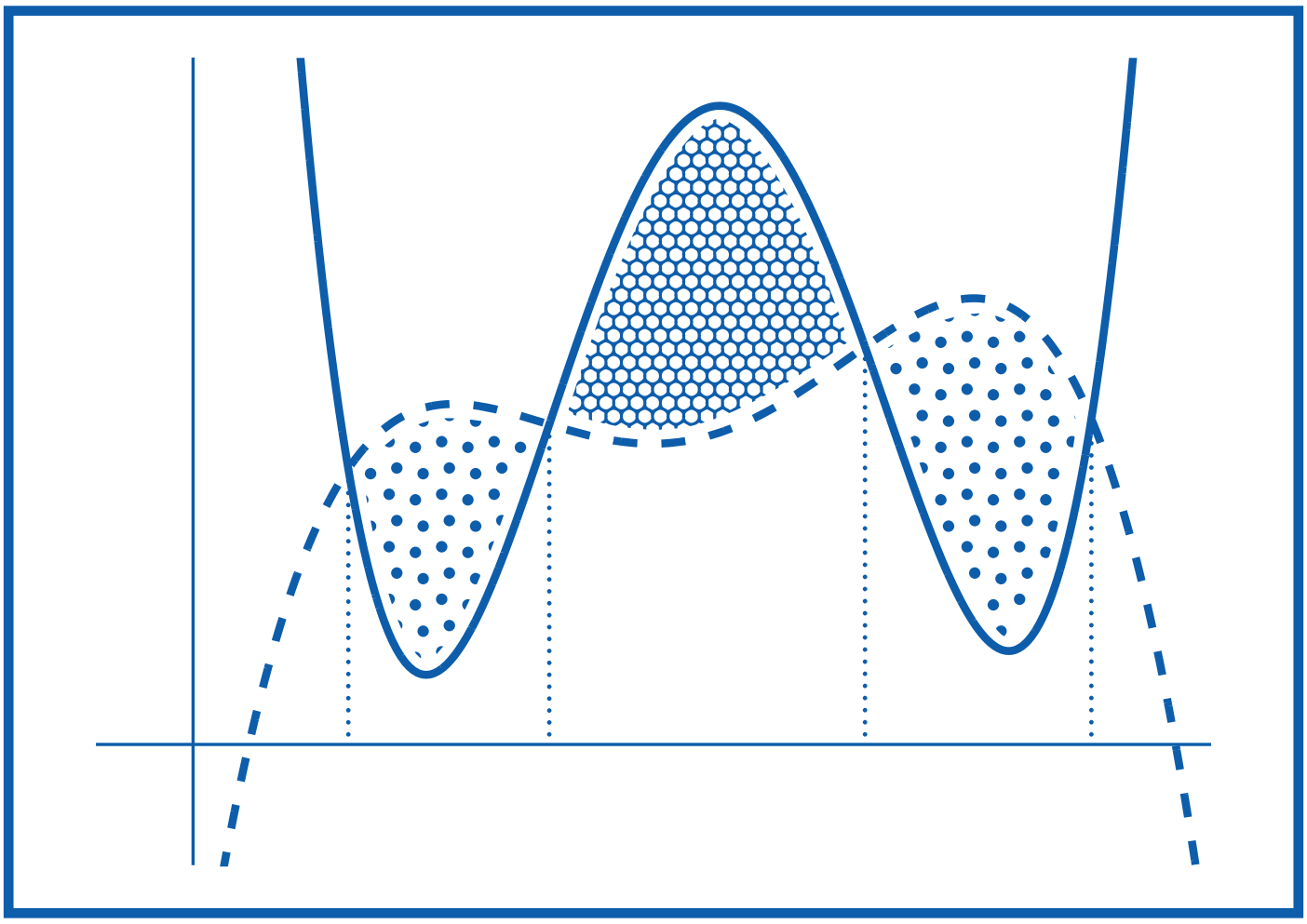
LEMA - Lehr- und Lernmaterialien
Hergestellt von der Abteilung für Inklusion und Lehrmittel (LMZ)
am Bundes-Blindeninstitut Wien

Grafikkatalog
12. Schulstufe (8. AHS)

Autor: Elisabeth Stanetty • Grafiken: Alexander Steiner, Angela Prendl



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1020 Wien, Tel.: 01/728 08 66-405, 406, lmz@bbi.at, www.bbi.at

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Stammfunktionen

Schulstufe 12

Mögliche Stammfunktionen (F)

diverser Funktionen (f)

Inhalt

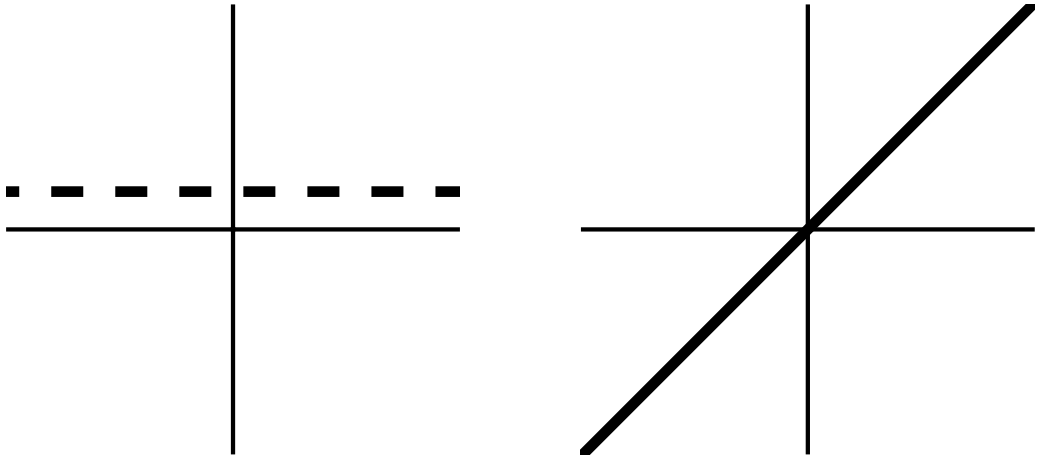
Stammfunktionen

f ... Funktion: - - - -

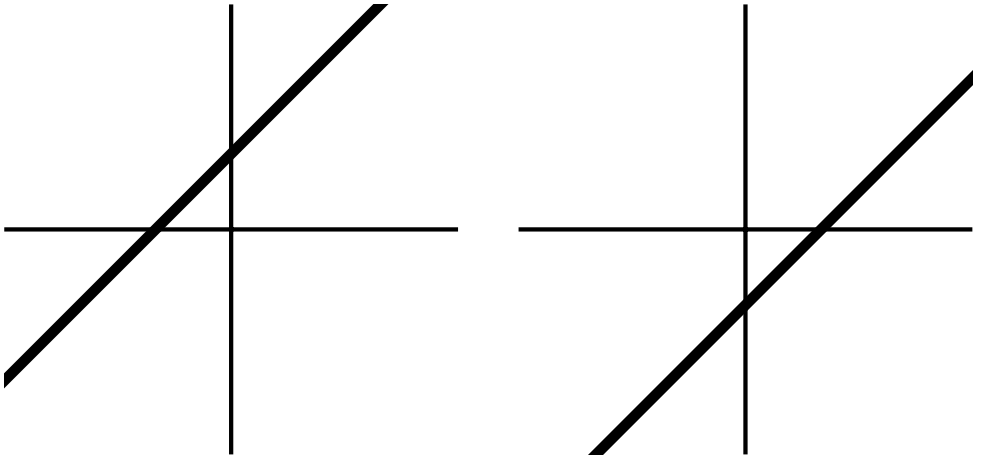
F ... eine mögliche Stammfunktion: ——

Ursprung Koordinatensystem: —┼—

f(x) =1

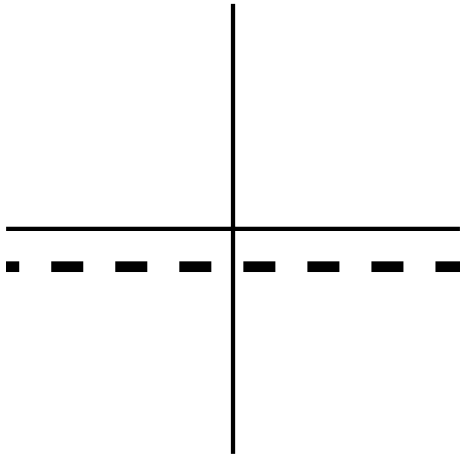


F(x) =x +C

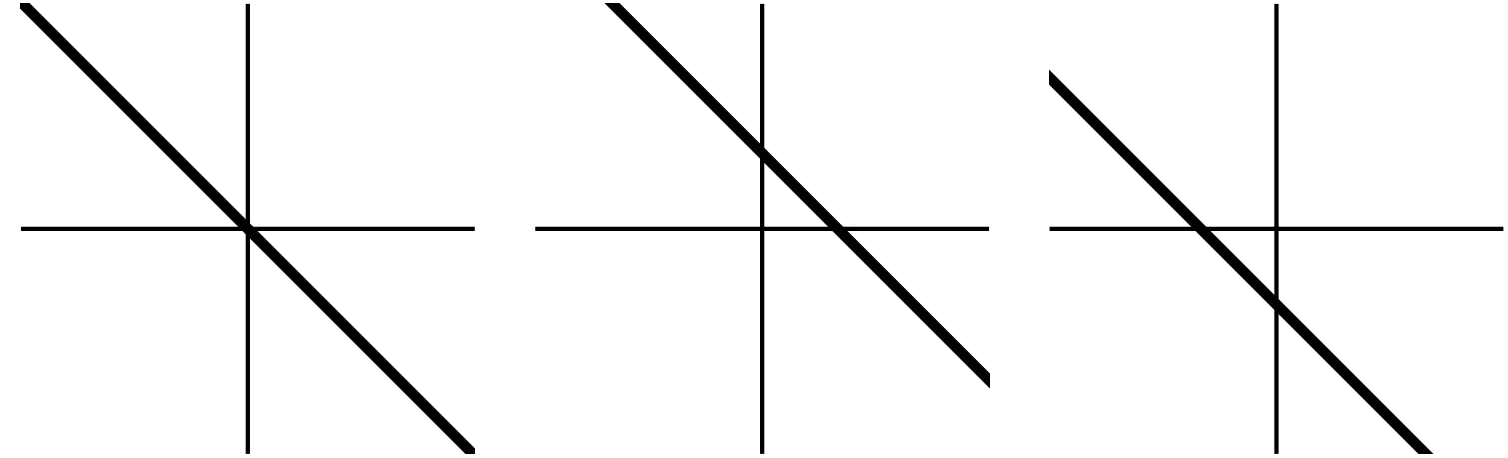


Stammfunktionen

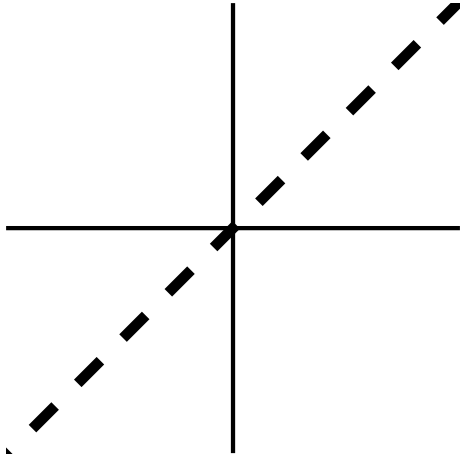
$f(x) = -1$



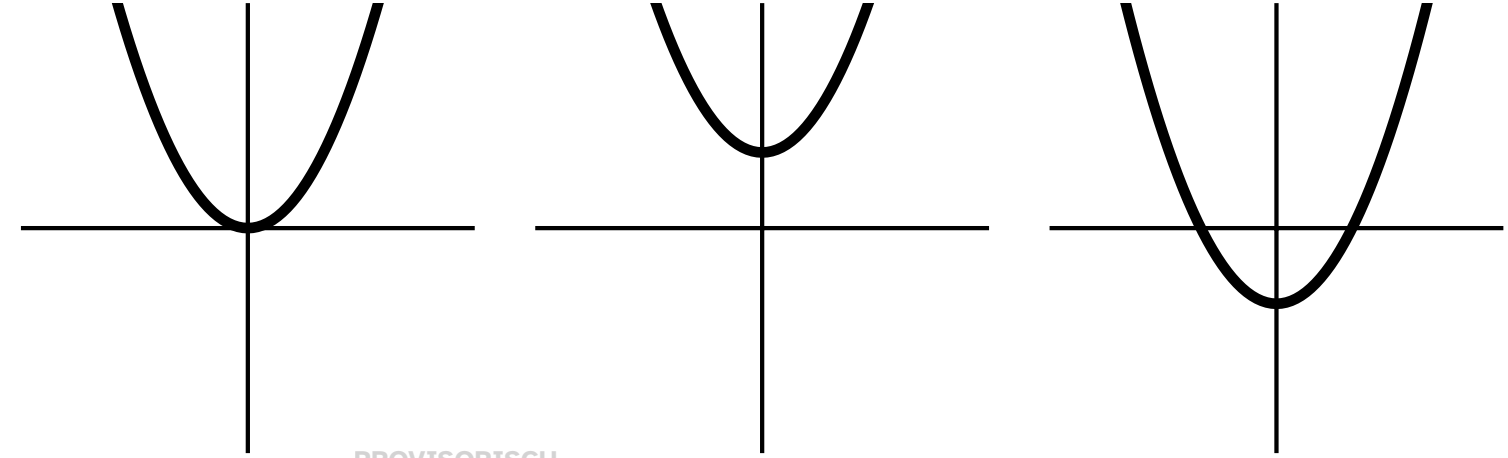
$F(x) = -x + C$



$f(x) = x$



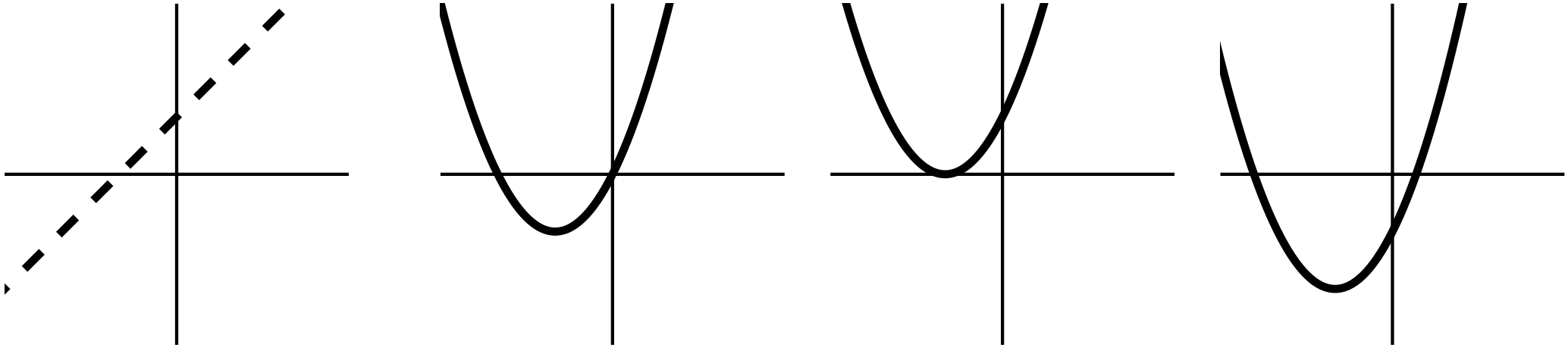
$F(x) = x^2/2 + C$



Stammfunktionen

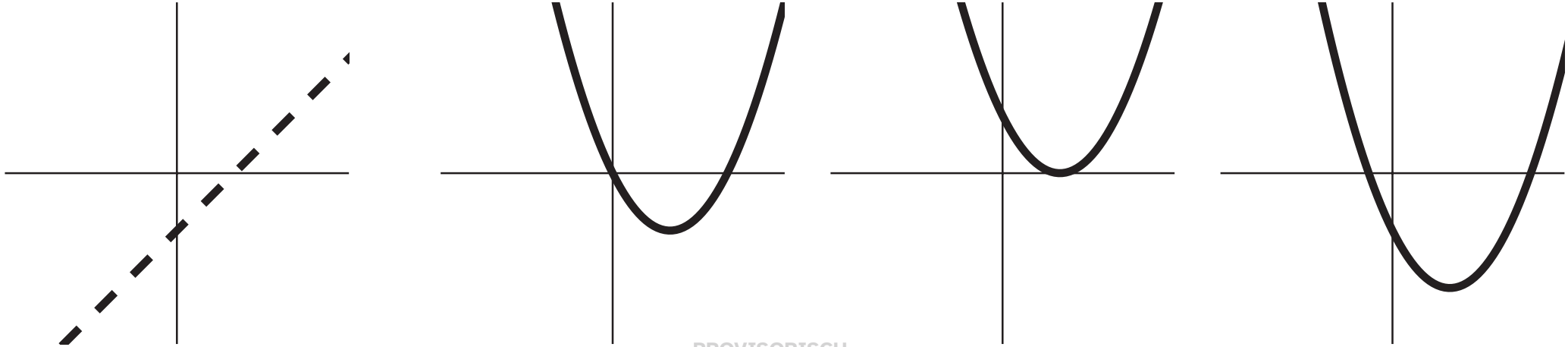
f(x) =x +2

F(x) =x^2/2 +2 *x +C



f(x) =x -2

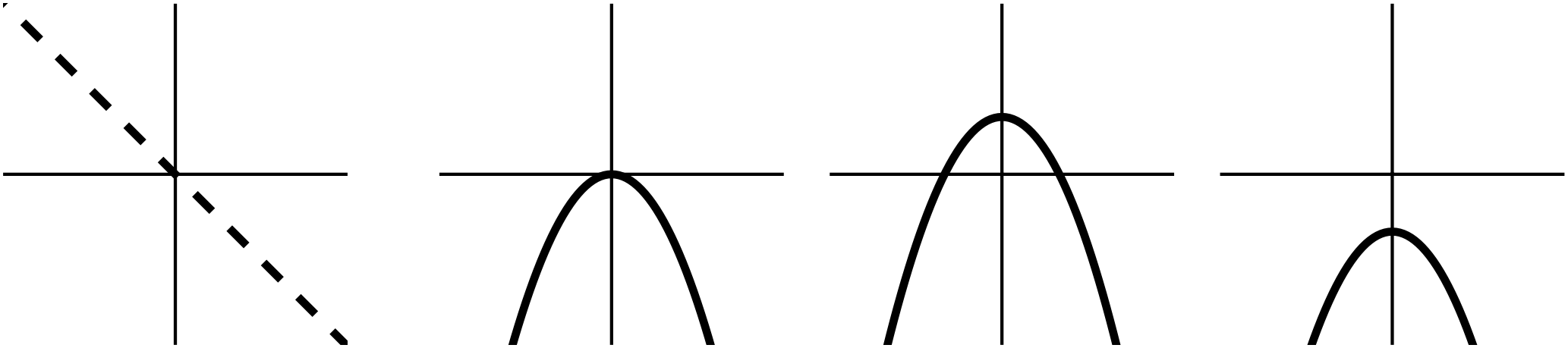
F(x) =x^2/2 -2 *x +C



Stammfunktionen

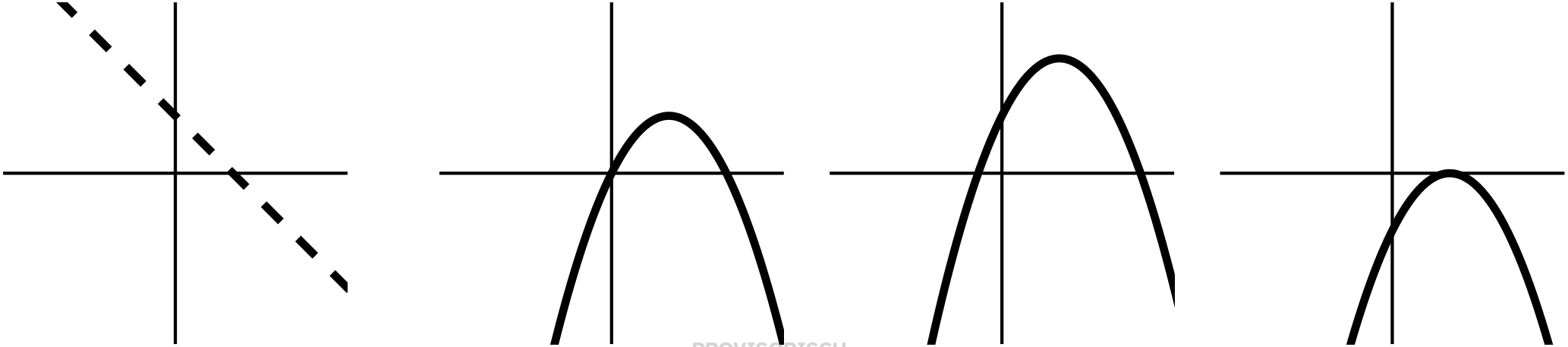
f(x) =-x

F(x) =-x^2/2 +C



f(x) =-x +2

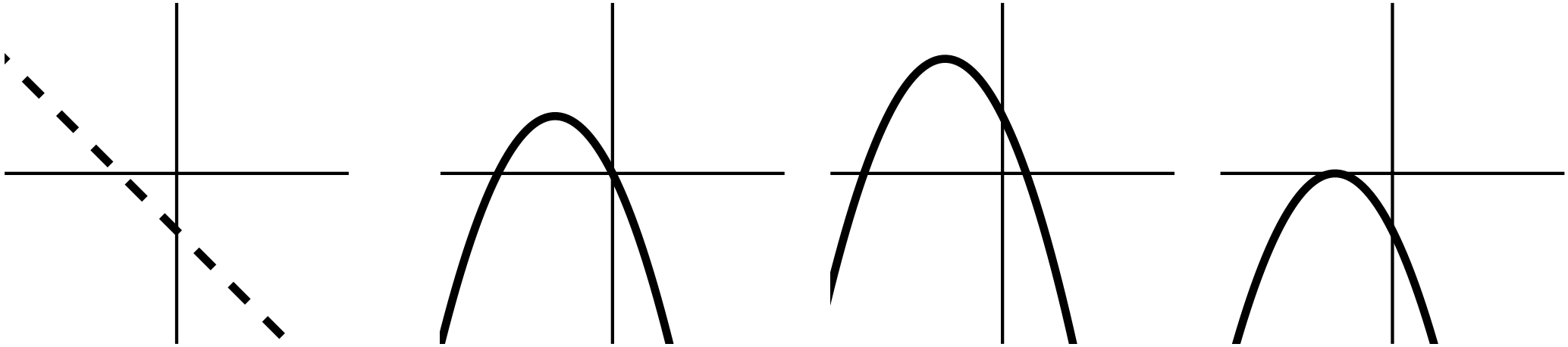
F(x) =-x^2/2 +2 *x +C



Stammfunktionen

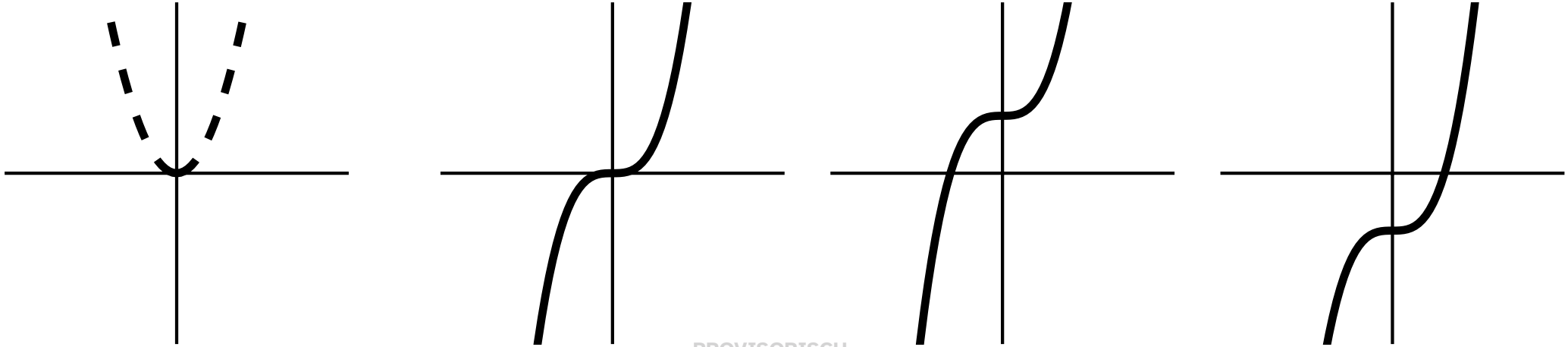
f(x) =-x -2

F(x) =-x^2/2 -2 *x +C



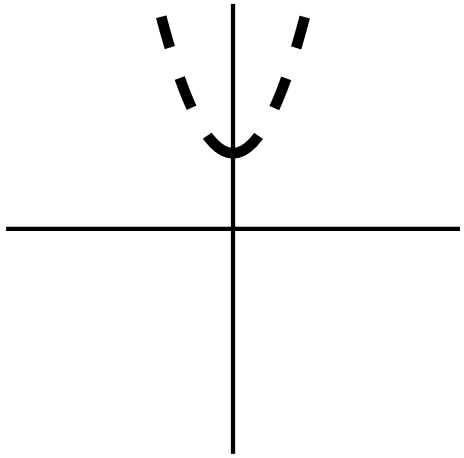
f(x) =x^2

F(x) =x^3/3 +C

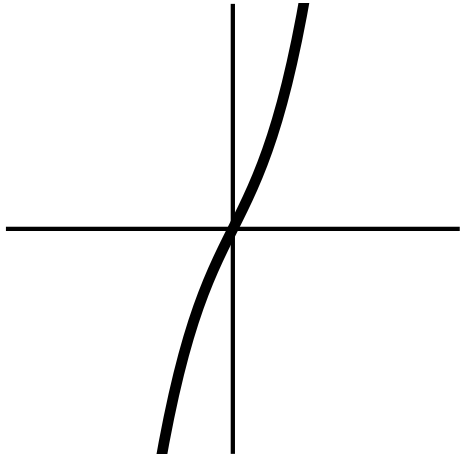


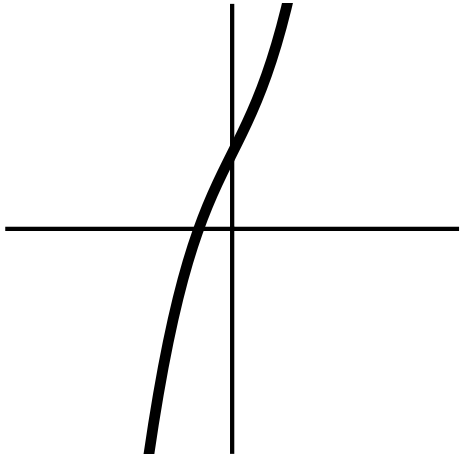
Stammfunktionen

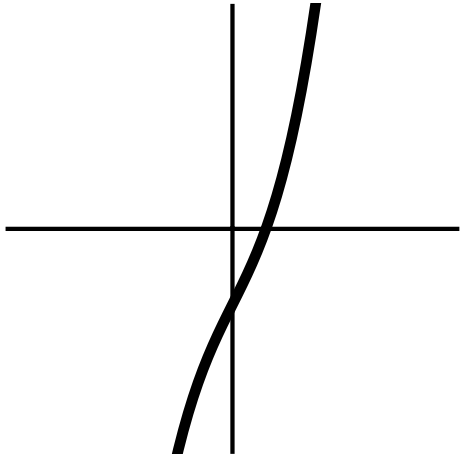
f(x) =x^2 +2



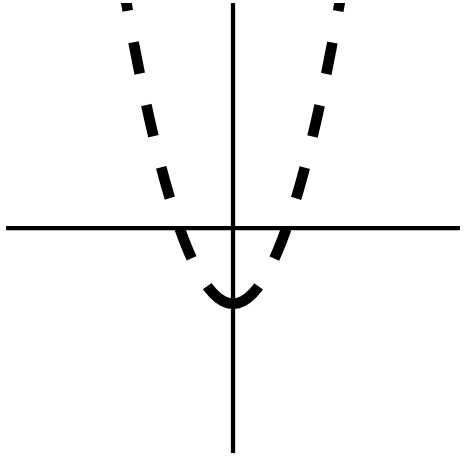
F(x) =x^3/3 +2 *x +C



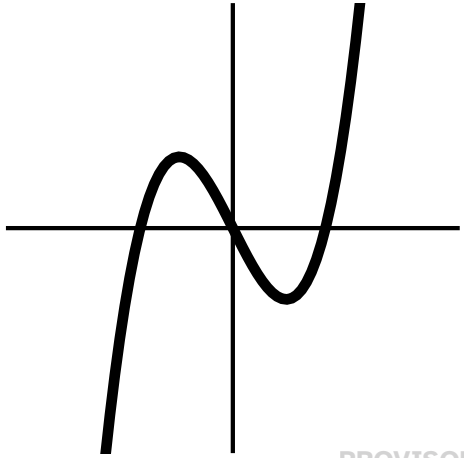


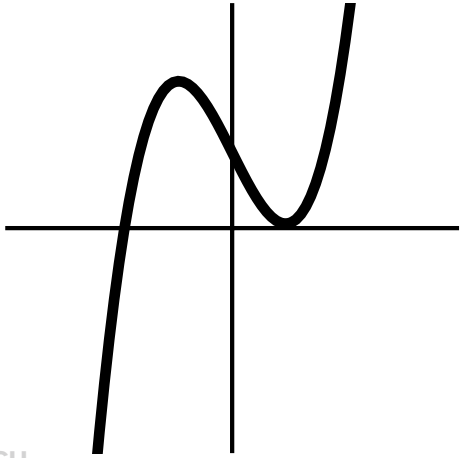


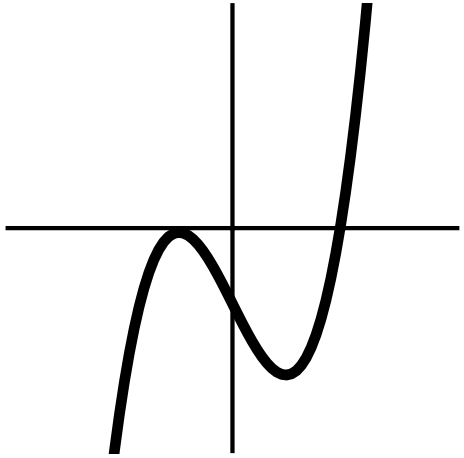
f(x) =x^2 -2



F(x) =x^3/3 -2 *x +C



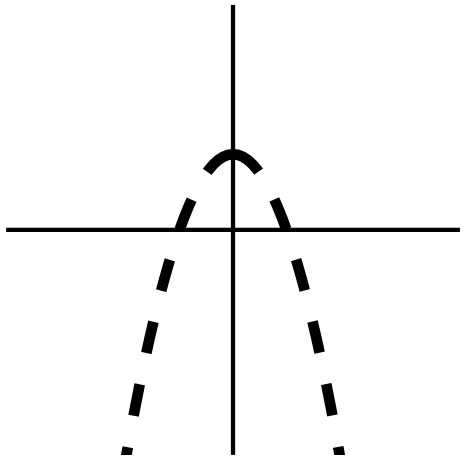


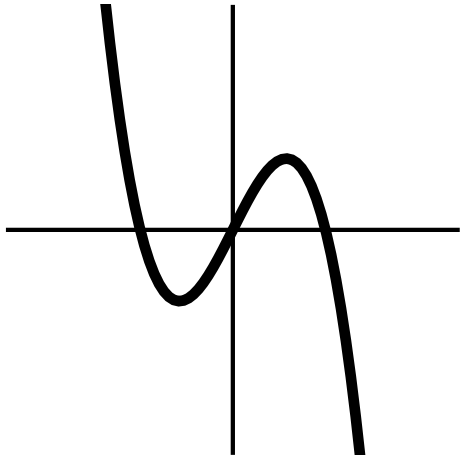


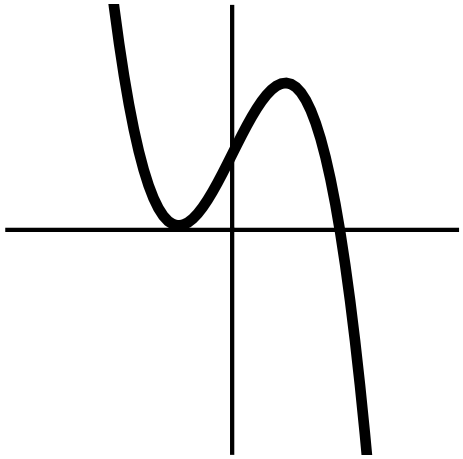
Stammfunktionen

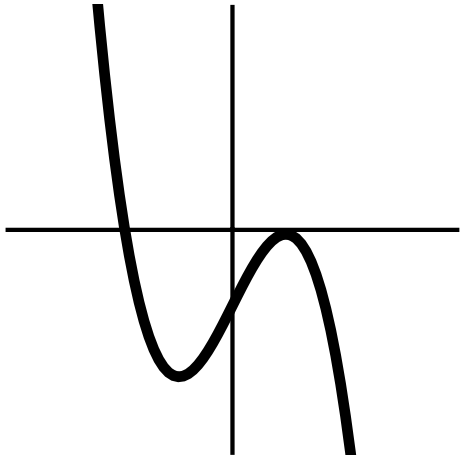
$f(x) = -x^2 + 2$

$F(x) = -x^3/3 + 2 \cdot x + C$



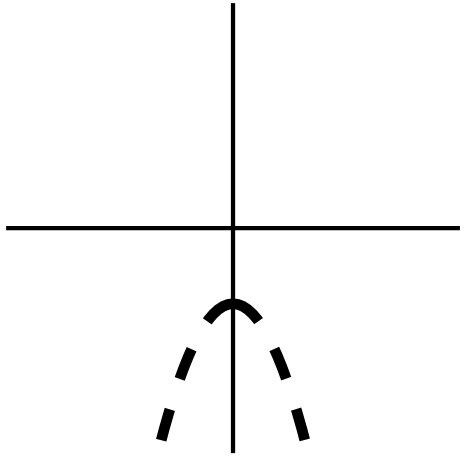


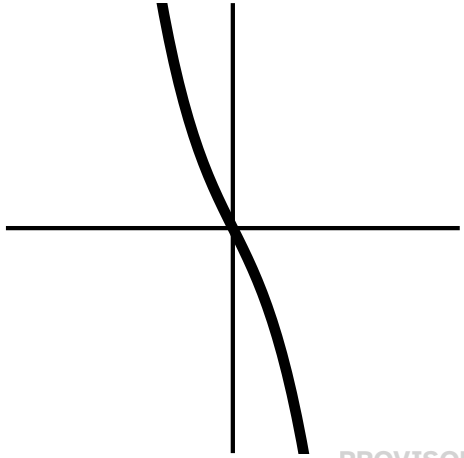


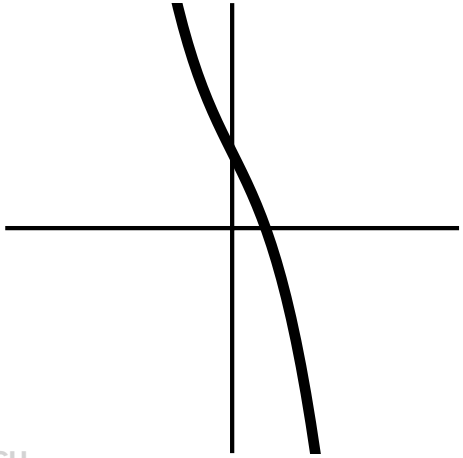


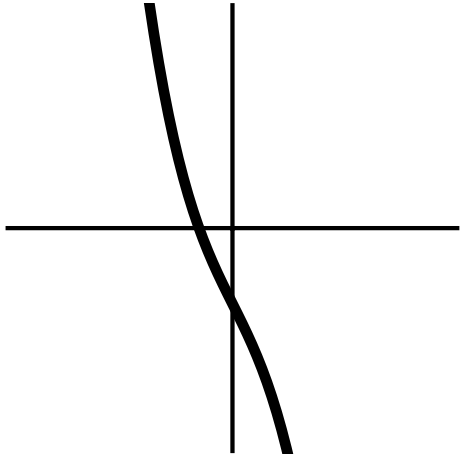
$f(x) = -x^2 - 2$

$F(x) = -x^3/3 - 2 \cdot x + C$





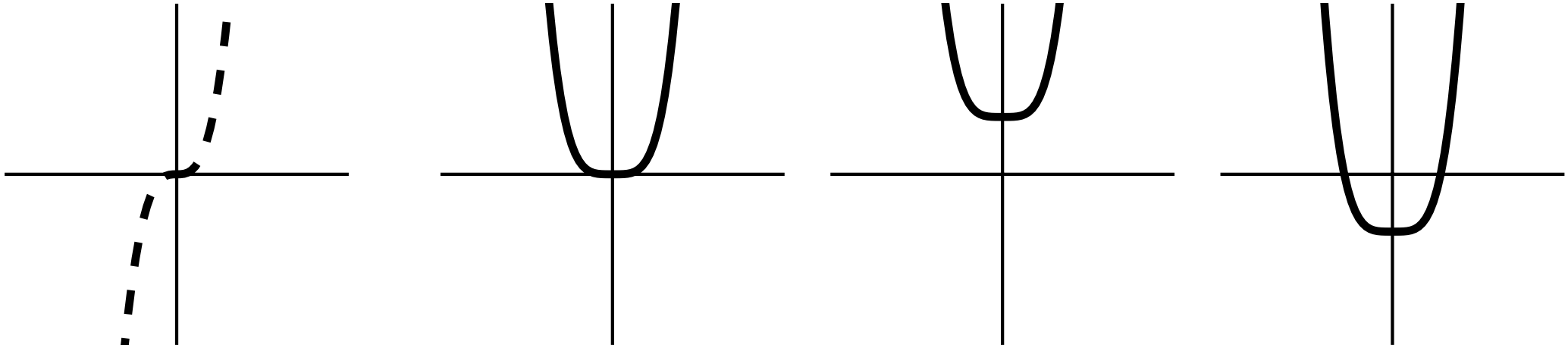




Stammfunktionen

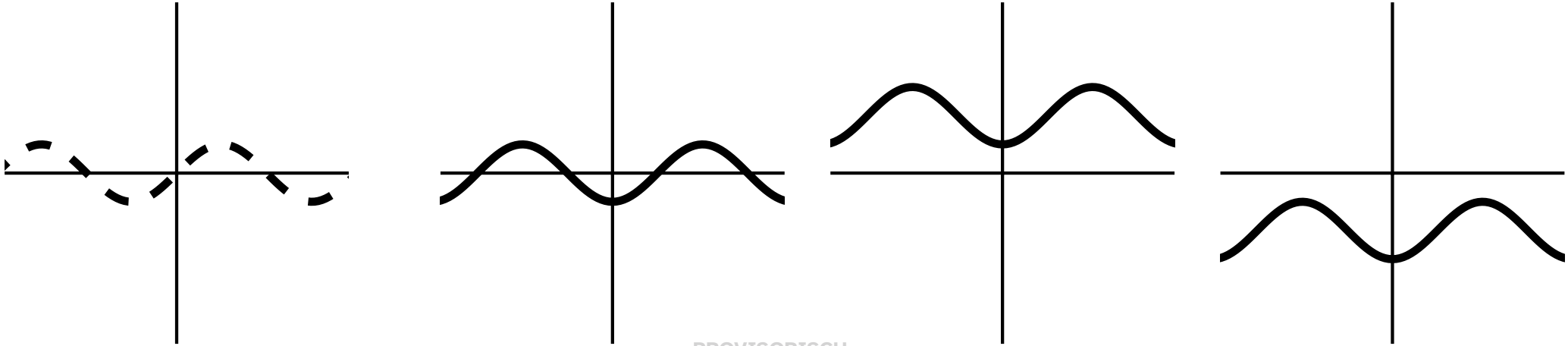
f(x) =x^3

F(x) =x^4/4 +C



f(x) ='sin(x)

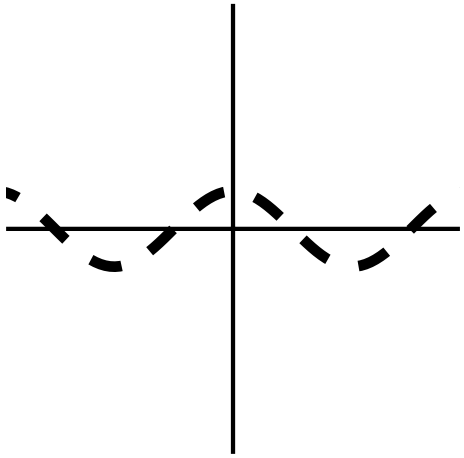
F(x) =-'cos(x) +C

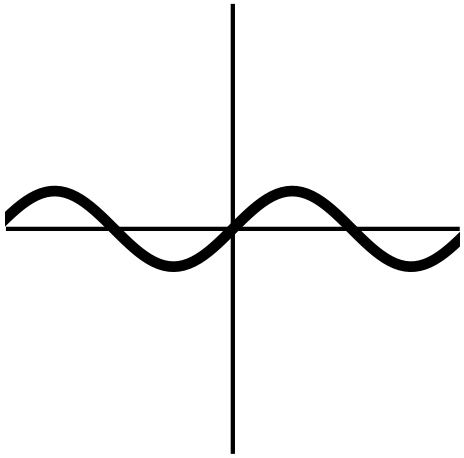


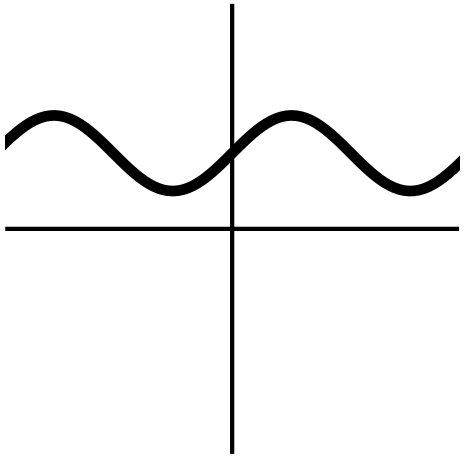
Stammfunktionen

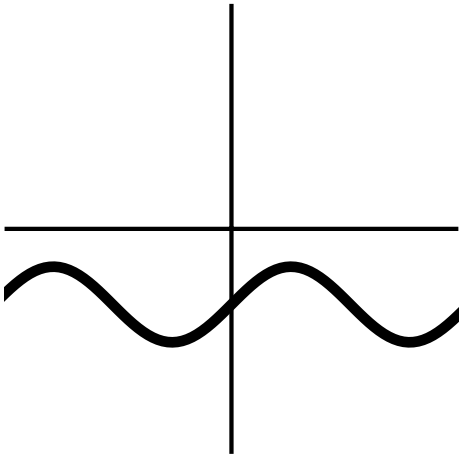
f(x) ='cos(x)

F(x) ='sin(x) +C



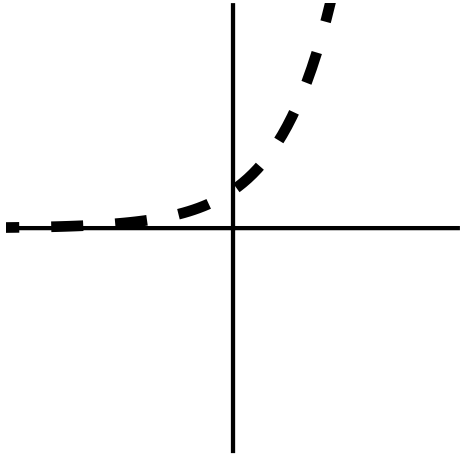


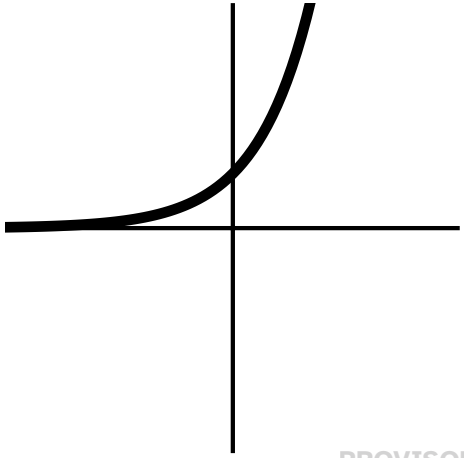


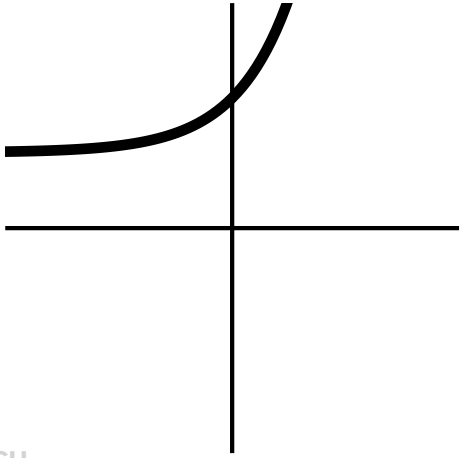


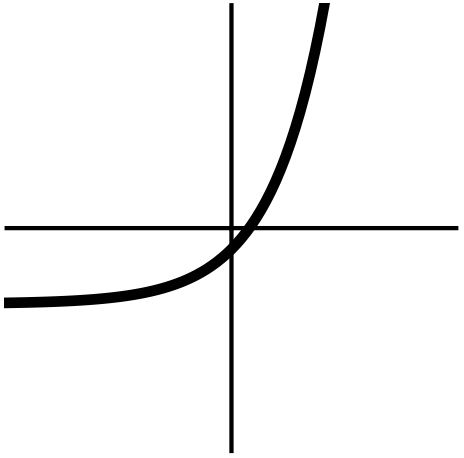
f(x) =2^x

F(x) =2^x/'ln(2) +C



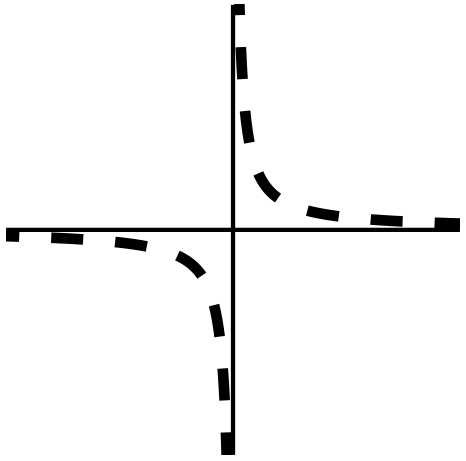




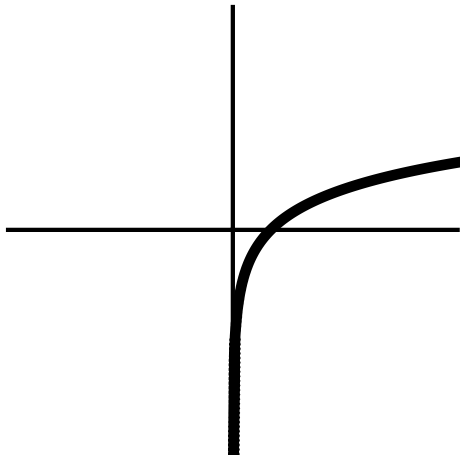


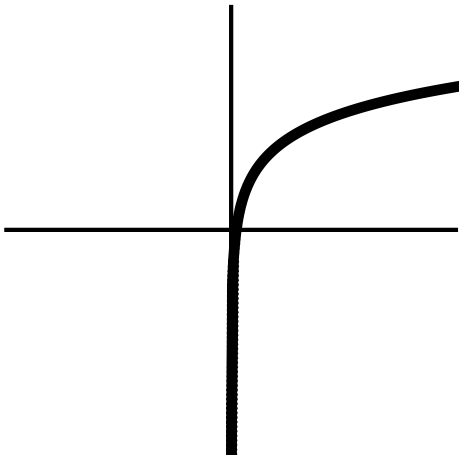
Stammfunktionen

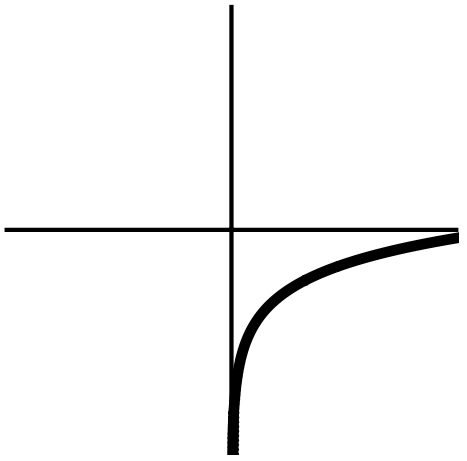
f(x) =1/x



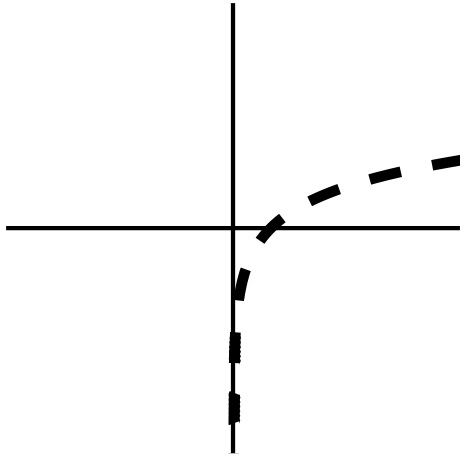
F(x) ='ln(x) +C



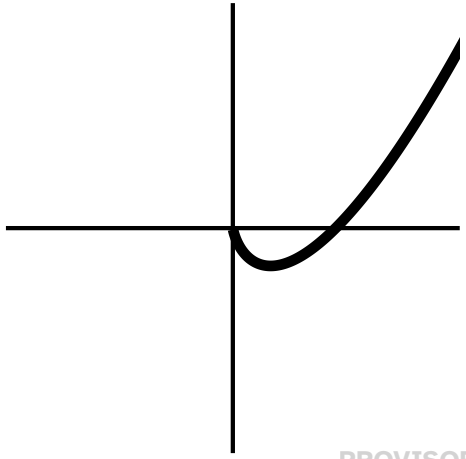


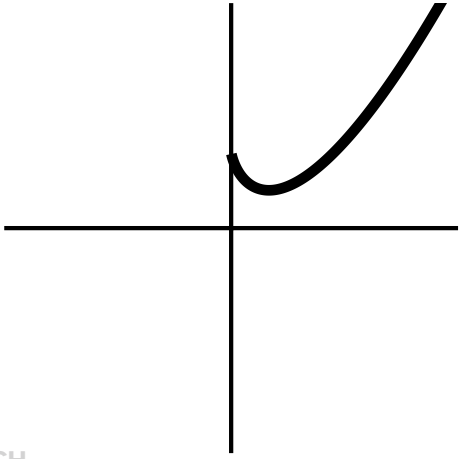


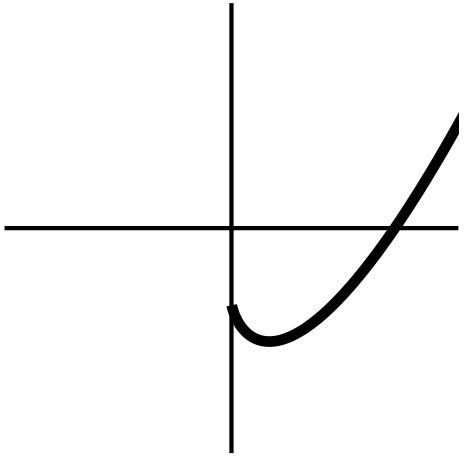
f(x) ='ln(x)



F(x) =x*'ln(x) -x +C







Integrale

Schulstufe 12

Darstellung und Berechnung von Flächen

Weglängen

Volumina

Inhalt

13 - 24 Volumsberechnungen

13 Kreis x^2 +y^2 =r^2

14 Kreisrotation um x-Achse

15 Kreisrotation um y-Achse

16 Ellipse b^2x^2 +a^2y^2 =a^2b^2

17 Ellipsenrotation um x-Achse

18 Ellipsenrotation um y-Achse

19 Hyperbel b^2x^2 -a^2y^2 =a^2b^2

20 Hyperbelrotation um x-Achse

21 Hyperbelrotation um y-Achse

22 Parabel y^2 =2px

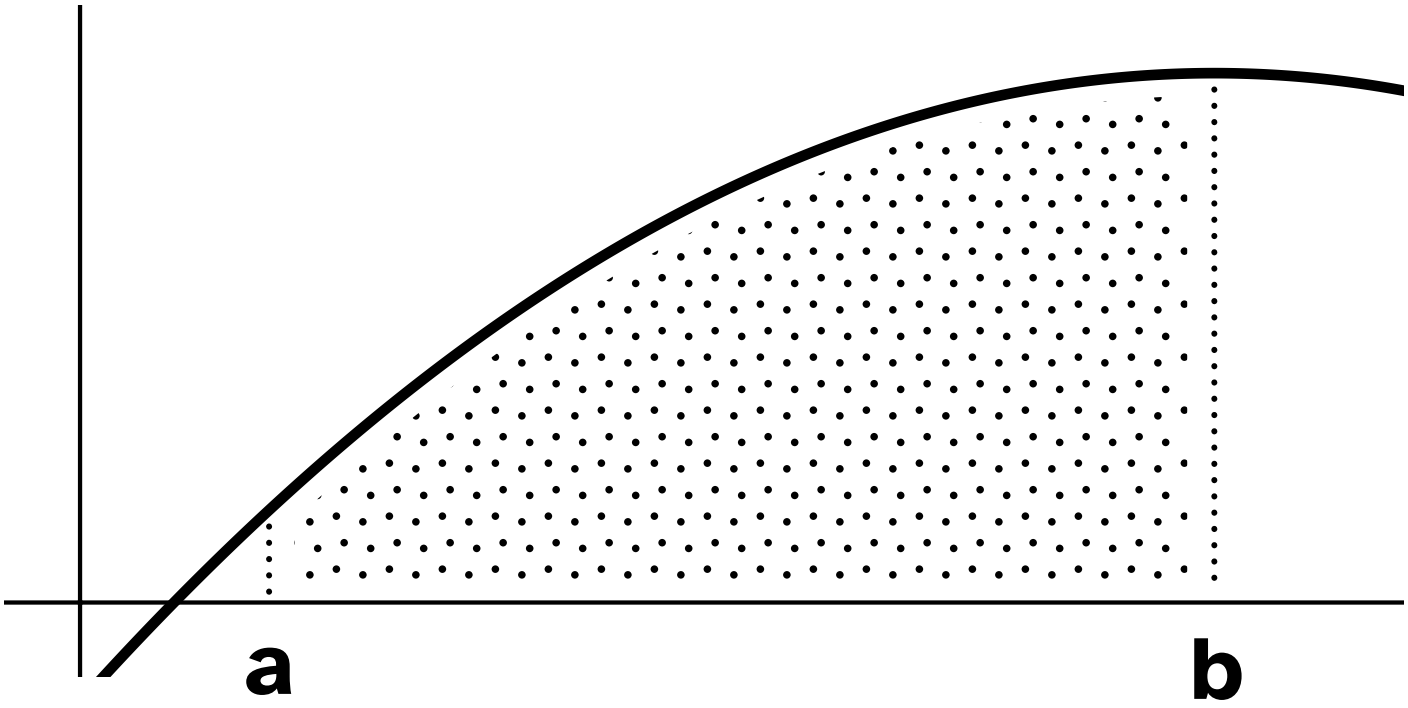
23 Parabelrotation um x-Achse

24 Parabelrotation um y-Achse

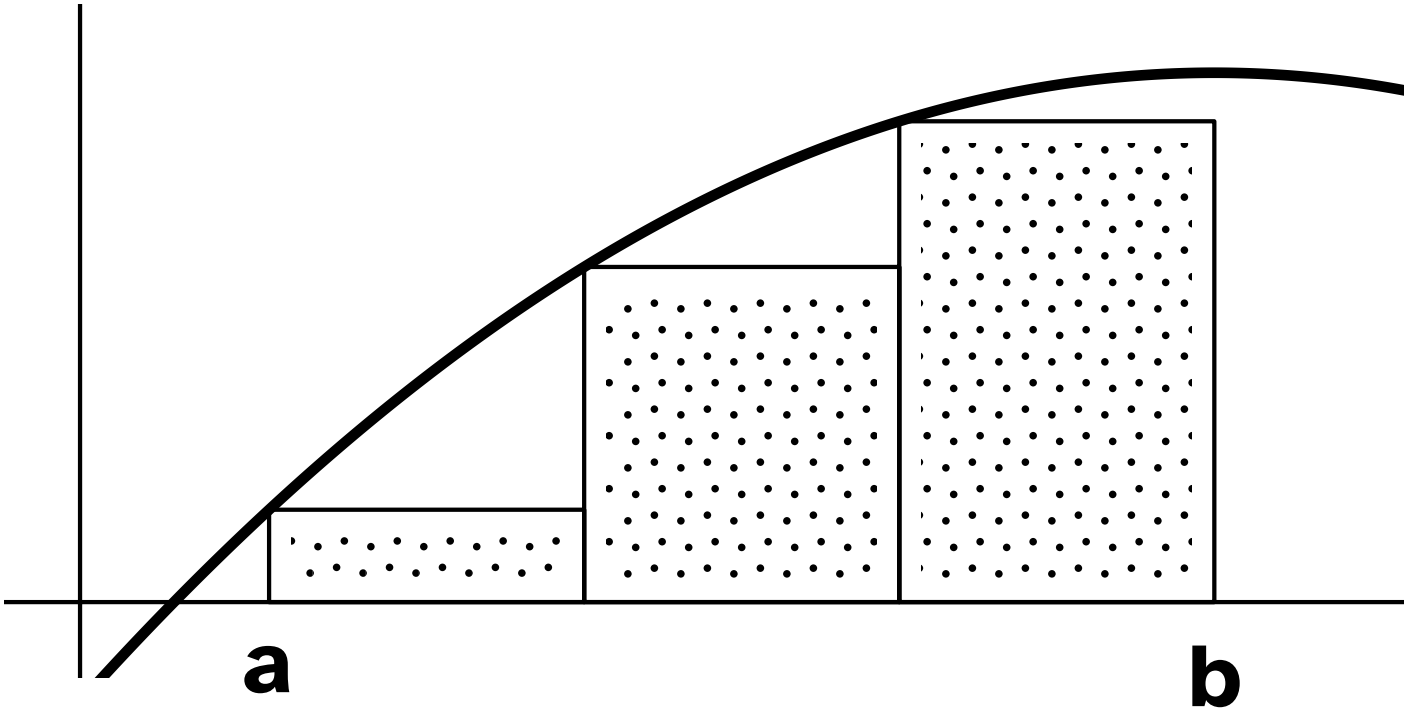
f ... Funktion: ———

a, b ... Grenzen

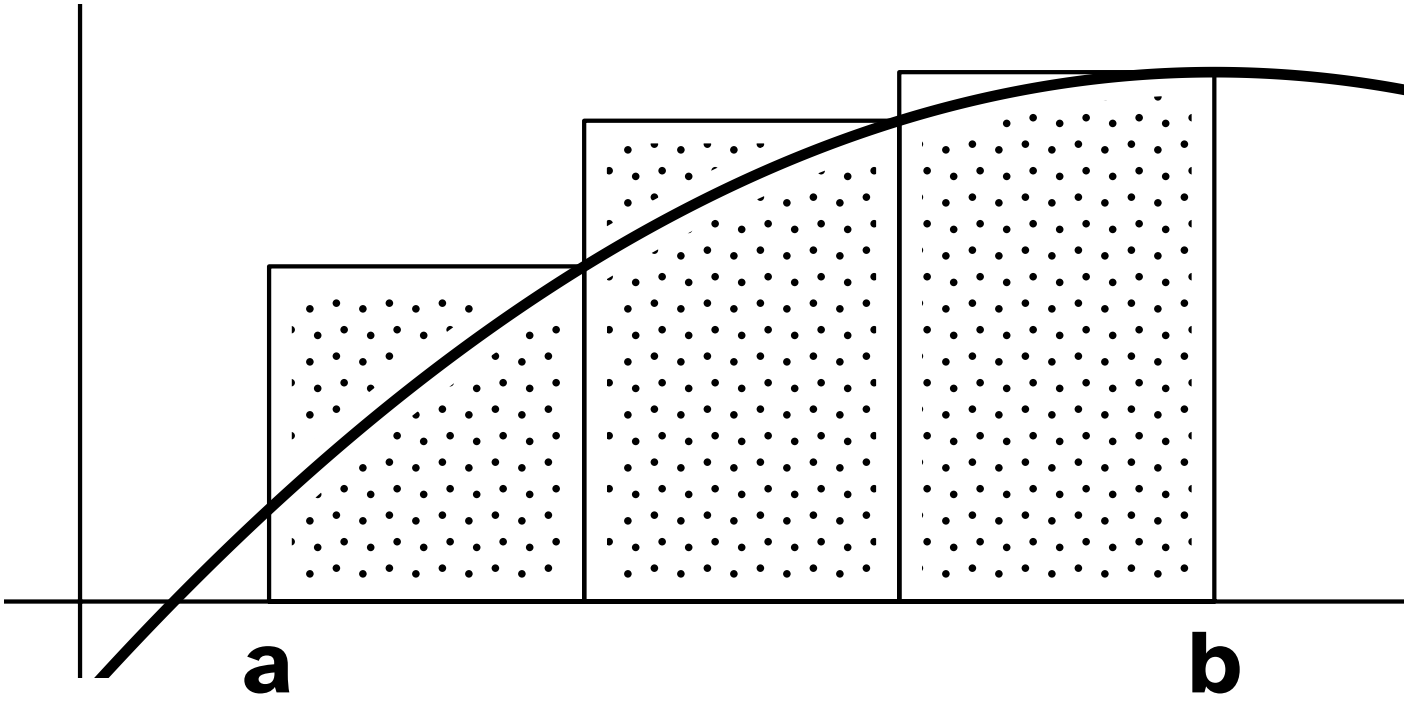
Integral



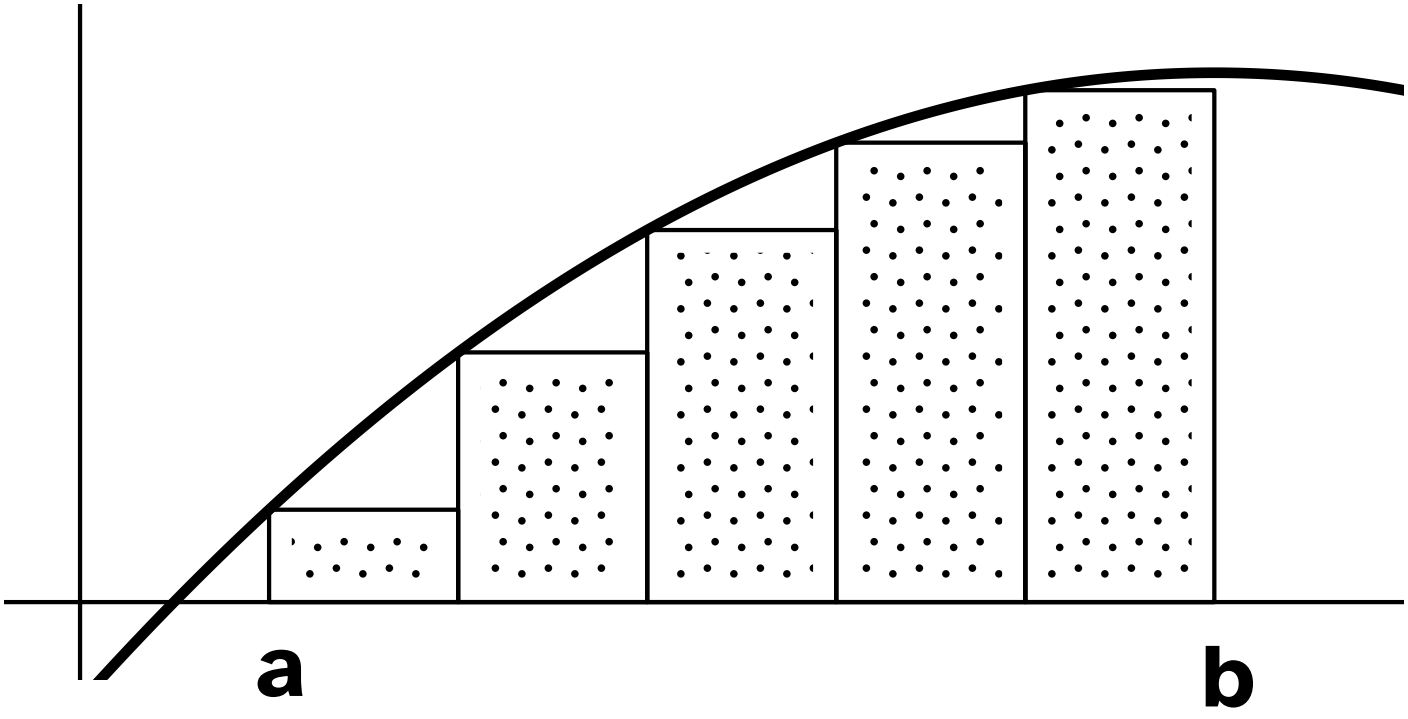
Untersumme, n =3



Obersumme, n =3



Untersumme, n =5



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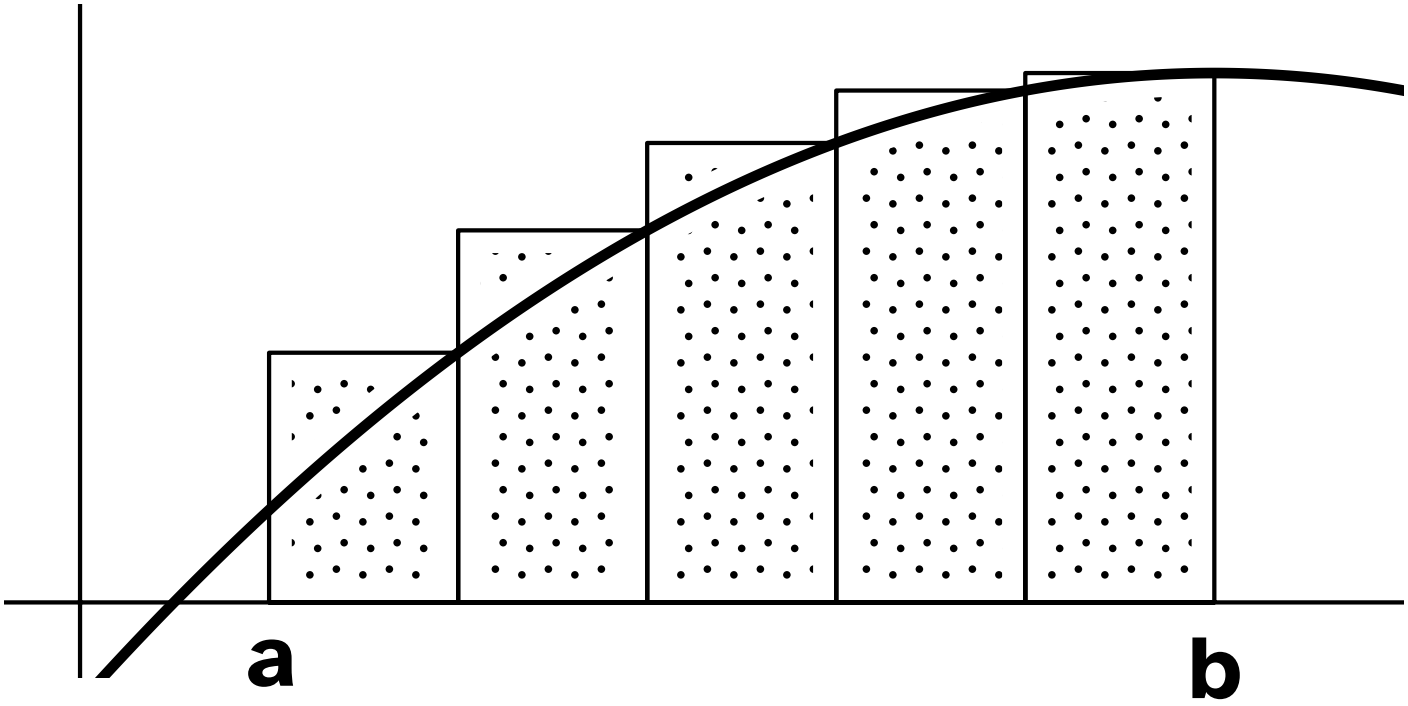
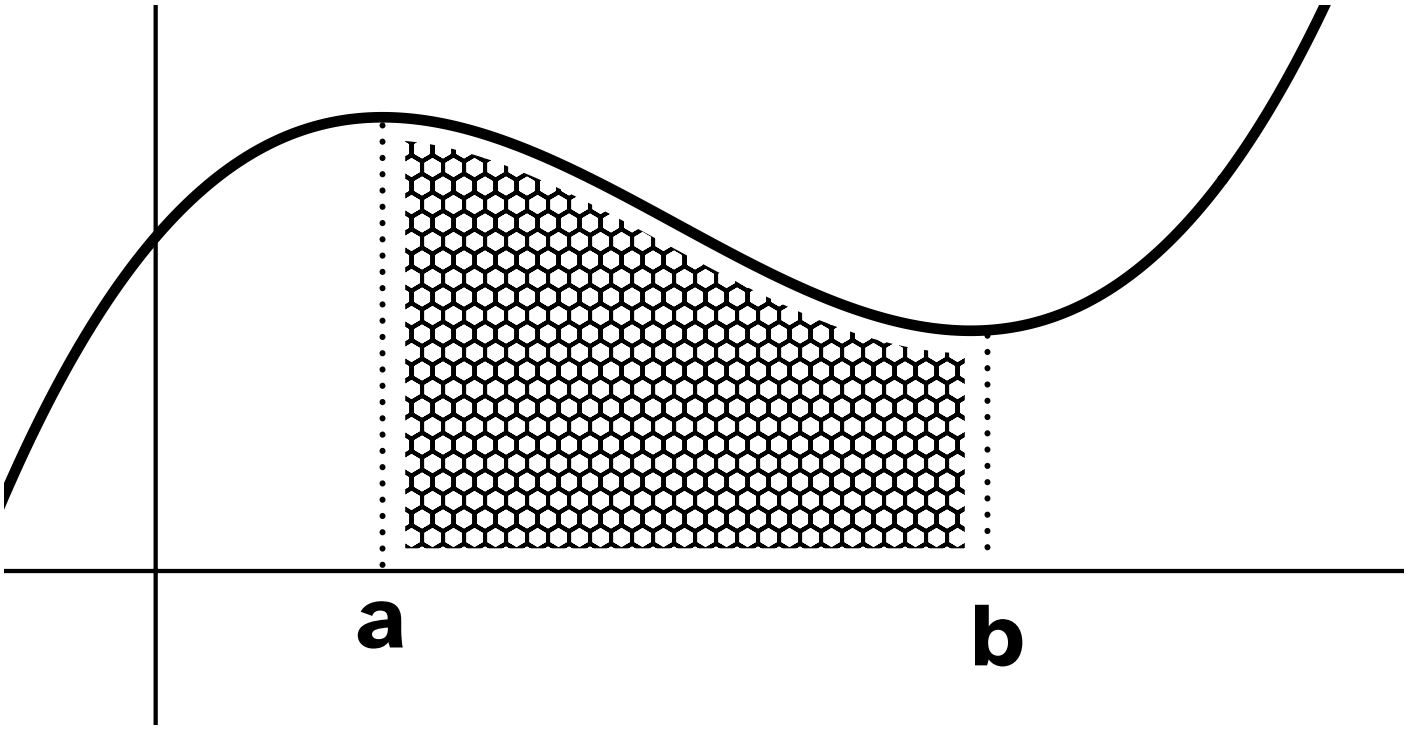
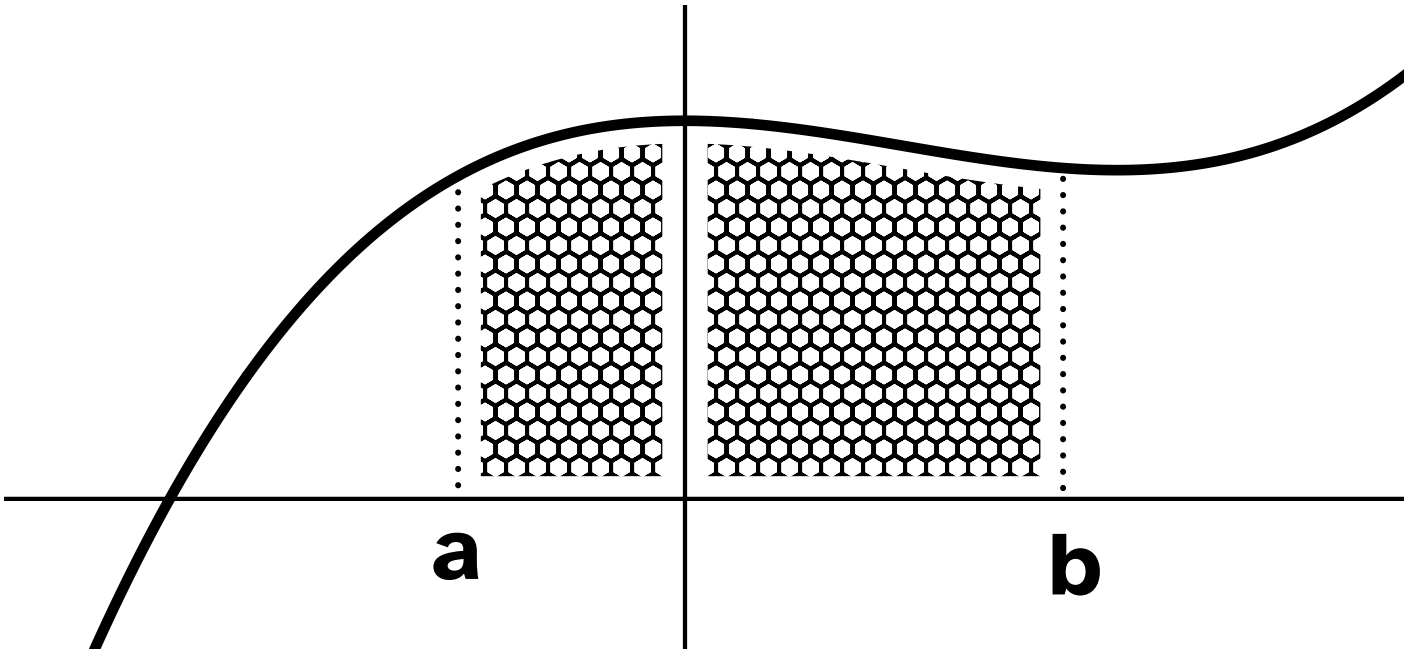


Int Flächen

'int[a; b](f(x) 'dx) =A

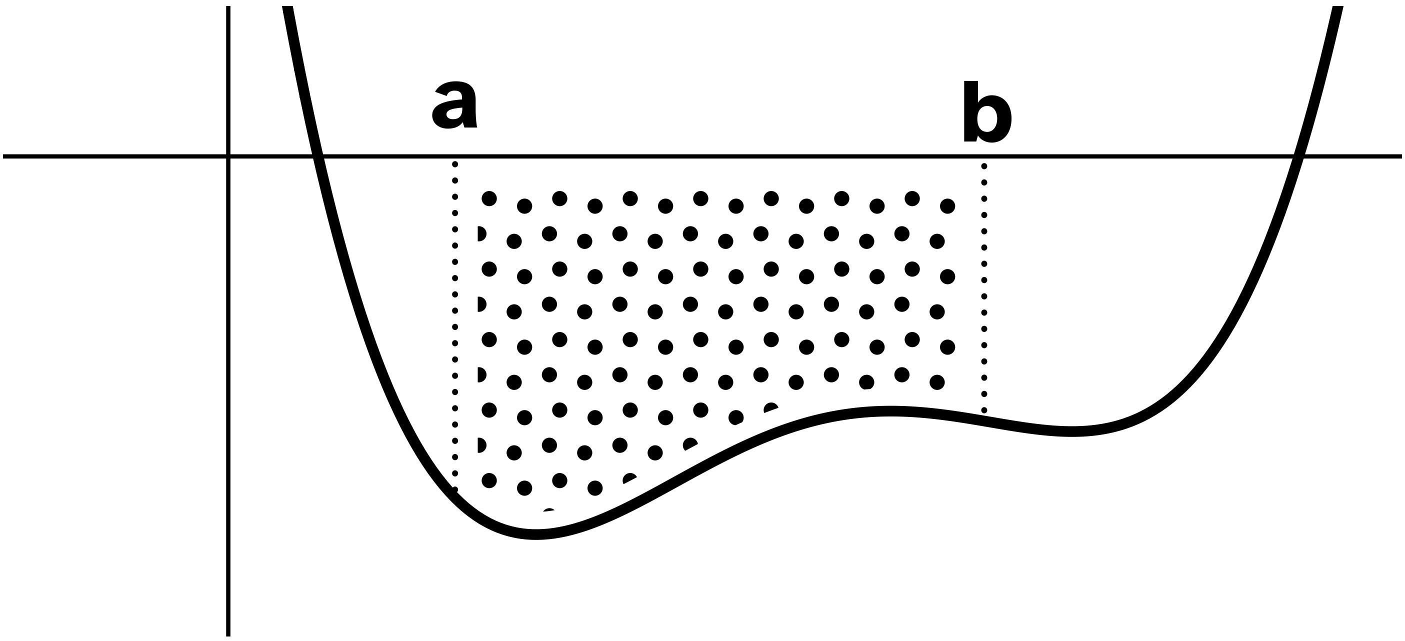
f: ——— ‖ A: 

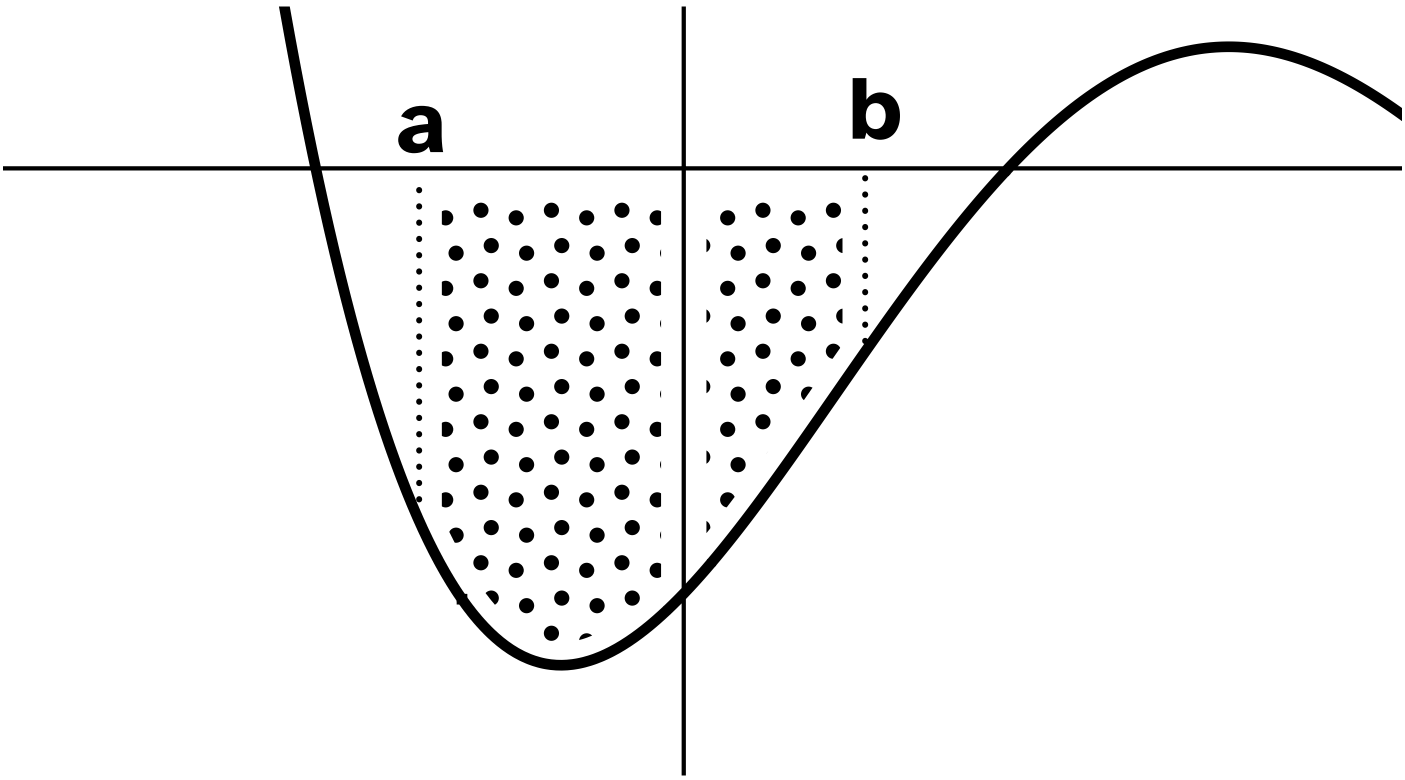




'int[a; b](f(x) 'dx) =-A

f: ——— ‖ A: ⁘⁘





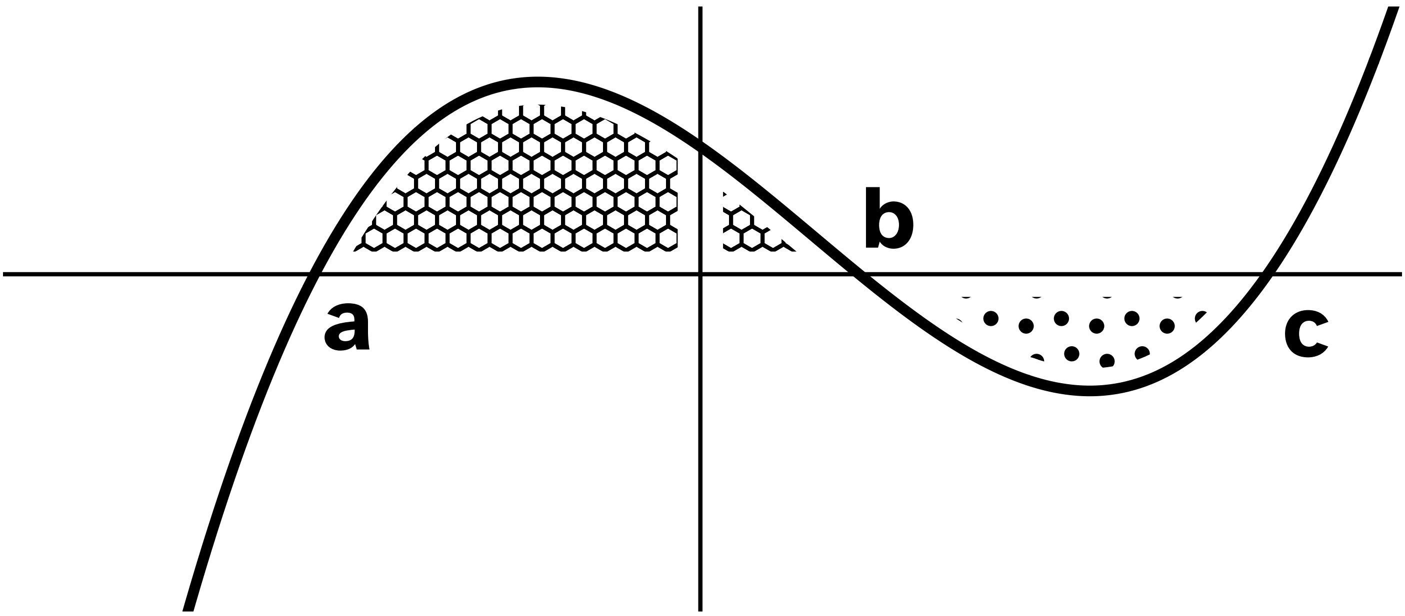
A_1 =|'int[a; b](f(x) 'dx)|

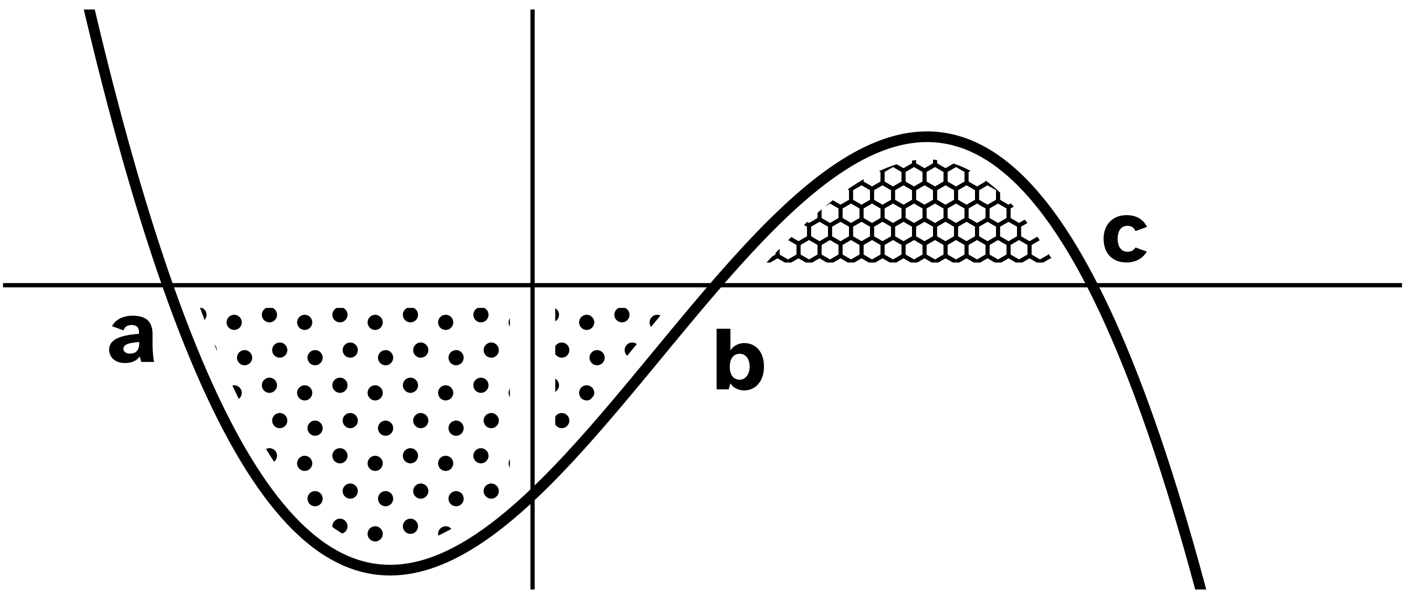
A_2 =|'int[b; c](f(x) 'dx)|

A =A_1 +A_2

f: ———

A_1, A_2: sowie

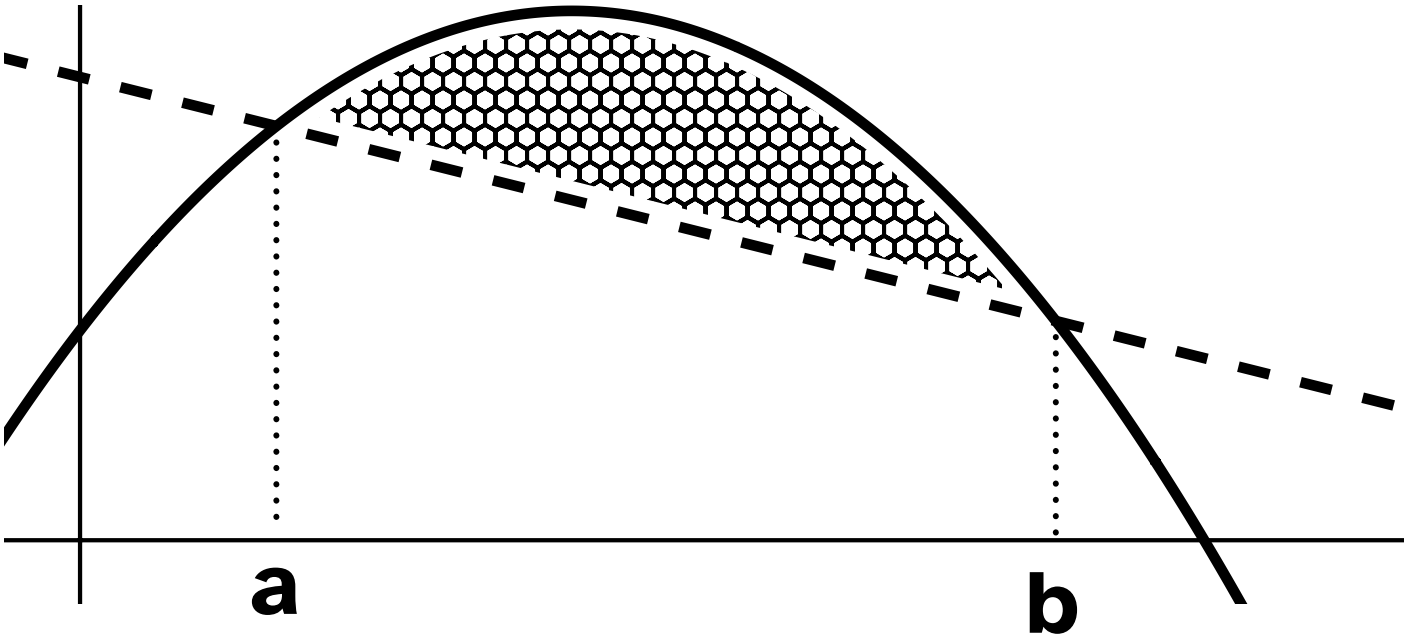


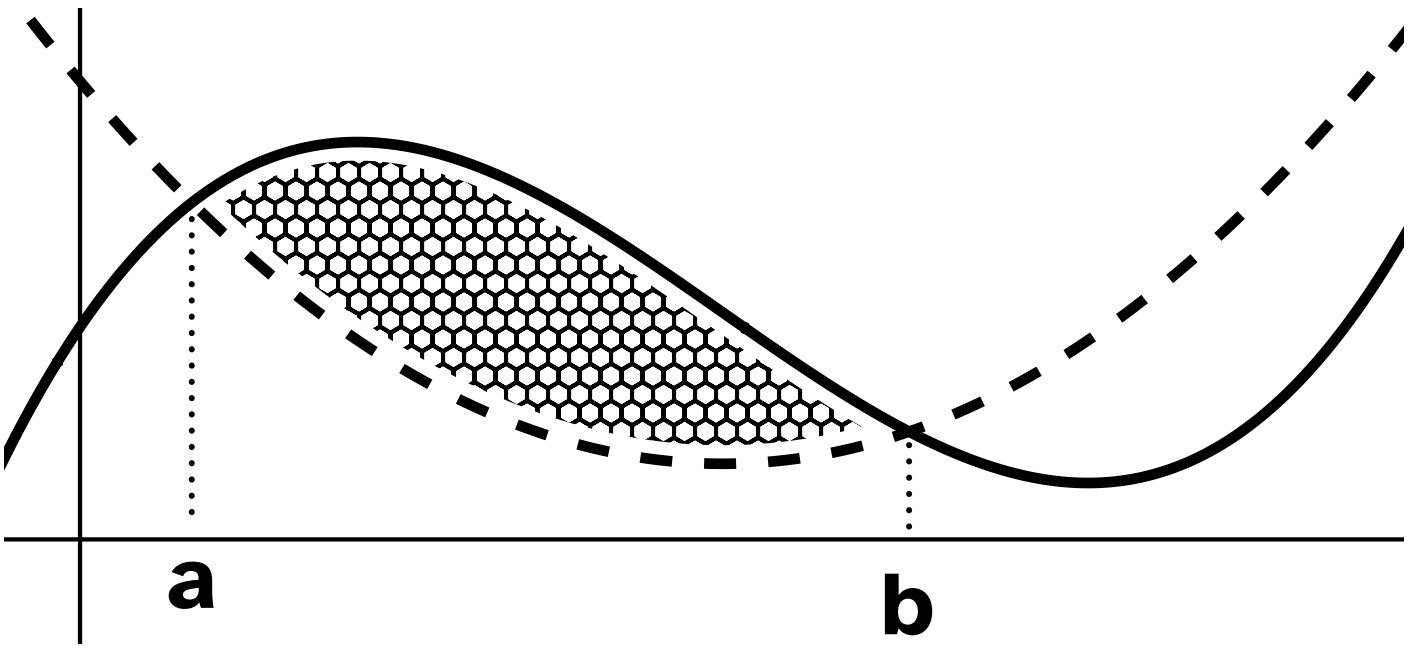


$$A = \int_a^b (f(x) - g(x))\,dx$$

f: ——— ‖ g: - - - -

A:

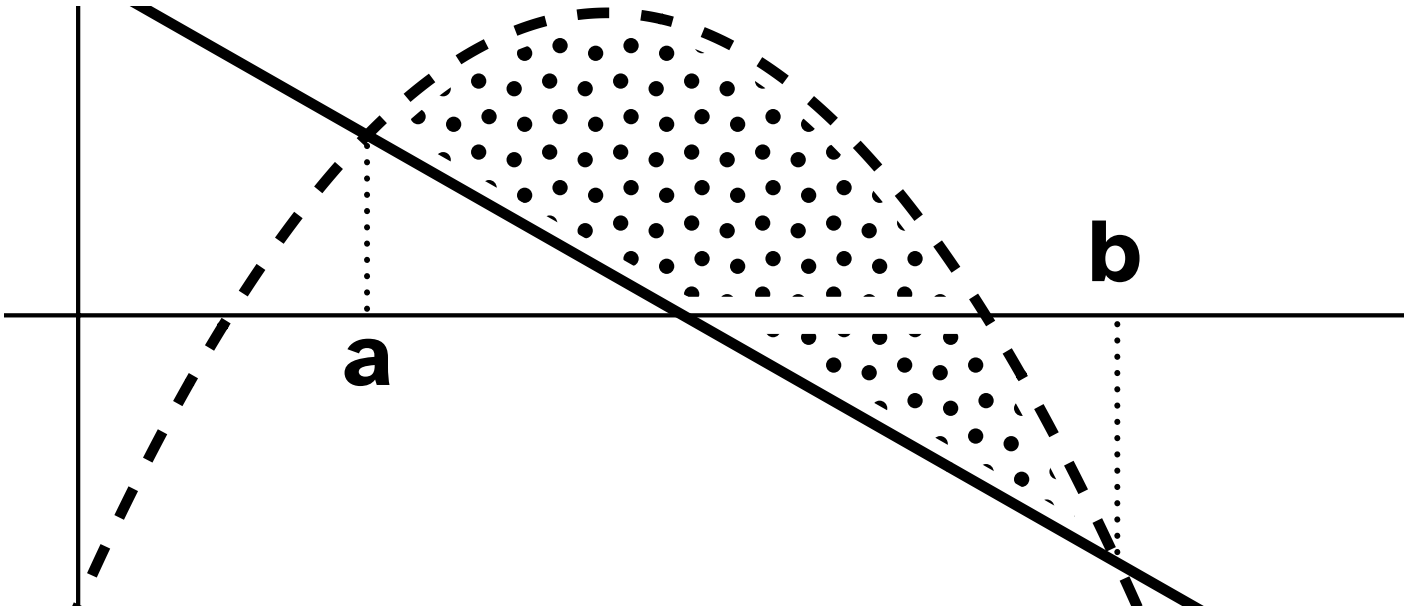


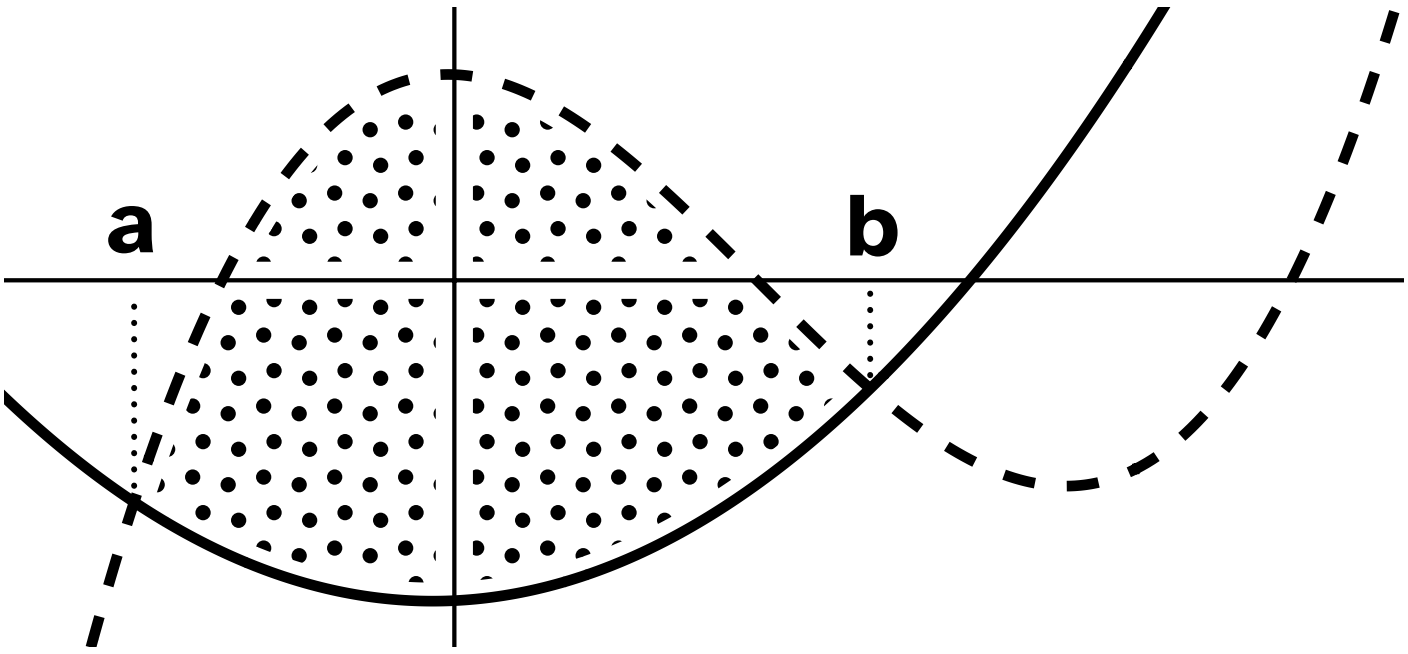


$A = \int_a^b (g(x) - f(x))\,dx$

f: ——— ‖ g: - - - -

A: ⁘



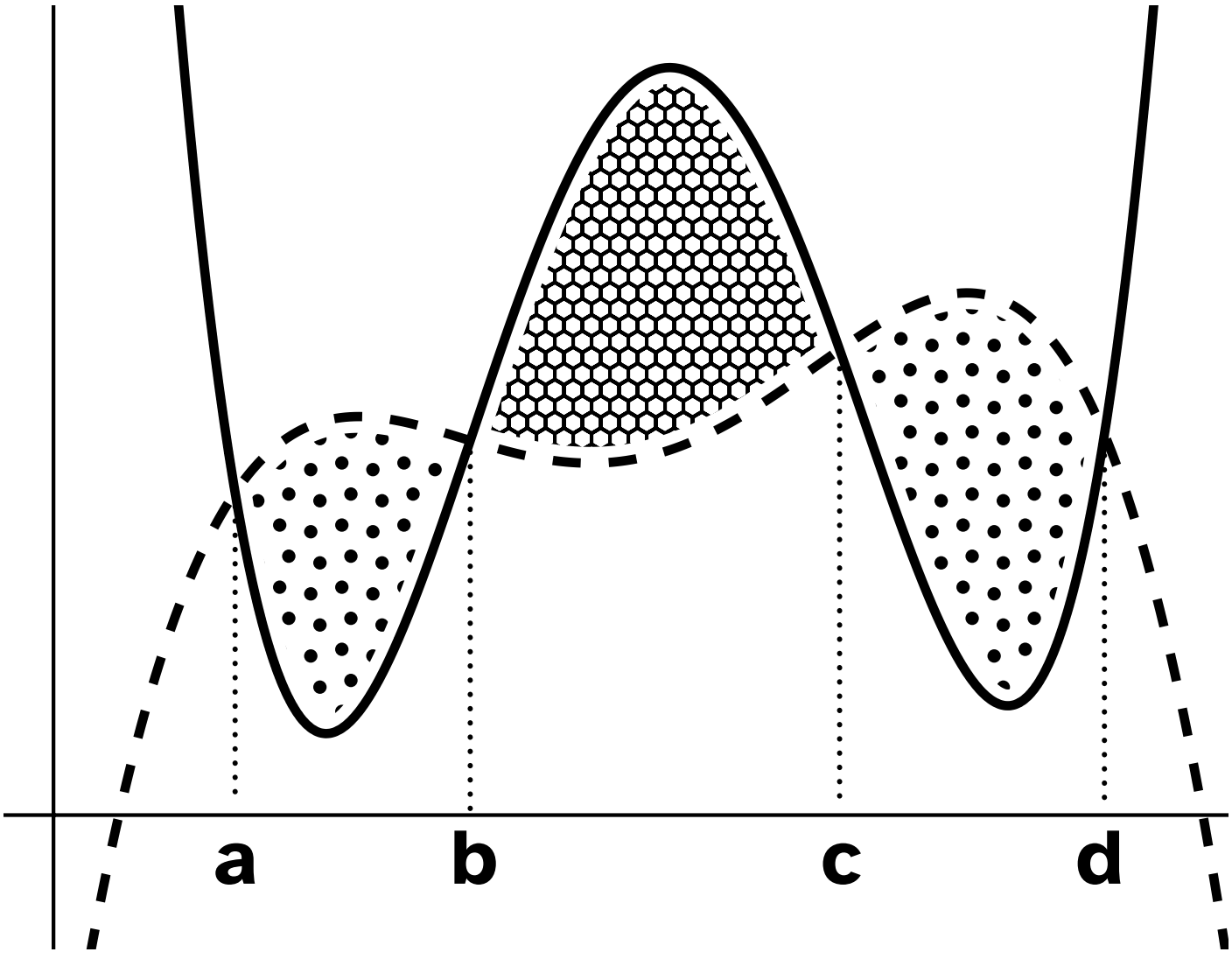


Int Flächen zw Fu

$$A = \int_a^b (g(x) - f(x))\,dx + \int_b^c (f(x) - g(x))\,dx + \int_c^d (g(x) - f(x))\,dx$$

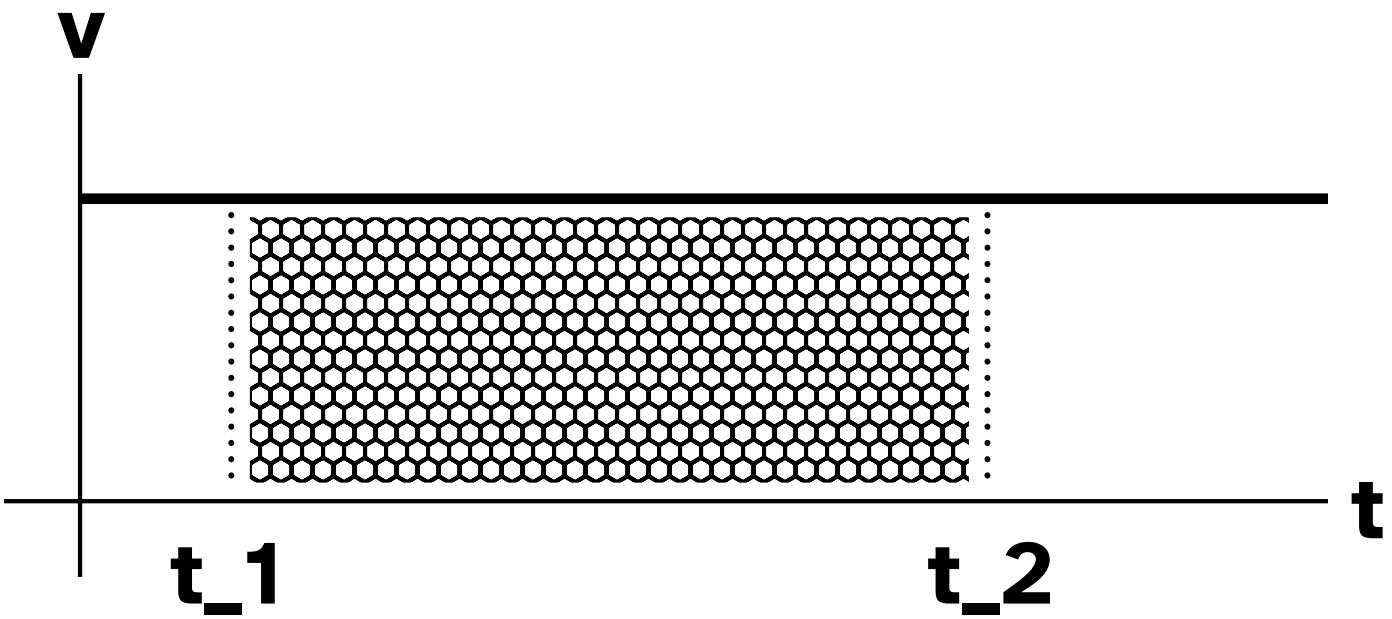
f: ——— ‖ g: – – – –

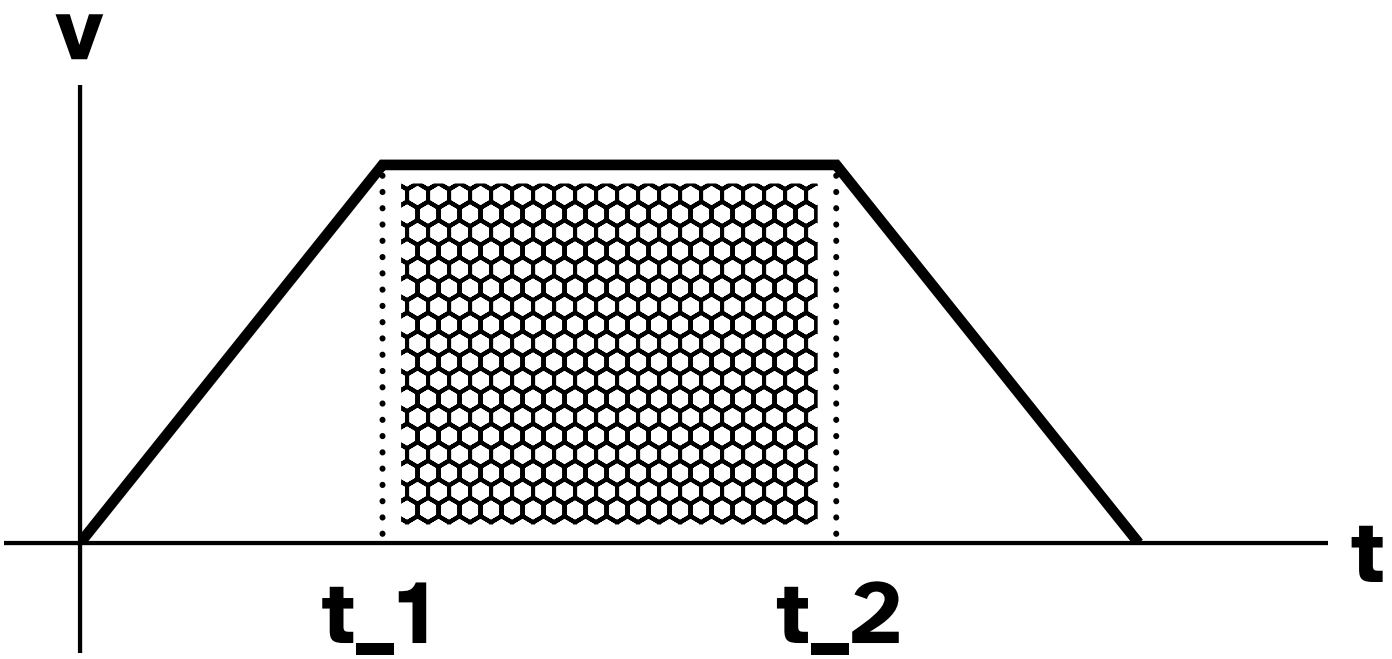
A: ▒▒▒ sowie ⁘⁘⁘



$s = \int_{t_1}^{t_2} v(t)\,dt$

v(t): ——— ‖ s:

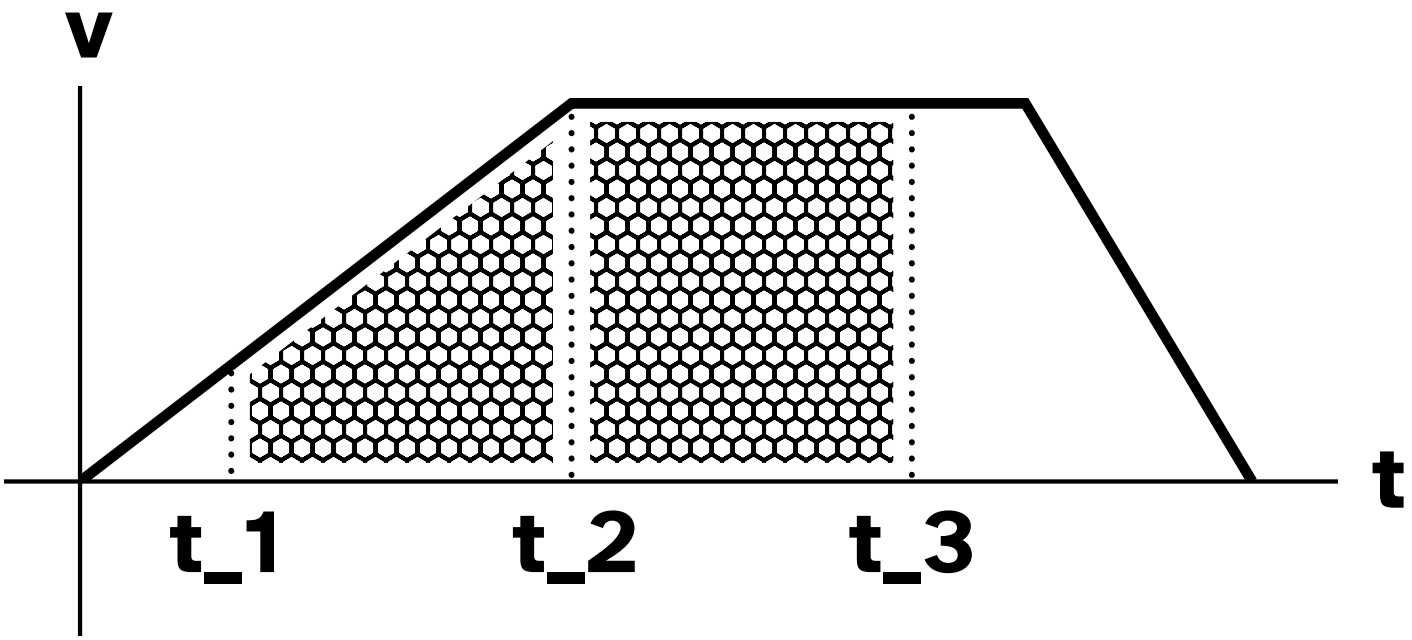


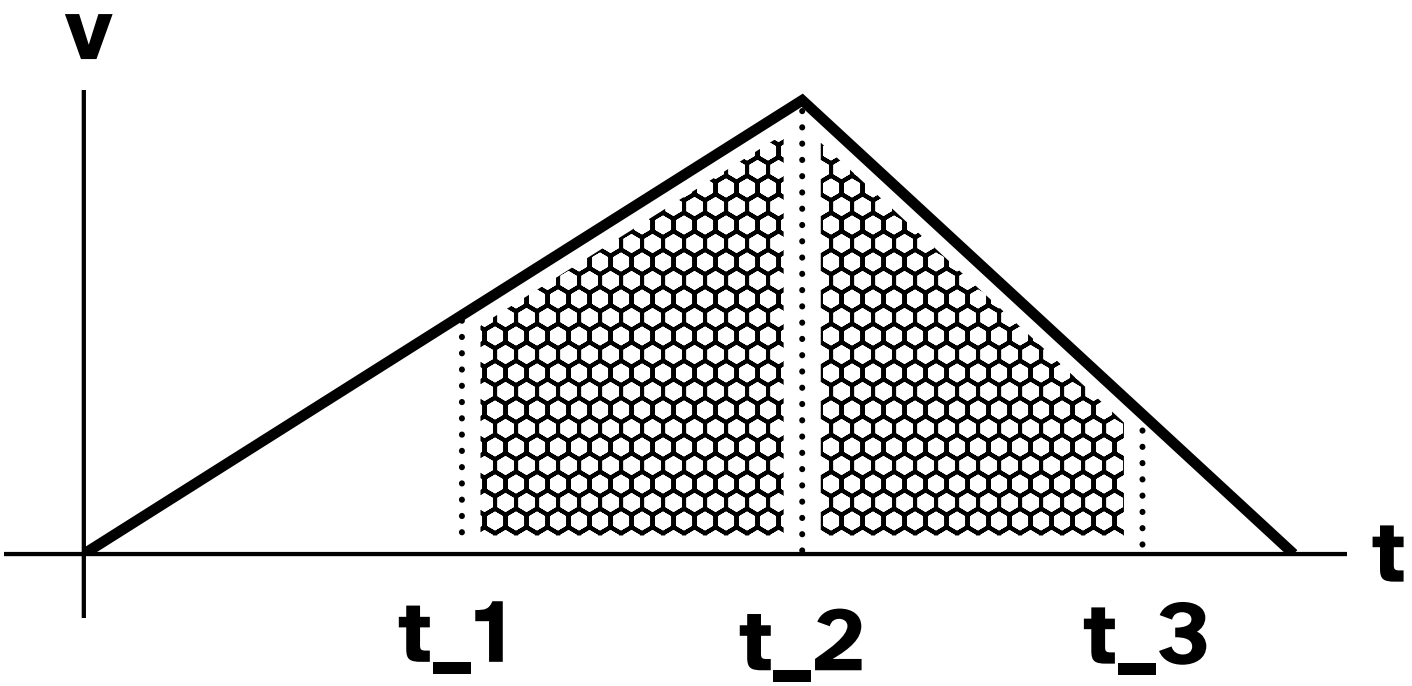


Int Weglängen

$$s = \int_{t_1}^{t_2} v(t)\,dt + \int_{t_2}^{t_3} v(t)\,dt$$

v(t): ——— ‖ s: ▒



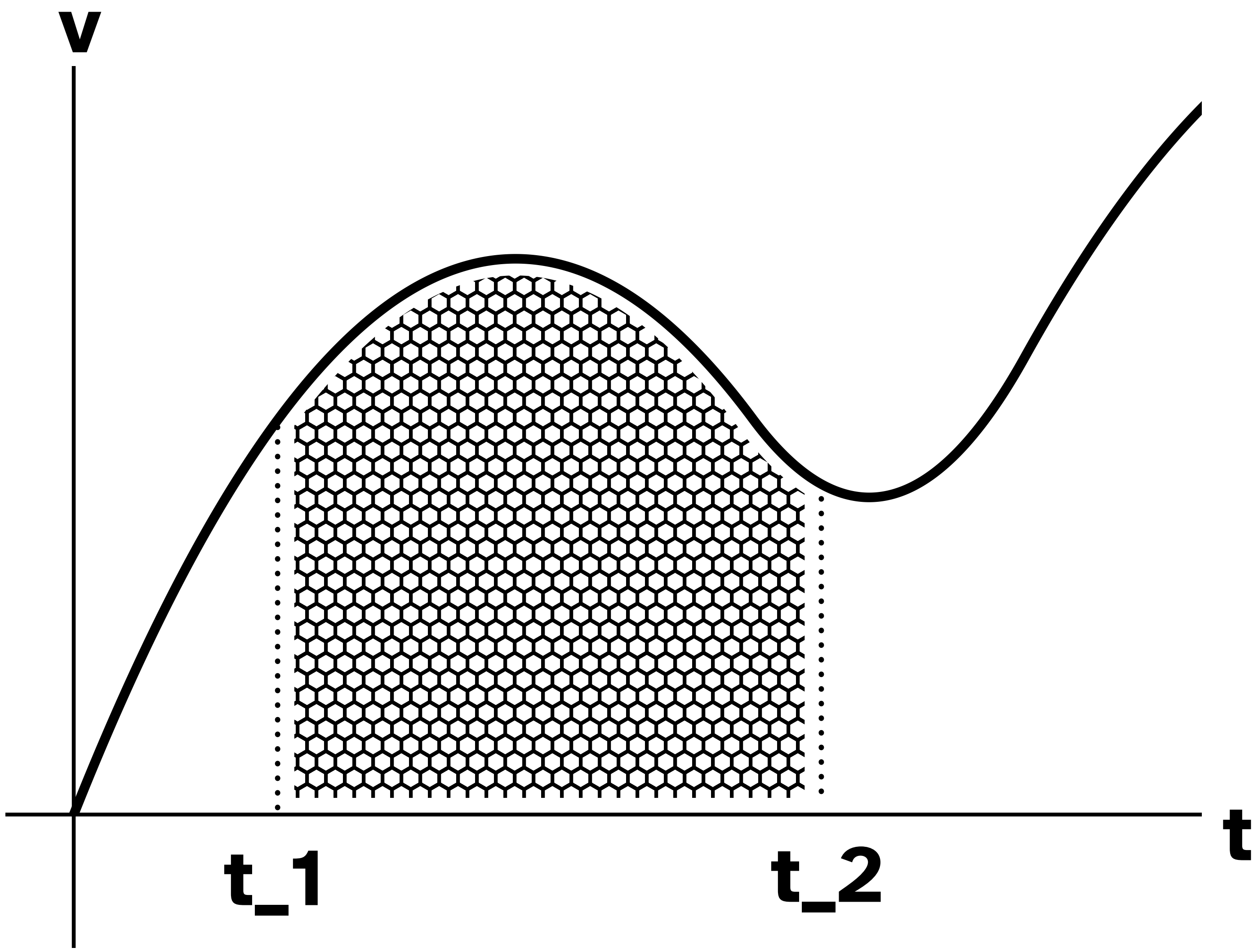


Int Weglängen

s ='int[t_1; t_2](v(t) 'dt)

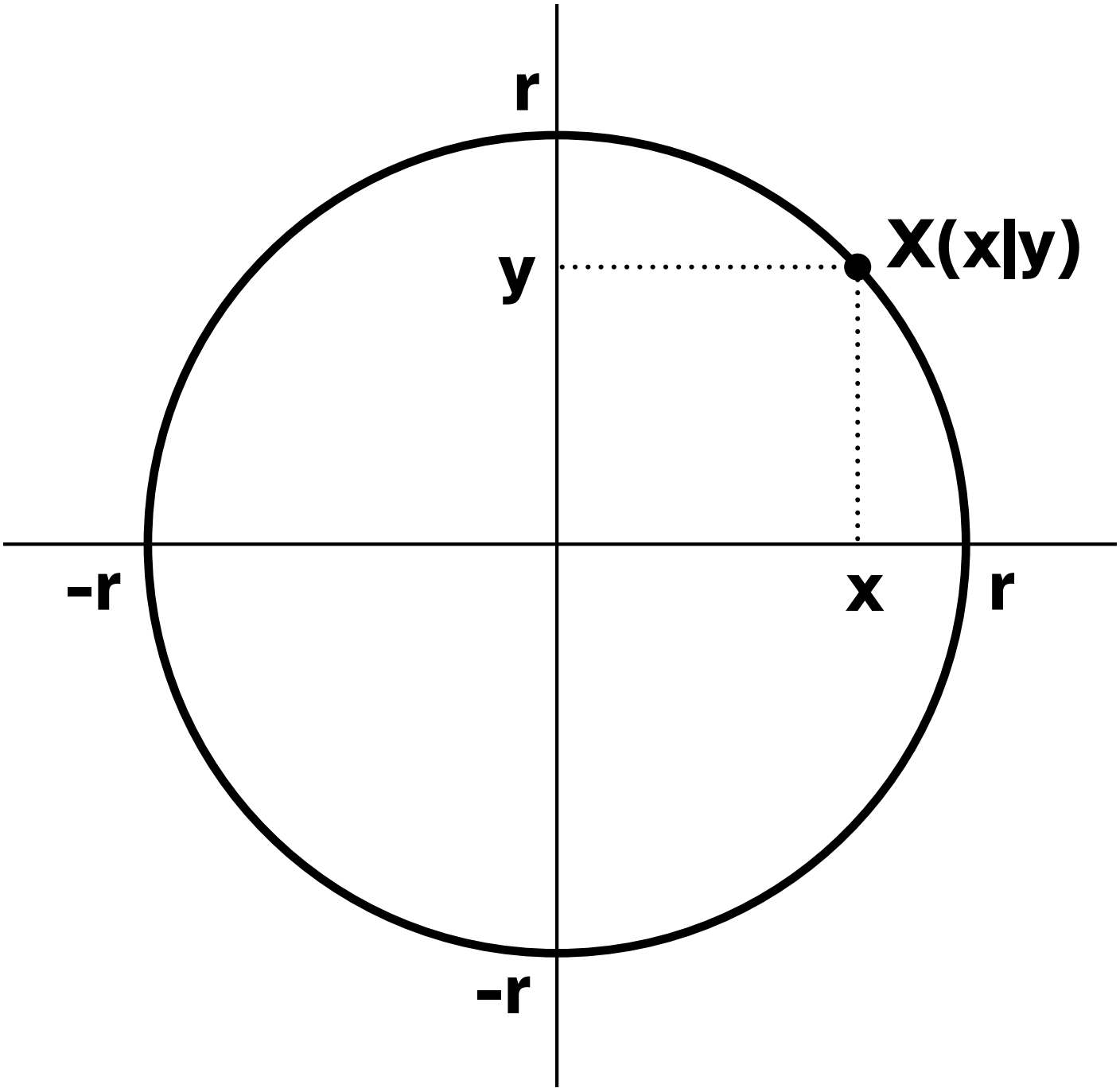
v(t):

s:



Int Volumina Kreis

k: x^2 +y^2 =r^2

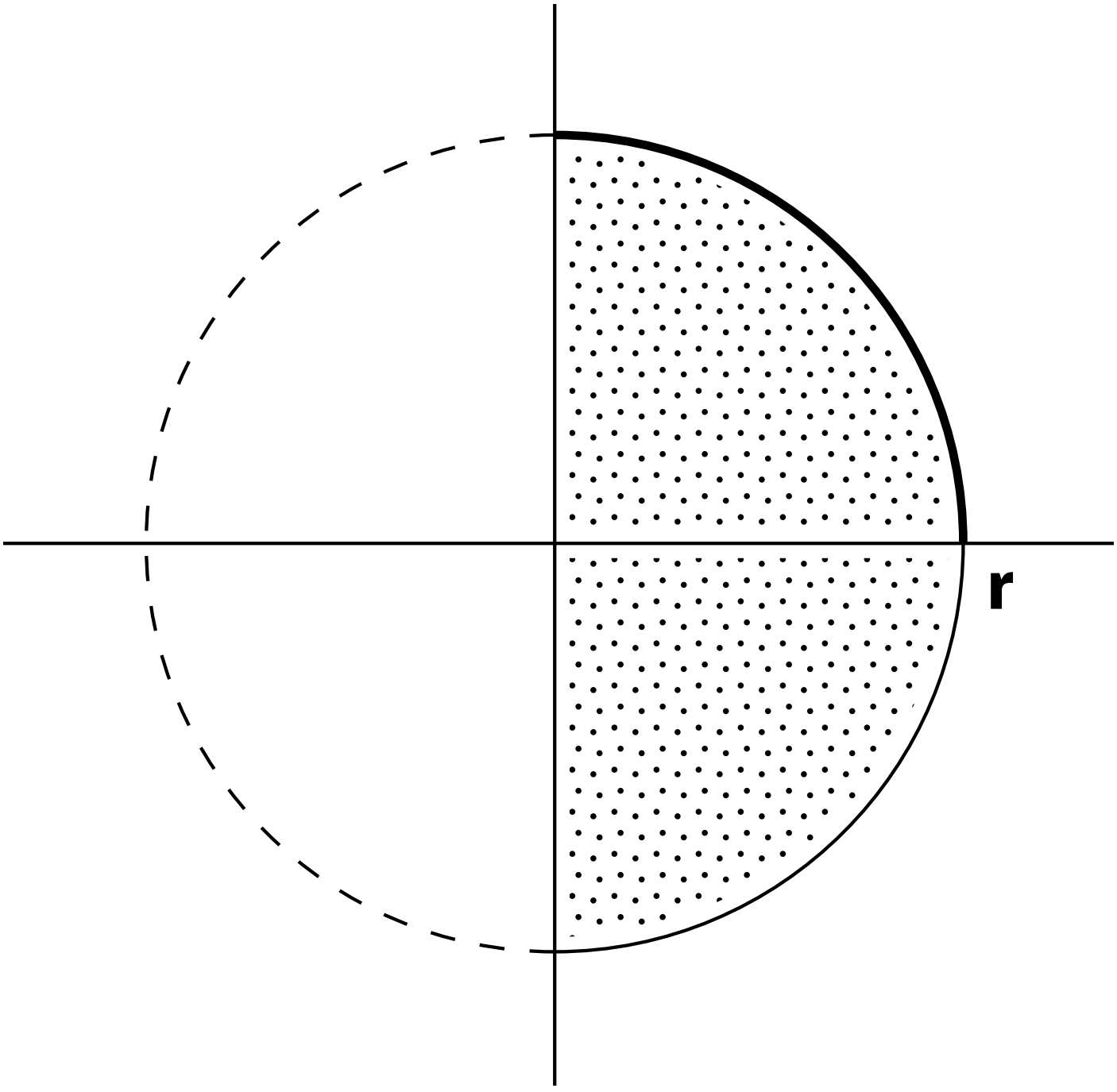


Int Volumina Kreis

$$y^2 = r^2 - x^2$$

$$V/2 =$$

$$\pi \int_0^r (y^2 \, dx)$$

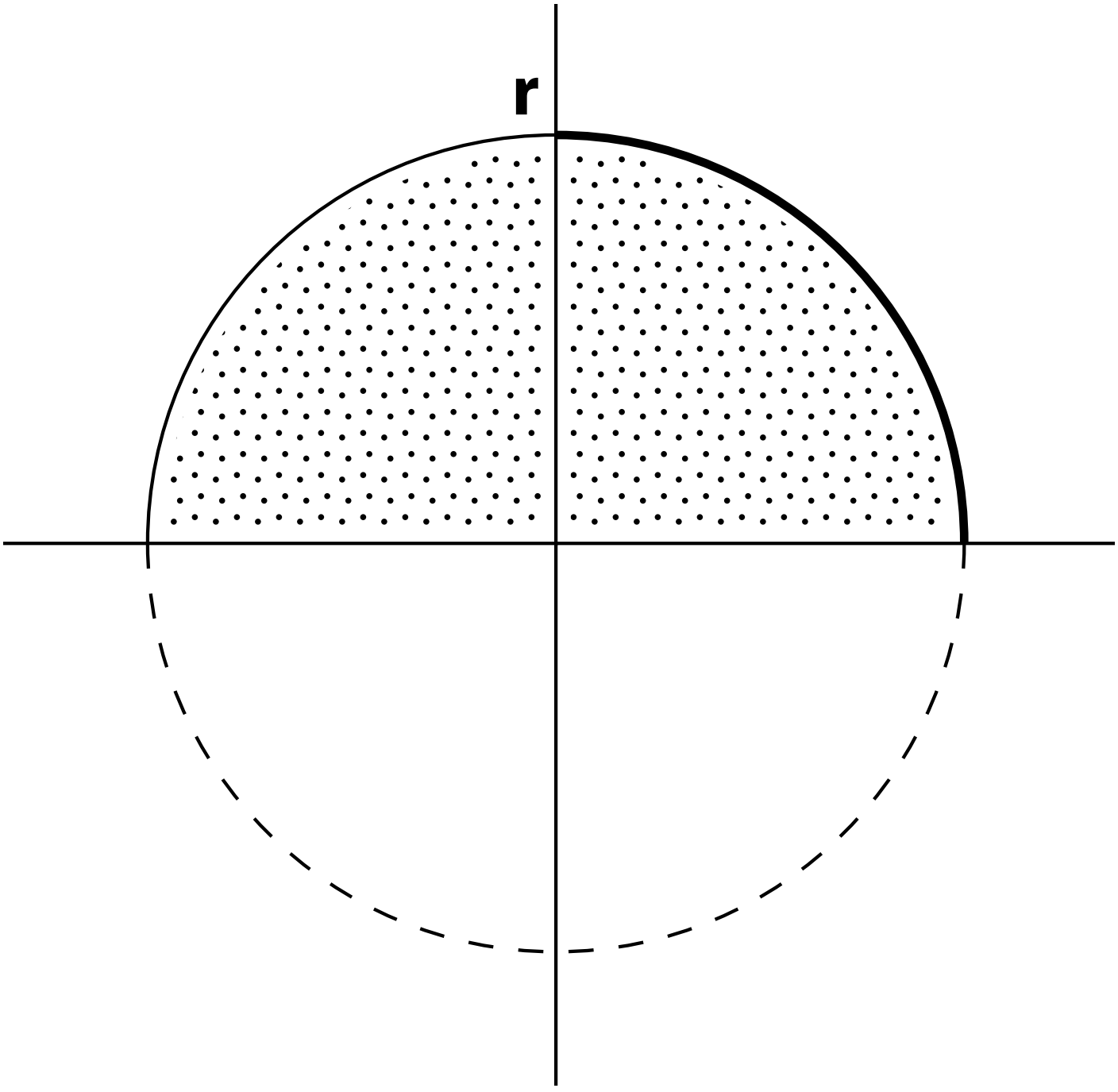


Int Volumina Kreis

$$x^2 = r^2 - y^2$$

$$V/2 =$$

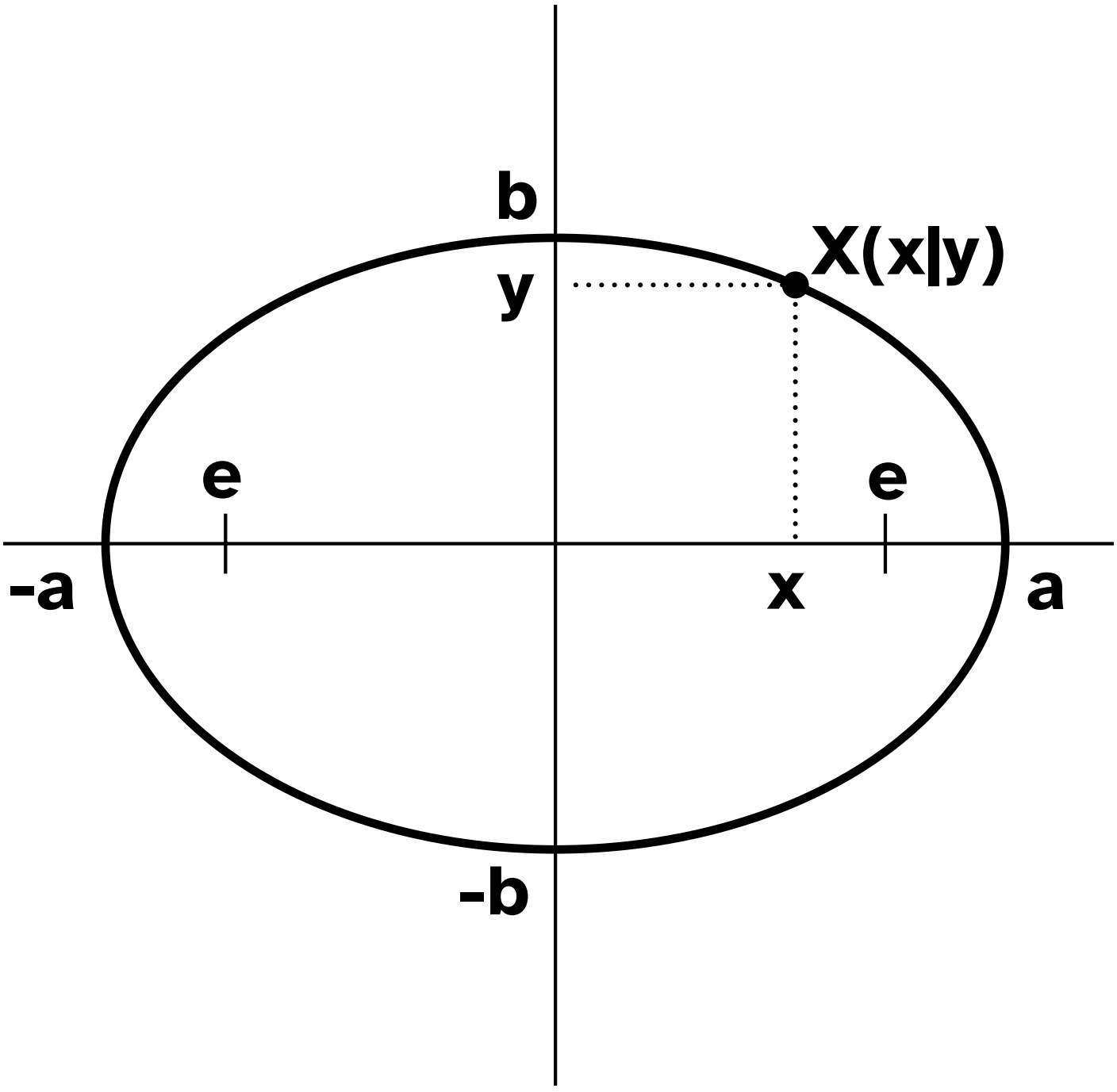
$$\pi \cdot \int_0^r (x^2 \, dy)$$



ell:

$$b^2 \cdot x^2 + a^2 \cdot y^2 = b^2 \cdot a^2$$

$$e^2 = a^2 - b^2$$

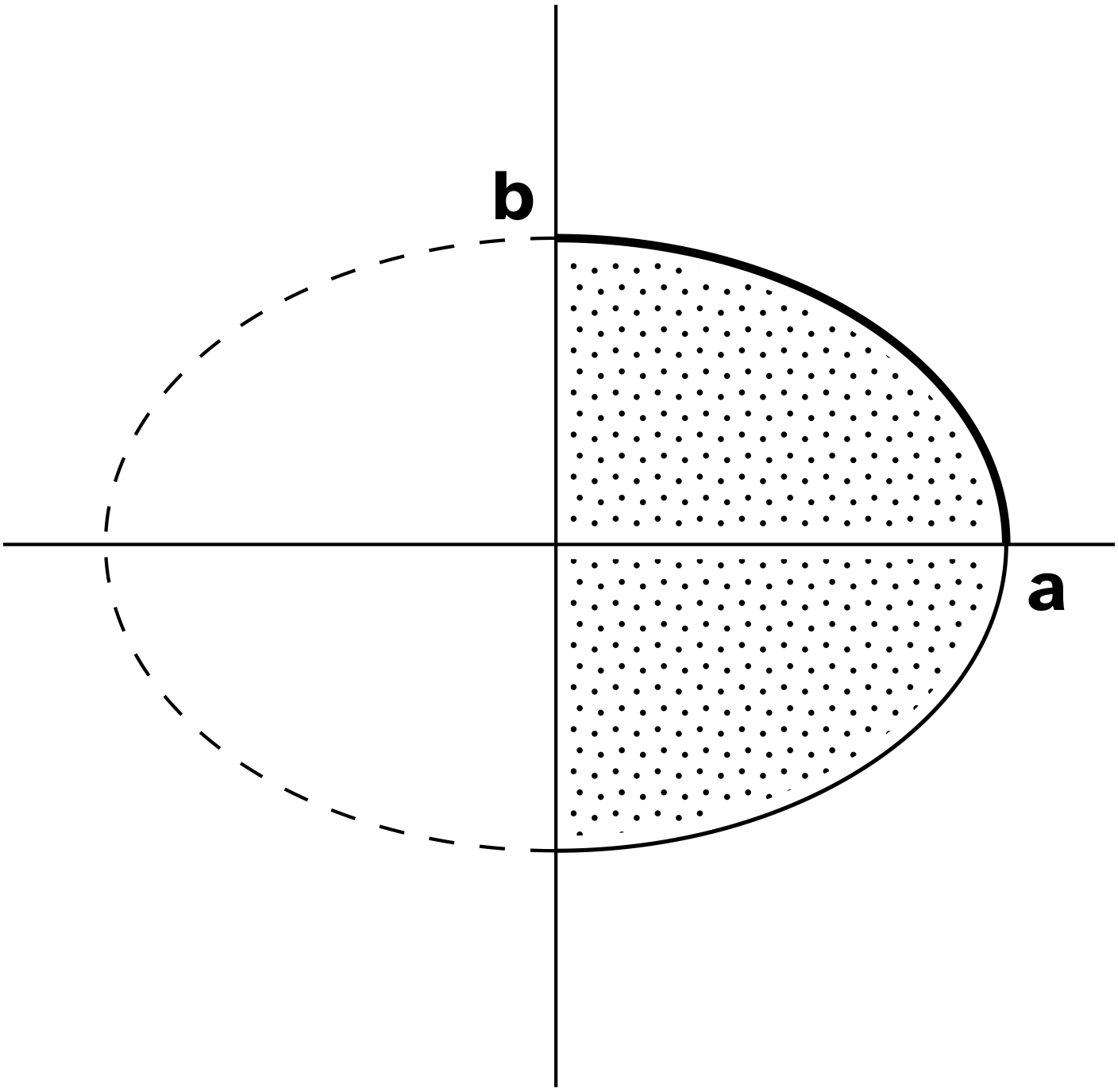


Int Volumina Ellipse 17/24

$$y^2 = b^2 - \frac{b^2}{a^2} \cdot x^2$$

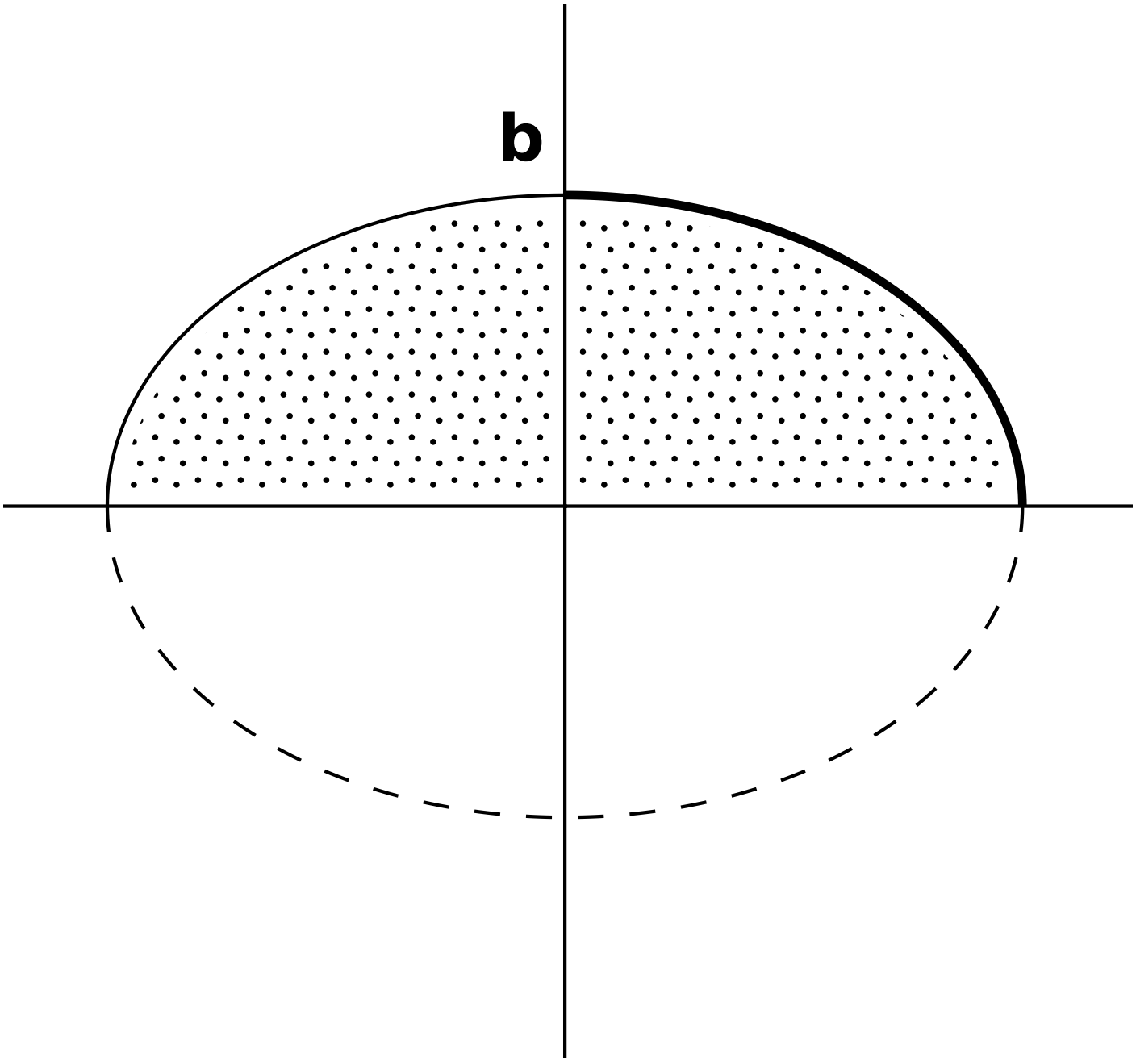
$$V/2 =$$

$$\pi \cdot \int_0^a (y^2 \, dx)$$



Int Volumina Ellipse 18/24

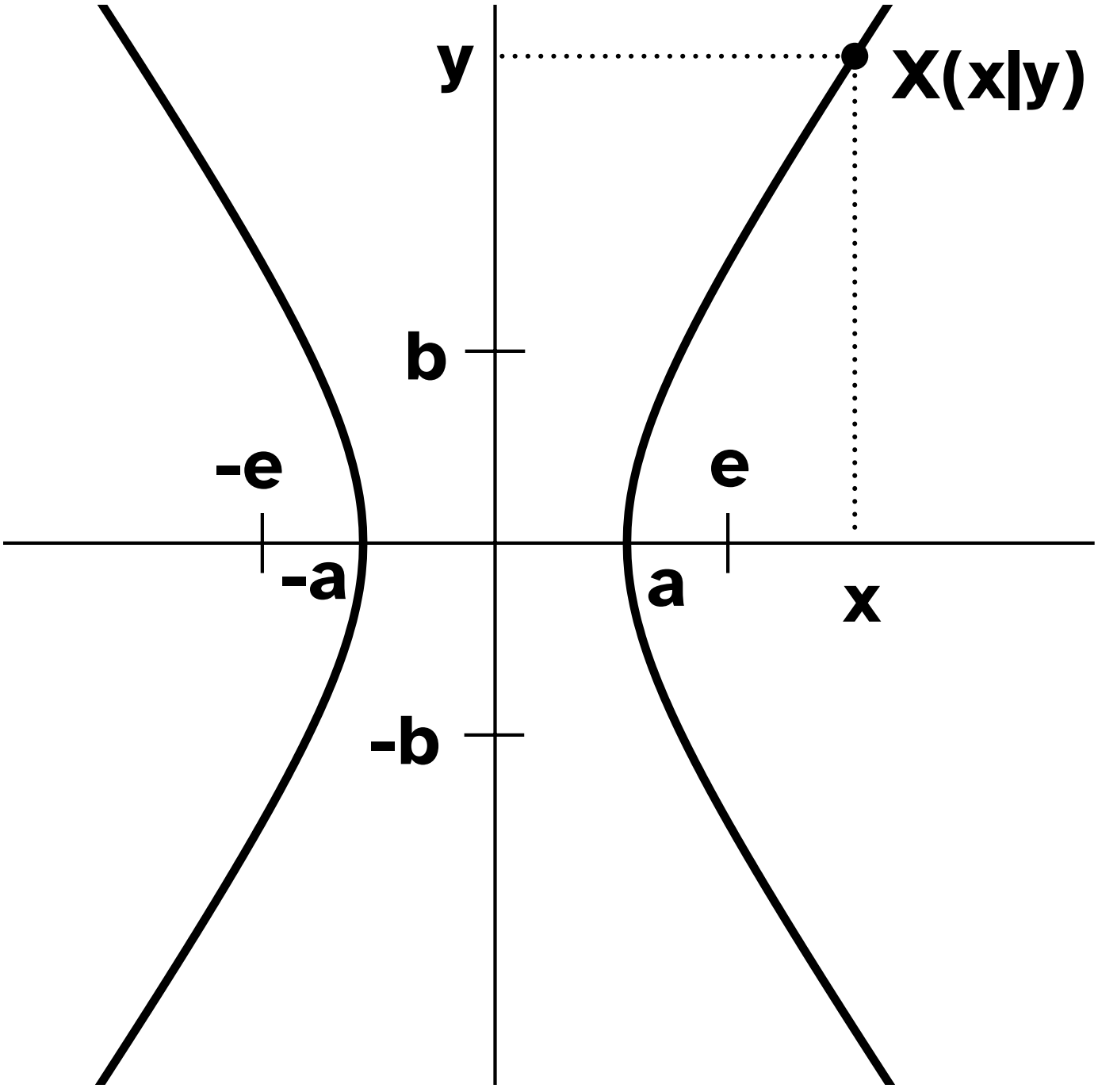
$$x^2 =$$
$$a^2 - a^2/b^2 \cdot y^2$$
$$V/2 = \pi \cdot \int_0^b (x^2 \, dy)$$



hyp:

$b^2 \cdot x^2 - a^2 \cdot y^2 = b^2 \cdot a^2$

$e^2 = a^2 + b^2$

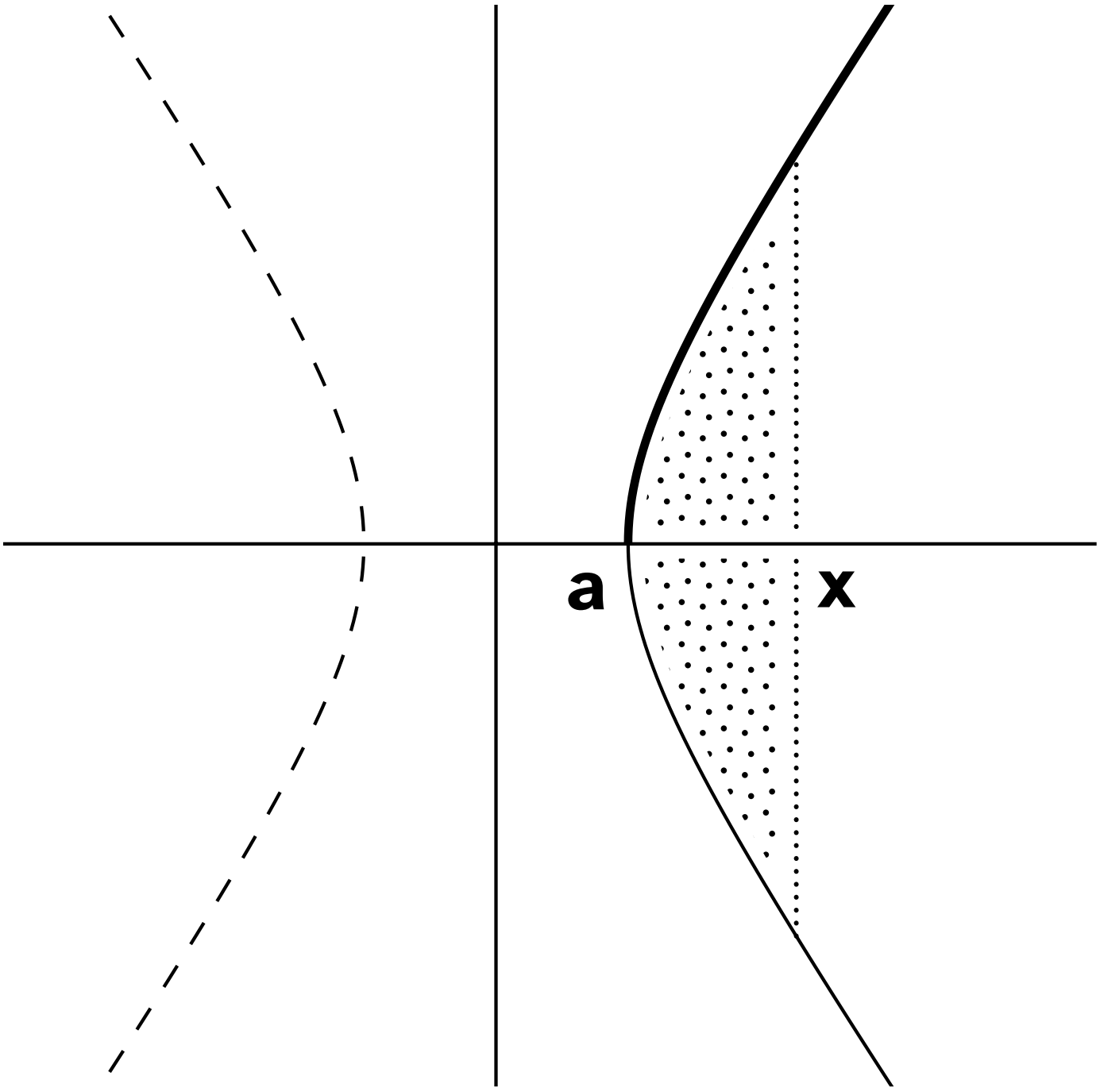


Int Volumina Hyperbel 20/24

$$y^2 = b^2 + \frac{b^2}{a^2} \cdot x^2$$

$$V/2 =$$

$$\pi \cdot \int_a^x (y^2 \, dx)$$

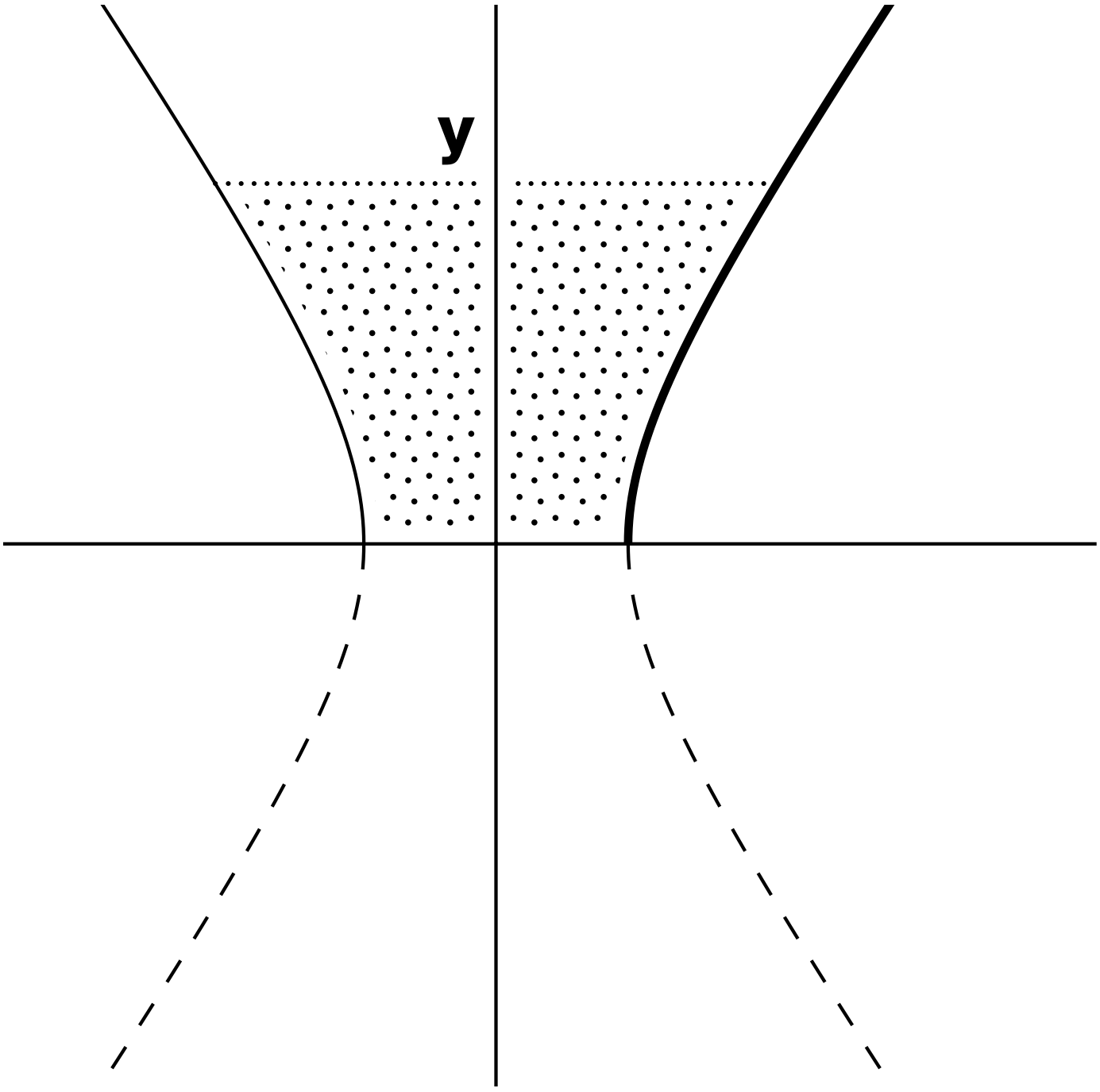


Int Volumina Hyperbel 21/24

$$x^2 = a^2 + \frac{a^2}{b^2} \cdot y^2$$

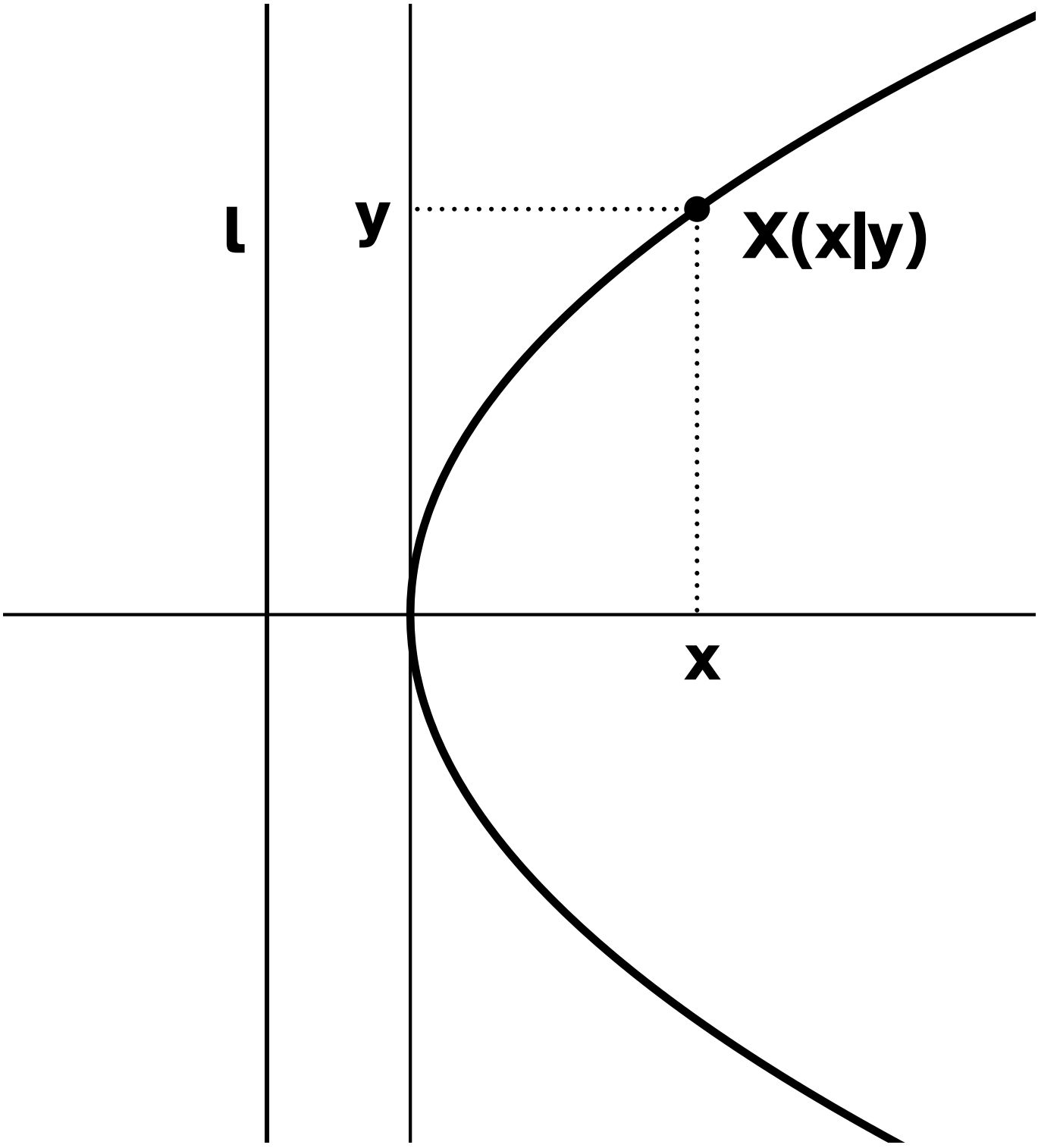
$$V/2 =$$

$$\pi \cdot \int_0^y (x^2 \, dy)$$



par: y^2 =2 *p *x

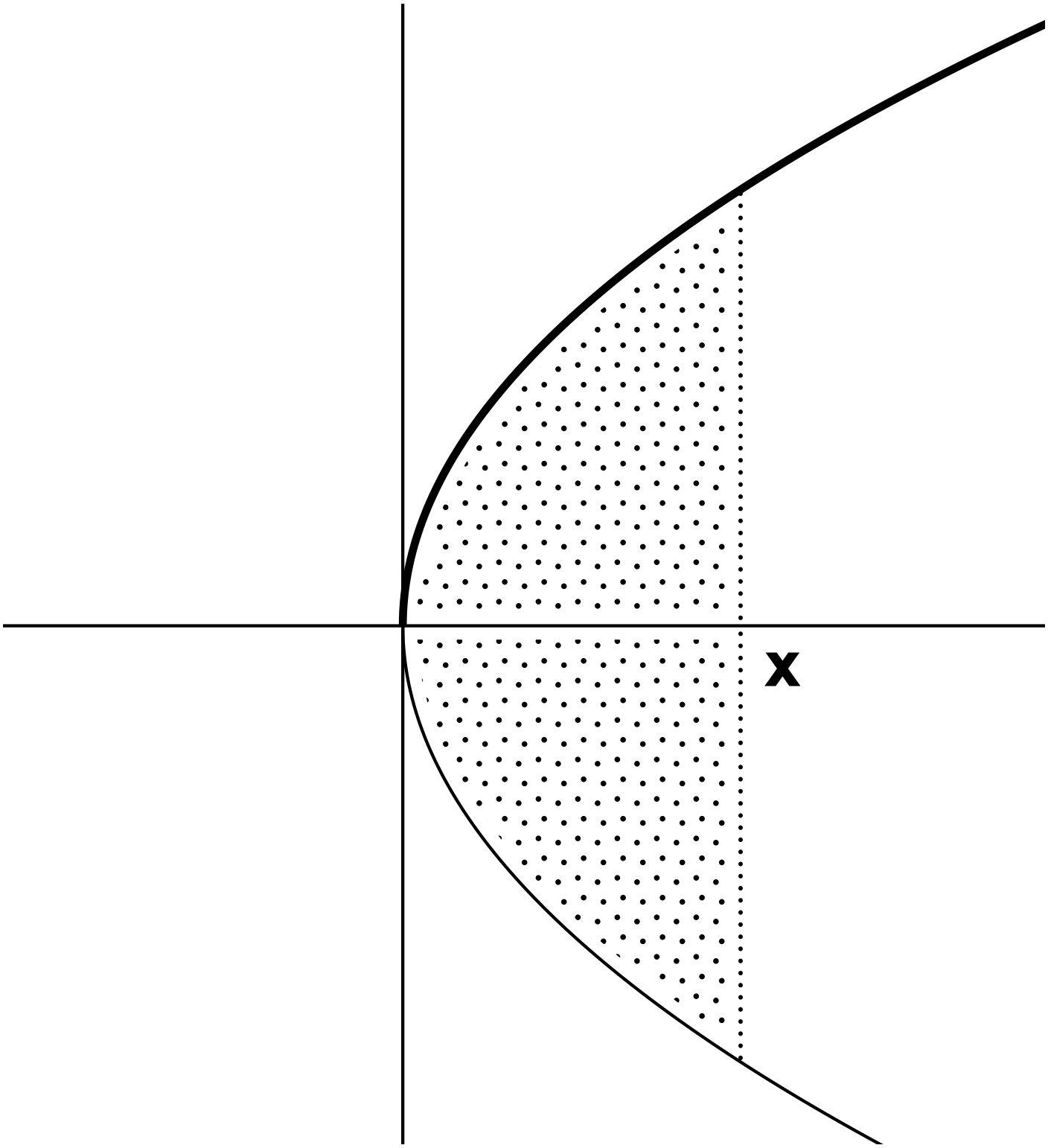
e =p/2; l =-p/2



Int Volumina Parabel 23/24

$$y^2 = 2 \cdot p \cdot x$$

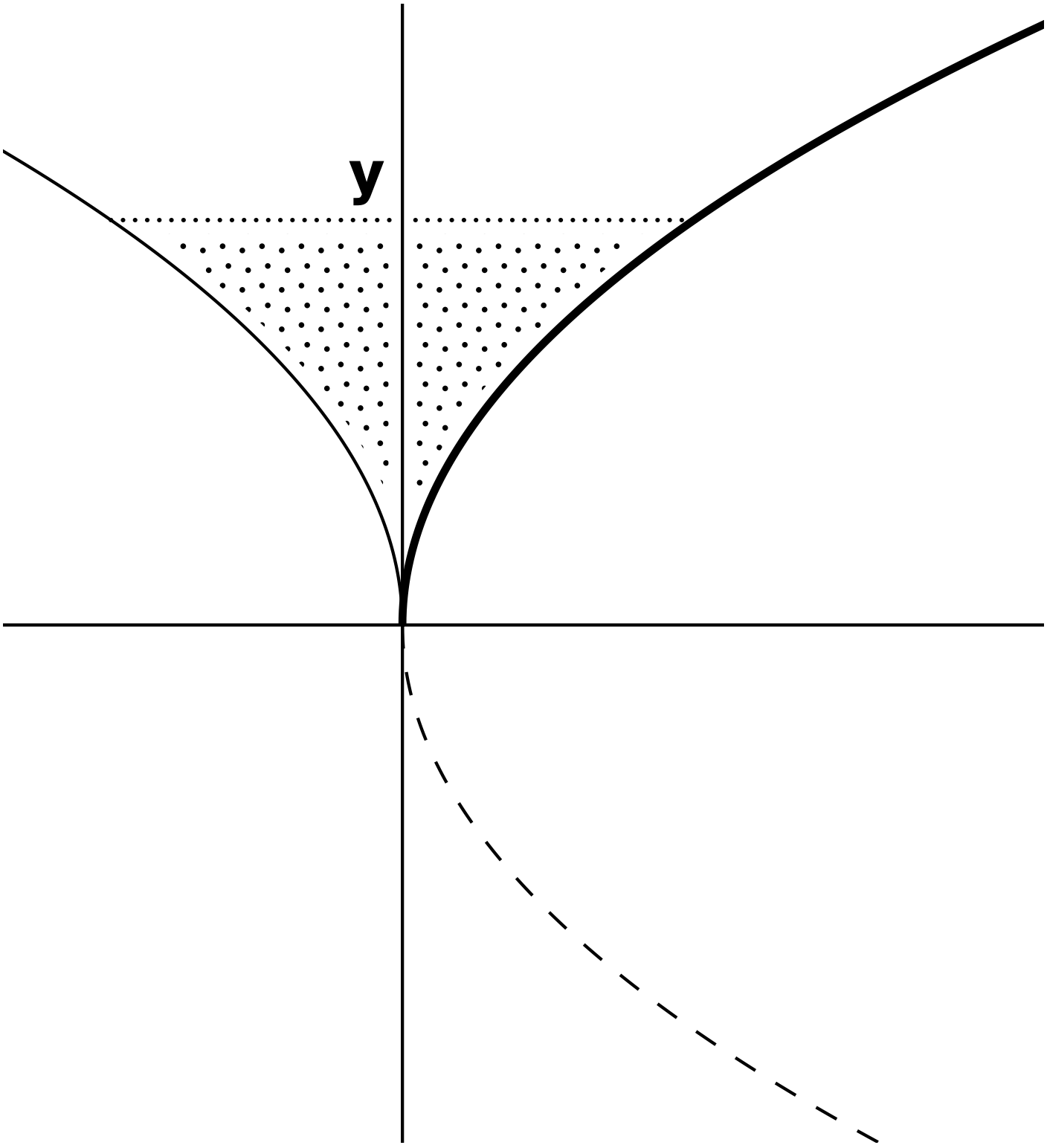
$$V = \pi \cdot \int_0^x (y^2 \, dx)$$



Int Volumina Parabel 24/24

$$x^2 = (y^2/(2 \cdot p))^2$$

$$V = \pi \cdot \int_0^y (x^2 \, dy)$$



Wirtschaft

Schulstufe 12

Kostenfunktionen

Grenzkostenfunktionen

Stückkostenfunktionen

Betriebsoptimum

Preis- und Erlösfunktionen

Gewinngrenzen

Gewinnmaximierung

Gewinnfunktionen

Cournot‘sche Punkt

Inhalt

7 Gewinngrenzen
(K_deg, K_s-förm, E_lin)

8 Gewinngrenzen
(K_lin, K_prog, E_quad)

9 Gewinngrenzen
(K_deg, K_s-förm, E_quad)

10 - 11 Gewinnmaximierung

10 Gewinnmaximierung x_m
(K_prog, E_lin)

11 Gewinnmaximierung x_m
(K_s-förm, E_lin)

12 - 15 Gewinnfunktionen

12 Gewinnfunktionen G
G =K -E
(K_lin, K_prog, E_lin)

13 Gewinnfunktionen G
(K_deg, K_s-förm, E_lin)

14 Gewinnfunktionen
(K_lin, K_prog, E_quad)

15 Gewinnfunktionen
(K_deg, K_s-förm, E_quad)

16 Cournot‘sche Punkt C
C (x_m/p(x_m))
(p(x), G, x_C =x_m)

Wirtschaft - Abkürzungen

Index

C: Cournot‘sche Punkt (liegt auf der Preisfunktion mit dem x-Wert: x_m

E: Erlösfunktion (p *x)

Eq: E_quad, quadrat. E

F: Fixkosten

G: Gewinnfunktion (Erlös minus Kosten)

GE: Geldeinheiten

H: Höchstpreis

K: Kostenfunktion:

Kl: K_lin, lineare K,

Kp: K_prog, progressive K

Kd: K_deg, degressive K

Ks: K_s-förm, S-förmige K

K': Grenzkostenfunktion
Preis pro zusätzlich erzeugter ME

K^-: Stückkostenfunktion
K/x

ME: Mengeneinheiten

p: Preis (fix)

p(x): von Nachfrage abhängiger Preis (Nachfragefunktion)

p_C: Preis bei Verkauf von Cournot‘scher Menge

W: Wirtschaft

x_C: Cournot‘sche Menge,
gewinnmaximale Produktionsmenge
eines Monopolisten
x_C =x_m

x_m: gewinnmaximale Produktionsmenge
(G'(x_(m) =0)

x_o: obere Gewinngrenze
(2. Break-Even-Point)

x_S: Sättigungsmenge

x_u: untere Gewinngrenze
(1. Break-Even-Point)

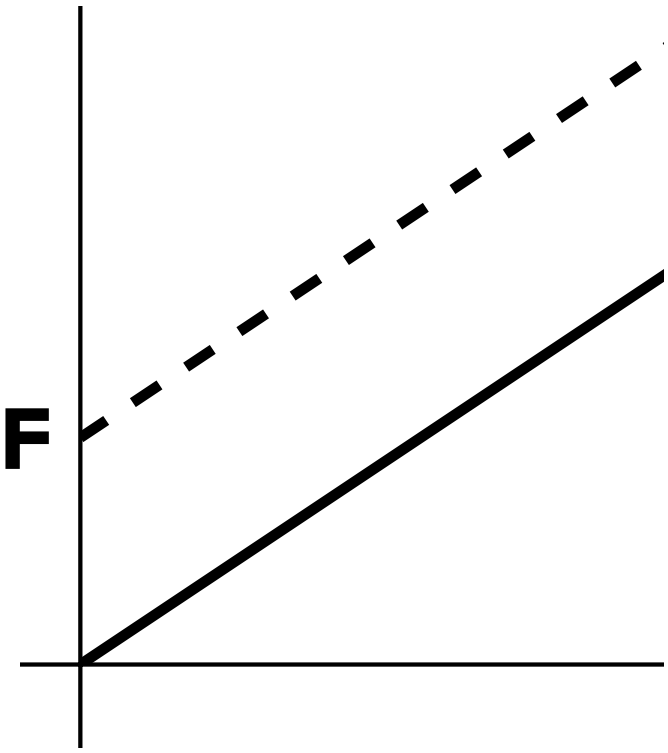
x_W Kostenkehre
Wendestelle von K

x_(opt): Produktionsmenge
mit minimalen Stückkosten
K'(x) =K^-

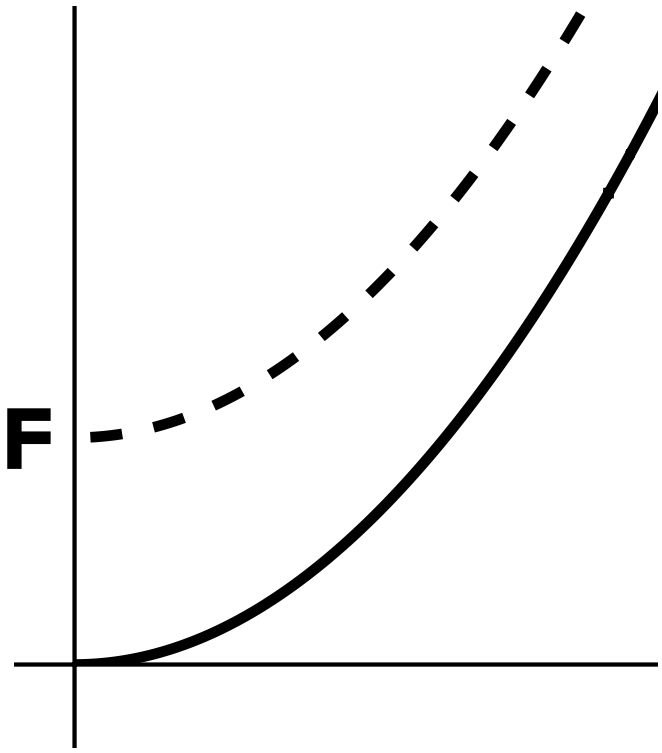
Wi Kostenfunktionen 1/16

K: ——— ‖ KV: - - - -

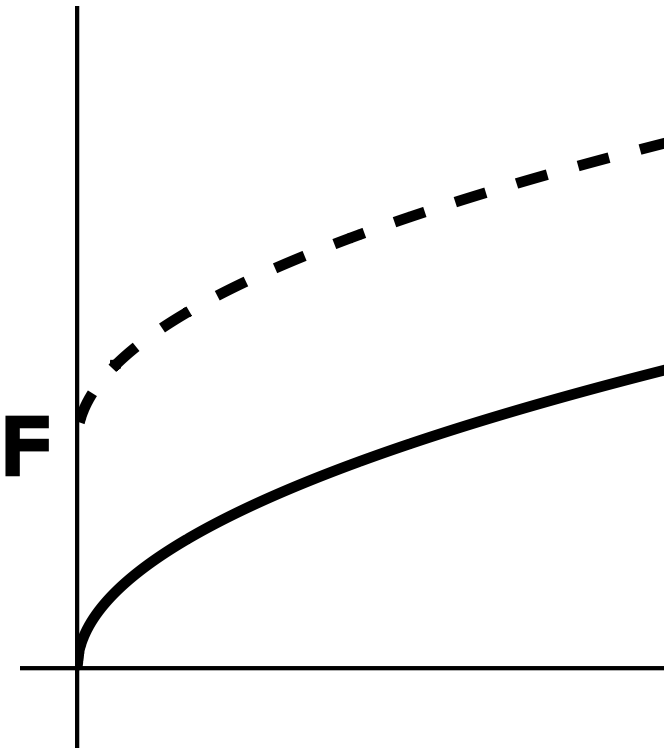
Kl; KVl



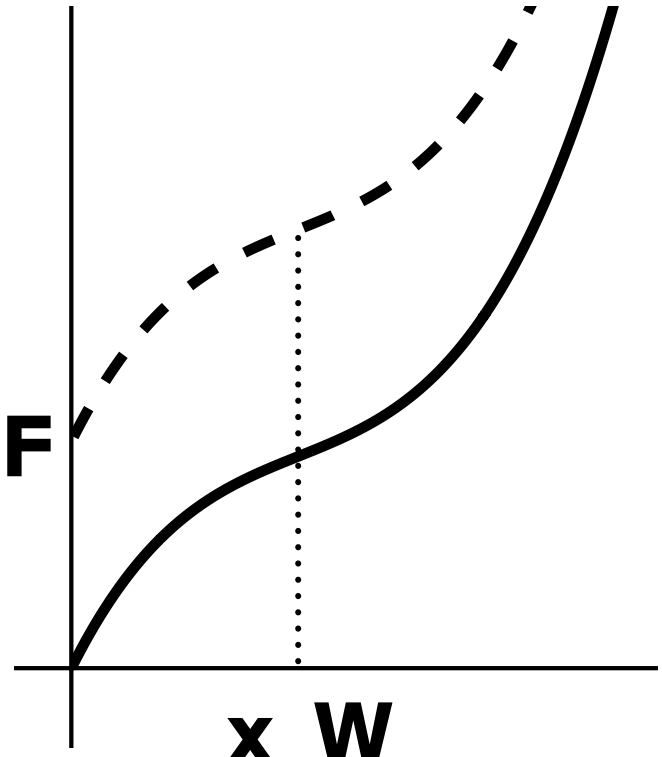
Kp; KVp



Kd; KVd



Ks; KVs



Ergänzungen zu Seite 1

Kl: linearer Kostenverlauf
F: Fixkosten
KVl: linearer Kostenverlauf ohne Fixkosten

Kp: progressiver Kostenverlauf
linksgekrümmt (pos. gekrümmt), strg mon st,
überproportional
K' >0; K" >0

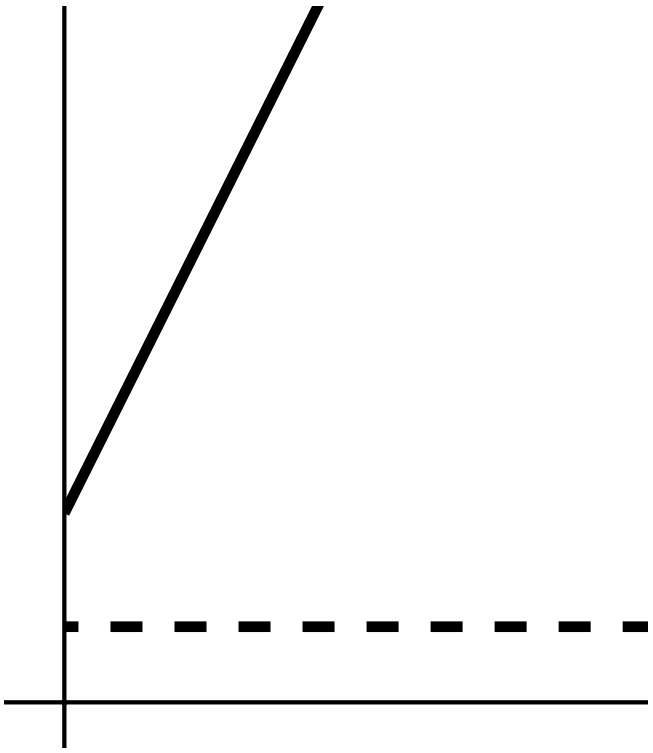
Kd: degressiver Kostenverlauf
rechtsgekrümmt (neg. gekrümmt), strg mon st,
unterproportional
K' >0; K" <0

Ks: S-förmiger Kostenverlauf
erst rechtsgekrümmt (neg. gekrümmt),
dann linksgekrümmt (pos. gekrümmt)
x_W =Kostenkehre (Wendestelle)
K"(x_W) =0

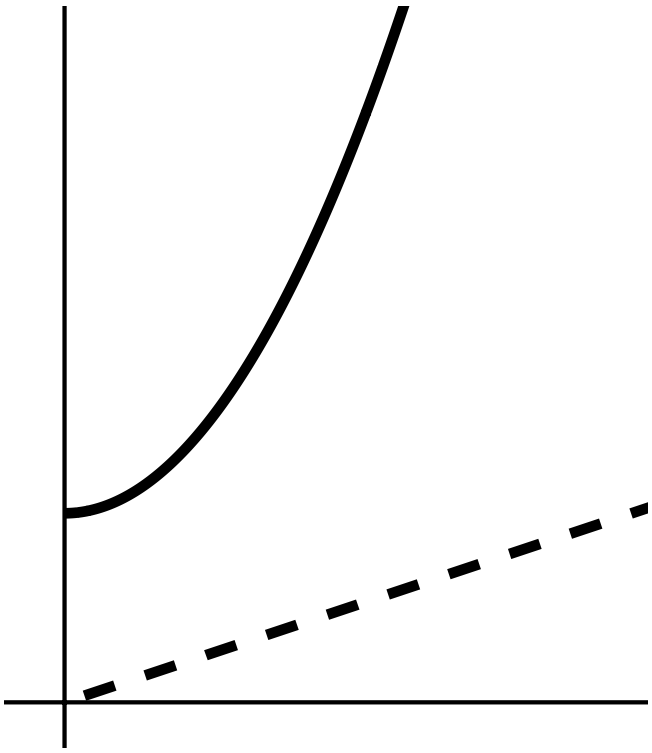
Wi Grenzkosten

K: ——— ‖ K': - - - -

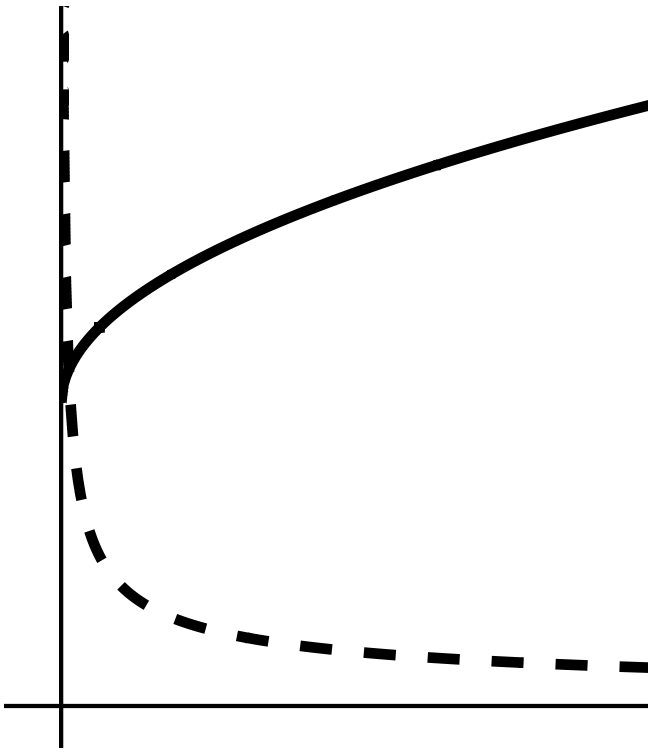
Kl; Kl'



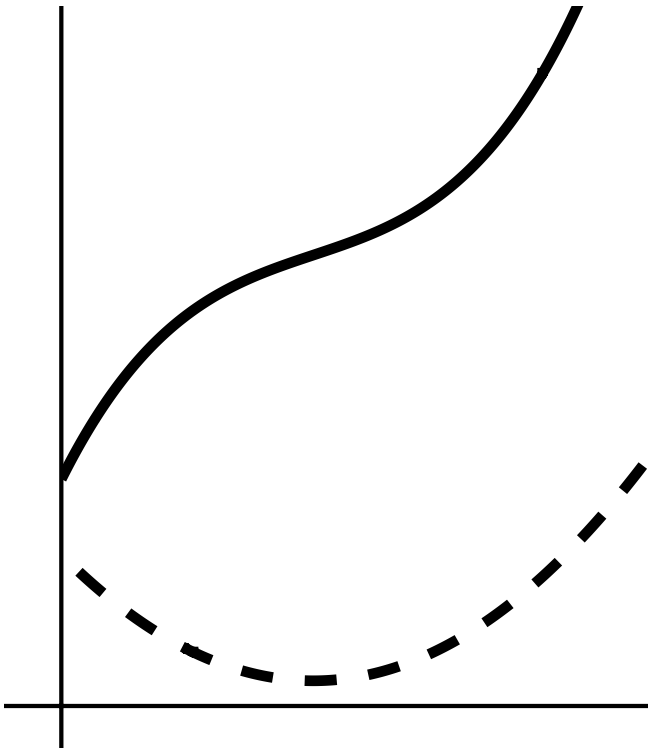
Kp; Kp'



Kd; Kd'



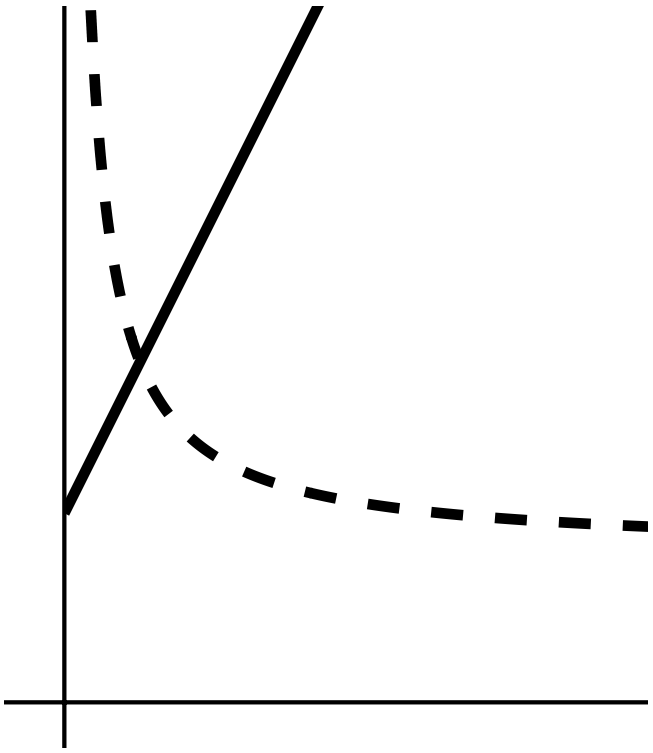
Ks; Ks'



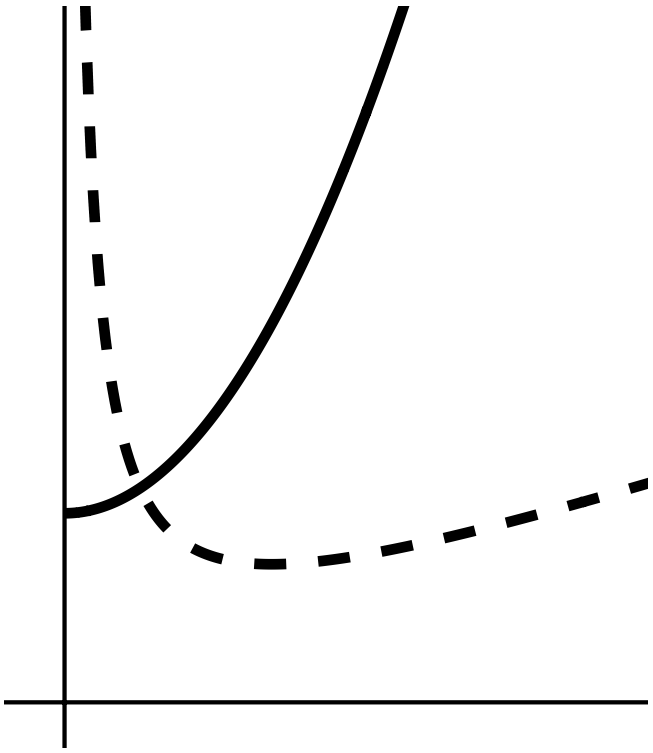
Wi Stückkosten

K: ——— ‖ K^- =K/x: - - - -

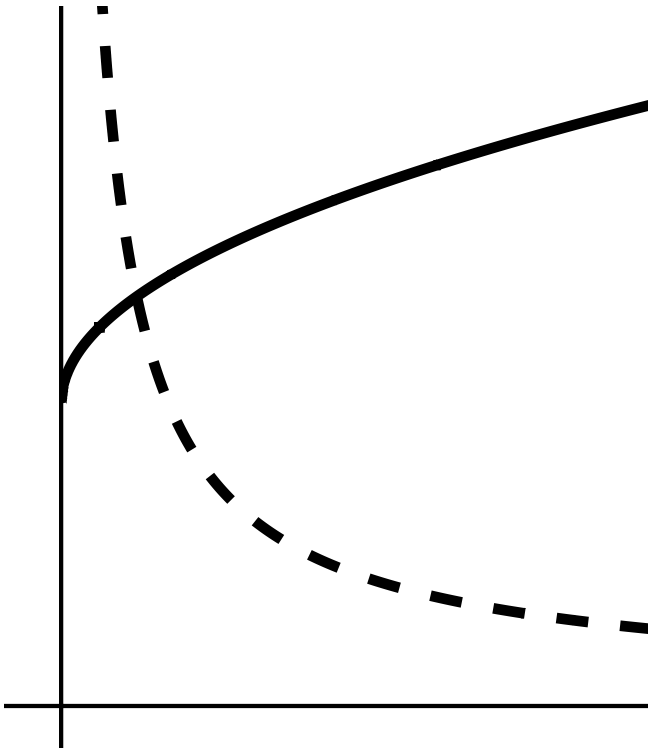
Kl; Kl/x



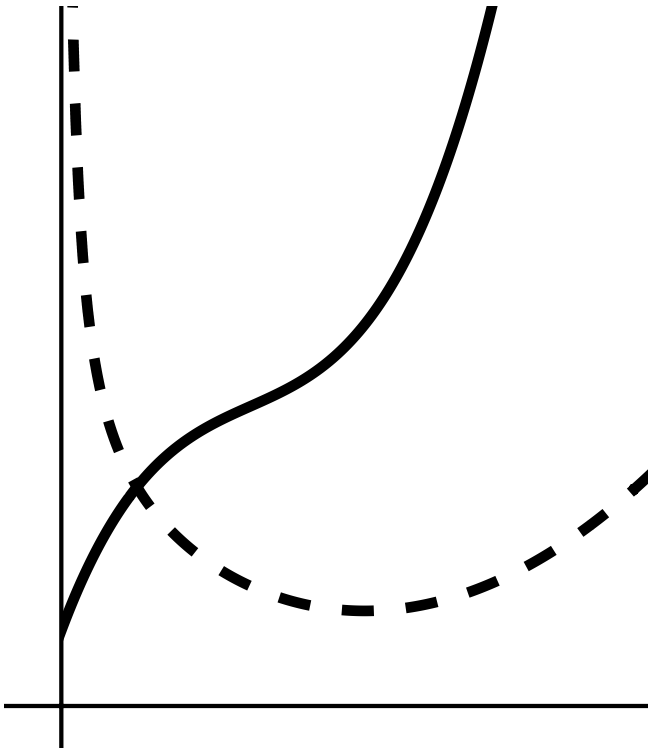
Kp; Kp/x



Kd; Kd/x



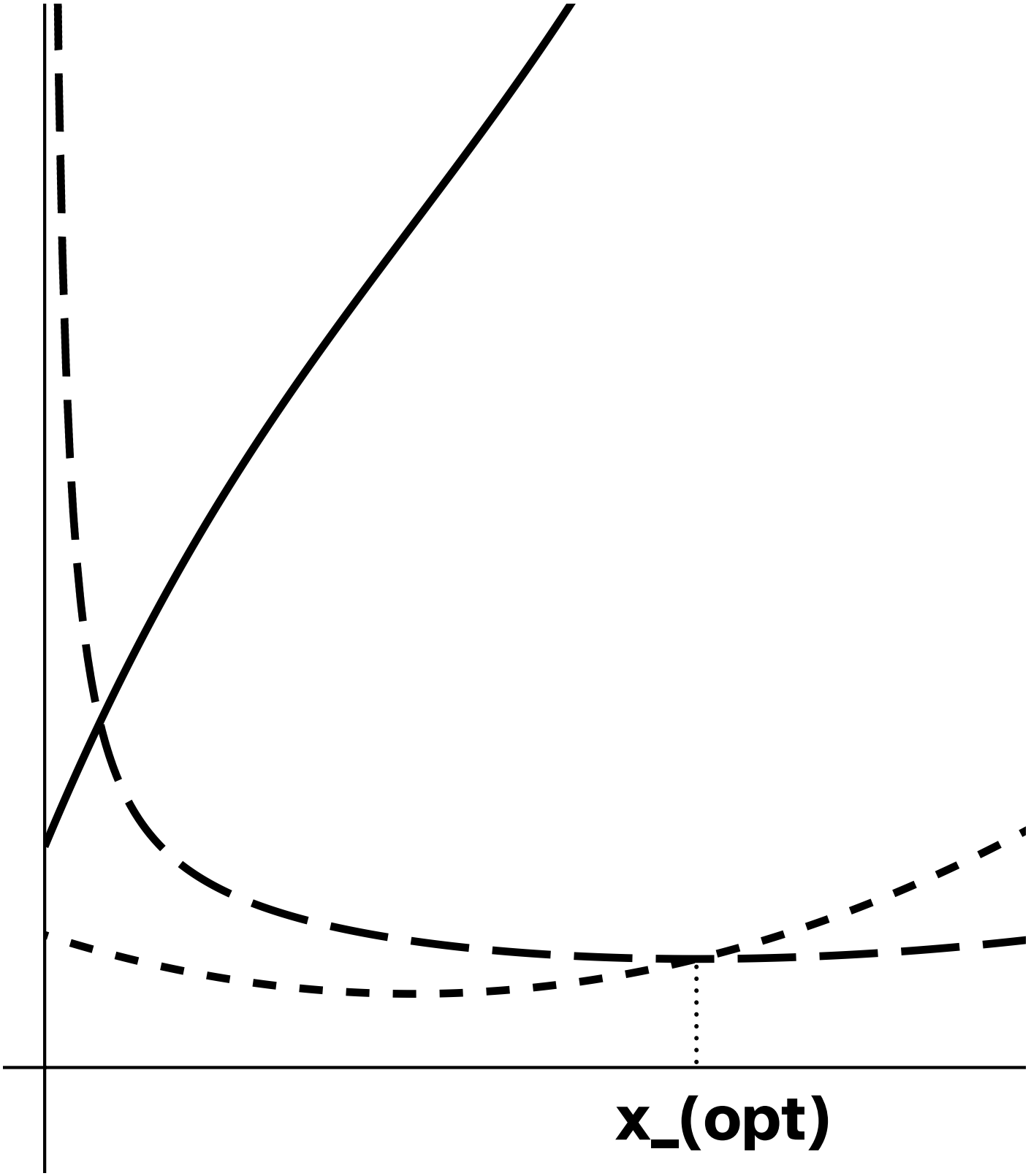
Ks; Ks/x



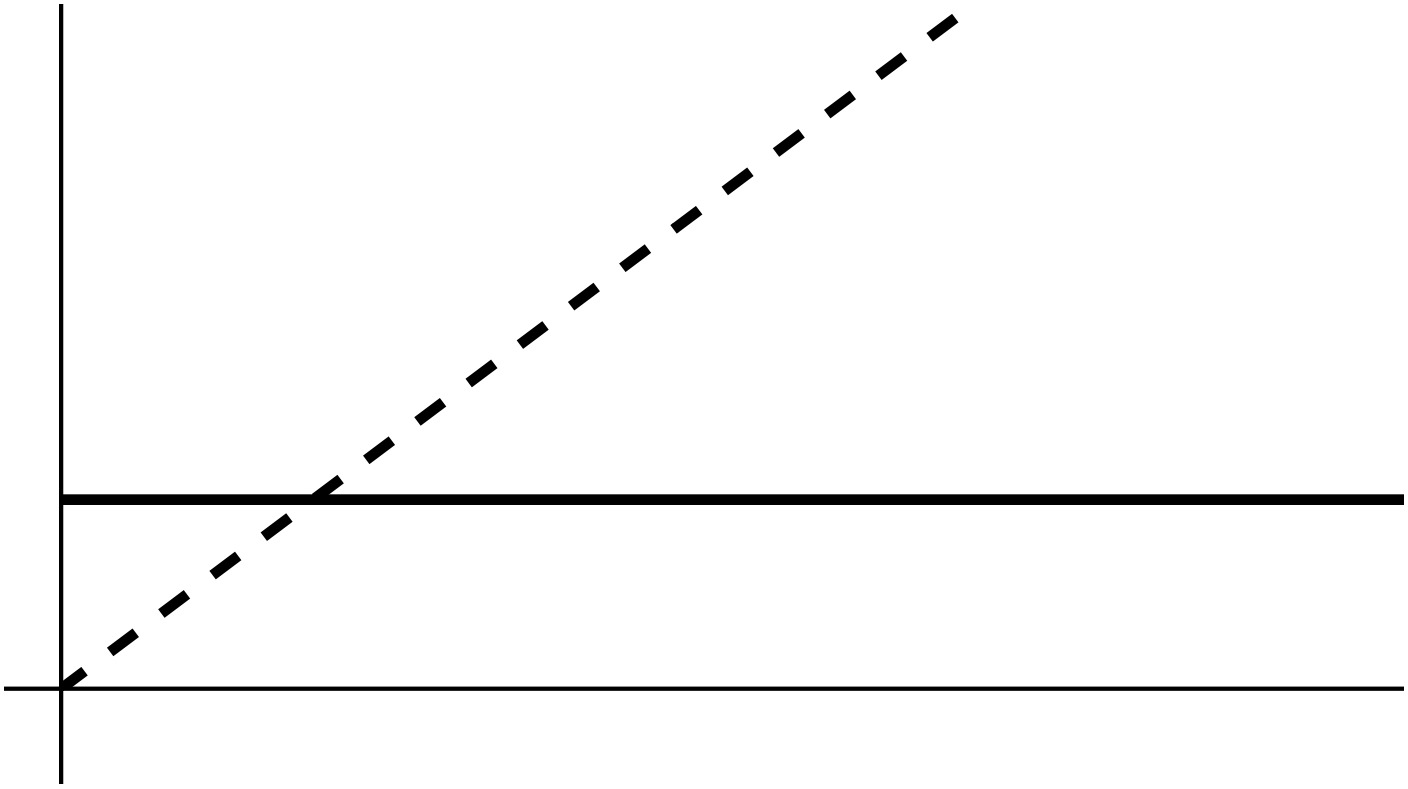
Wi Betriebsoptimum 4/16

Ks: ———

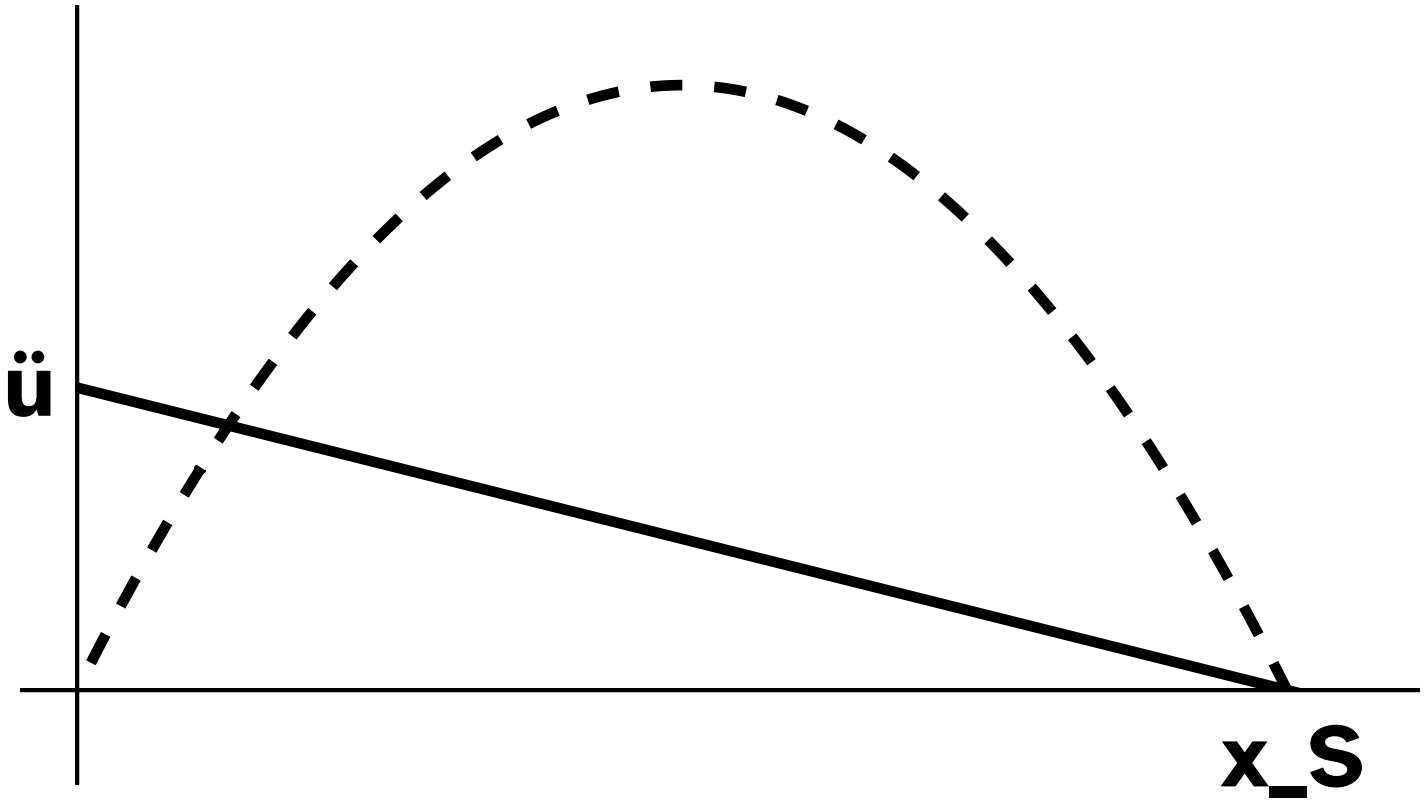
Ks': - - - - ‖ **Ks/x:** — —



p: ——— ‖ E: - - - -



p(x): ——— ‖ Eq: - - - -



Ergänzungen zu Seite 5

Fixpreis: p

Erlös linear steigend

Nachfragefunktion: p(x)

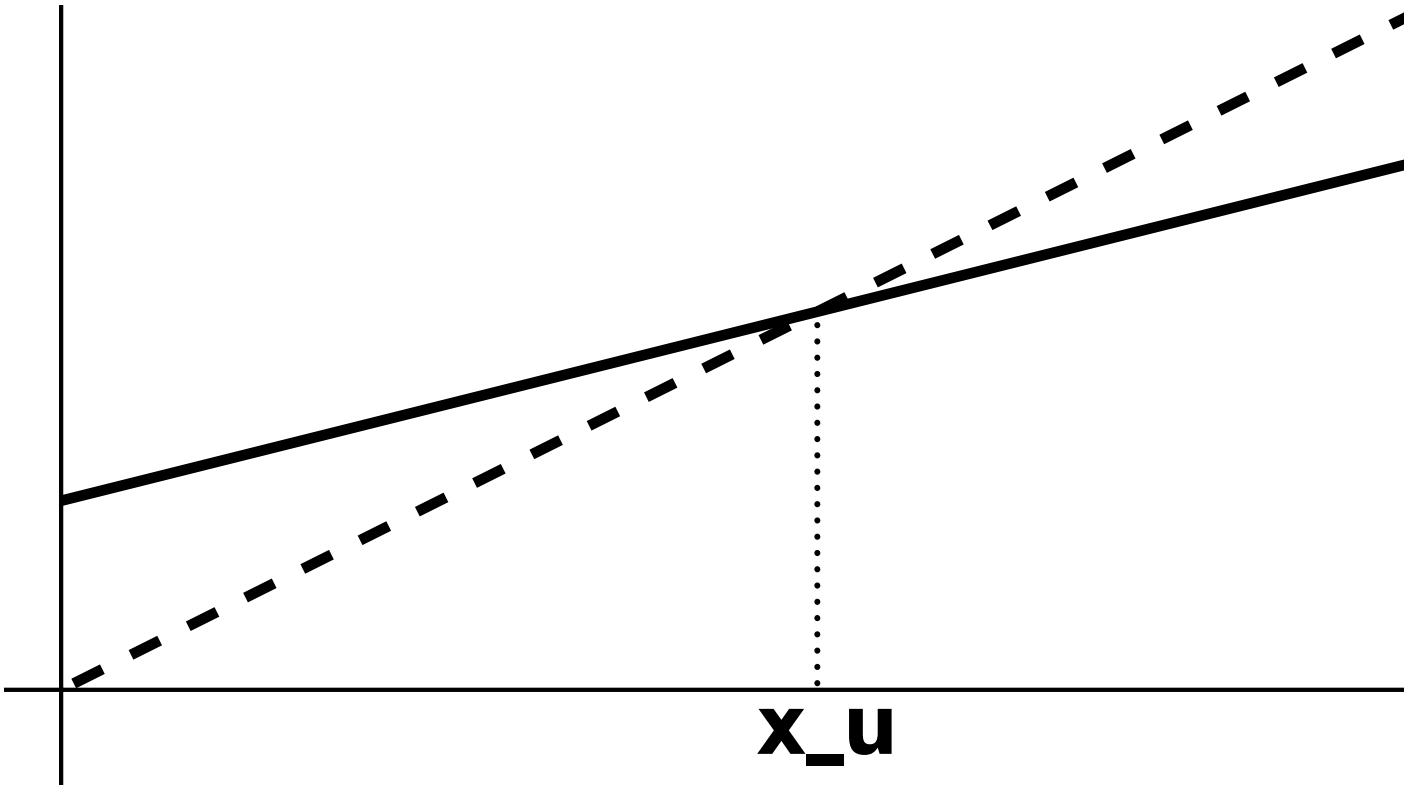
H(0|p(0))

Höchstpreis H =Schnittpunkt mit der y-Achse

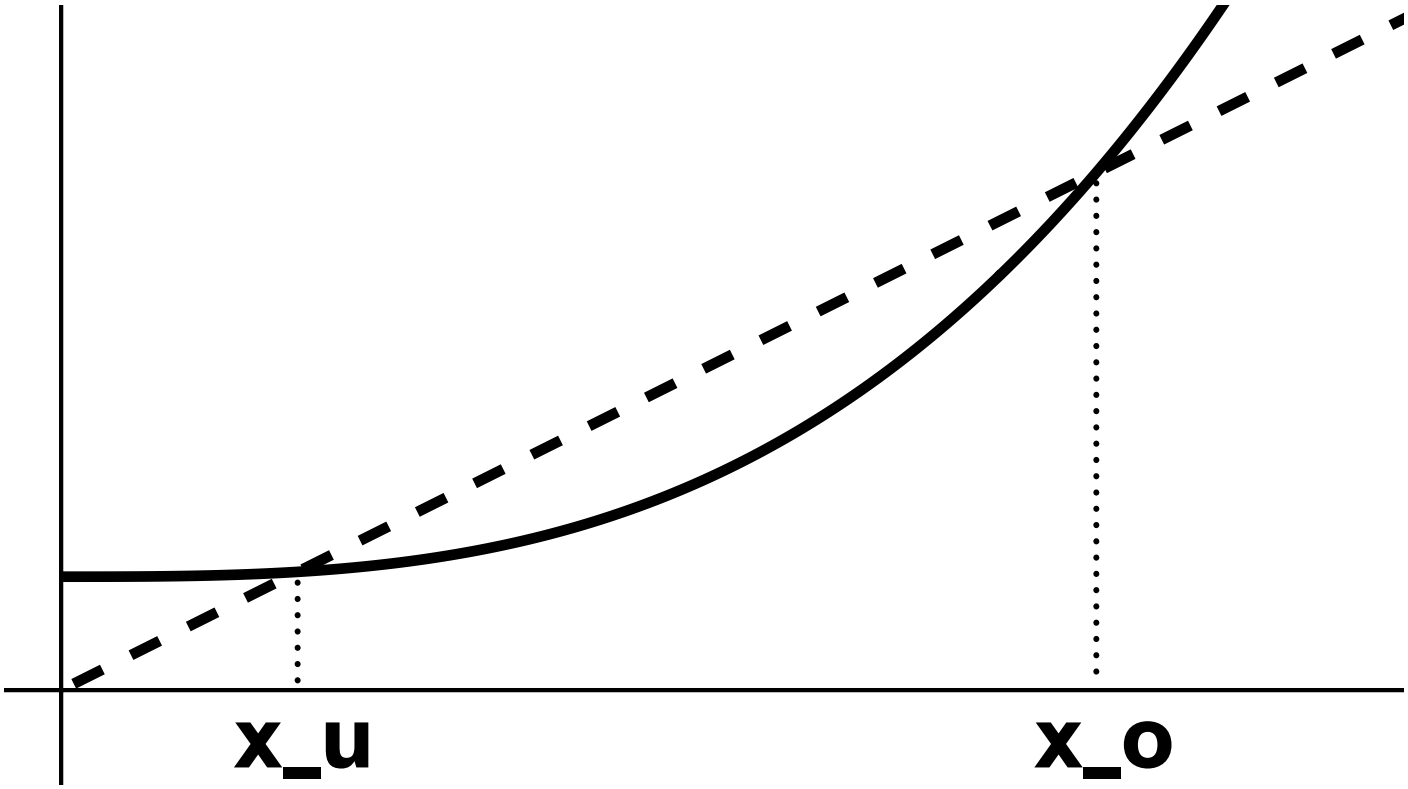
S(x_S|0)

Sättigungsmenge x_S =Schnittstelle mit der x-Achse

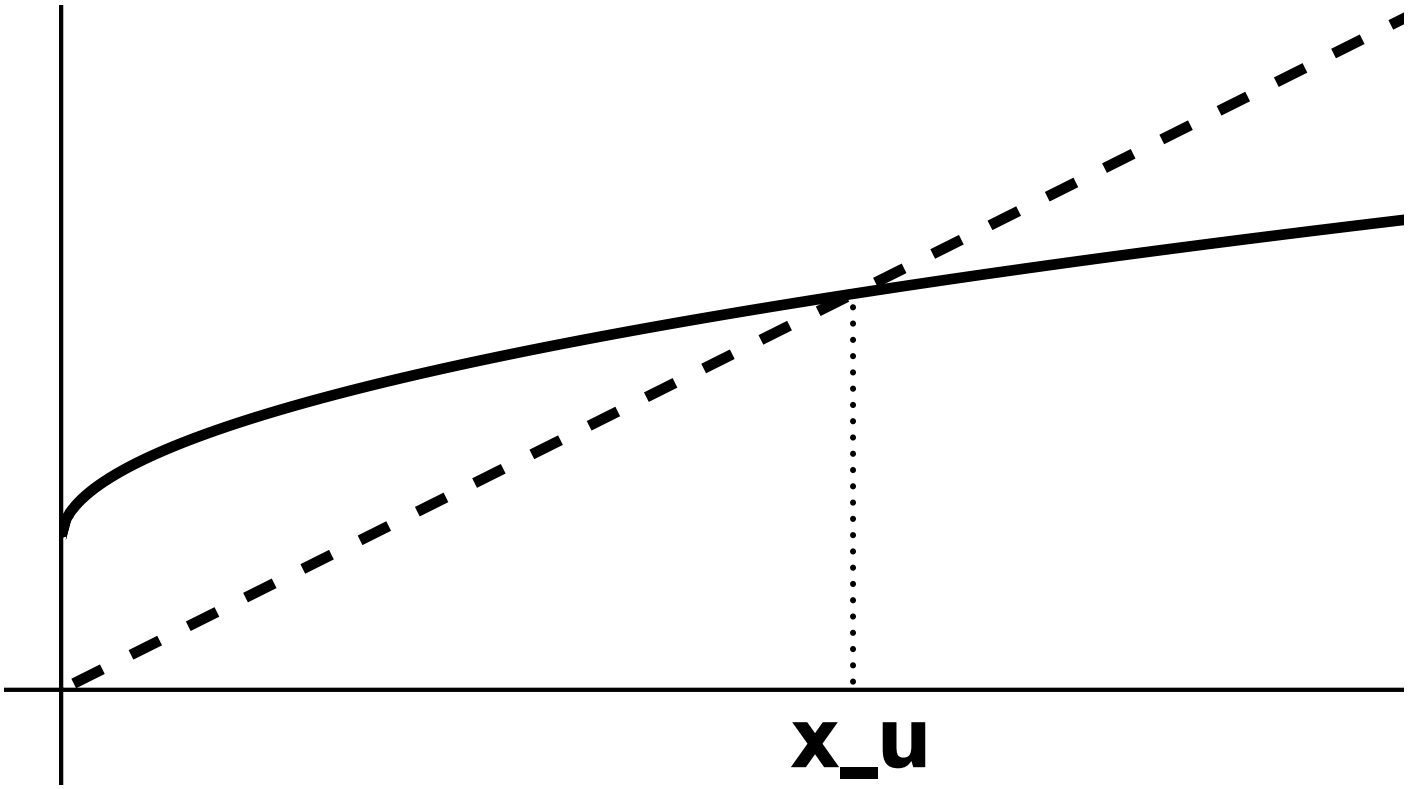
Kl: ——— ‖ E: - - - -



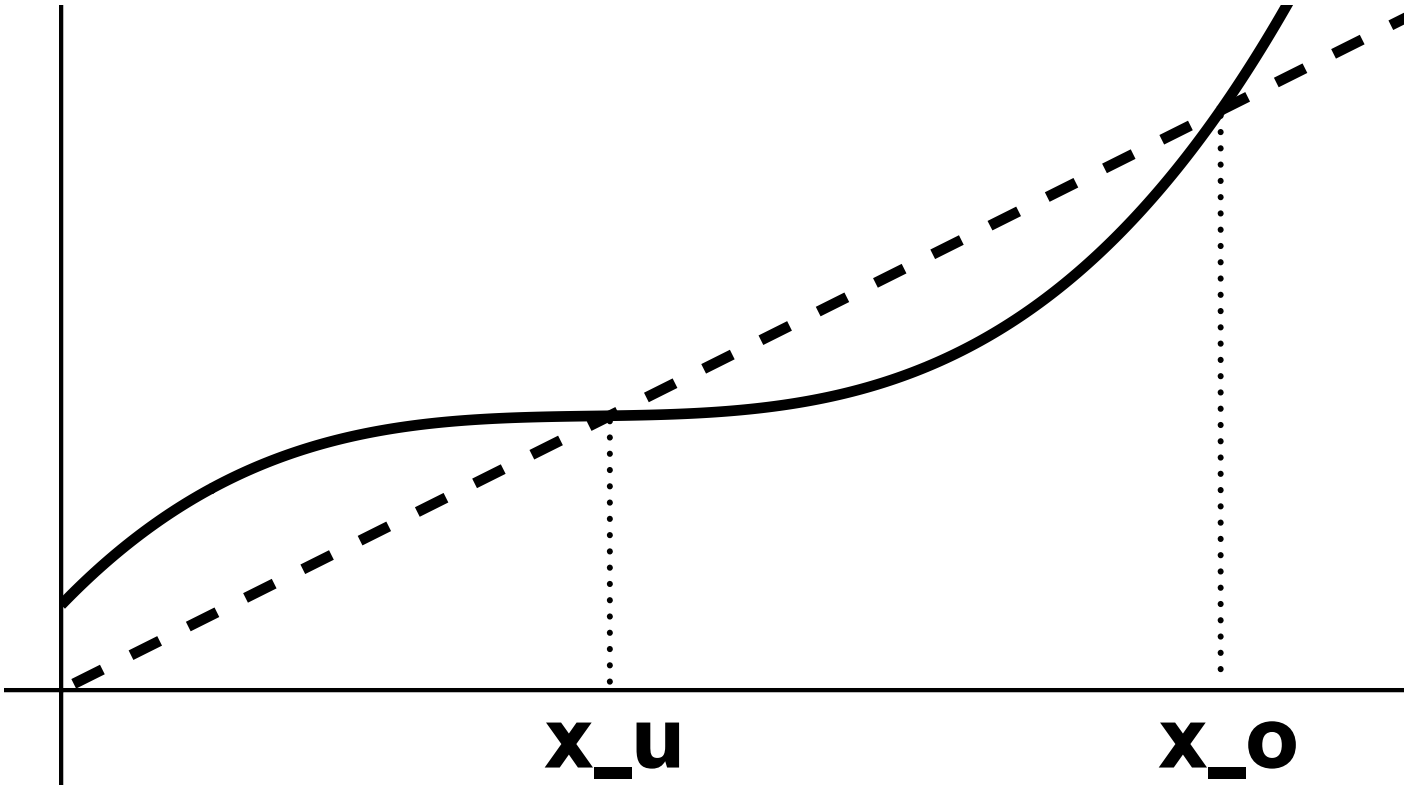
Kp: ——— ‖ E: - - - -



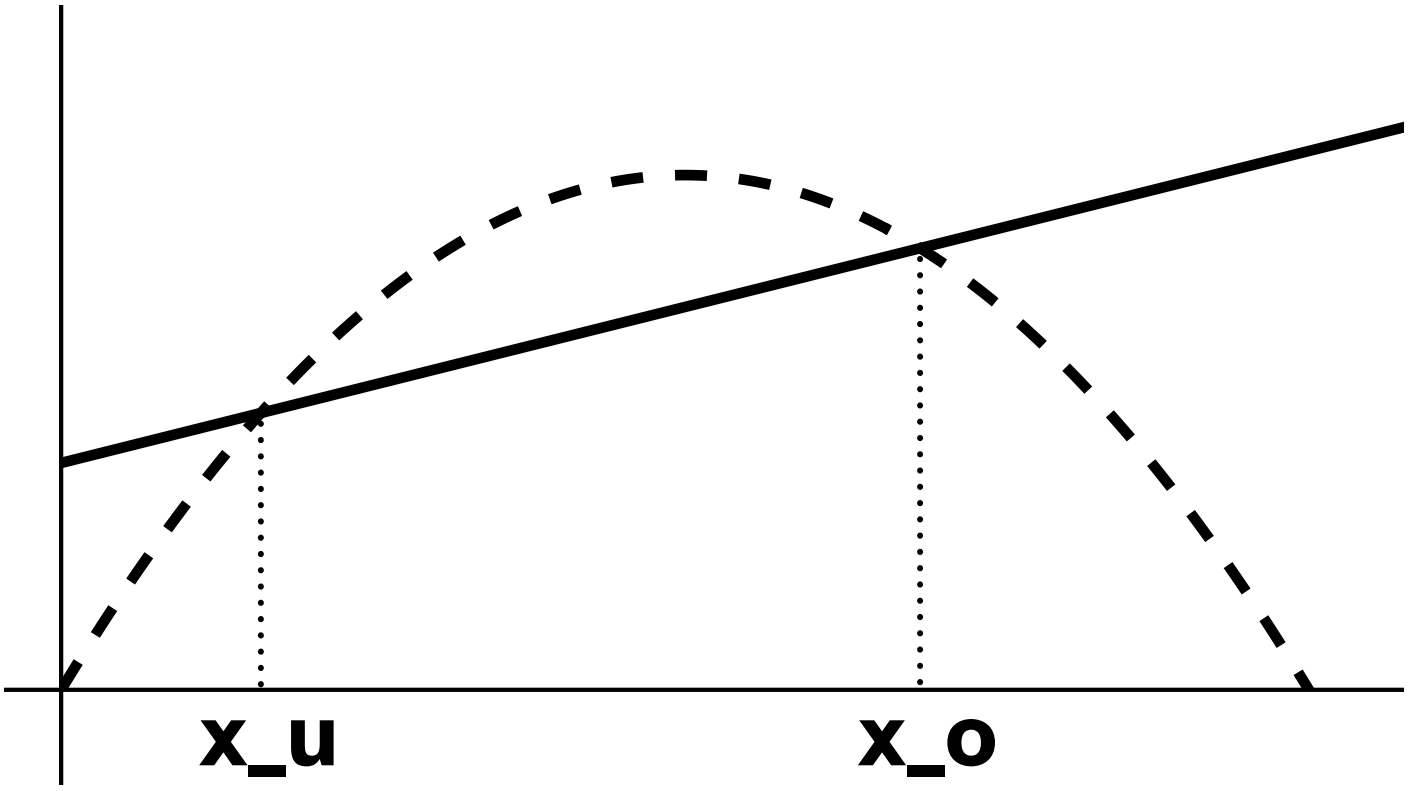
Kd: ——— ‖ E: - - - -



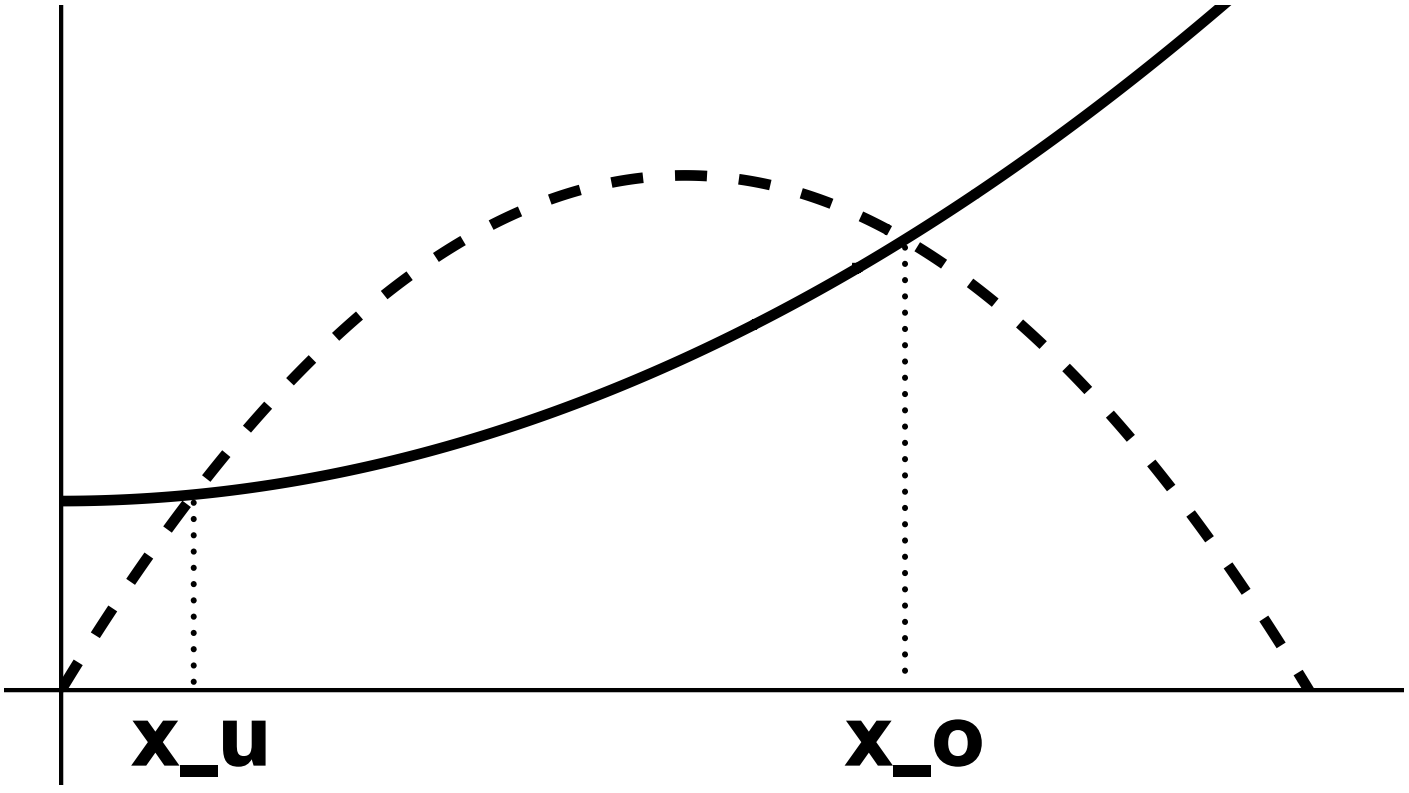
Ks: ——— ‖ E: - - - -



Kl: ——— ‖ Eq: - - - -

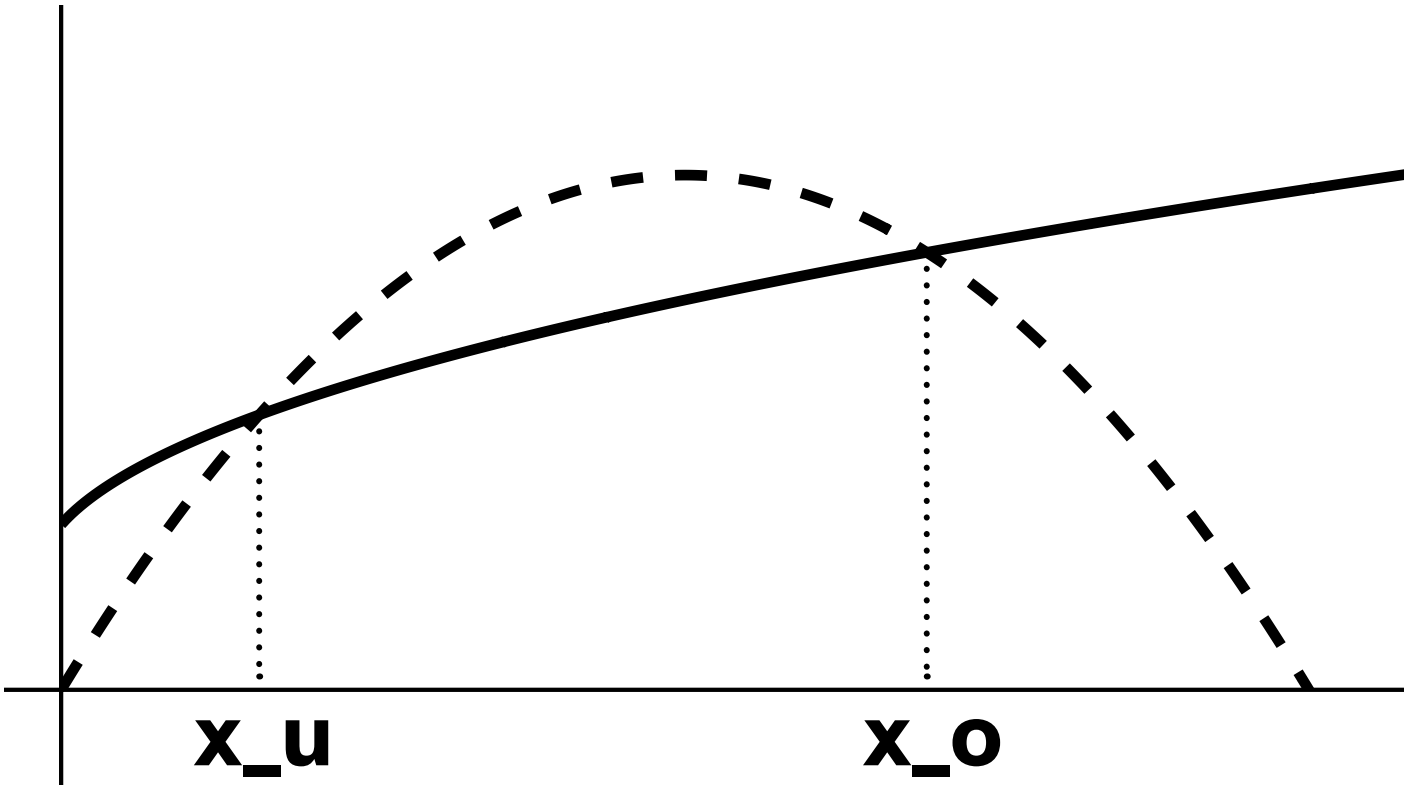


Kp: ——— ‖ Eq: - - - -

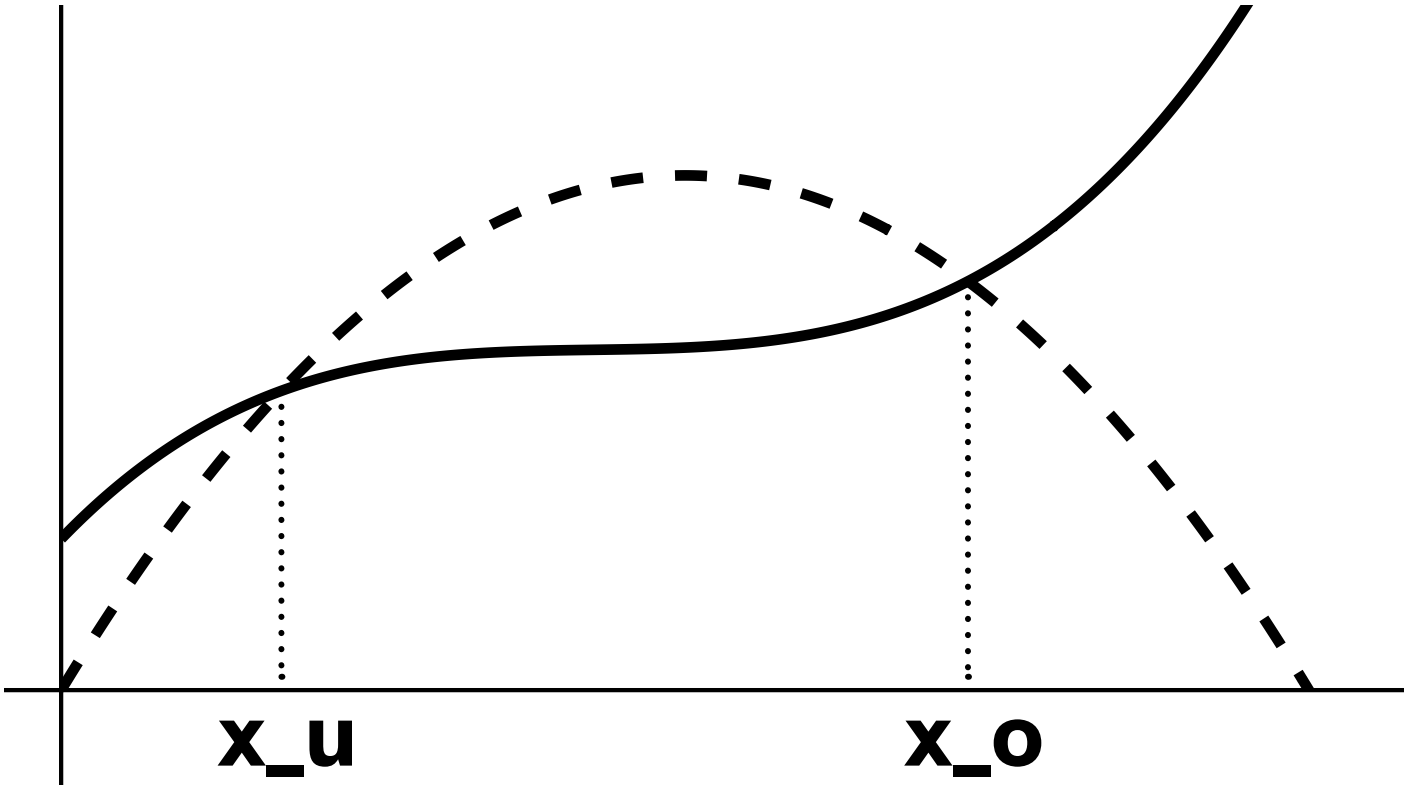


Wi Gewinngrenzen

Kd: ——— || Eq: - - - -



Ks: ——— || Eq: - - - -

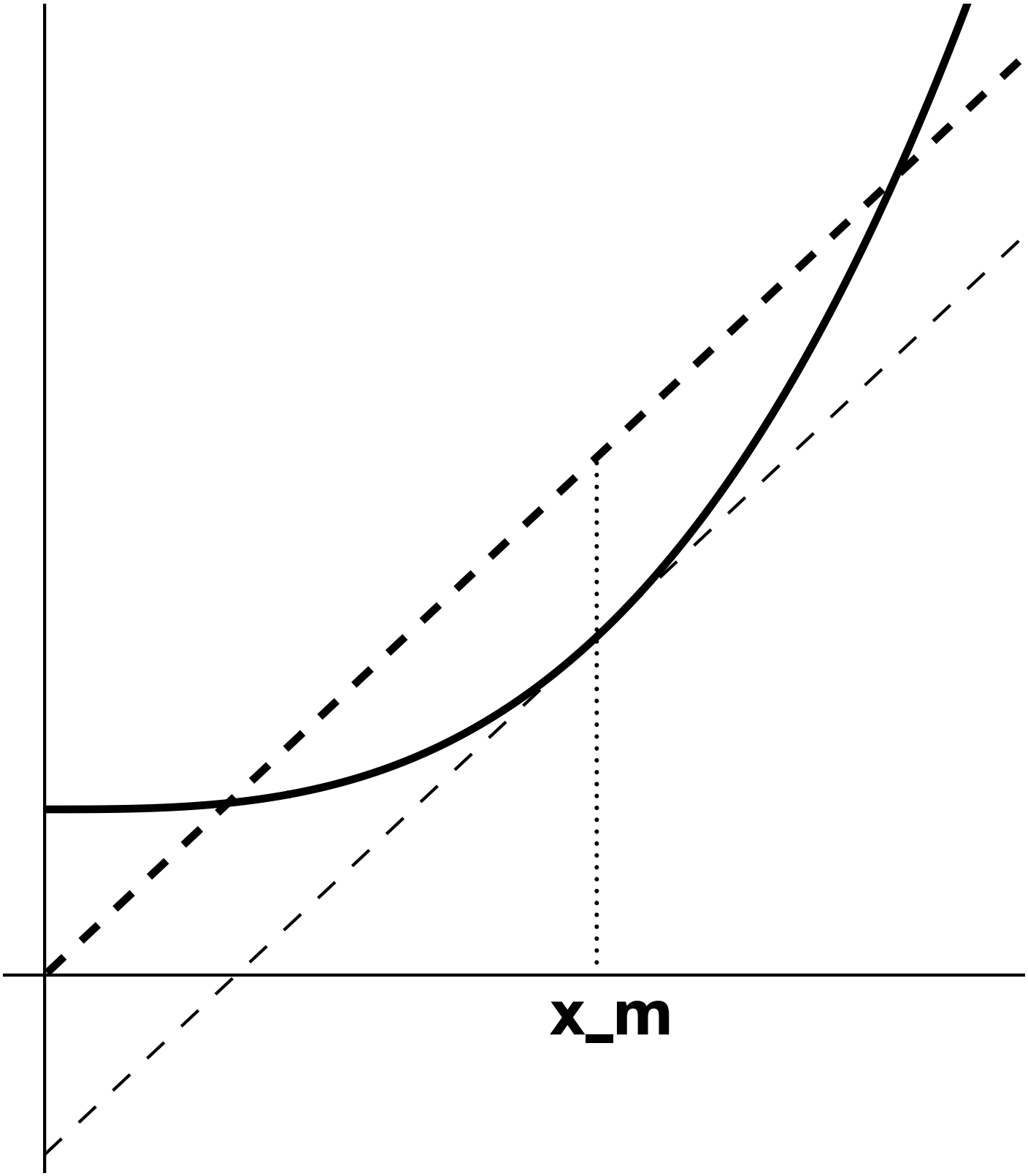


Wi Gewinnmaximierung 10/16

Kp: ———

E =p *x: - - - -

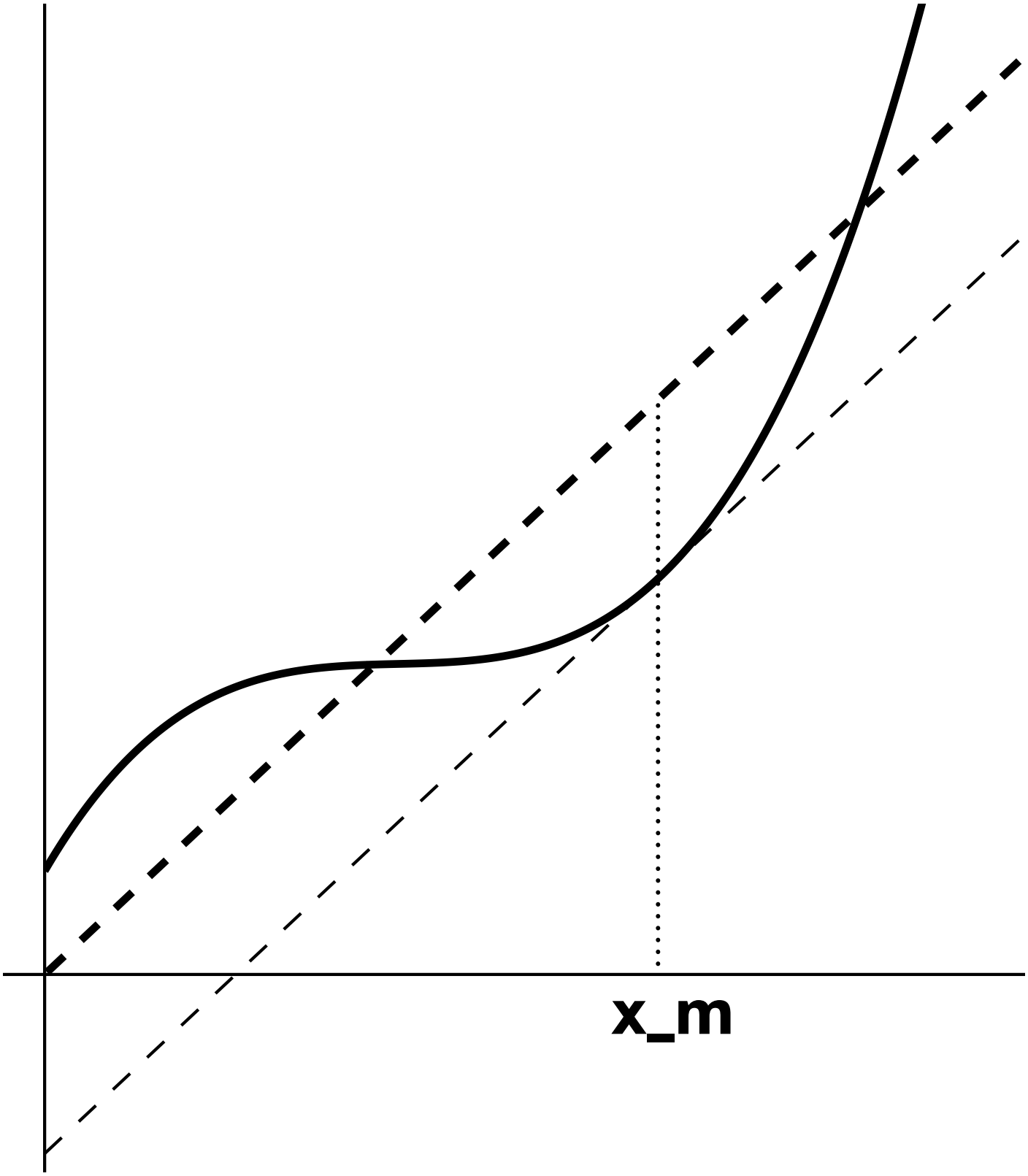
E‖: – – – –



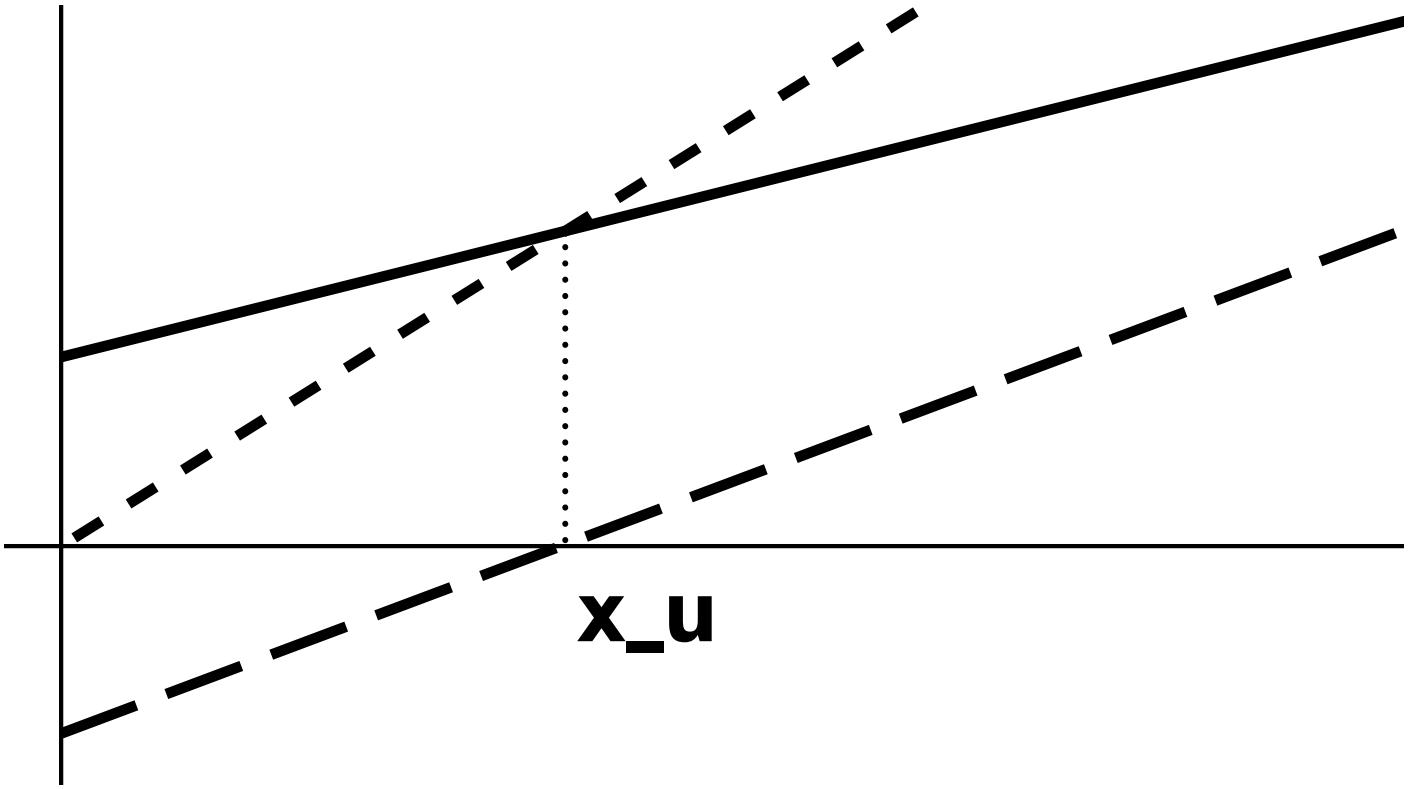
Wi Gewinnmaximierung 11/16

Ks: ———

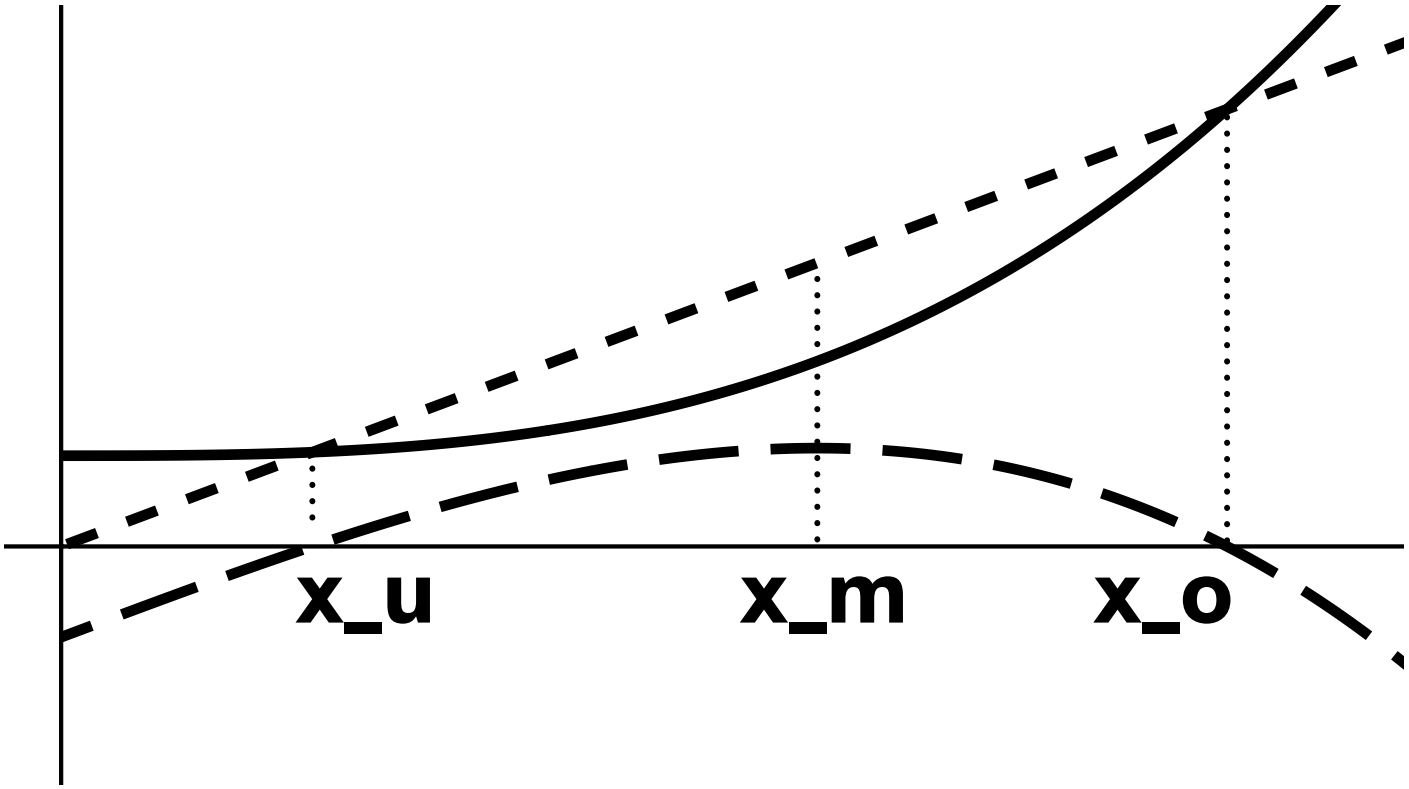
E =p *x: - - - - ‖ **E‖: – – – –**

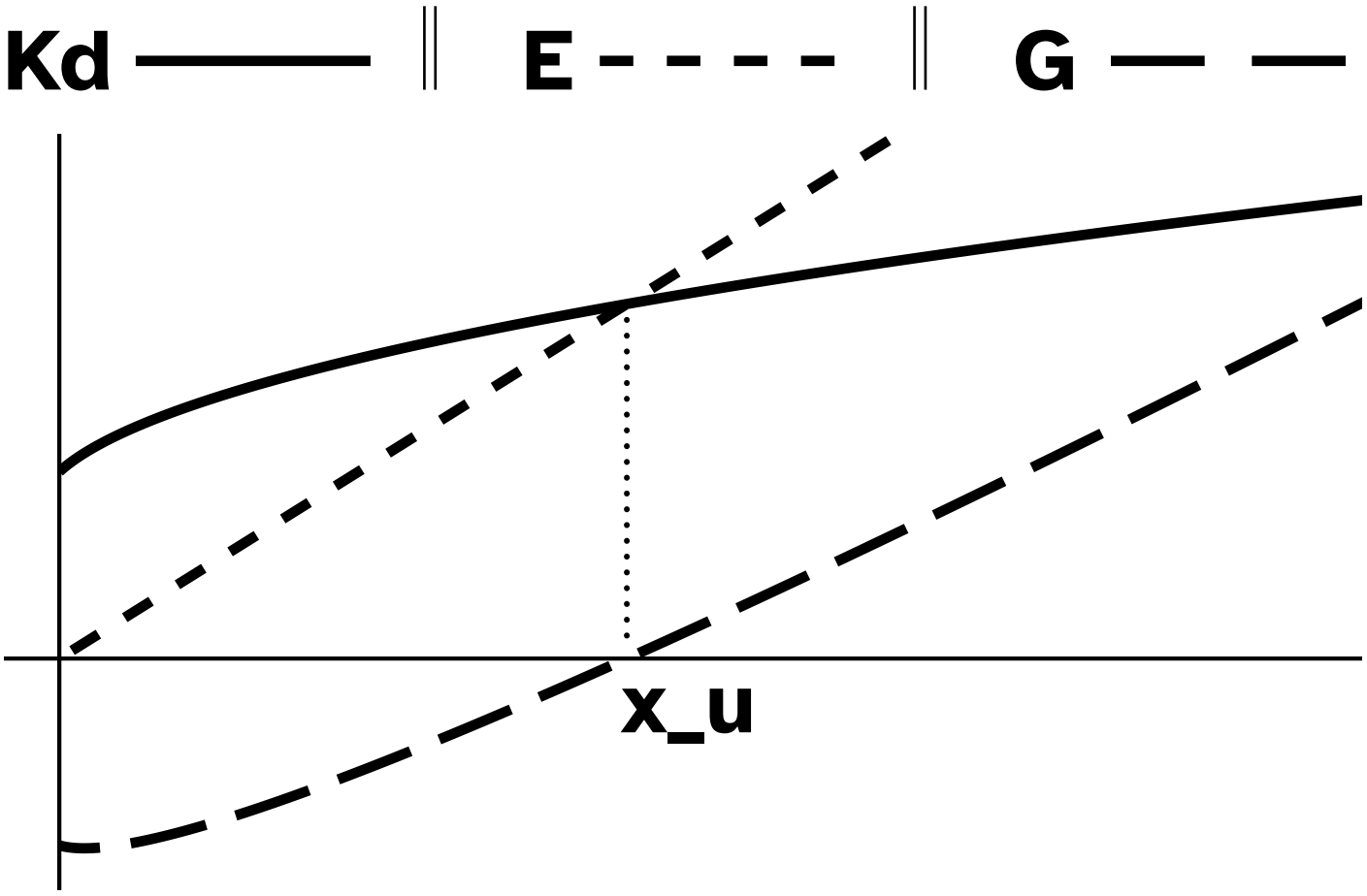


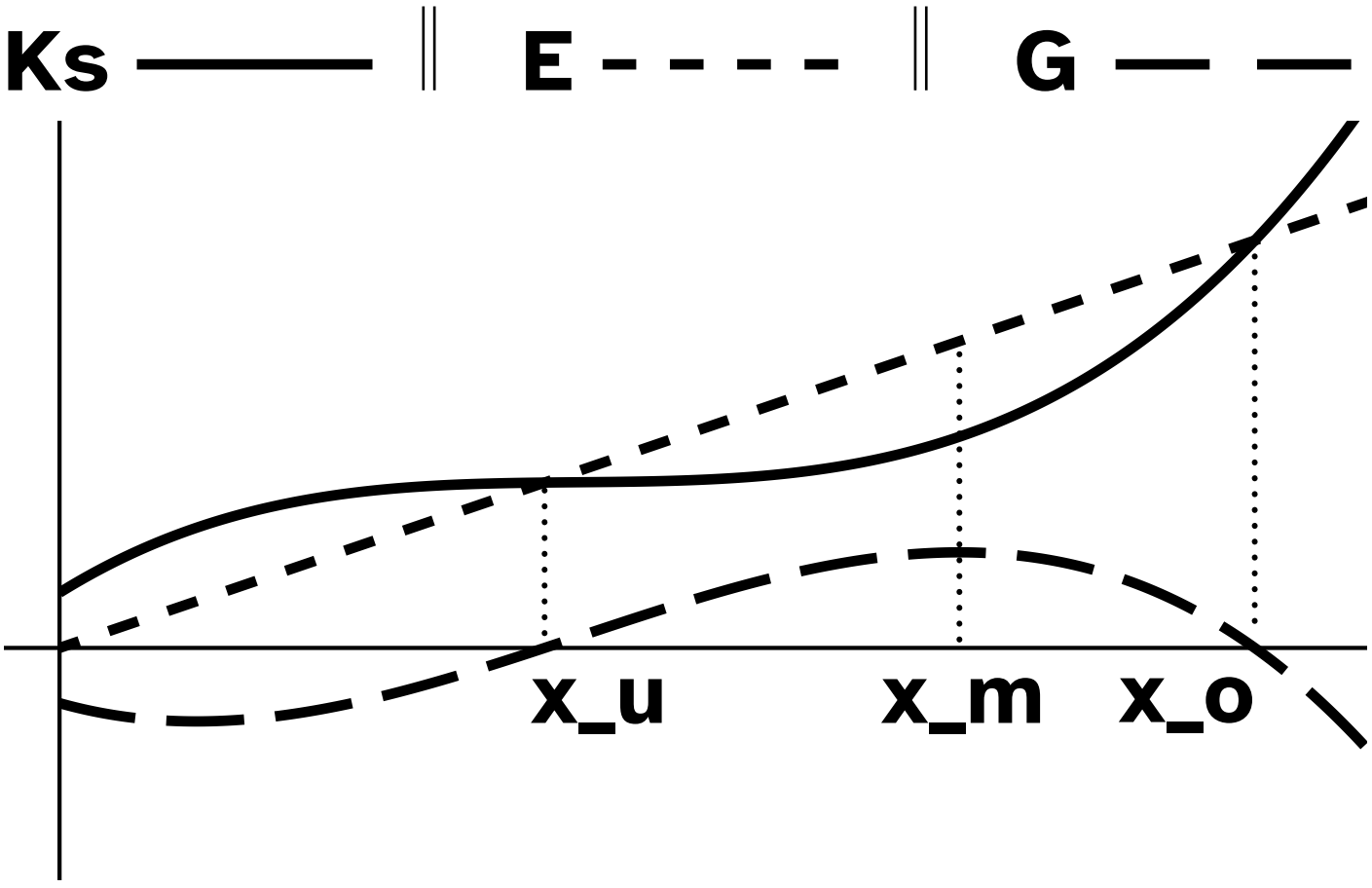
Kl ——— ‖ E - - - - ‖ G — —

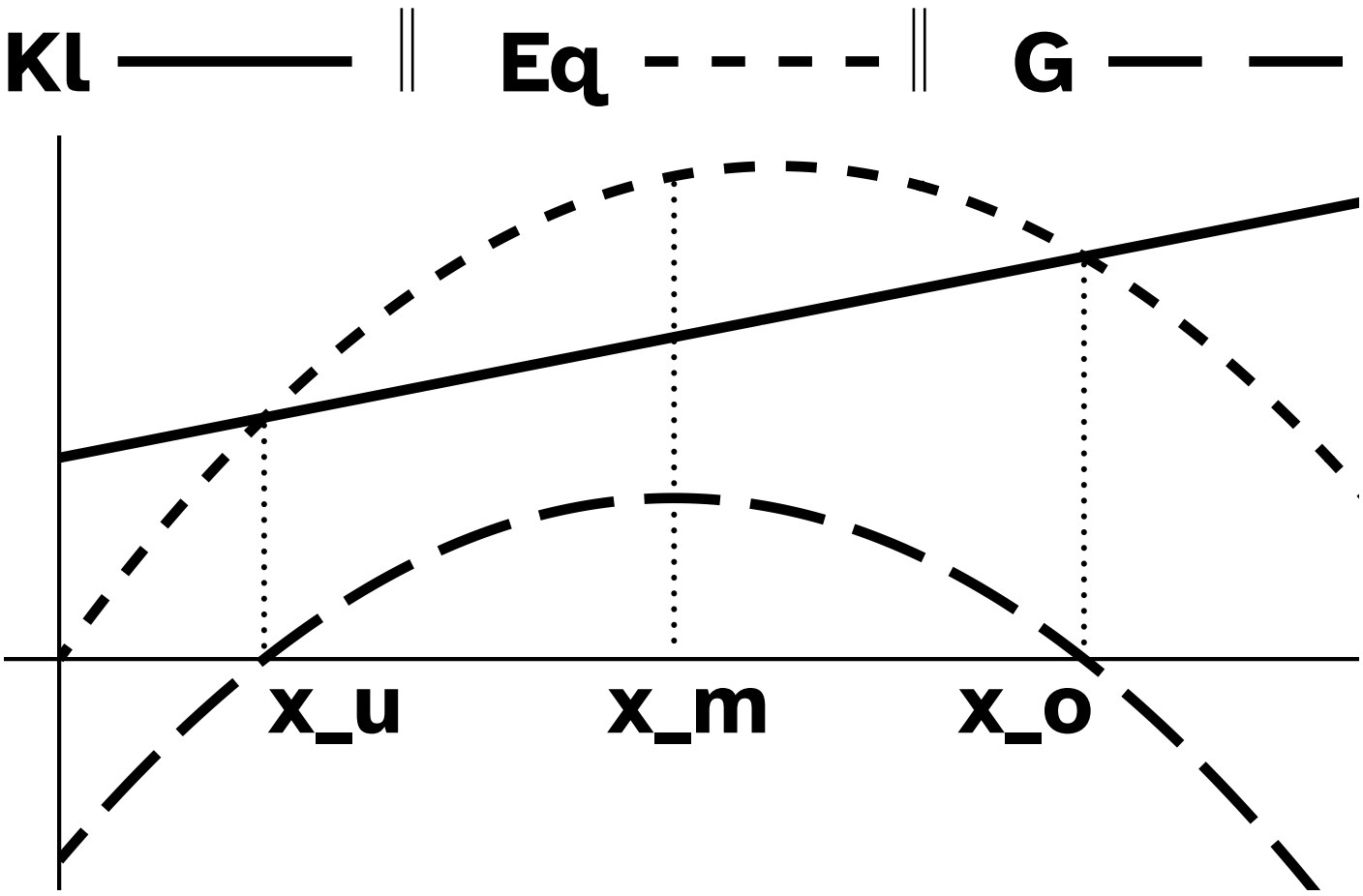


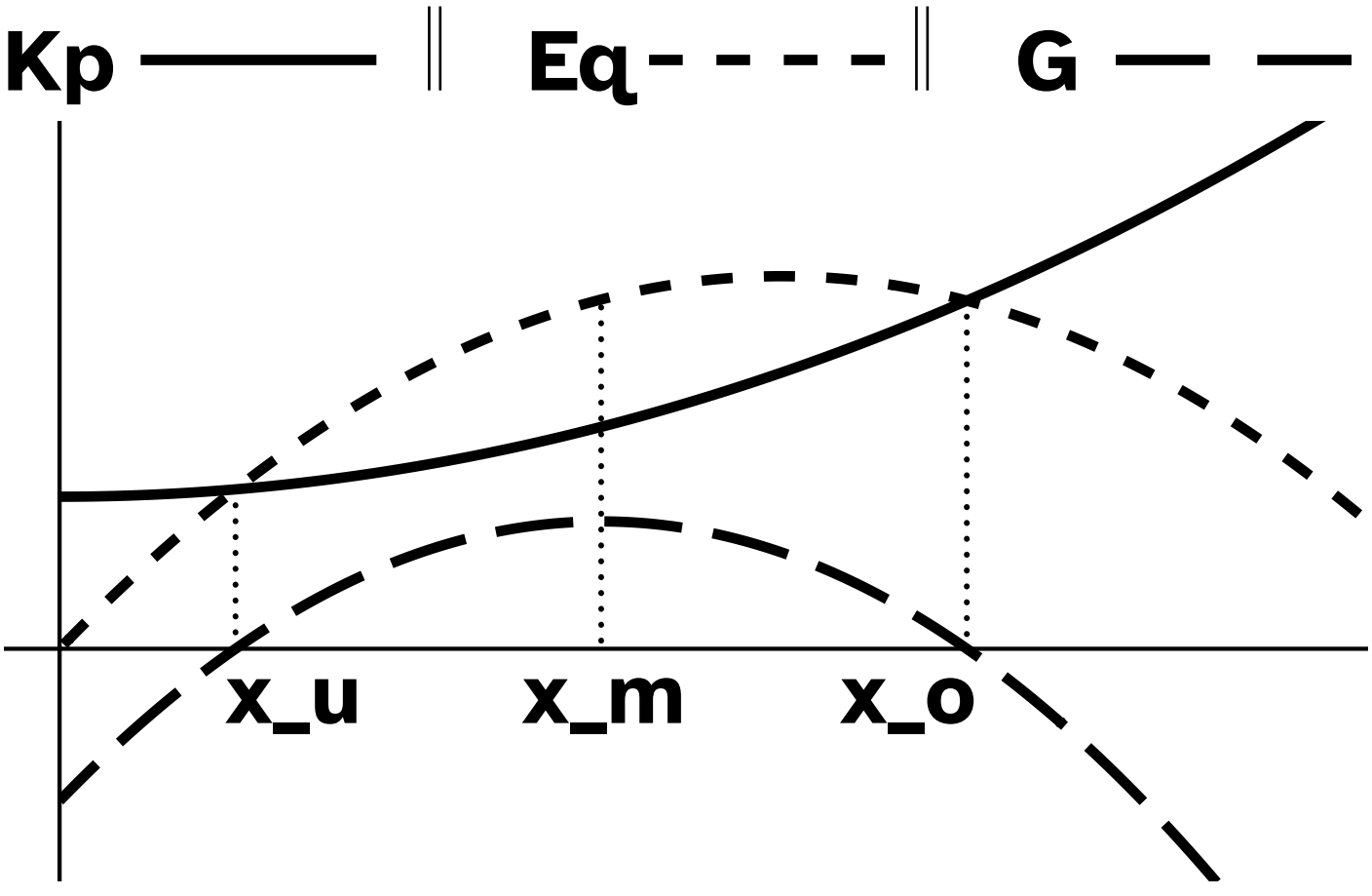
Kp ——— ‖ E - - - - ‖ G — —

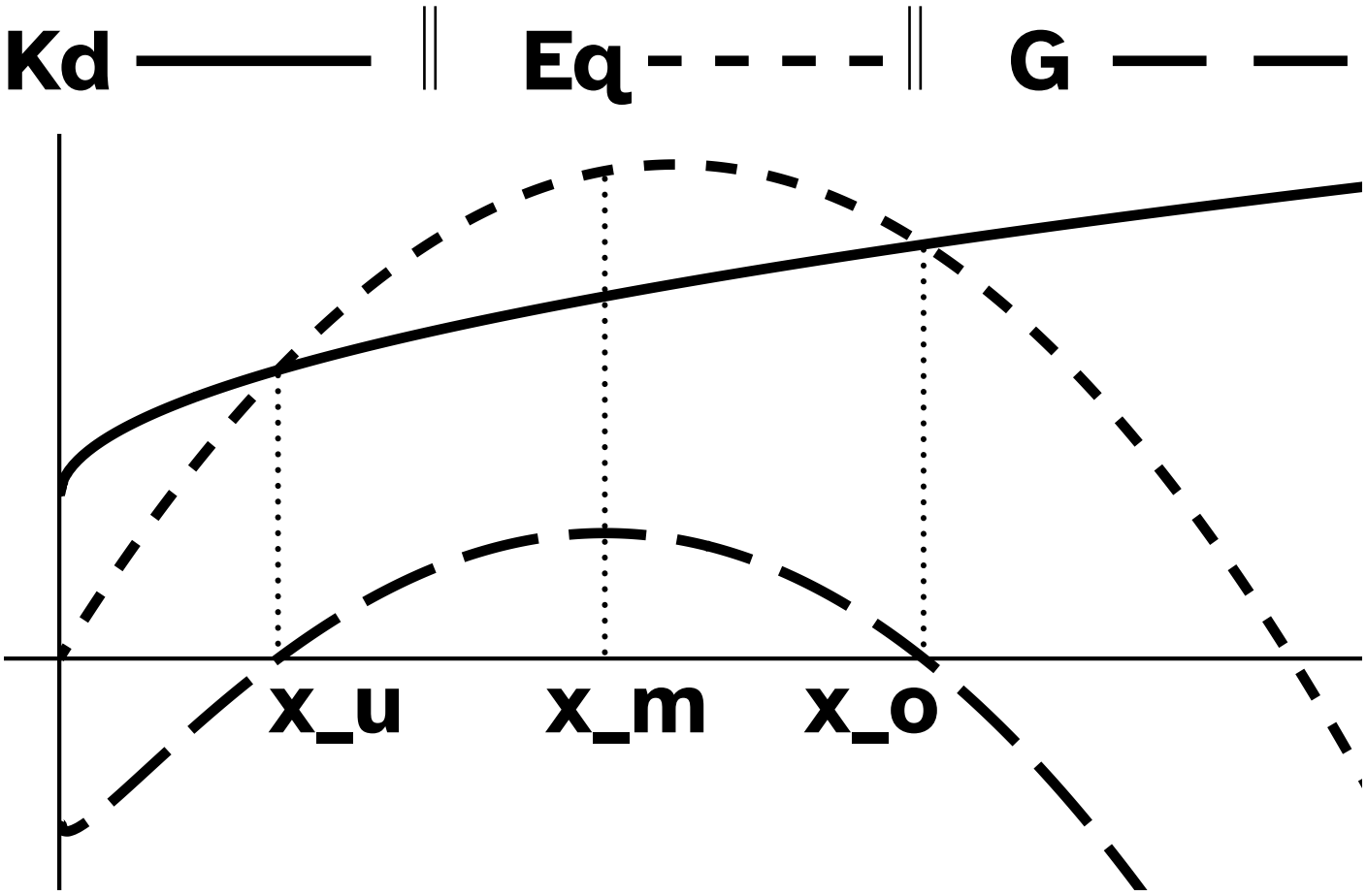


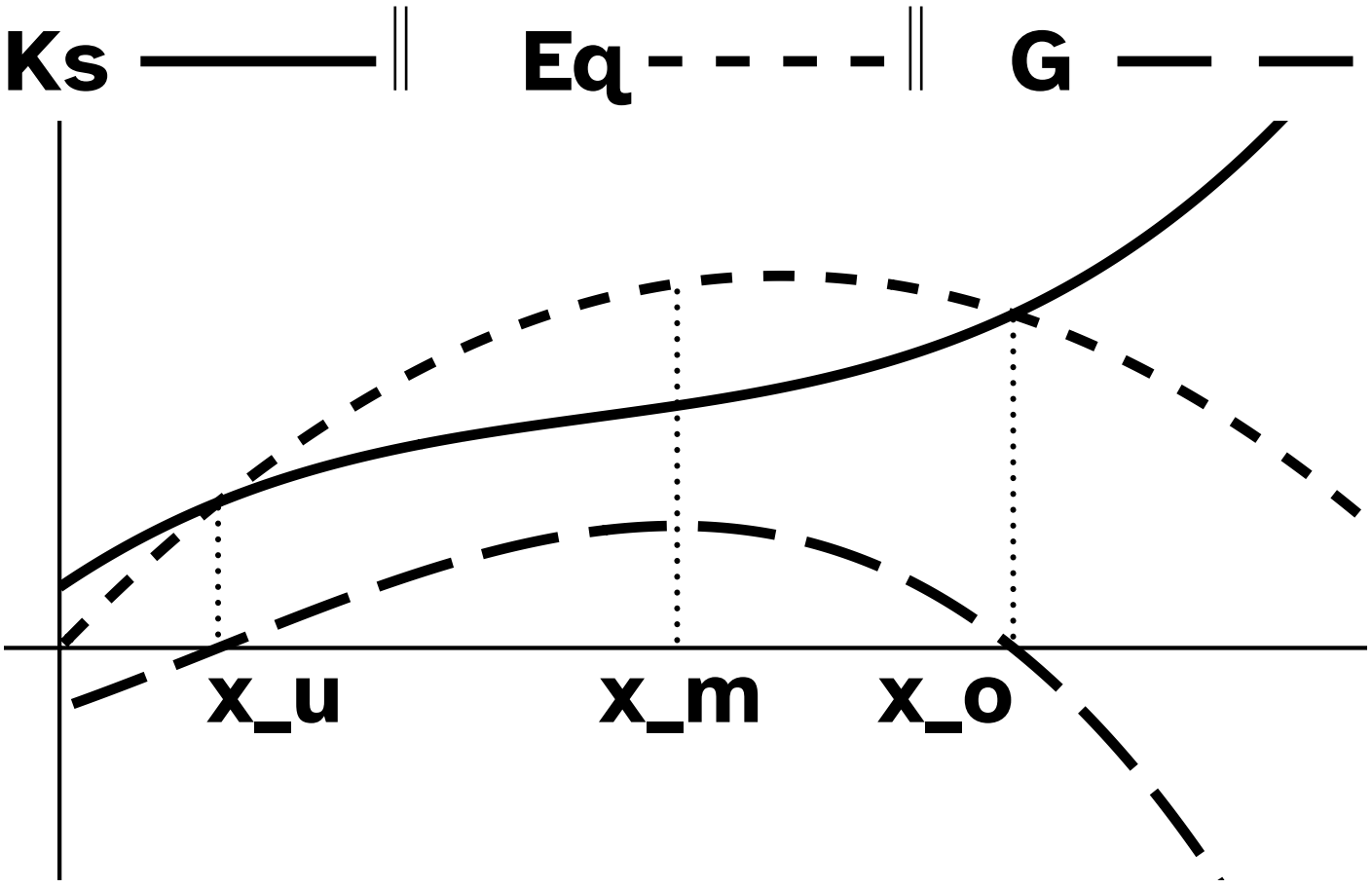




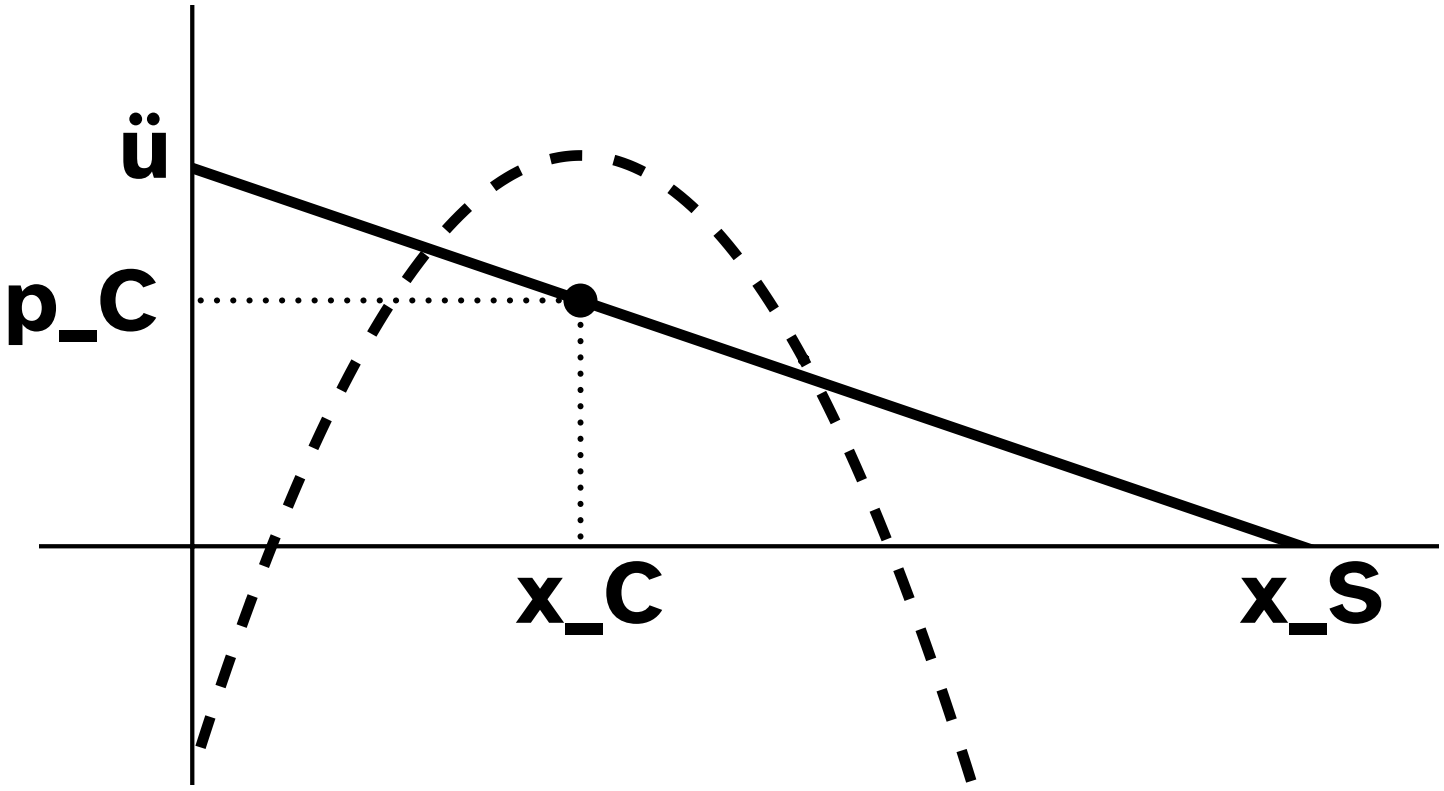




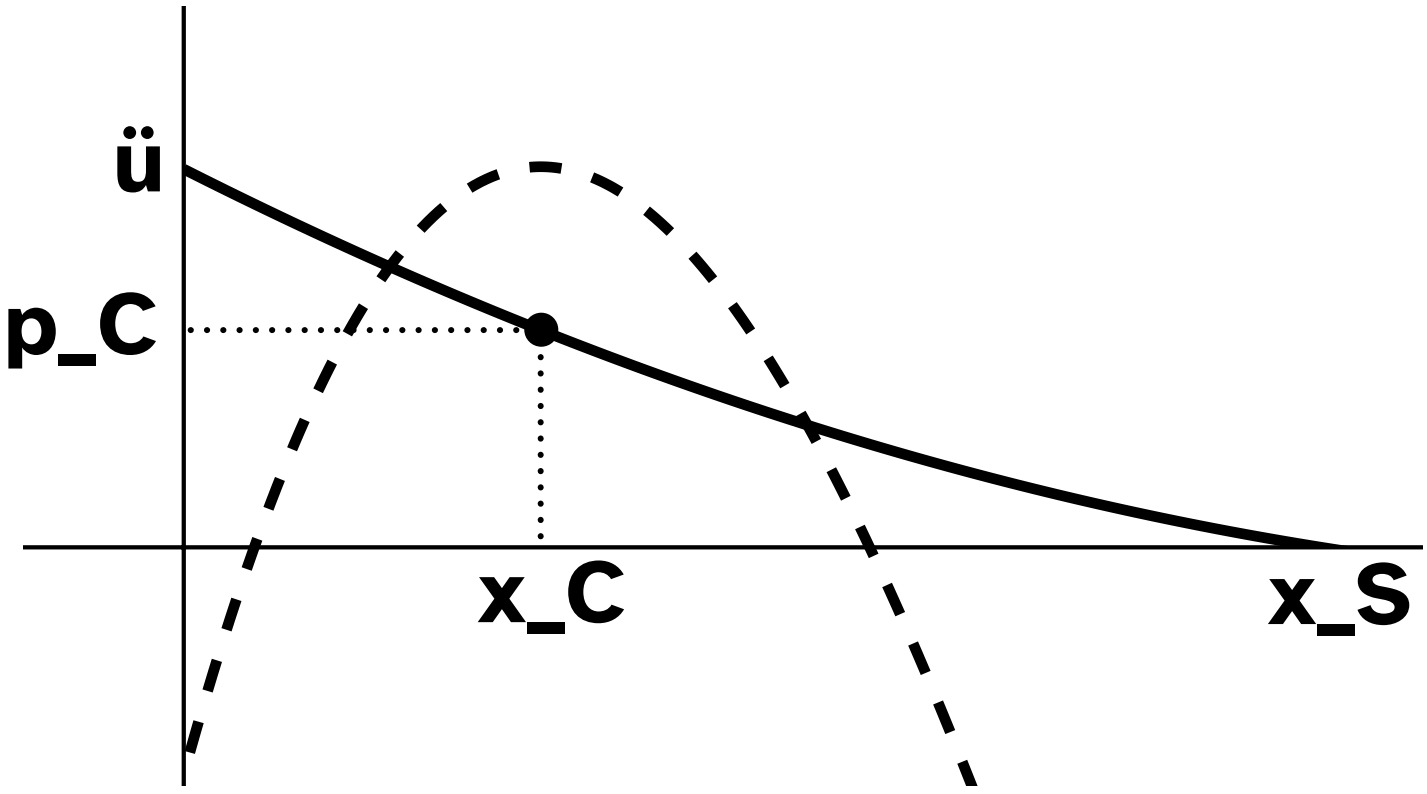




p(x): ——— ‖ G: - - - -



p(x): ——— ‖ G: - - - -



Ergänzungen zu Seite 16

C(x_C|p_C)

Cournot‘scher Punkt

x_C: gewinnmaximale Produktionsmenge eines Monopolisten

Normalverteilung

Schulstufe 12

Histogramm

Dichtefunktion f

Verteilungsfunktion F

standardisierte Dichtefunktion 'ph

standardisierte Verteilungsfunktion 'Ph

Inhalt

10 Standard. Verteilungsfkt 'Ph

'Ph(z) =P(X <='my +z *'si)

11 Standard. Verteilungsfkt 'Ph

1 -'Ph(z) ='Ph(-z)

12 12 Standard. Verteilungsfkt 'Ph

2 *'Ph(z) -1

13 Standard. Verteilungsfkt 'Ph

'Ph(z), z <0

14 Standard. Verteilungsfkt 'Ph

1 -'Ph(z), z >0

15 Standard. Verteilungsfkt 'Ph

'Ph(z_2) -'Ph(z_1), z >0

16 Standard. Verteilungsfkt 'Ph

'Ph(z_2) -'Ph(z_1)

17 Standard. Verteilungsfkt 'Ph

'Ph(z_2) -'Ph(z_1), z <0

18 Unterschiedl. Standardabweichungen

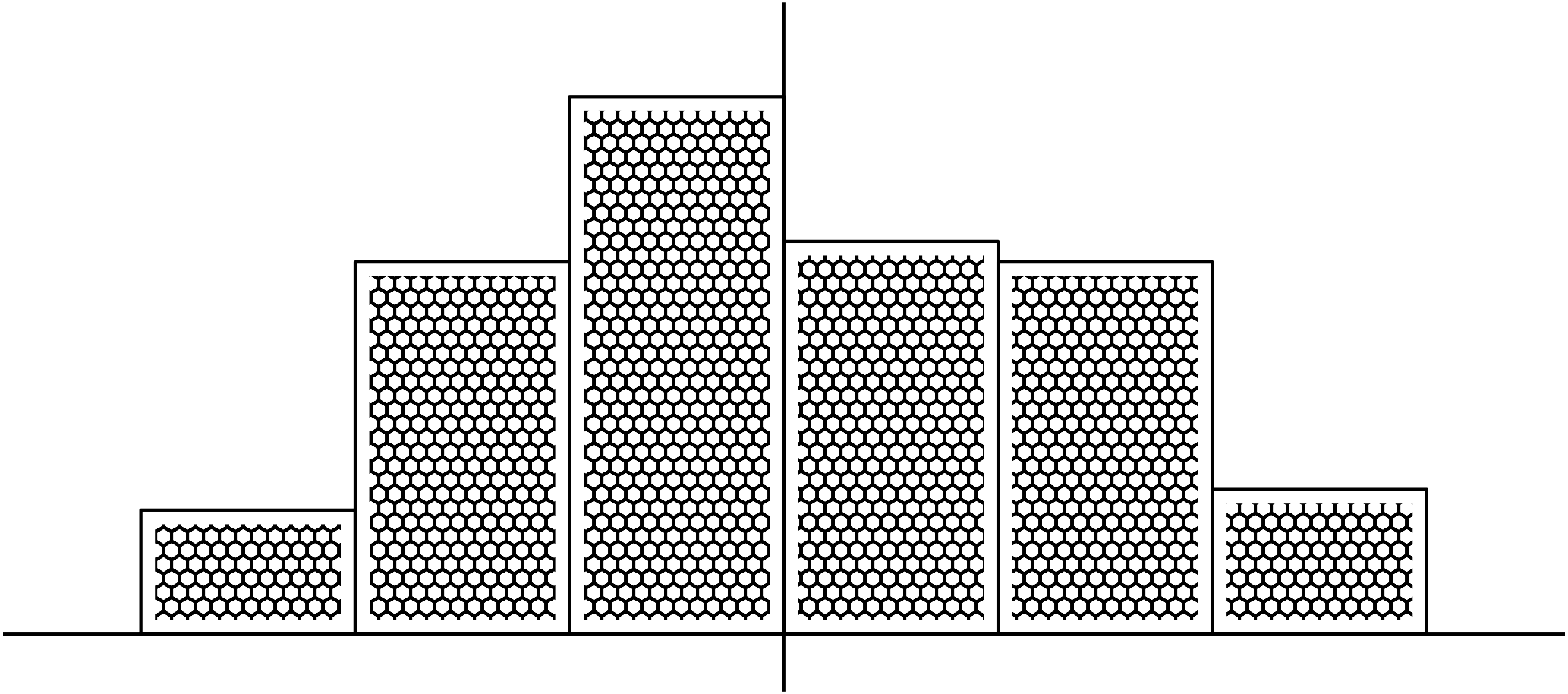
Normalverteilung - Abkürzungen

Index

f:	(Wahrscheinlichkeits-) Dichtefunktion von X
F:	Verteilungsfunktion von X F(a) =P(X <=a)
P:	Wahrscheinlichkeit
P(a_i):	Wahrscheinlichkeit für a_i
X:	diskrete Zufallsvariable
z:	(x -'my)/'si
'my:	Erwartungswert
'ph:	(Wahrscheinlichkeits) Dichtefunktion der Standardnormalverteilung von X
'Ph:	Verteilungsfunktion der Dichtefunktion der Standardnormalverteilung von X
'si:	Standardabweichung

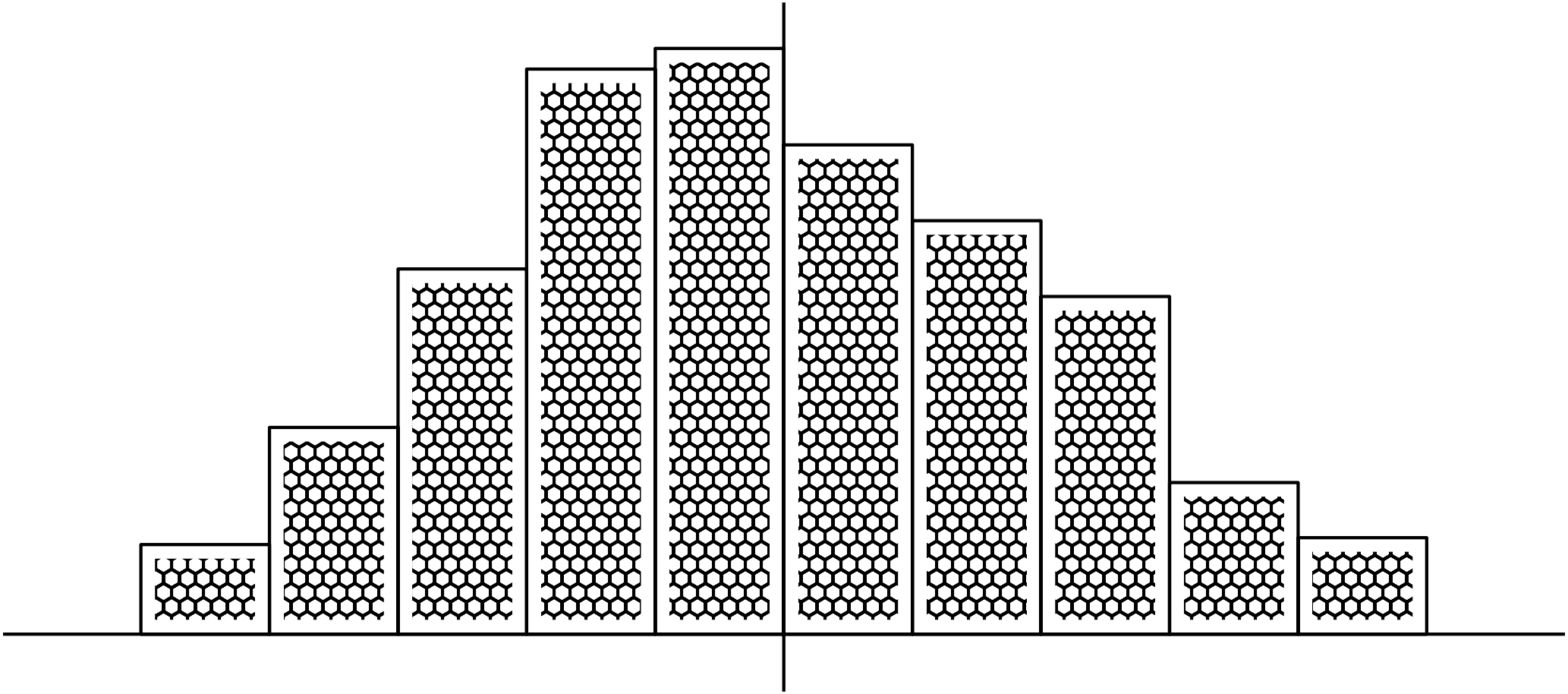
NV Histogramm

Histogramm, n =6



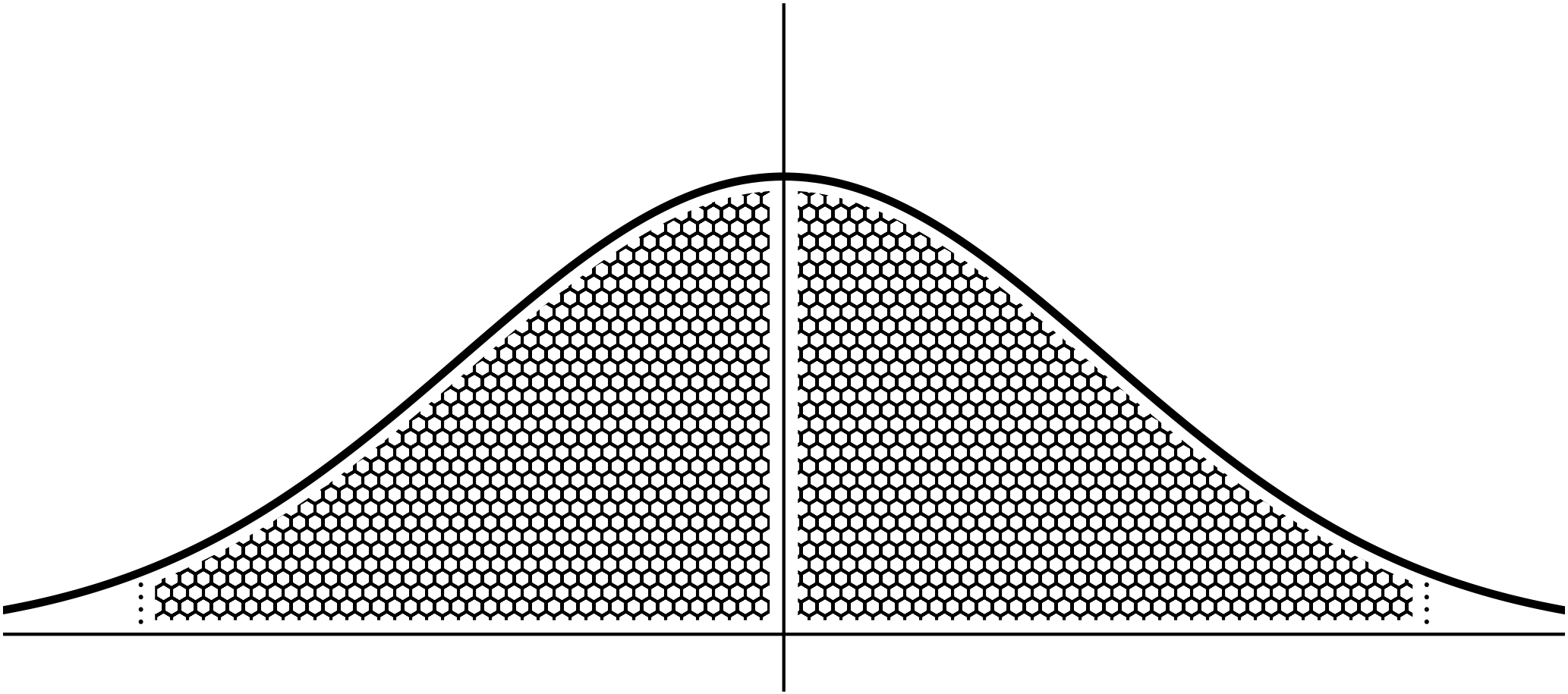
NV Histogramm

Histogramm, n =10



NV Histogramm

Verteilungsfunktion

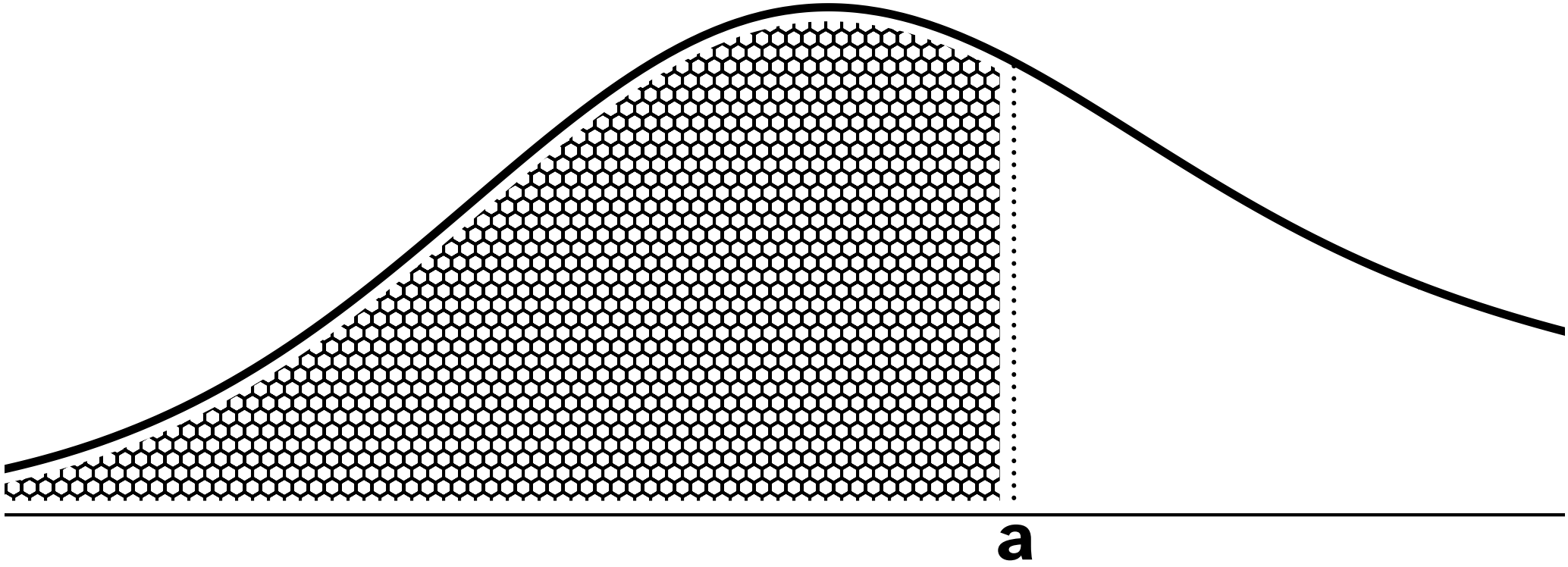


NV allg Verteilungsfunktion

P(X <=a) =F(a)

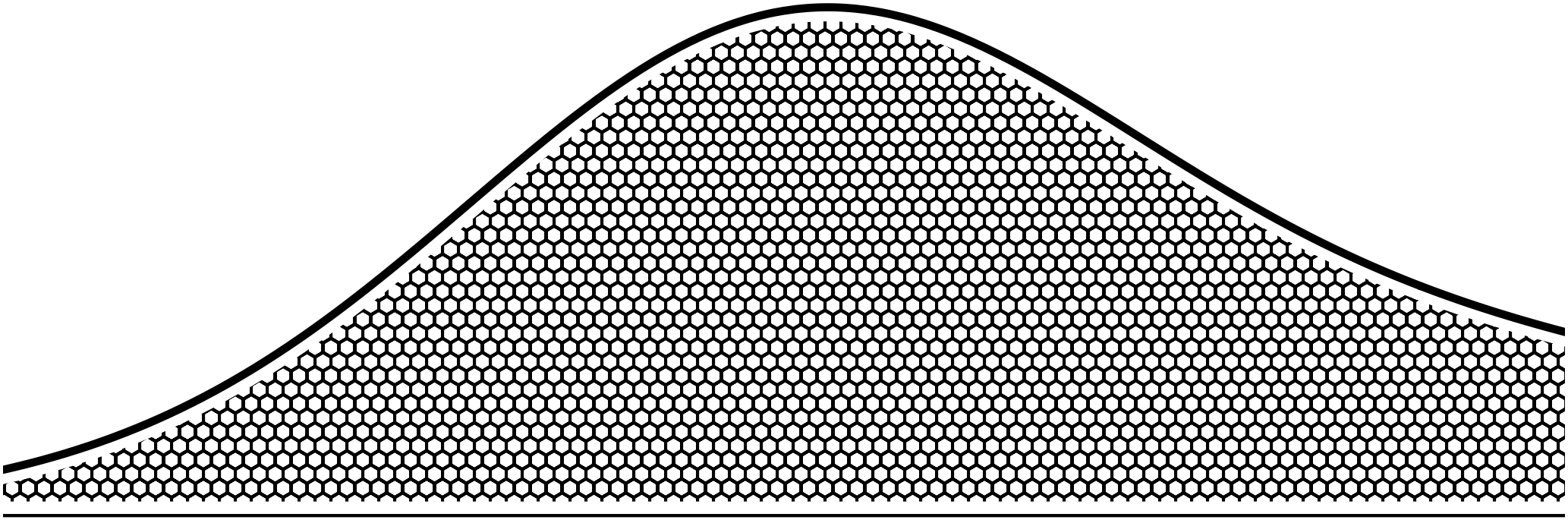
allg. Dichtefkt. f: ———

allg. Verteilungsfkt. F:



NV allg Verteilungsfunktion

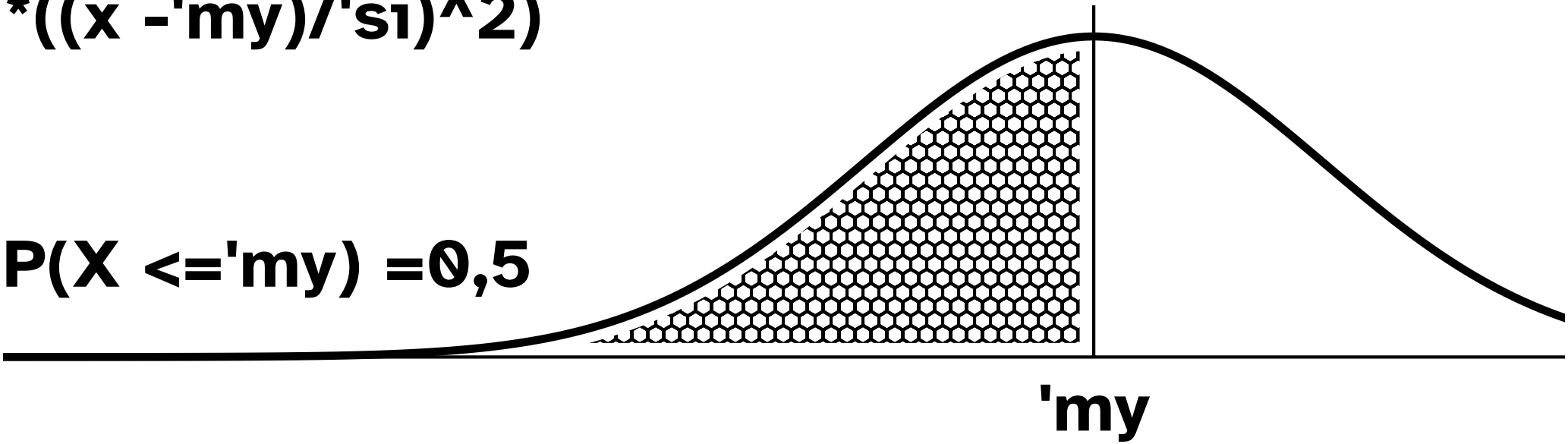
P(-'ue <x <+'ue) =1



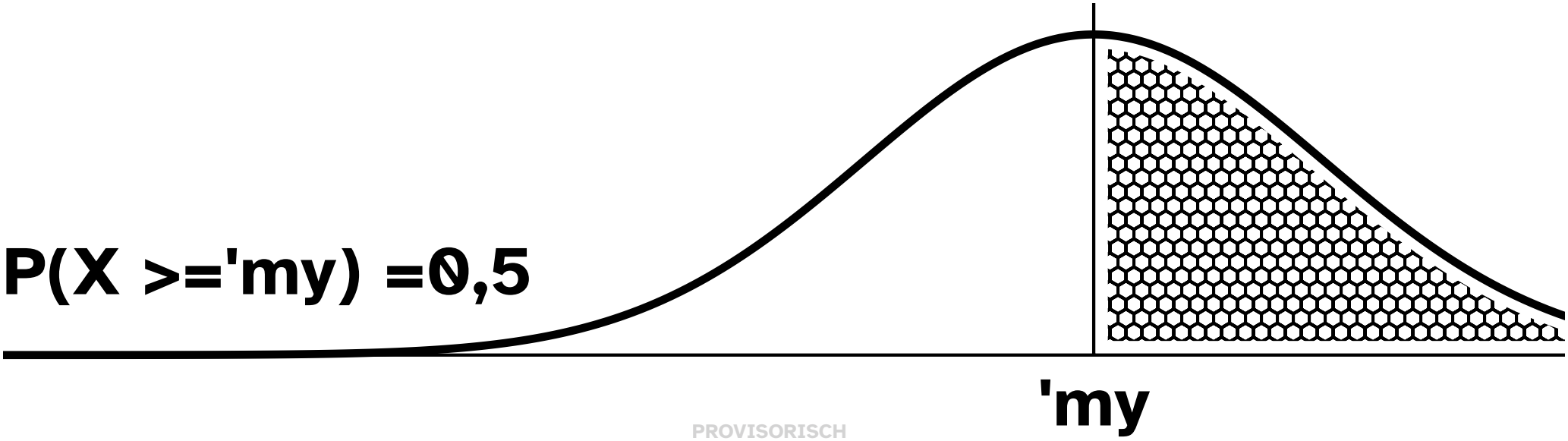
NV Dichtefunktion

f(x) =1/('si *'w(2 *'pi)) *'e^((-1/2) *((x -'my)/'si)^2)

P(X <='my) =0,5

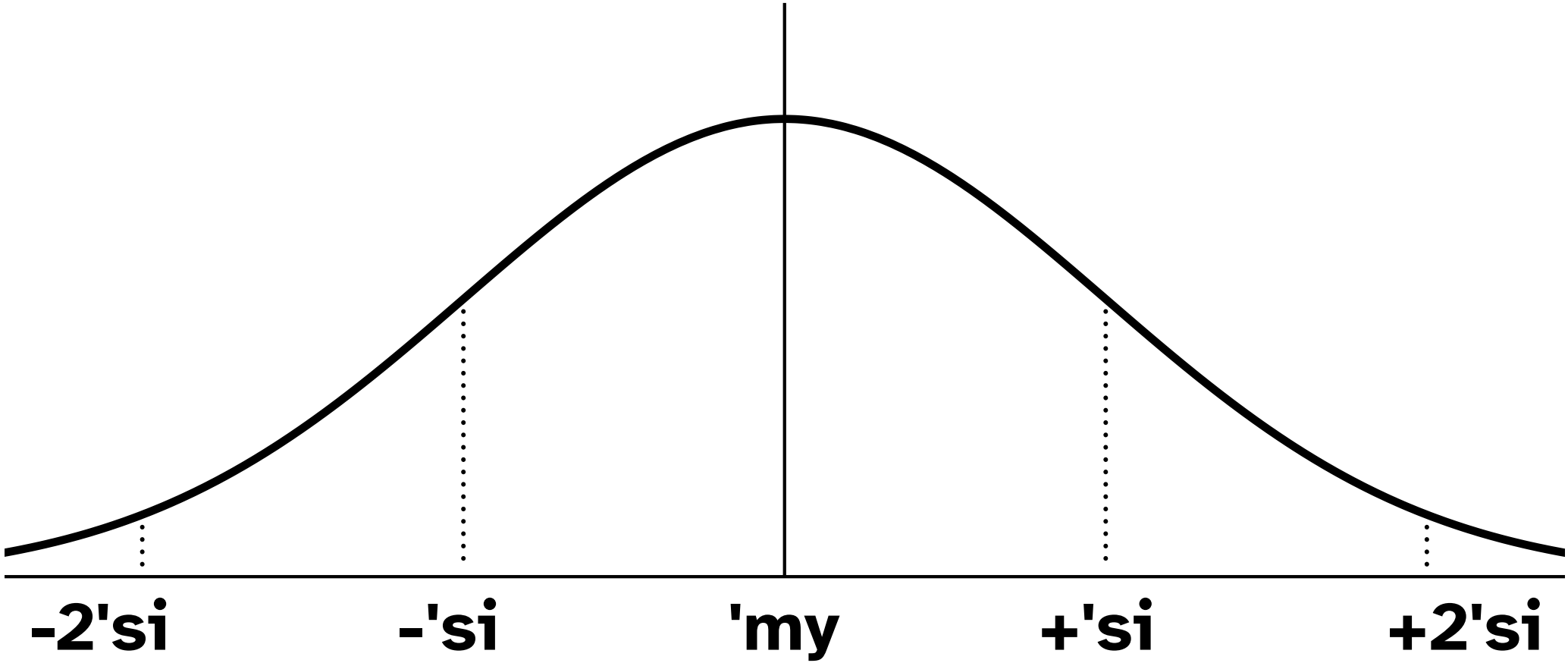


P(X >='my) =0,5



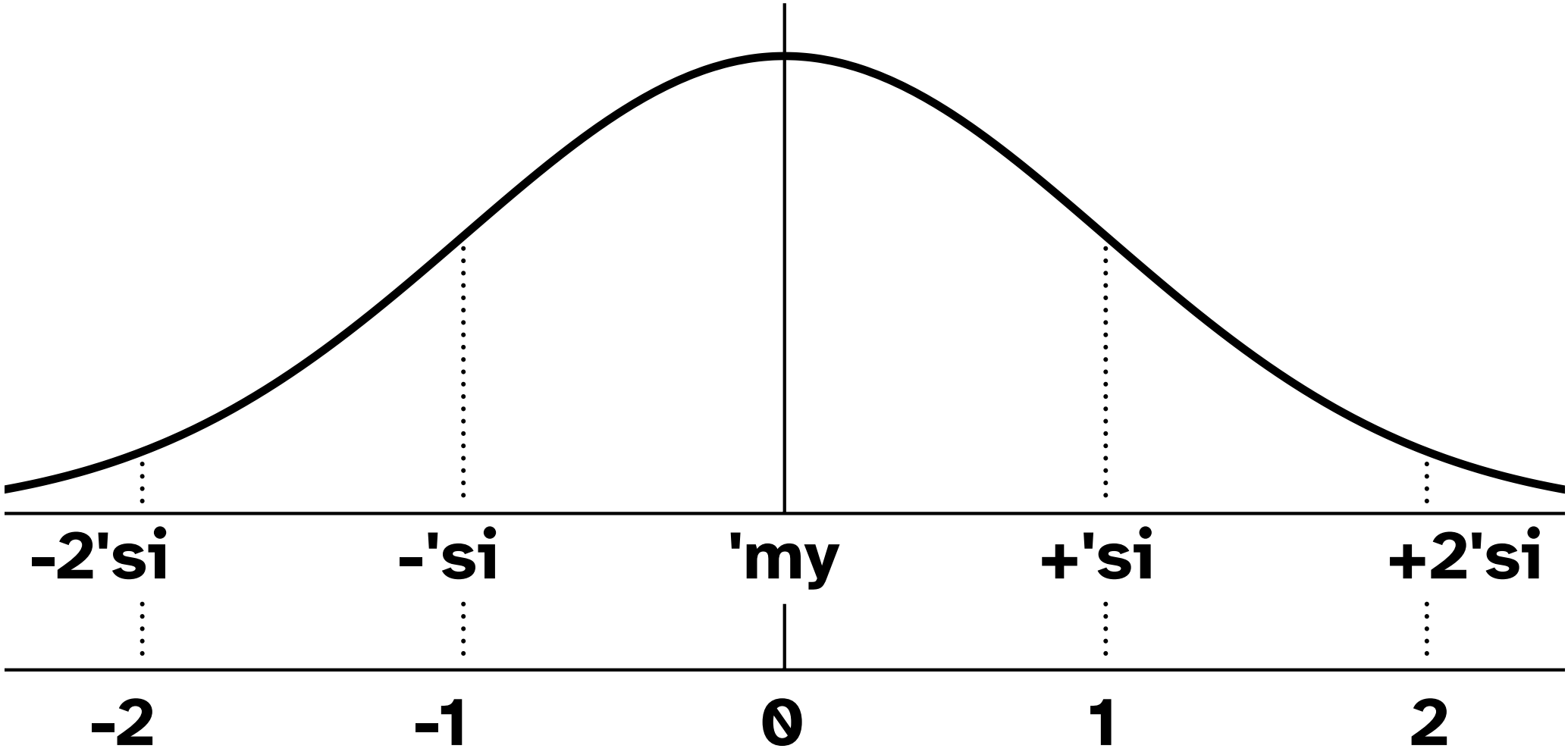
NV Dichtefunktion

$$f(x) = \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot e^{\left(\frac{-1}{2}\right) \cdot \left(\frac{x - \mu}{\sigma}\right)^2}$$



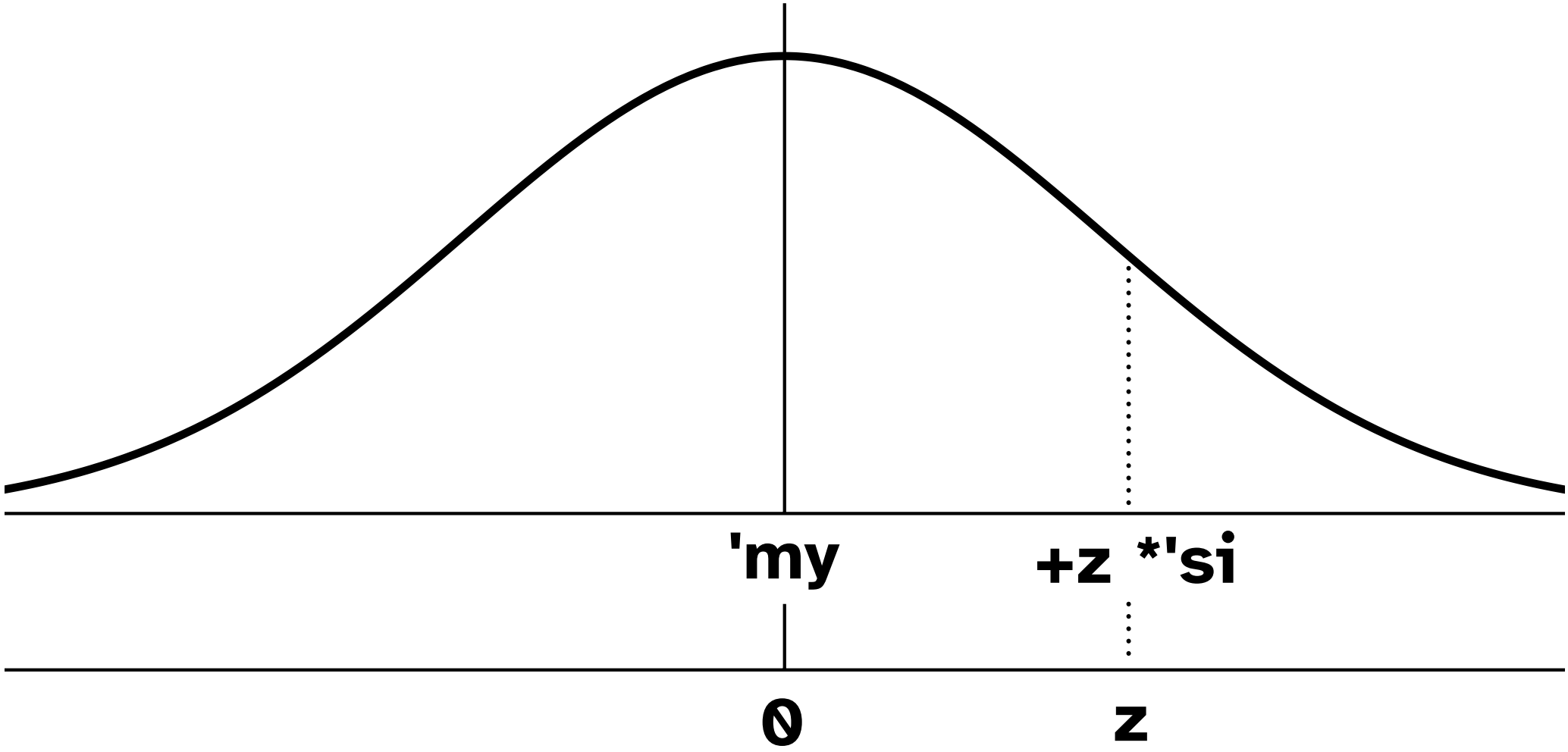
NV Dichtefunktion Standard

'ph(z) =1/('w(2 *'pi)) *'e^(-z^2/2)



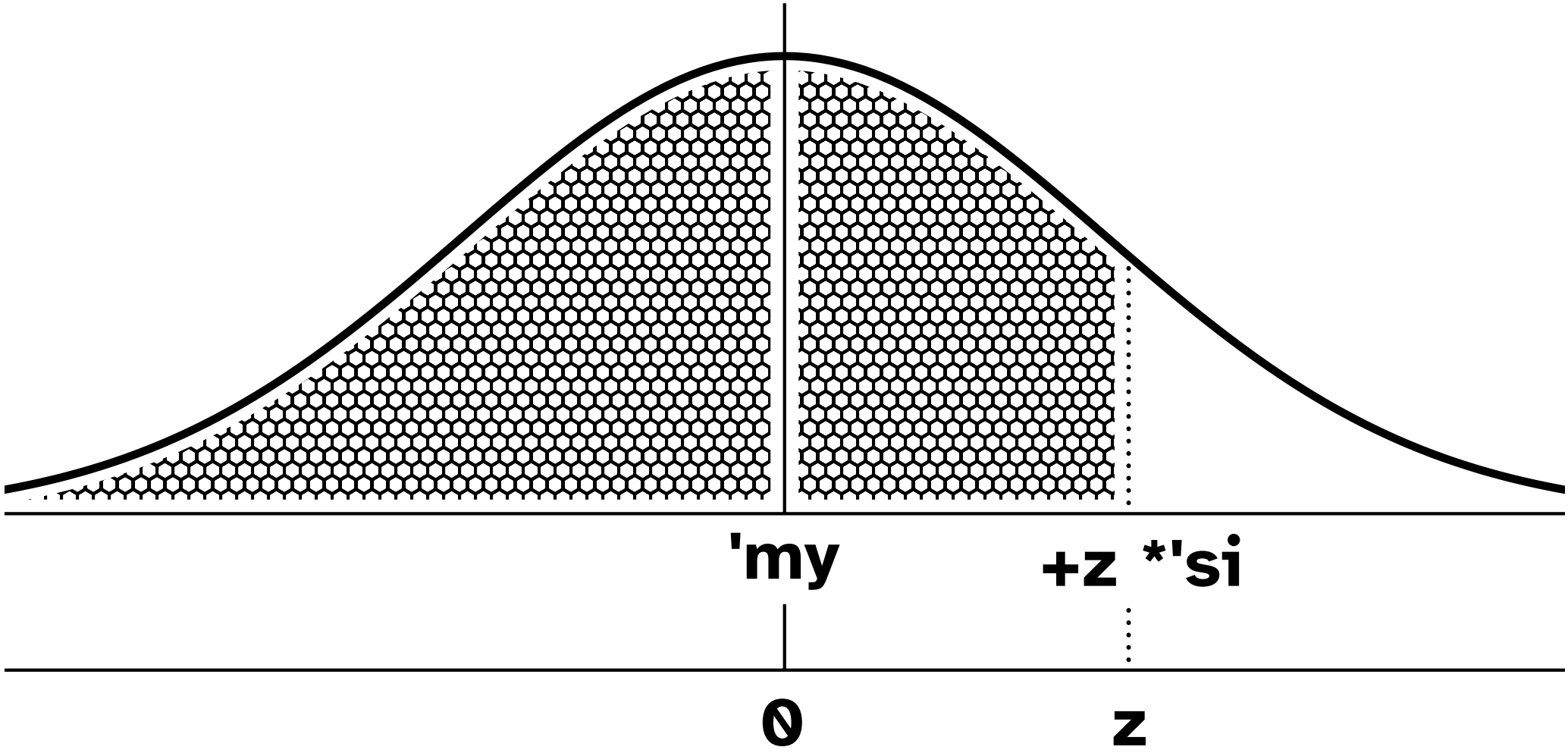
NV Dichtefunktion Standard

'ph(z) =f('my +z *'si)



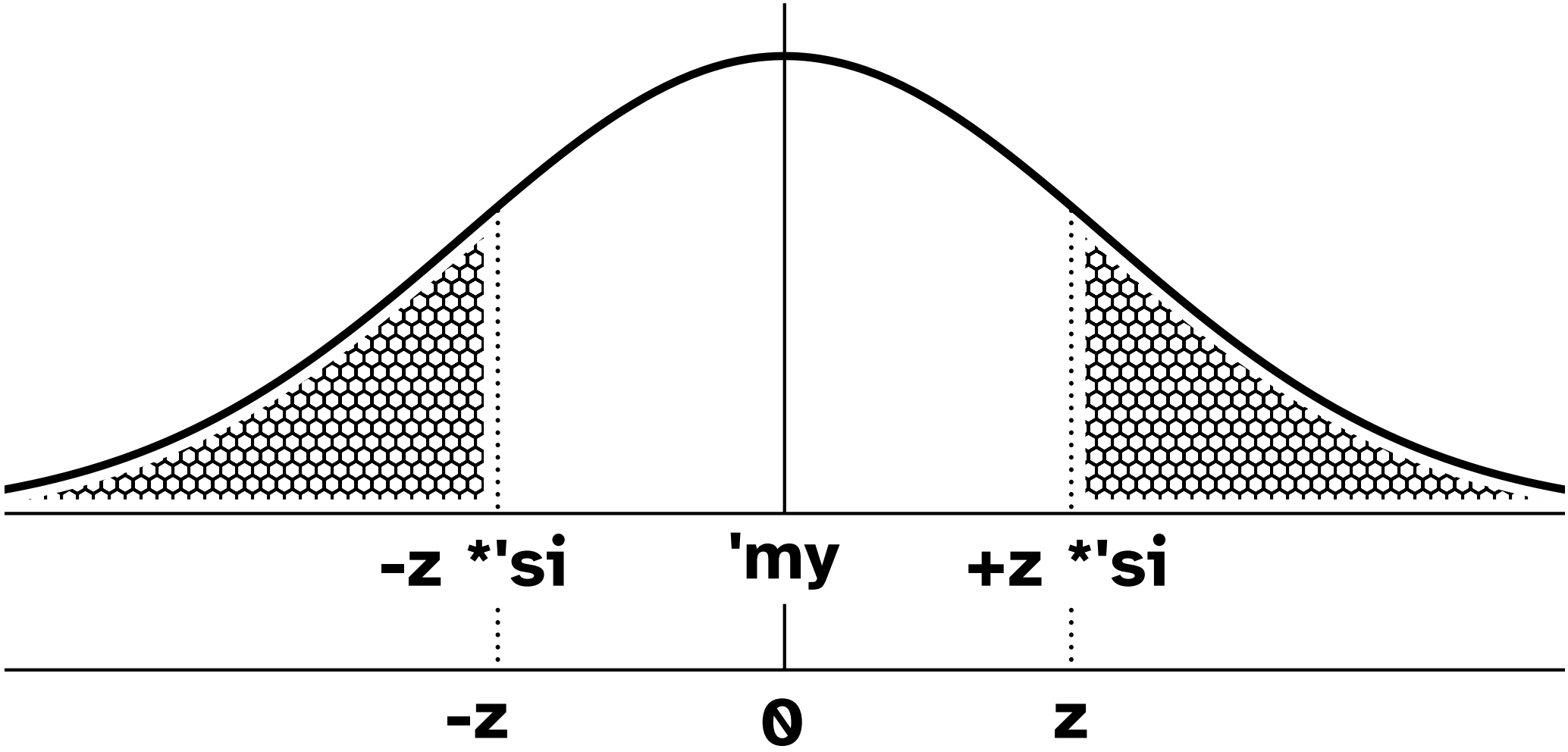
NV Verteilungsfunktion Standard

'Ph(z) =P(X <='my +z *'si)



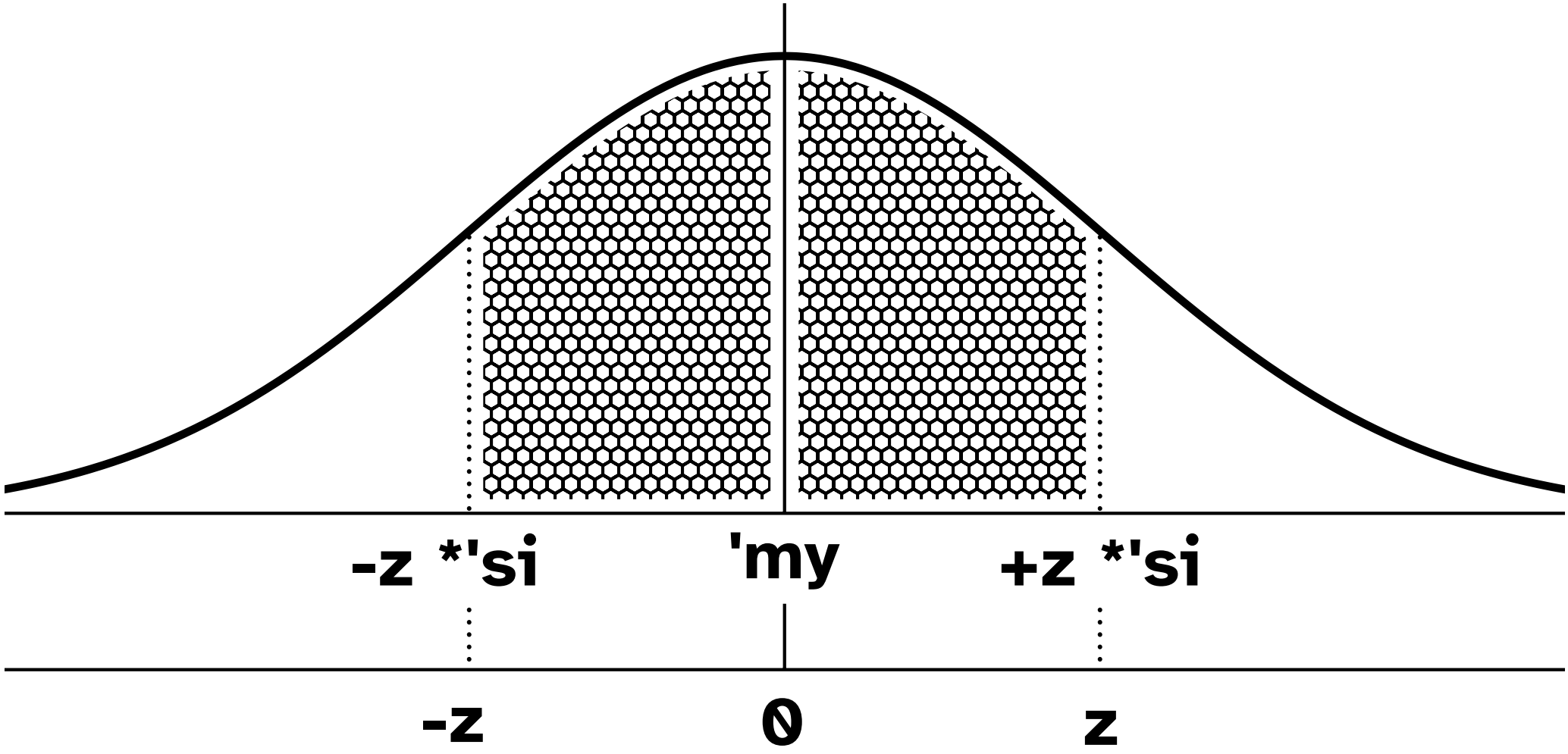
NV Verteilungsfunktion Standard

1 -'Ph(z) ='Ph(-z)



NV Verteilungsfunktion Standard

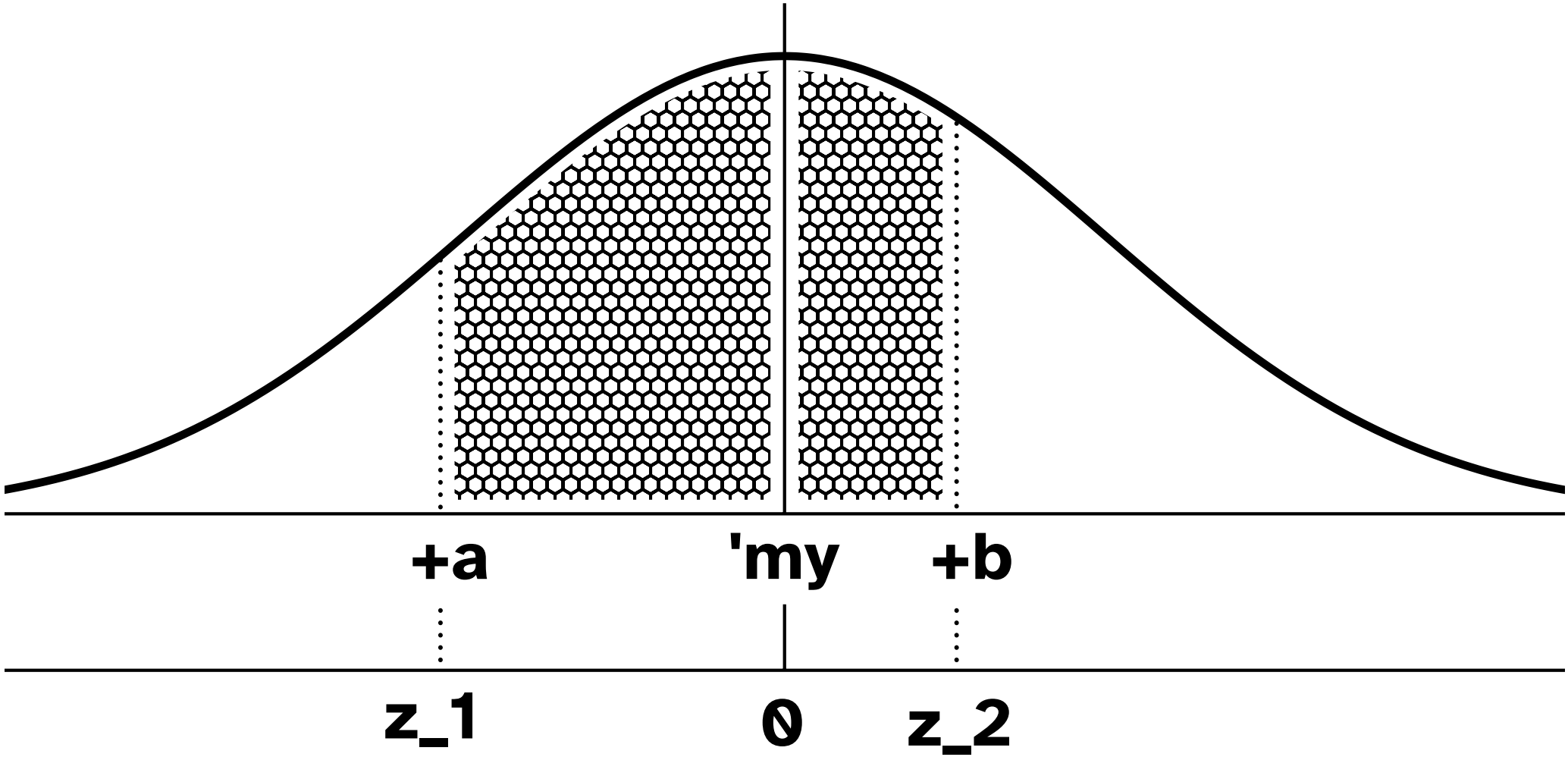
2*'Ph(z)-1:



NV Verteilungsfunktion Standard

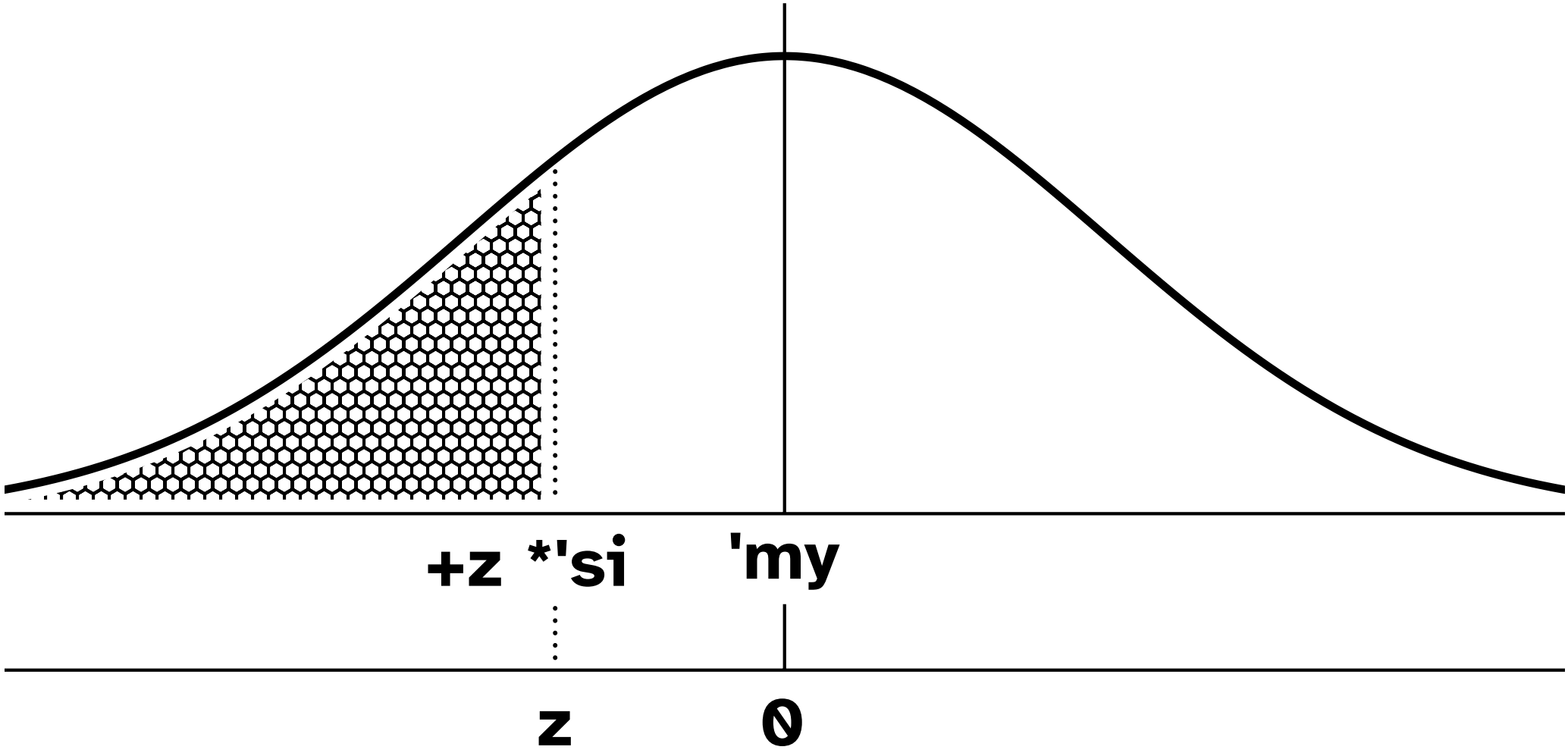
'Ph(z_2) -'Ph(z_1):

a =z_1 *'si ‖ b =z_2 *'si



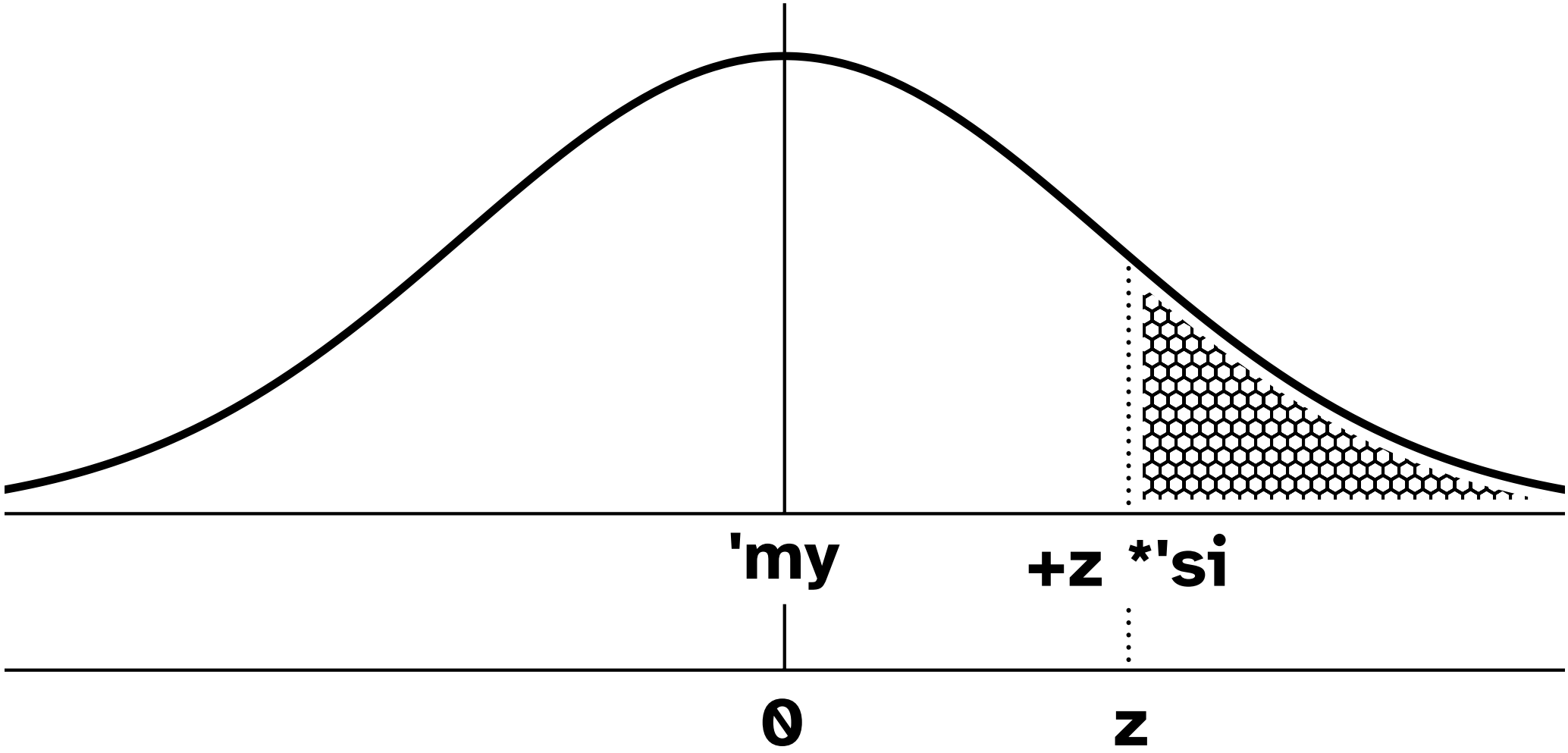
NV Verteilungsfunktion Standard

'Ph(z):



NV Verteilungsfunktion Standard

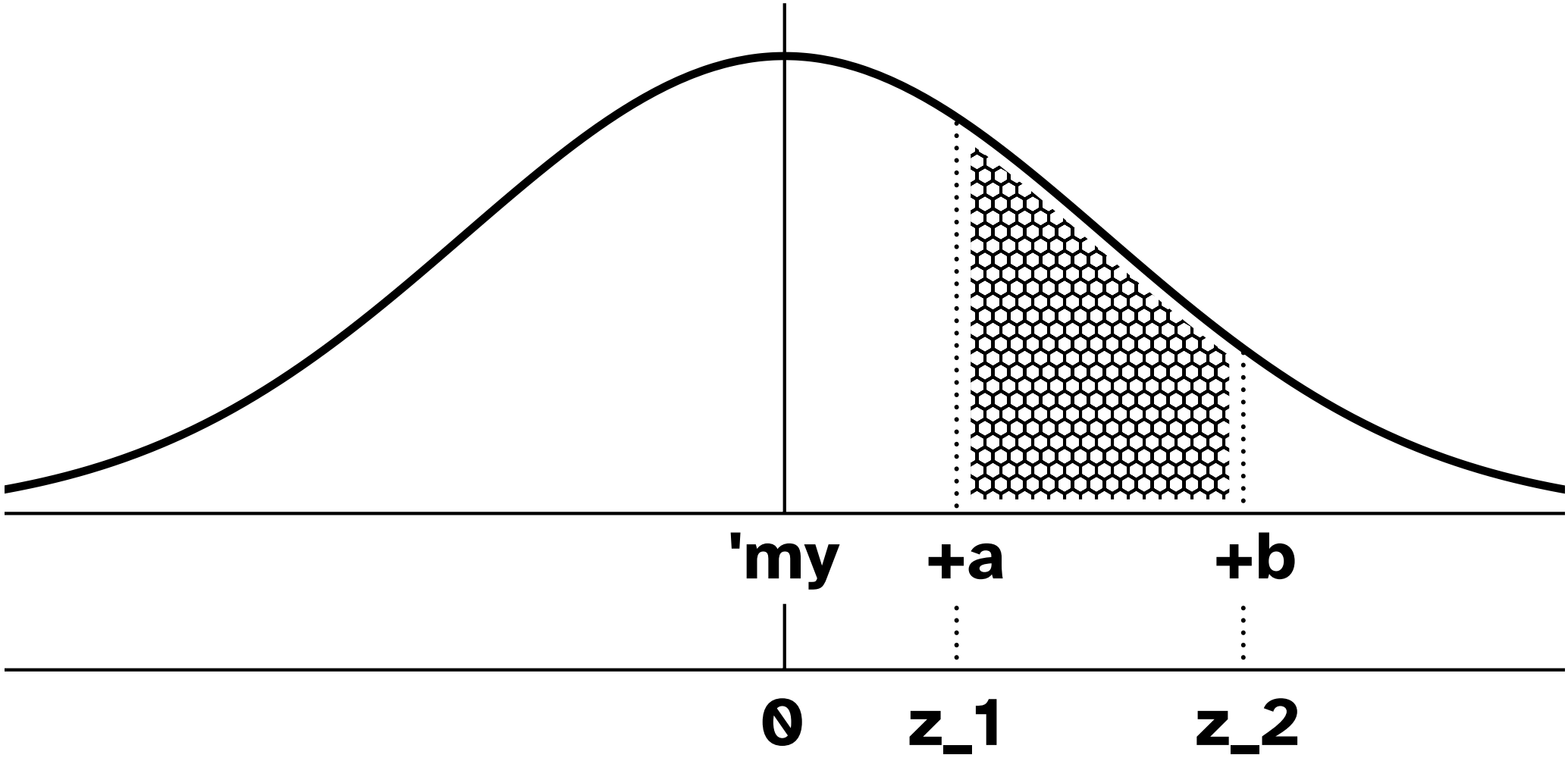
1-'Ph(z):



NV Verteilungsfunktion Standard

'Ph(z_2) -'Ph(z_1):

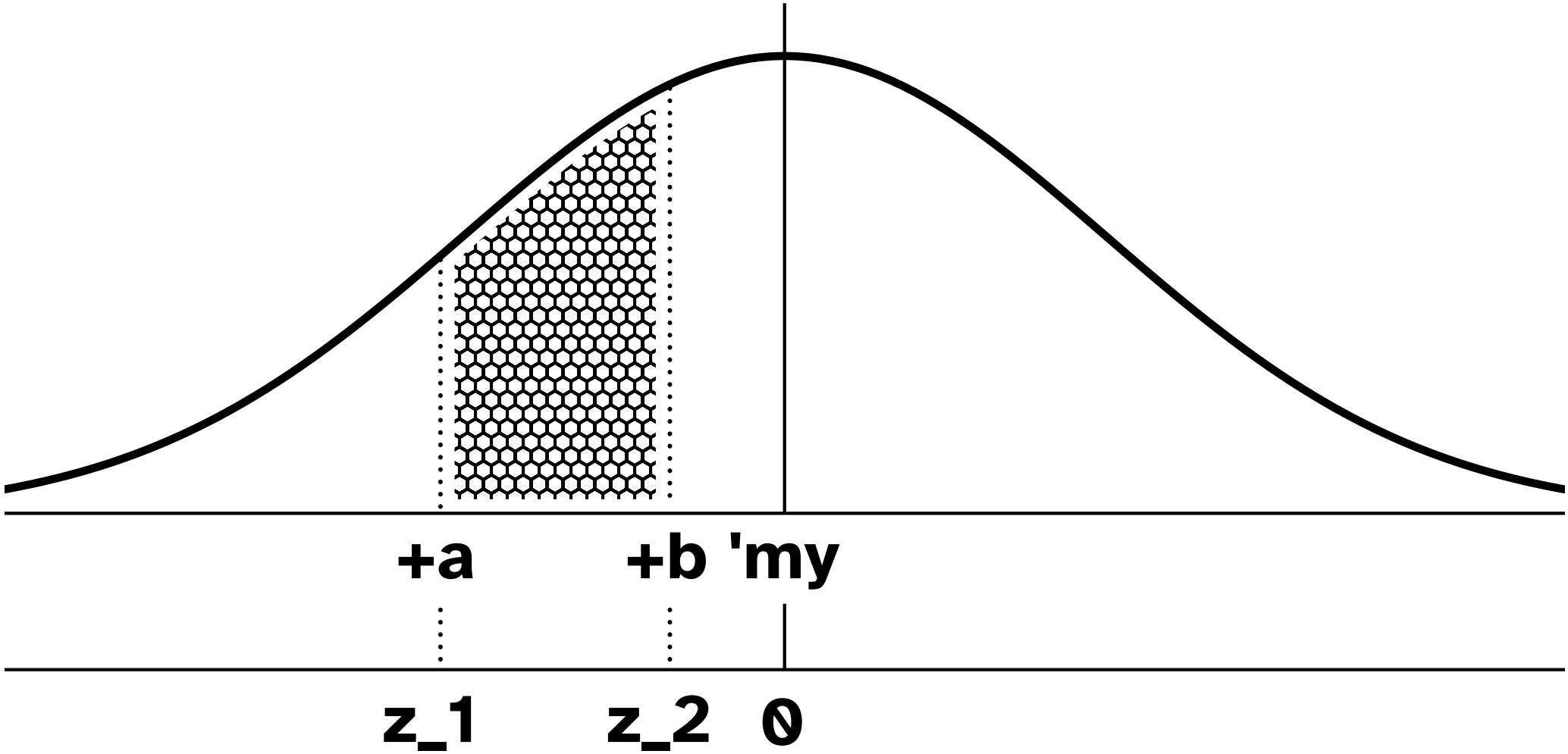
a =z_1 *'si ‖ b =z_2 *'si



NV Verteilungsfunktion Standard

'Ph(z_2) -'Ph(z_1):

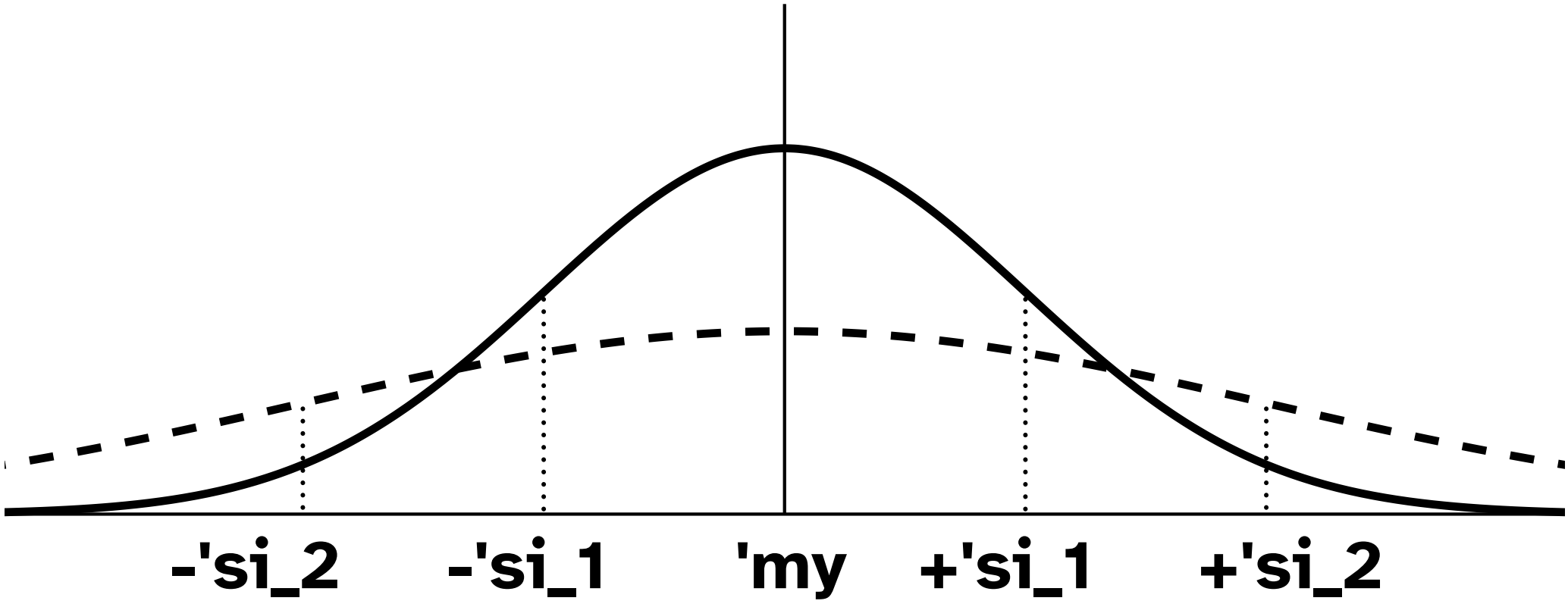
a =z_1 *'si ‖ b =z_2 *'si



NV Dichtefunktionen Vergleich

'my_1 = 'my_2 ‖ 'si_1 < 'si_2

ph_1(x): ——— ‖ ph_2(x): - - - -



Reele Funktionen

Schulstufe 12

Typische Funktionen (TF)

- Potenzfunktionen
- Polynomfunktionen
- Exponentialfunktionen

Vergleichen von Funktionen (VF)

Änderungsmaße (ÄM)

Inhalt

Typische reelle Funktionen

Vergleichen von reellen Funktionen

10 Mult: f(b *x)

b >1; 0 <b <1;

11 Add: f(x) +d

d >0; d <0;

12 Add: f(x +c)

c >0; c <0

Änderungsmaße

13 f(x) =k *x +d

f(x +1) =f(x) +k

f(x +h) =f(x) +k *h

14 f(x) =k *x +d

Steigungsdreiecke

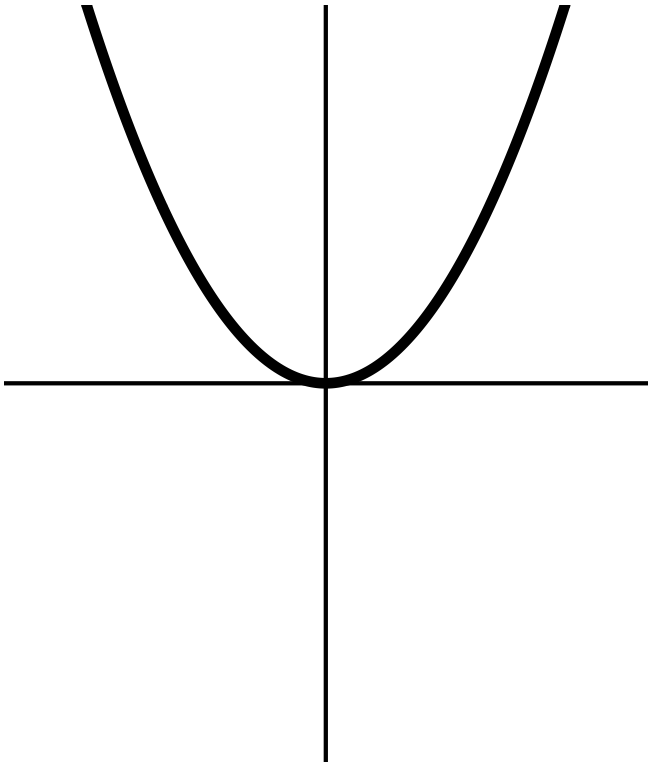
15 f(x) =c *a^x

f(x +1) =c *a^(x +1) =c *a^x *a

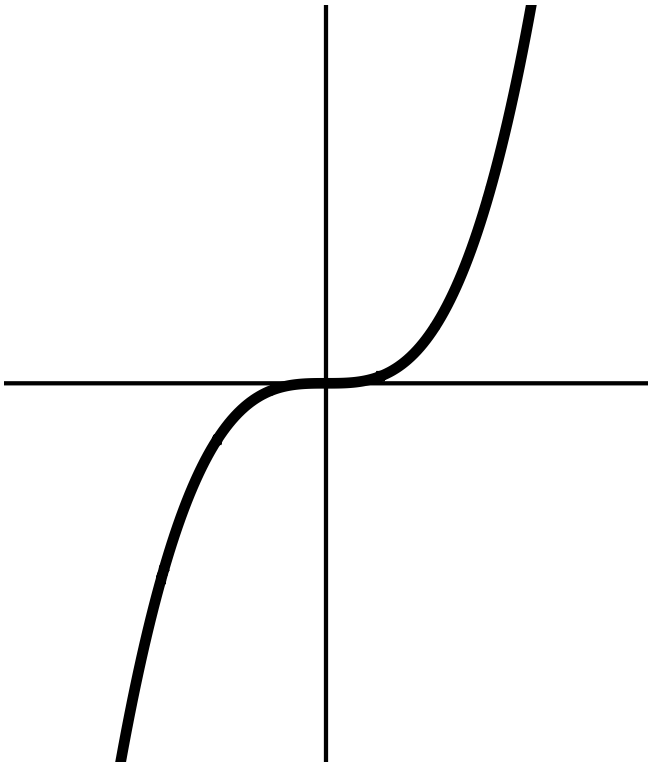
f(x +h) =c *a^(x +h) =c *a^x *a^h

RF TF Potenzfkt

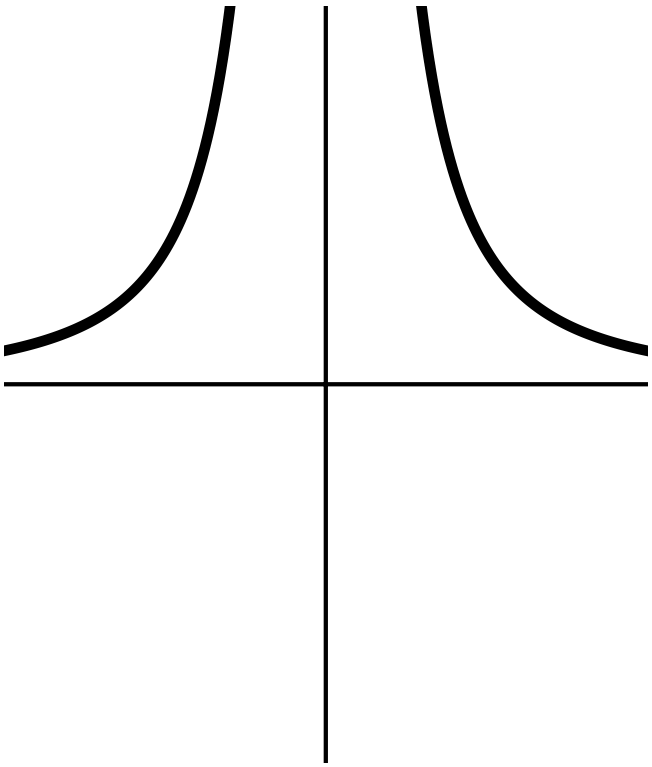
f(x) =x^2



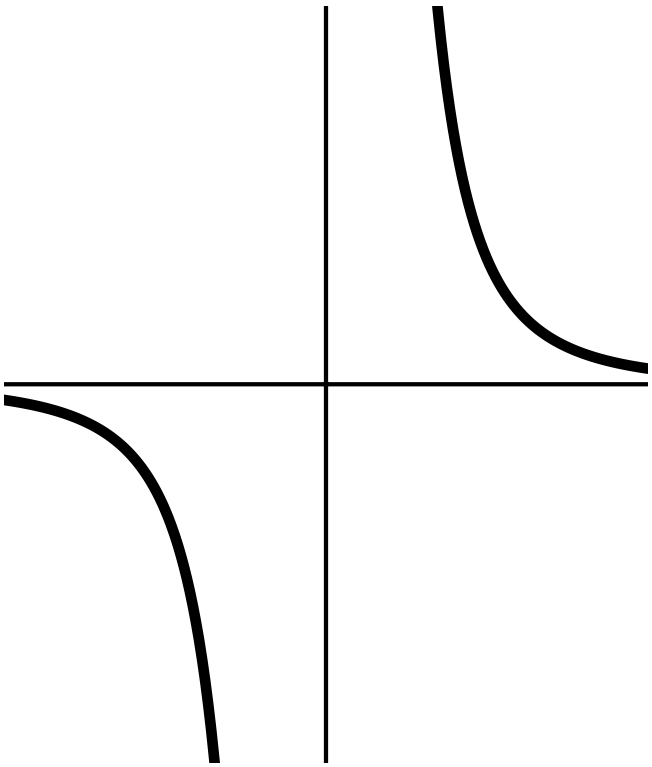
f(x) =x^3



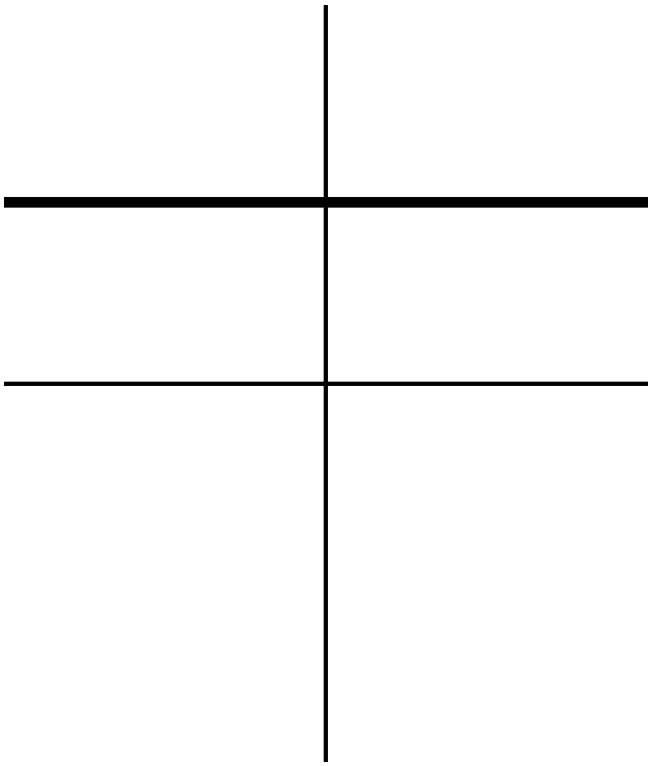
f(x) =1/x^2

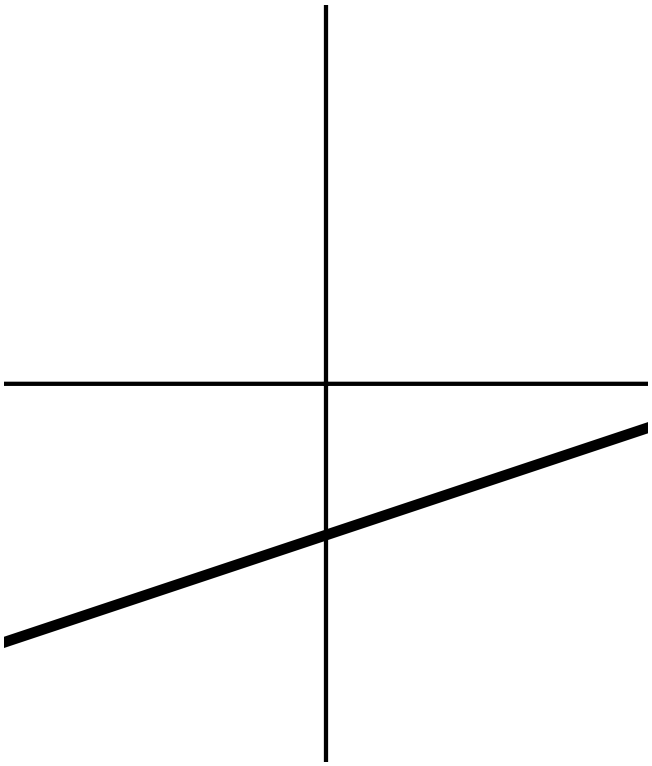


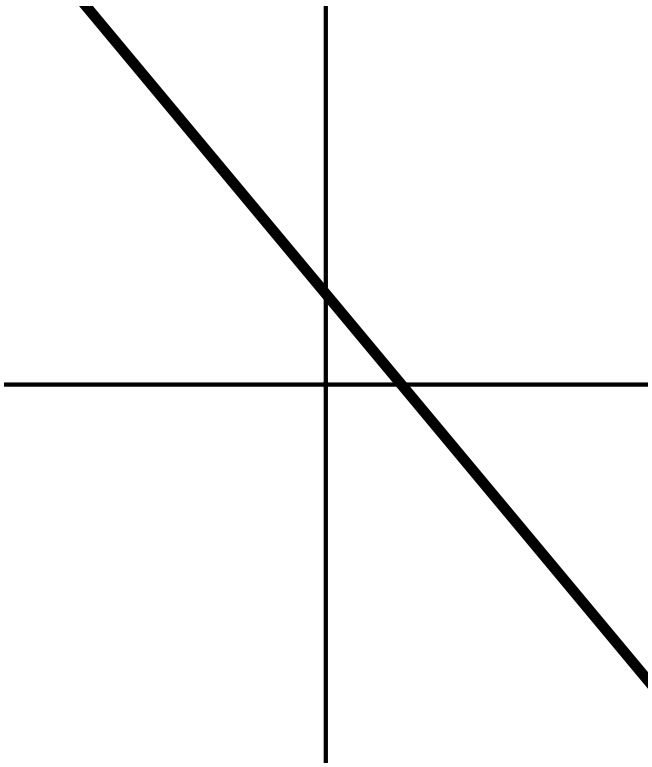
f(x) =1/x^3

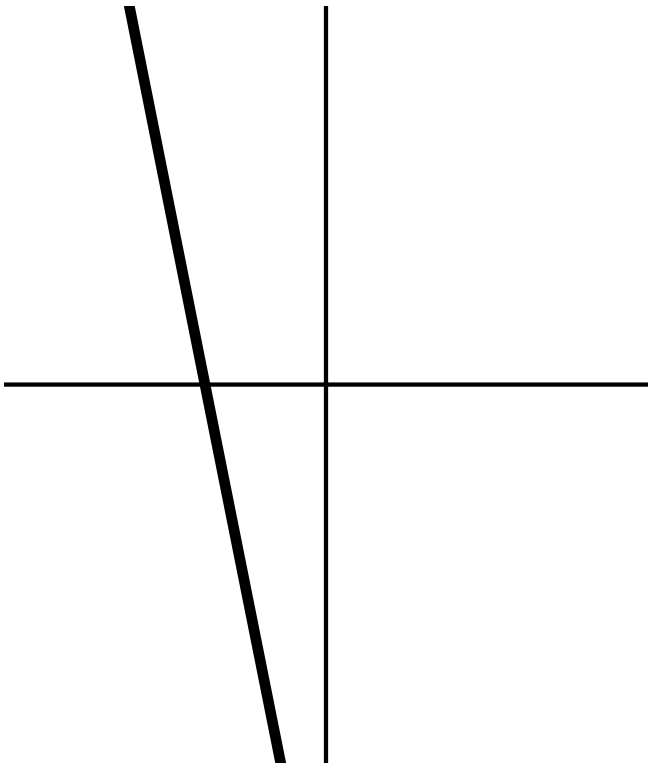


Grad 1: Gerade

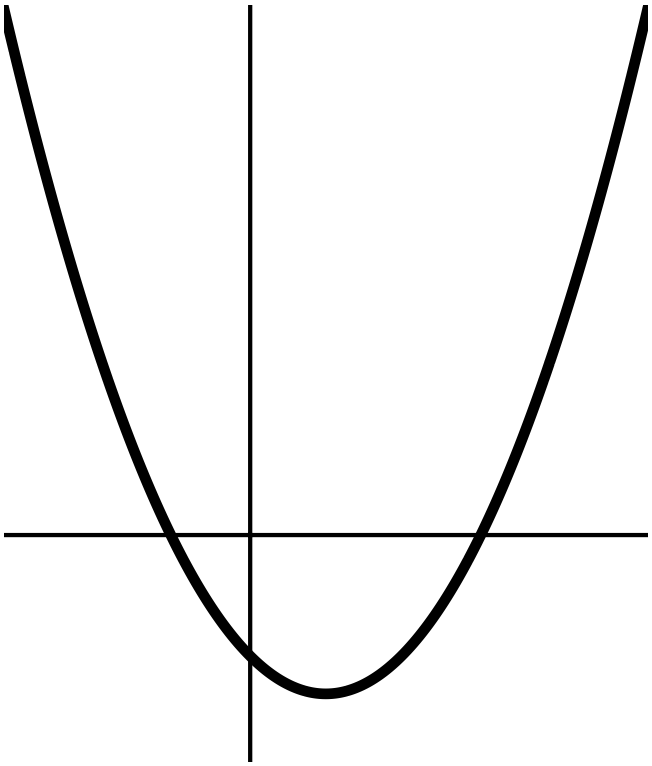


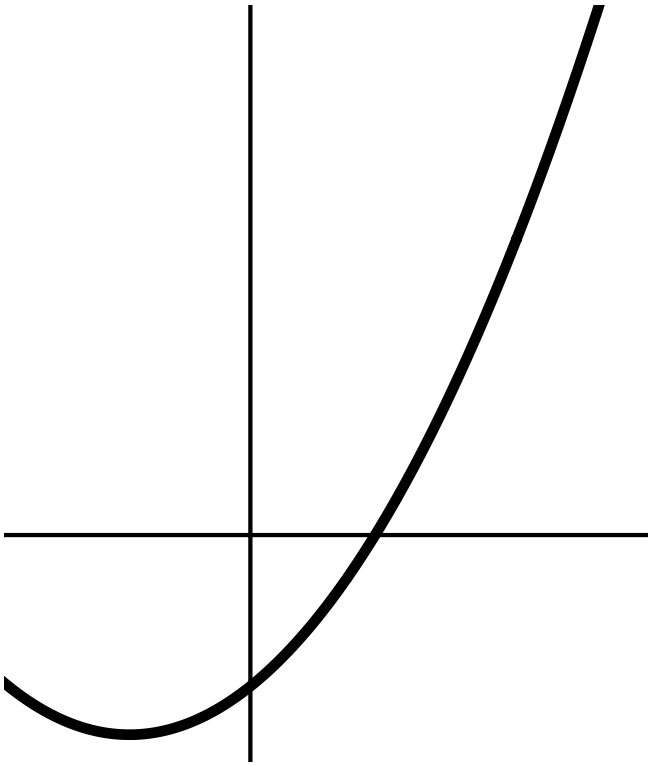


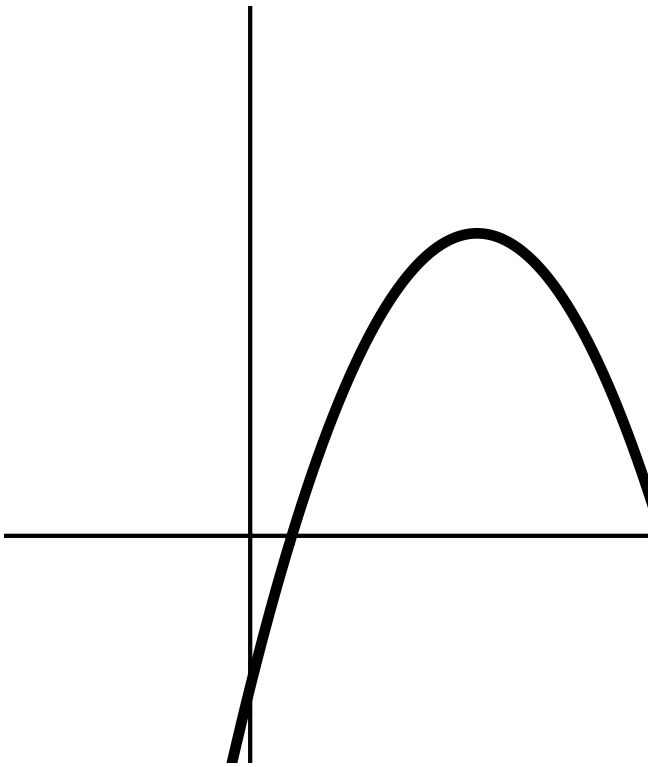


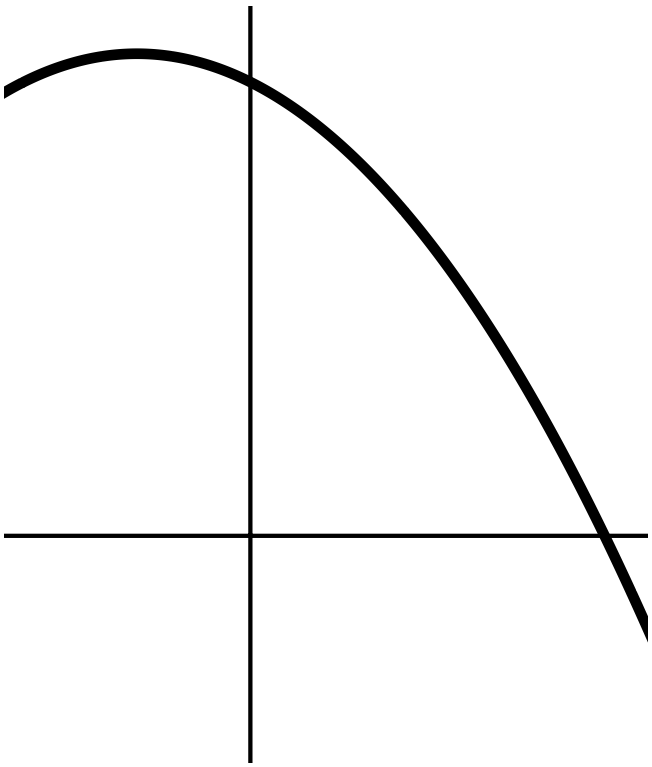


Grad 2: Parabel

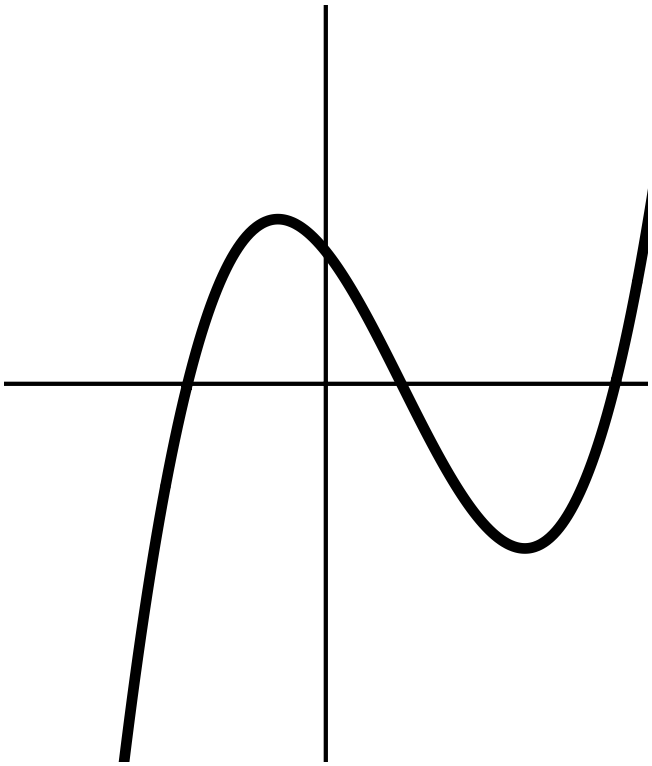


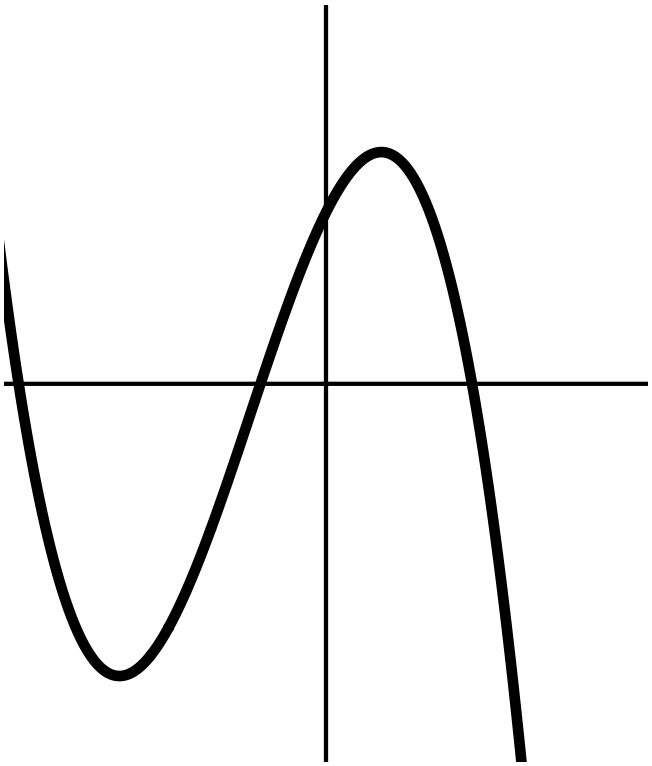


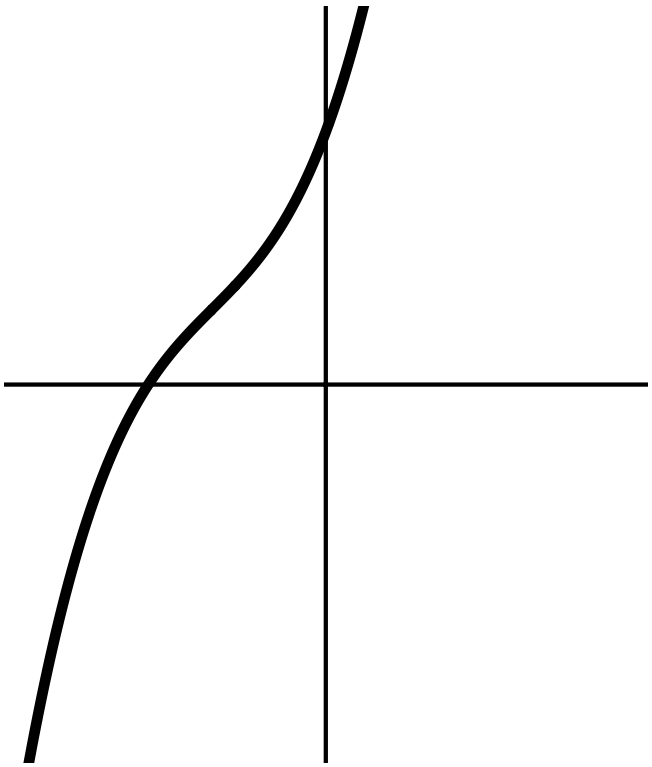


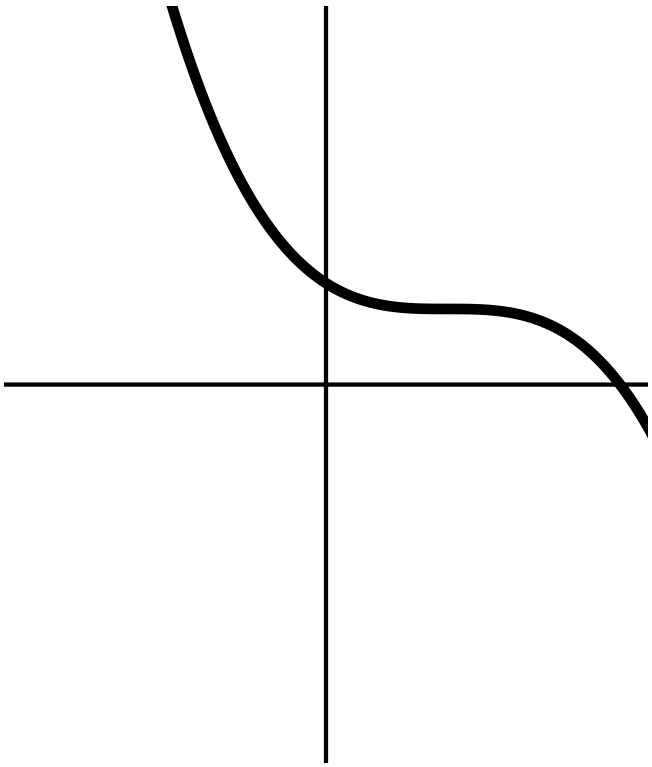


Grad 3: "S-Kurve" mit "Entartungen"

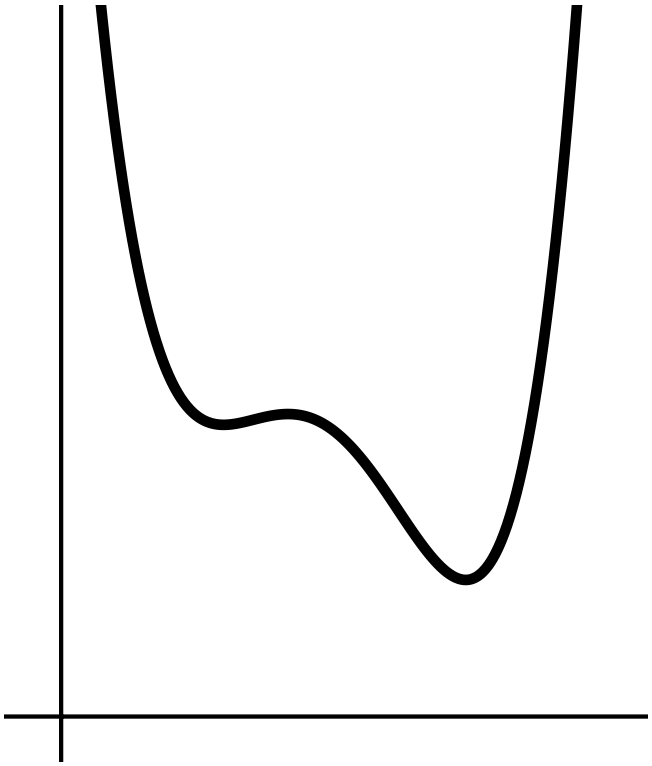


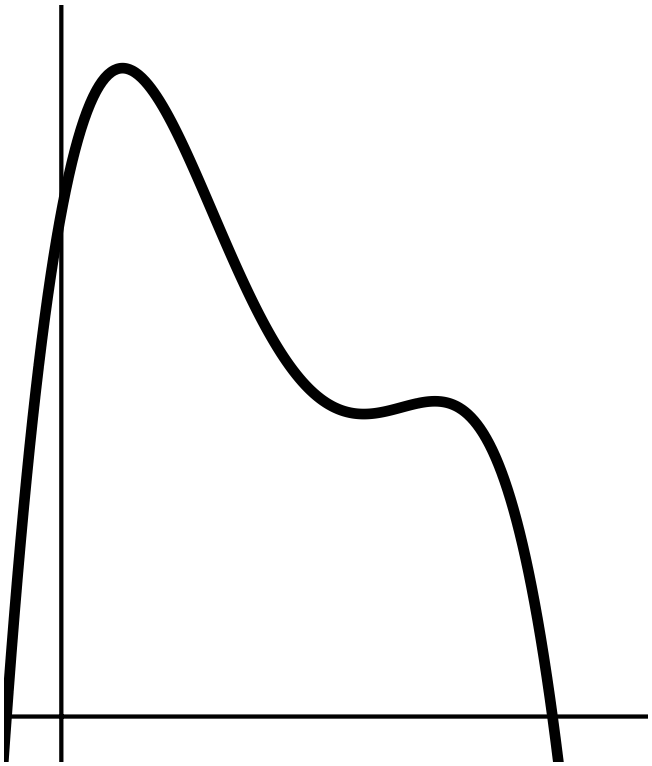


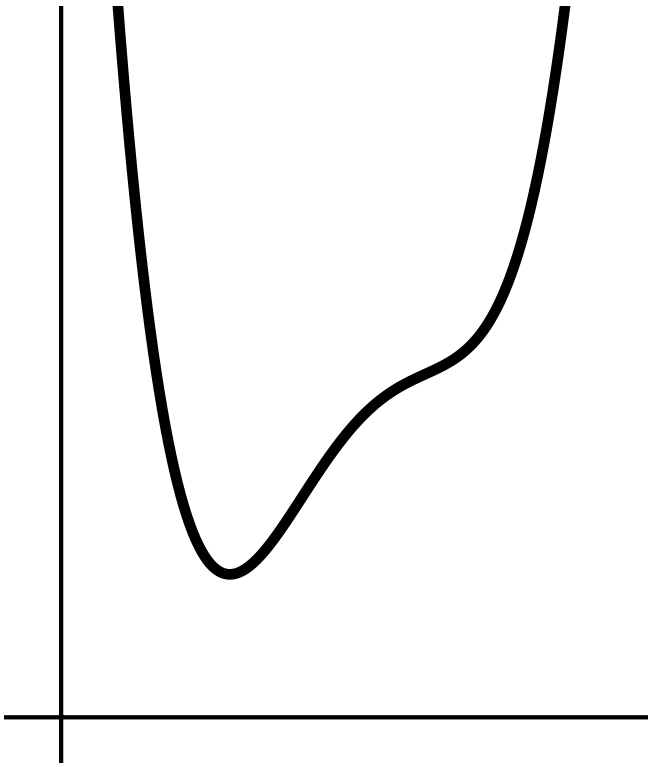


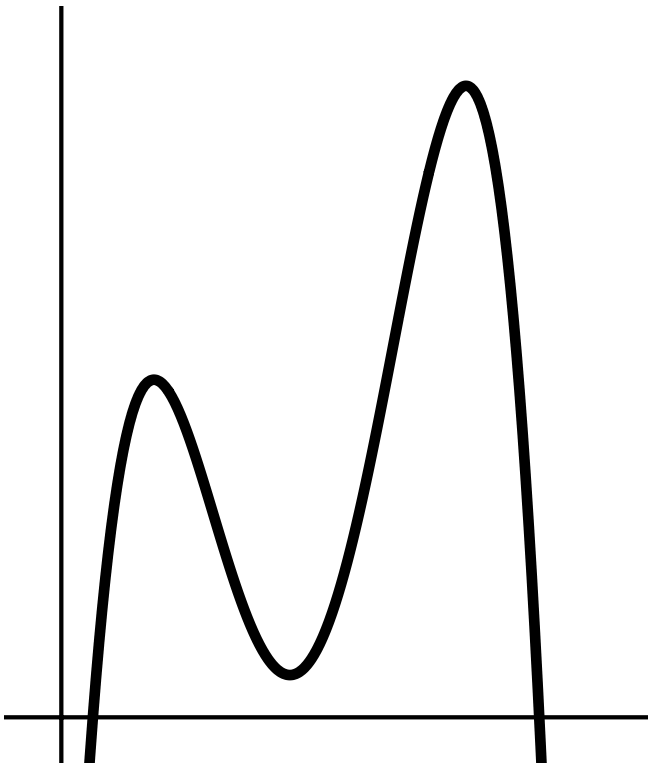


Grad 4: "Doppel-S-Kurve"

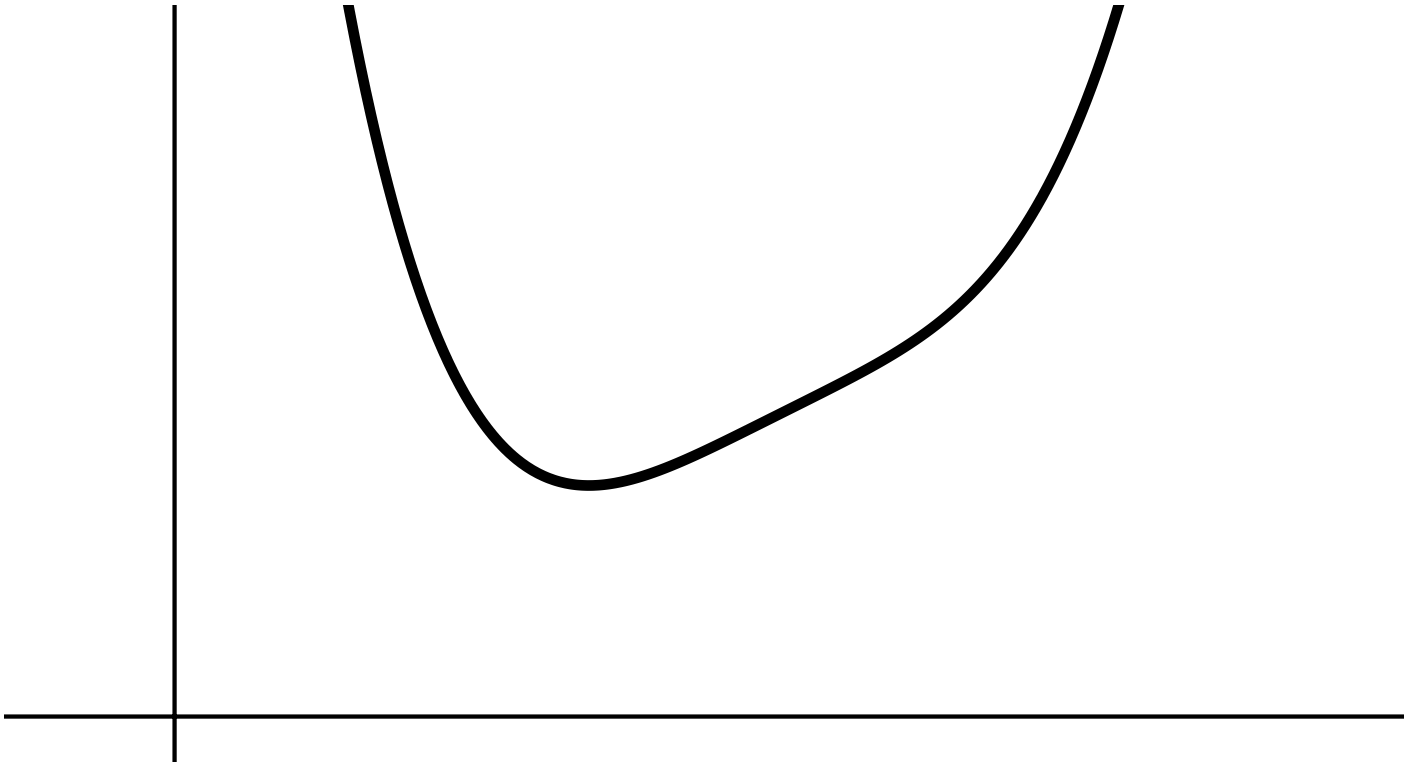


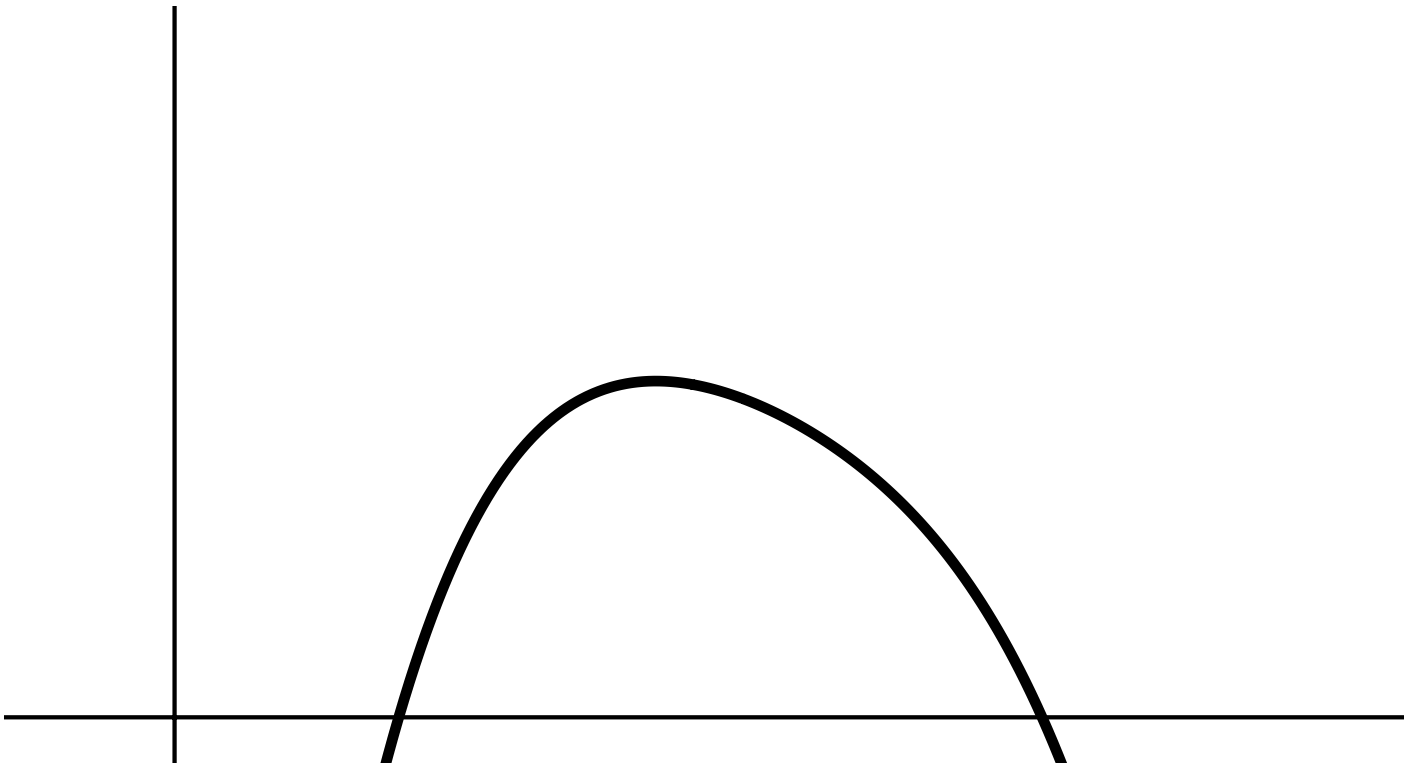






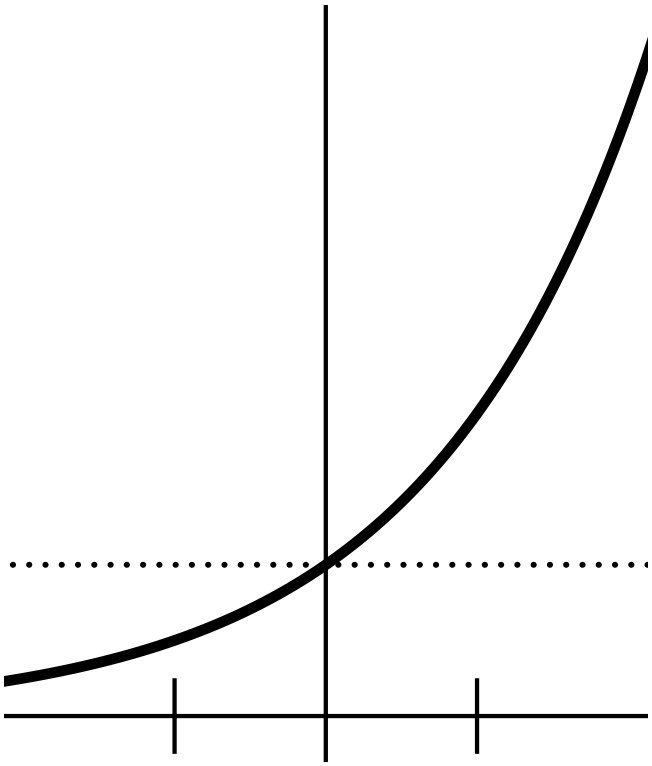
Grad 4: "Doppel-S-Kurve" mit "Entartungen"



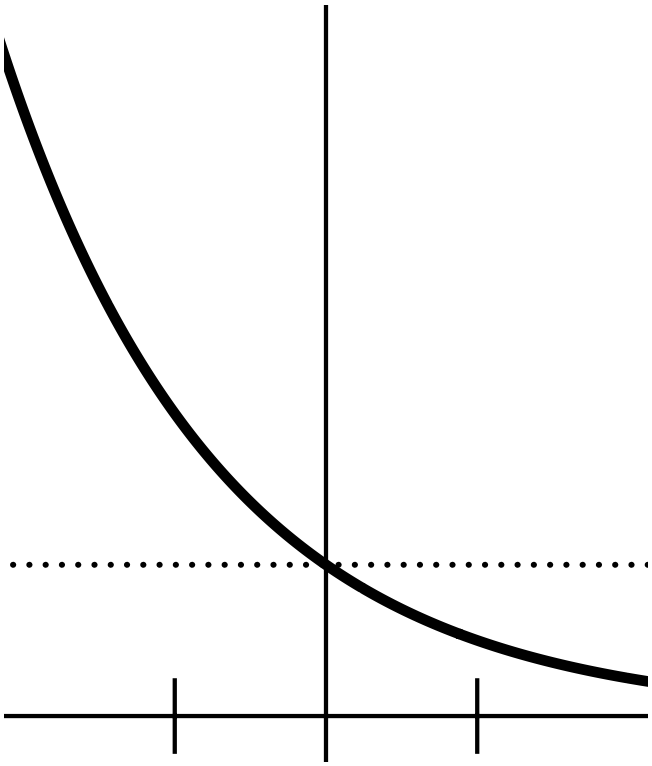


RF TF Exponentialfkt 7/16

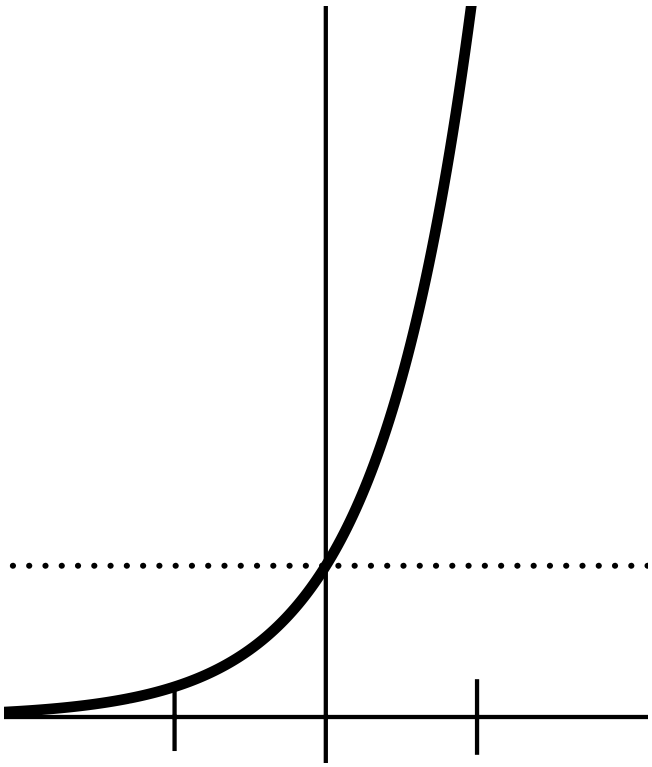
f(x) =2^x



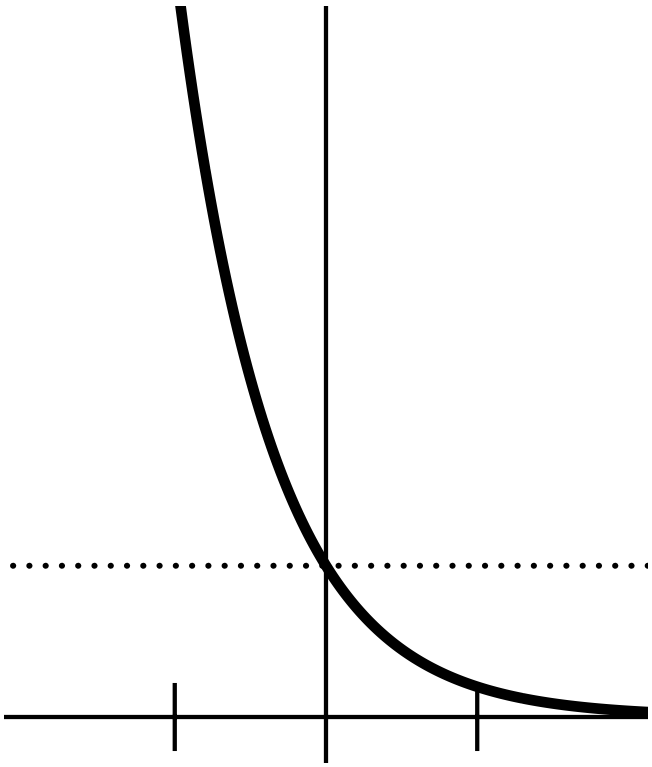
f(x) =(1/2) ^x



f(x) =5^x

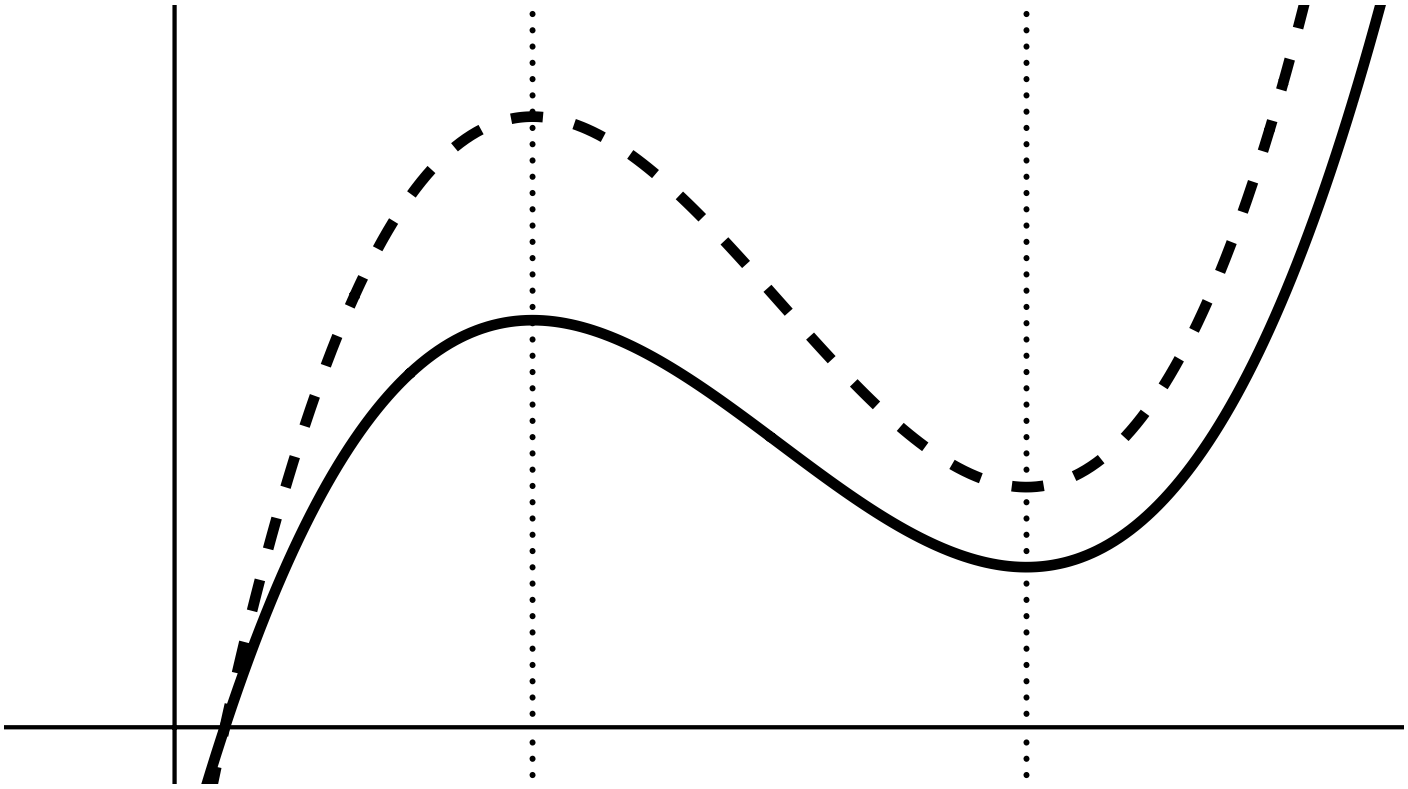


f(x) =(1/5) ^x

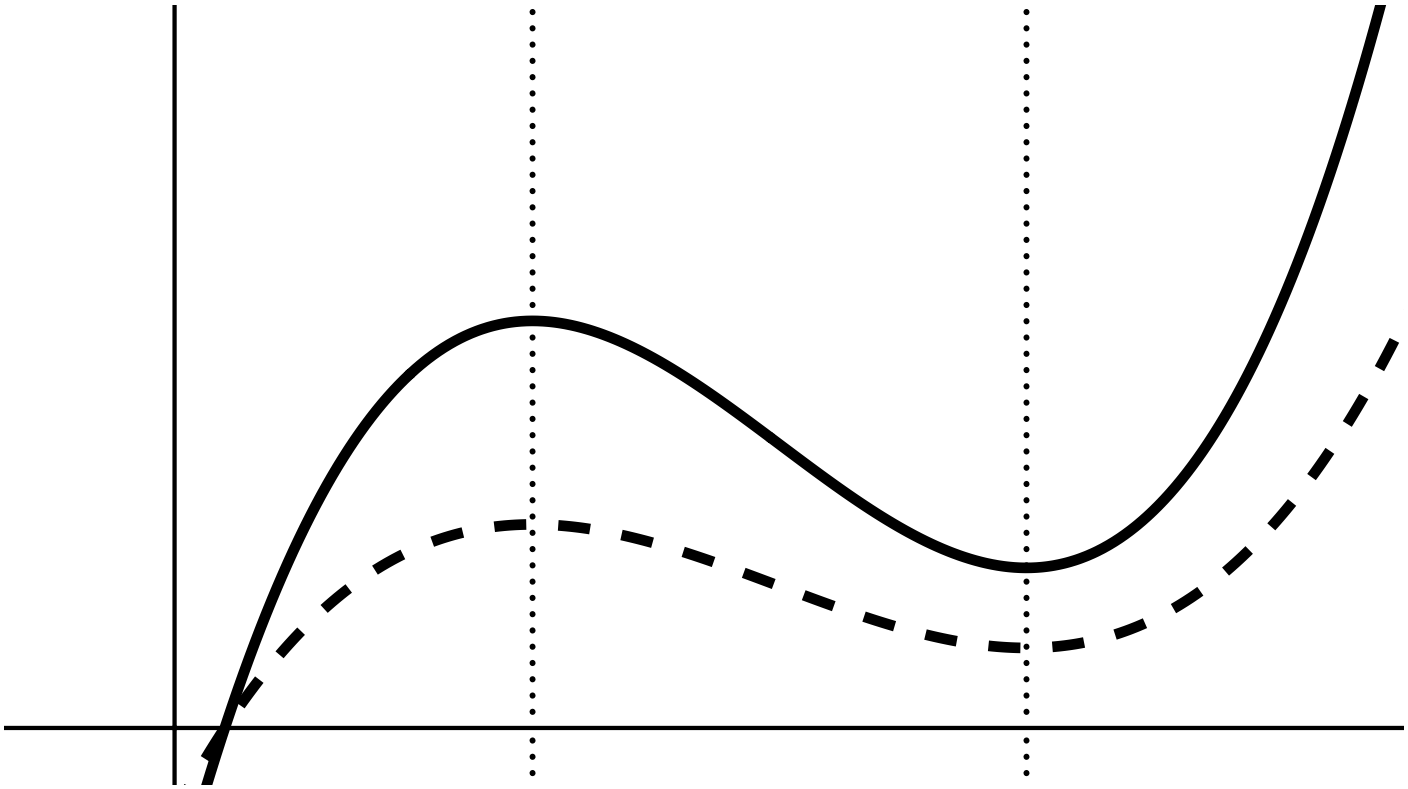


RF VG Mult. a *f(x)

f: ——— ‖ g: a >1 - - - -

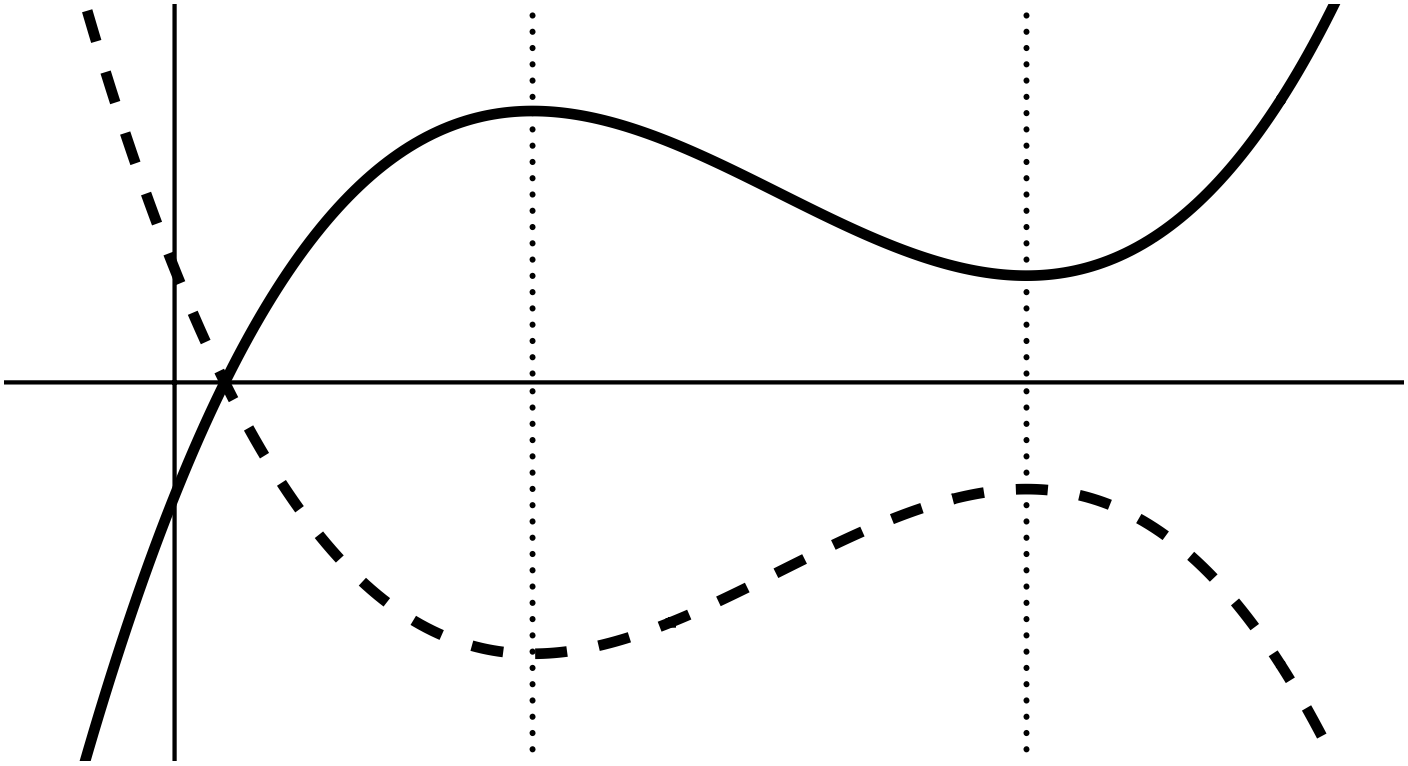


f: ——— ‖ h: 0 <a <1 - - - -

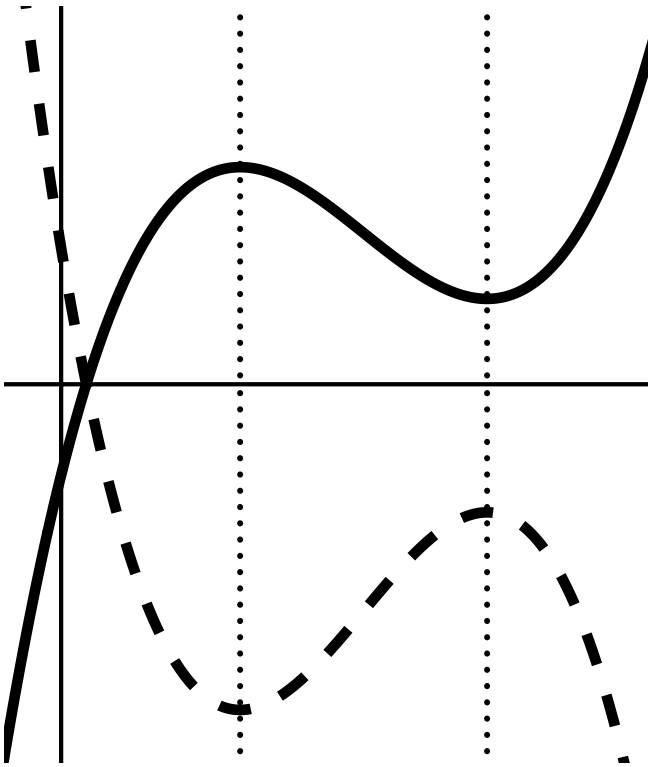


RF VG Mult. a *f(x)

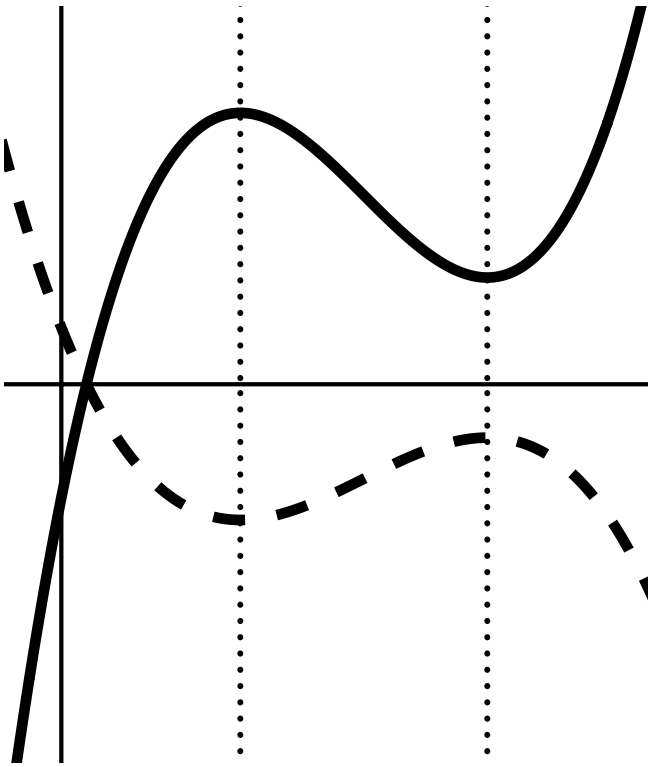
f: ——— ‖ g: a =-1 - - - -



h: a <-1

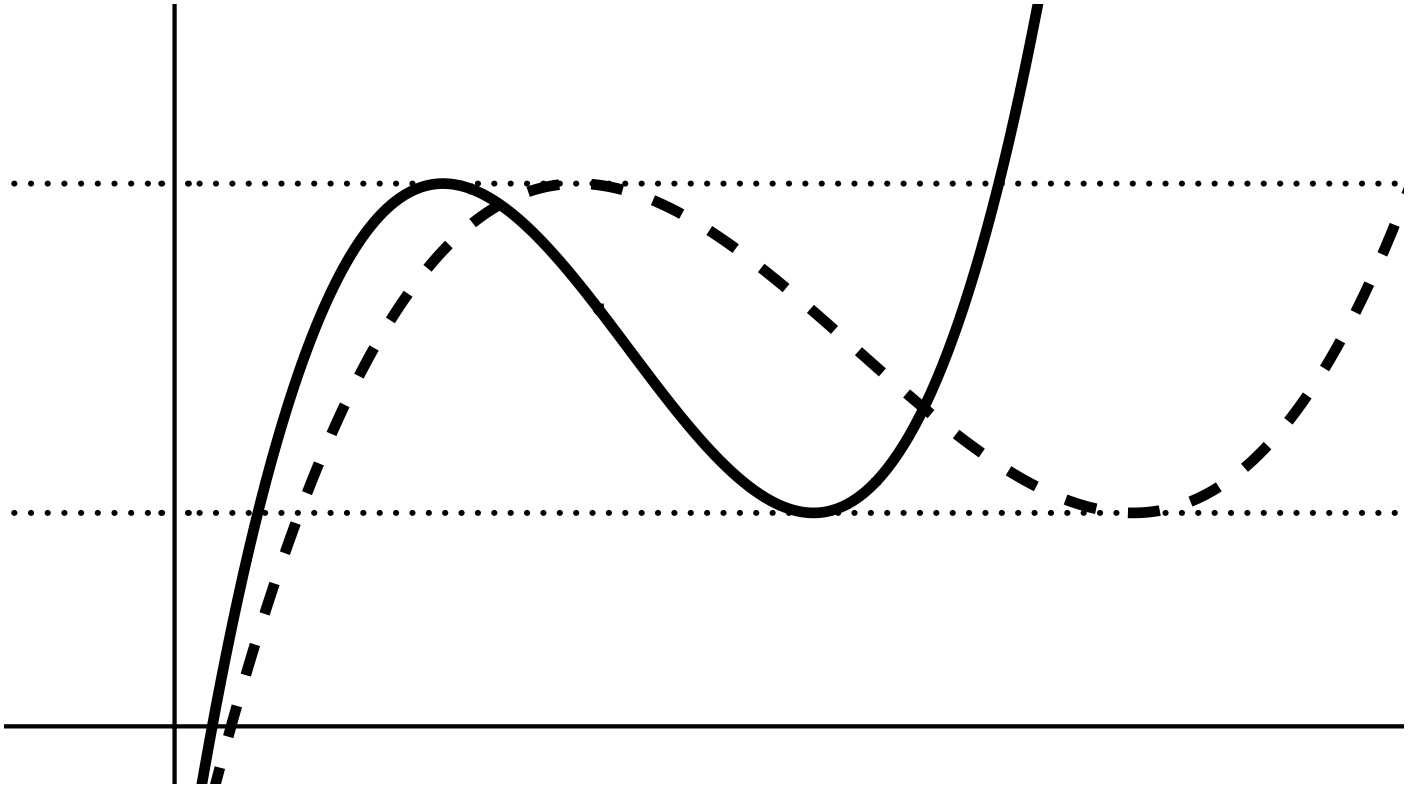


i: -1 <a <0

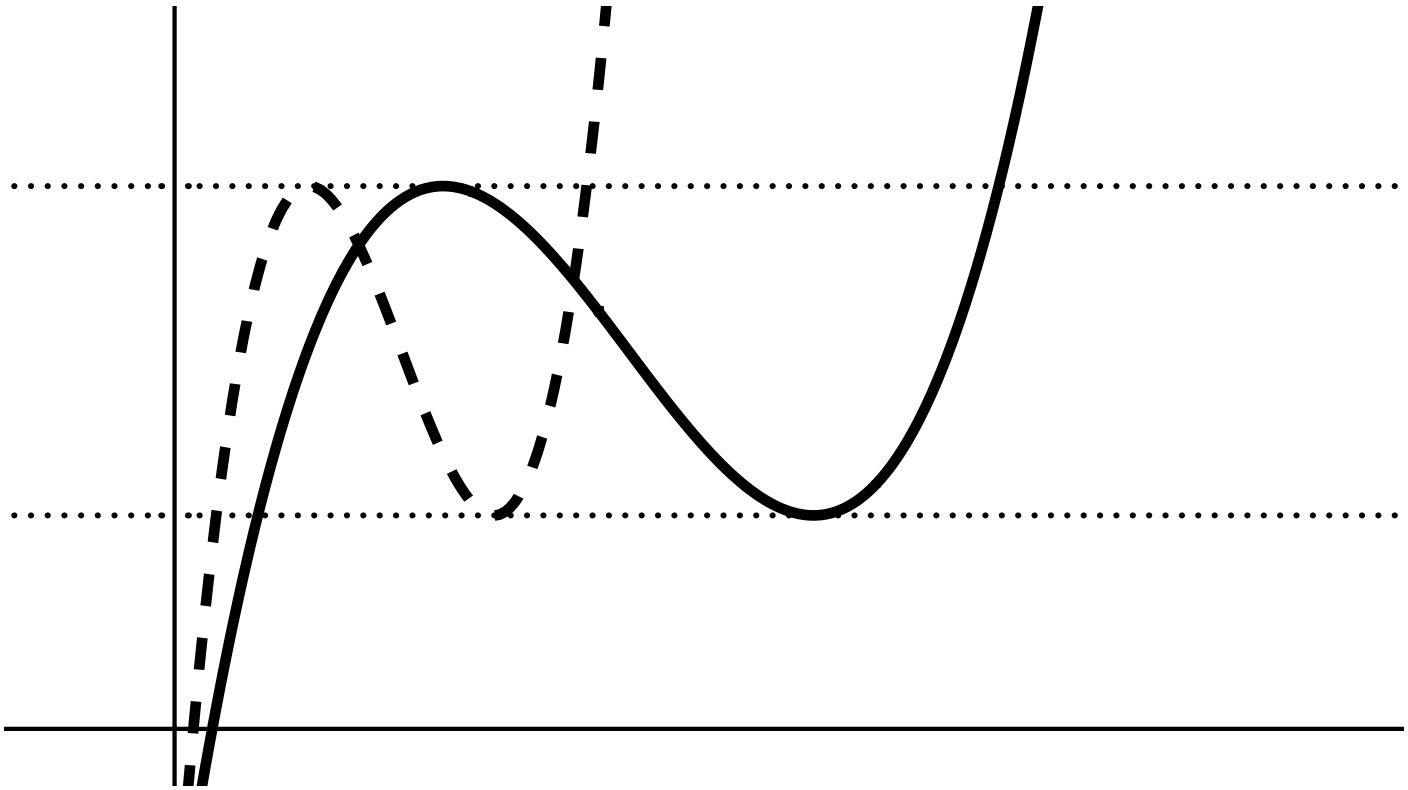


RF VG Mult. f(b *x)

f: ——— ‖ g: b >1 - - - -

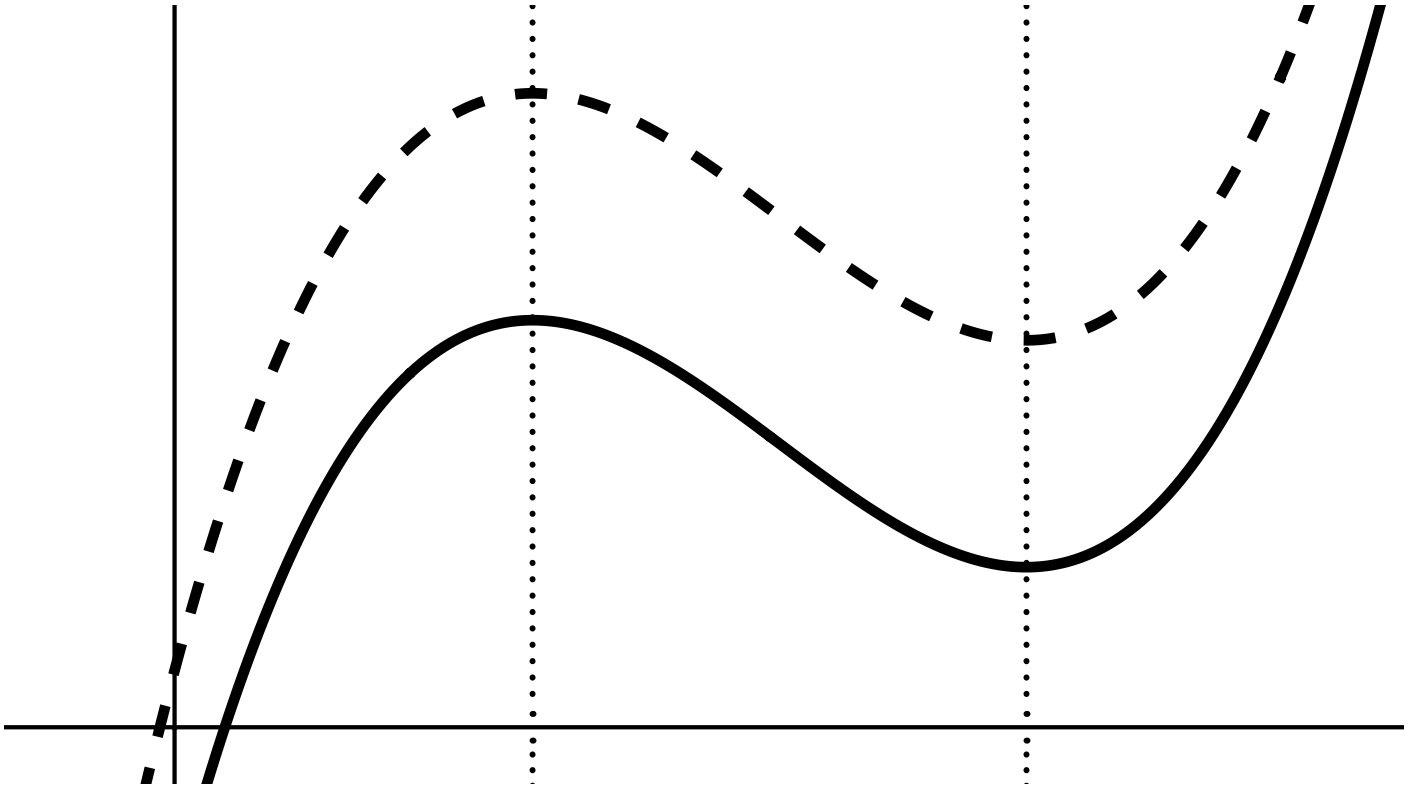


f: ——— ‖ h: 0 <b <1 - - - -

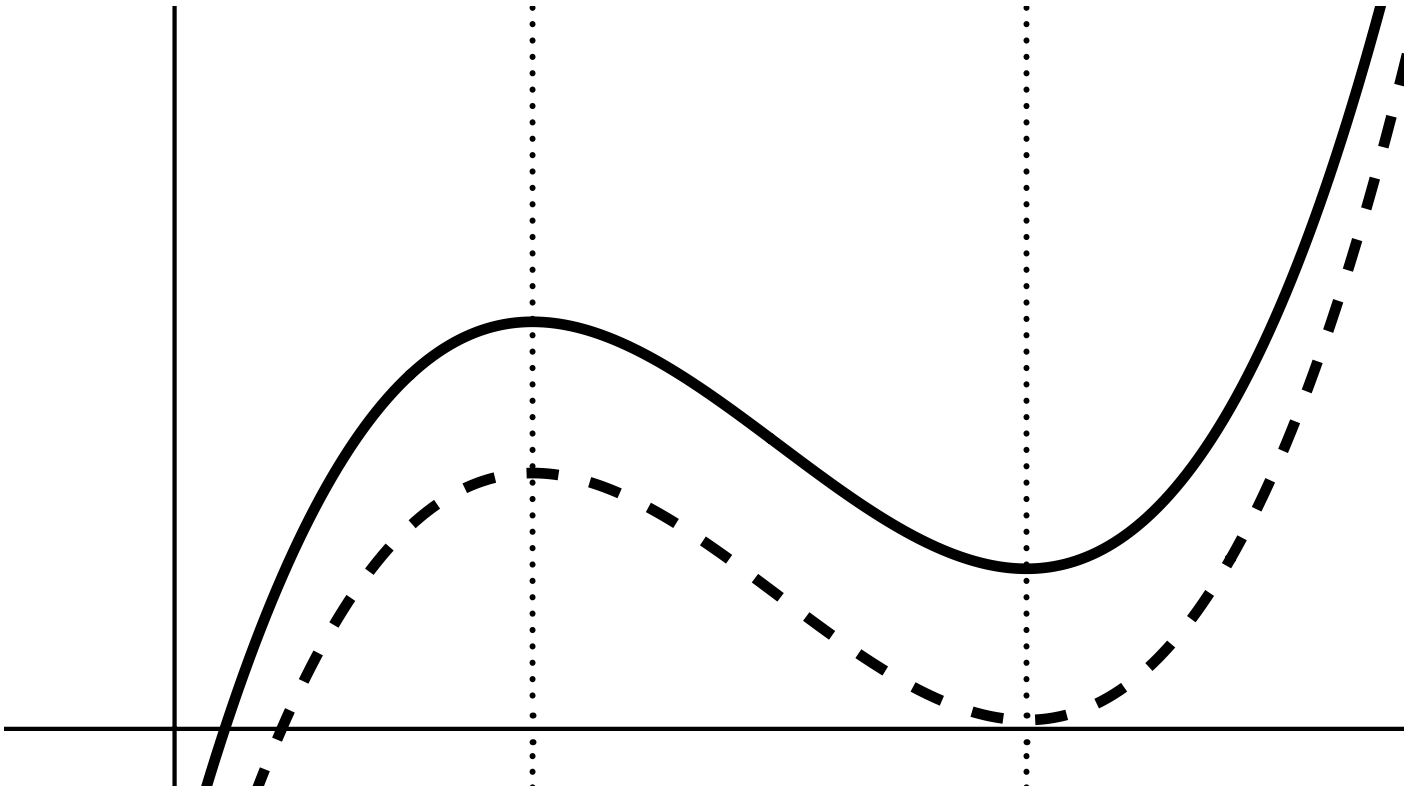


RF VG Add. f(x) +d

f: ——— ‖ g: d >0 - - - -

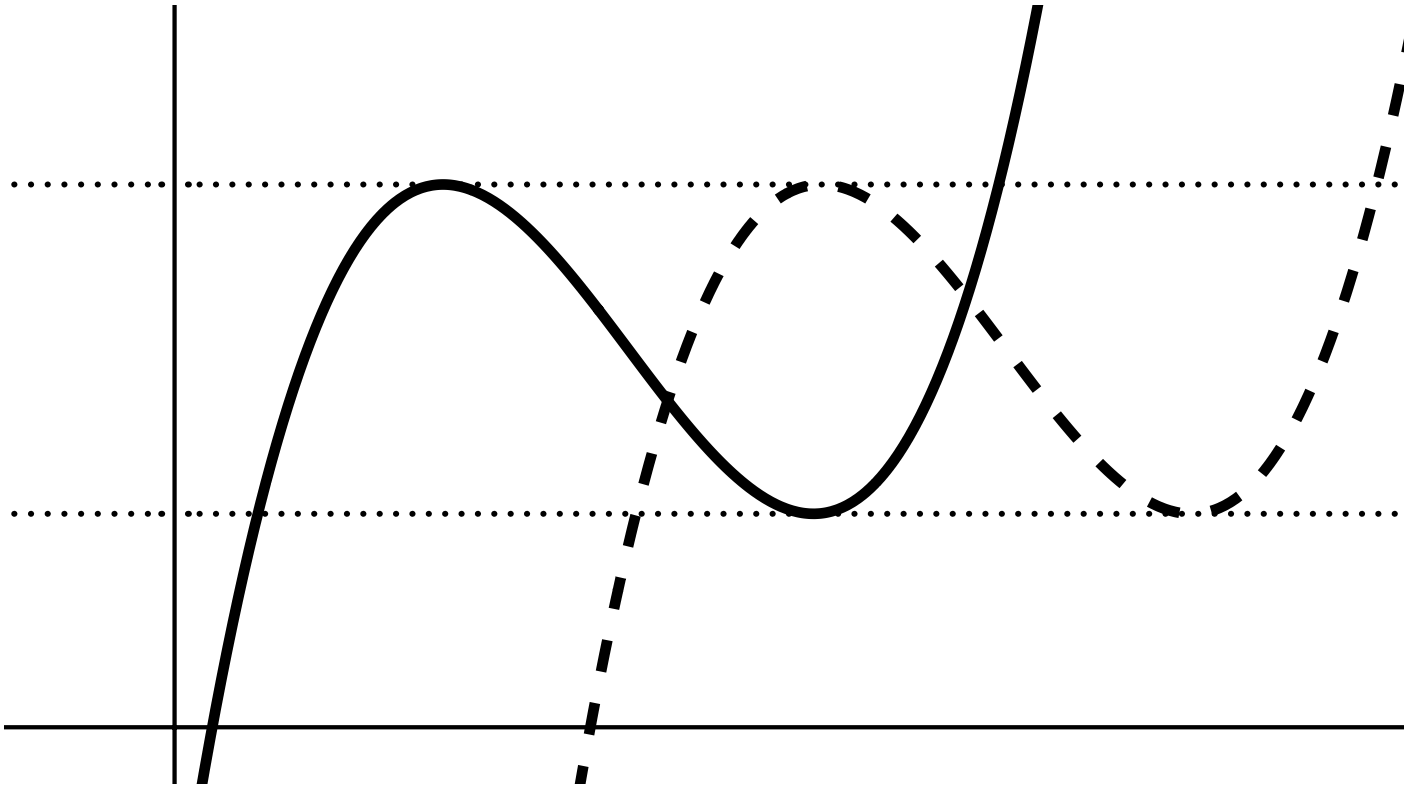


f: ——— ‖ h: d <0 - - - -

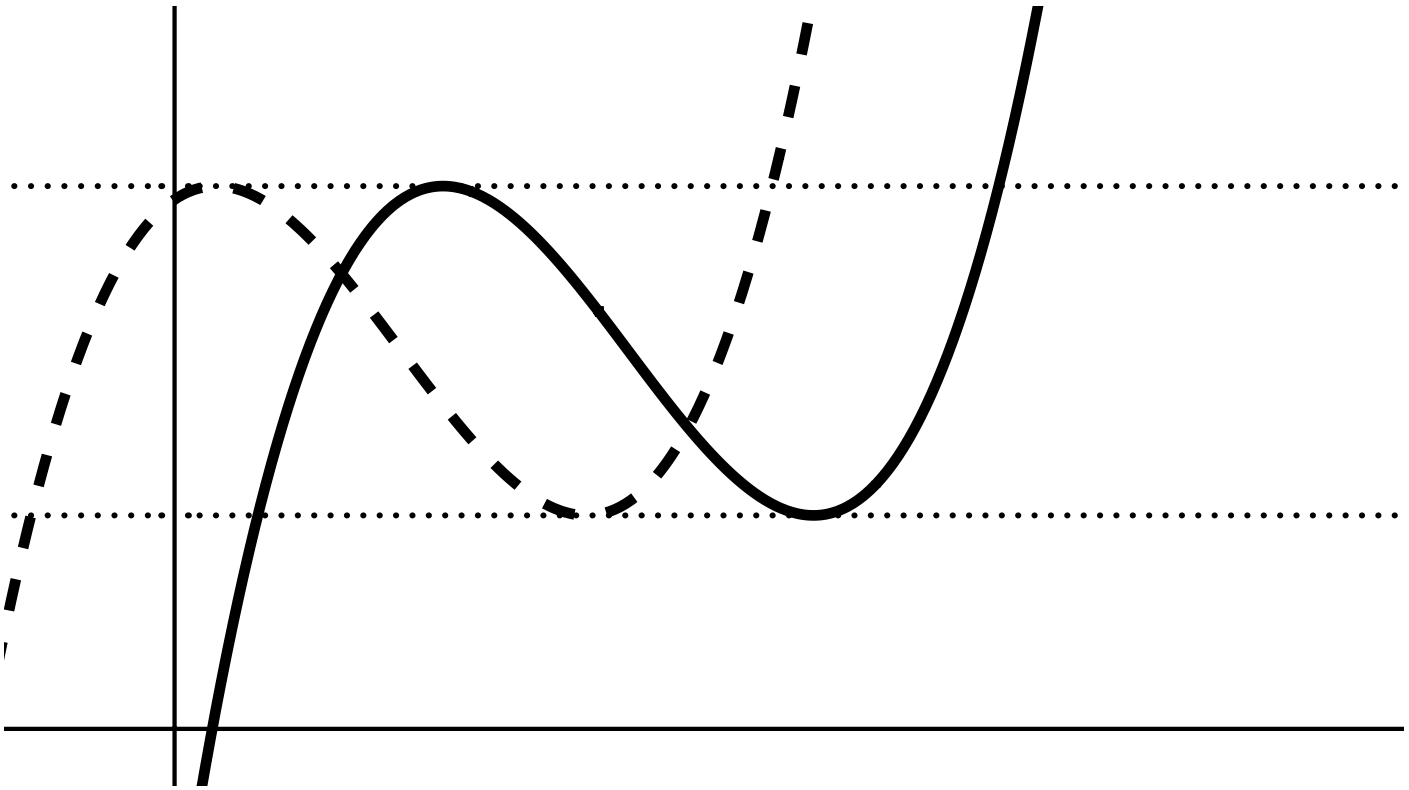


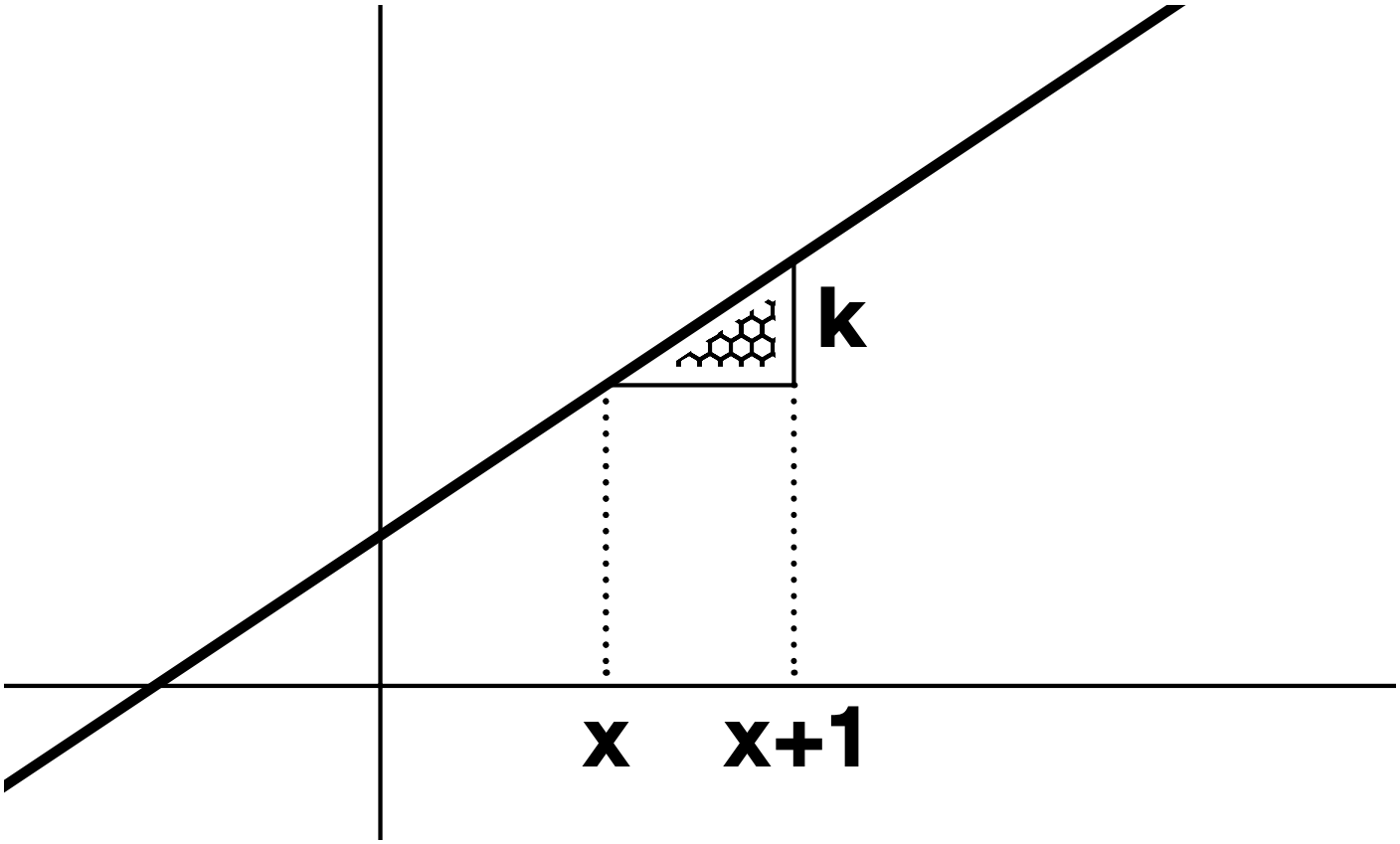
RF VG Add. f(x +c) 12/16

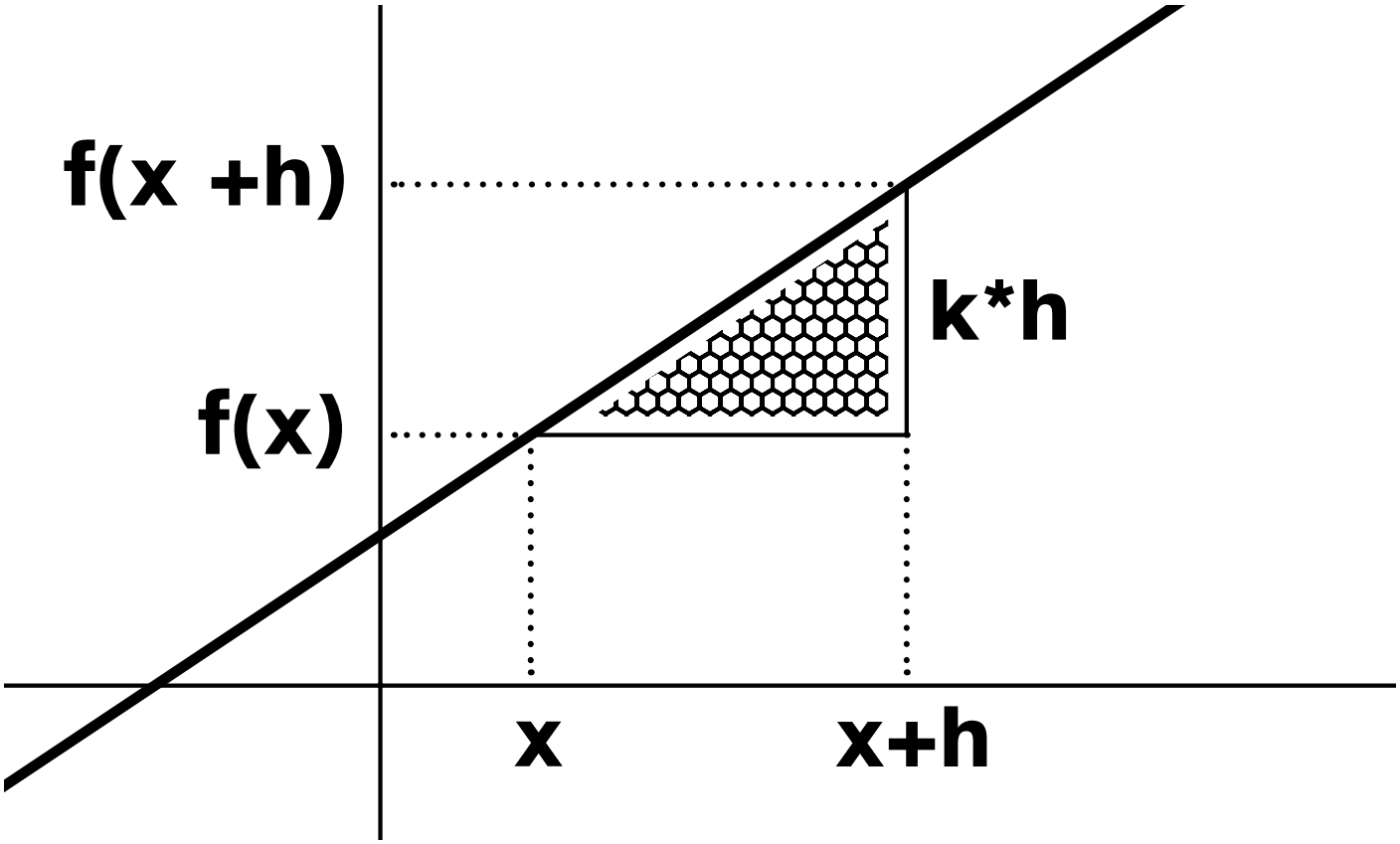
f: ——— ‖ g: c >0 - - - -



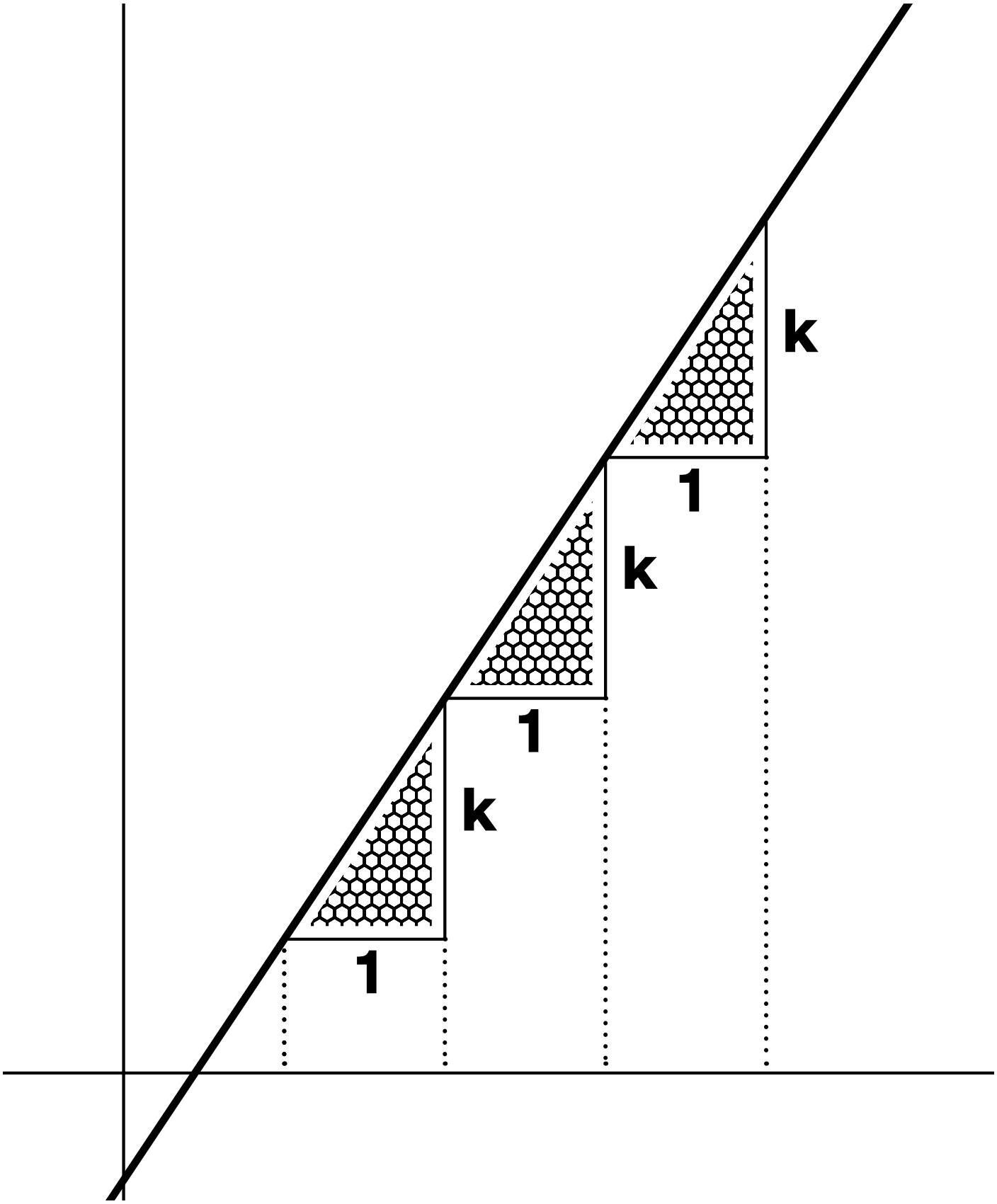
f: ——— ‖ h: c <0 - - - -

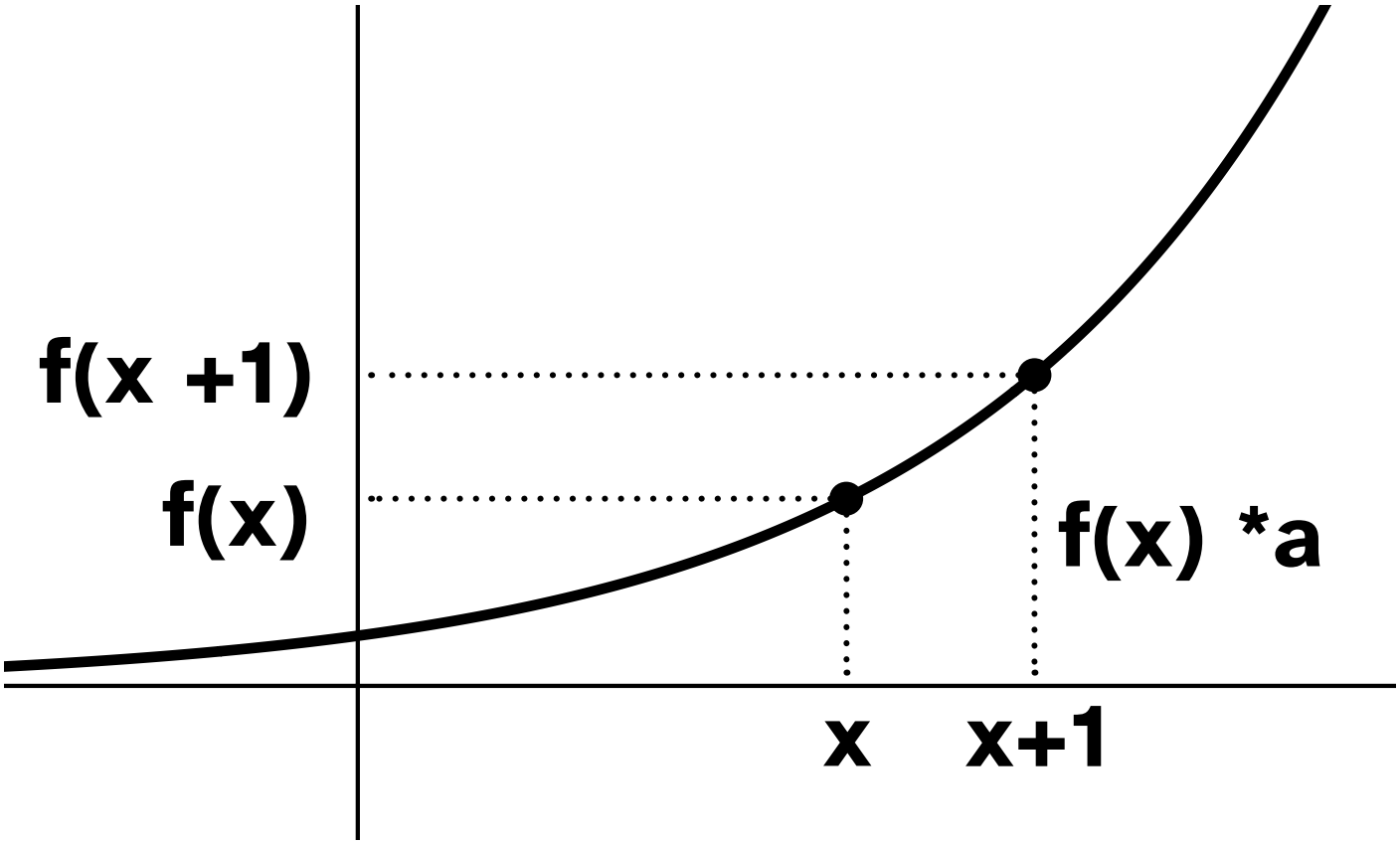


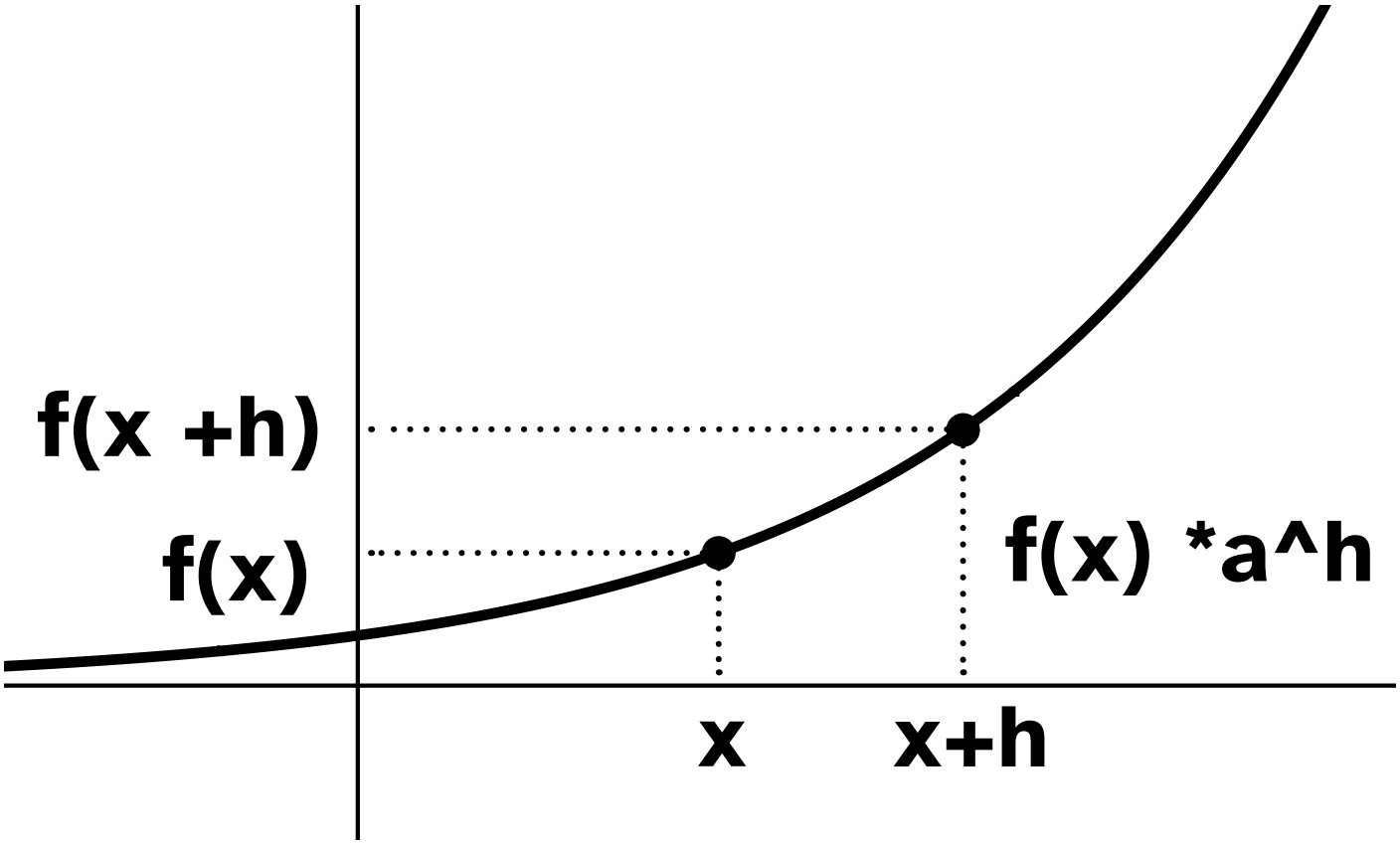




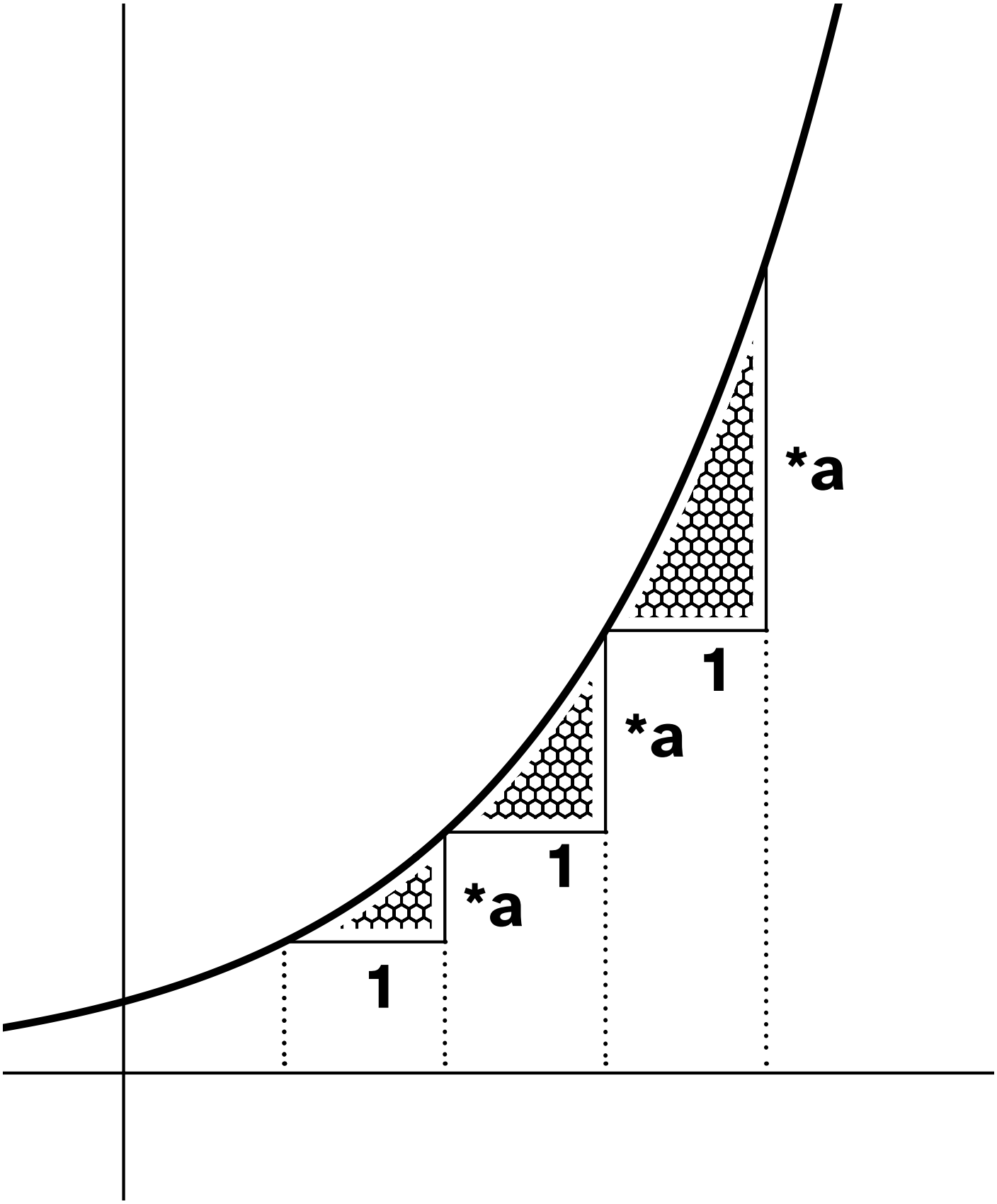
RF ÄM k *x +d







RF ÄM c *a^x



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