

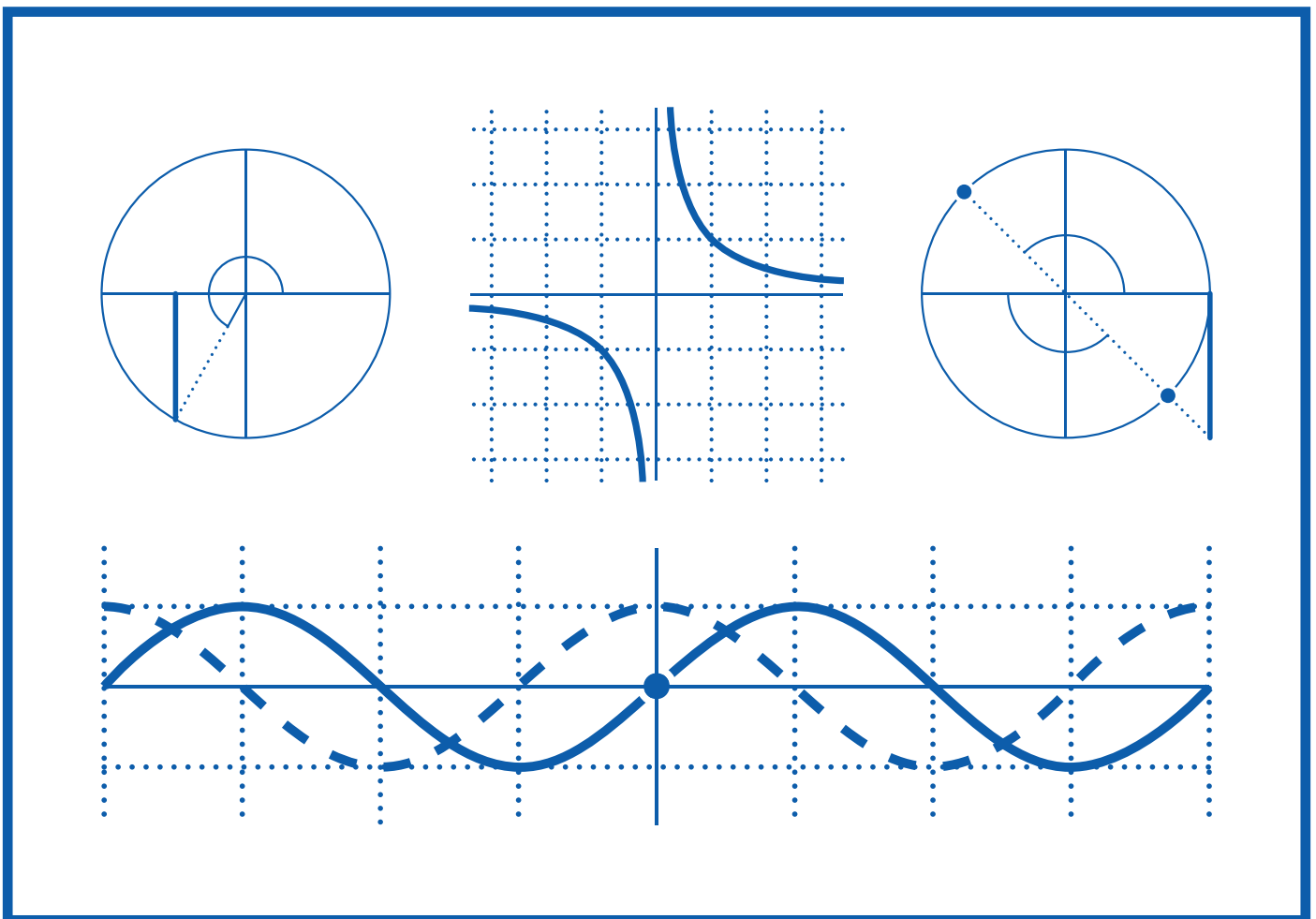


LEMA - Lehr- und Lernmaterialien
Hergestellt von der Abteilung für Inklusion und Lehrmittel (LMZ)
am Bundes-Blindeninstitut Wien

Grafikkatalog

10. Schulstufe (6. AHS)

Autor: Elisabeth Stanetty • Grafiken: Tomáš Batha





10

Grafikkatalog

10. Schulstufe (6. AHS)

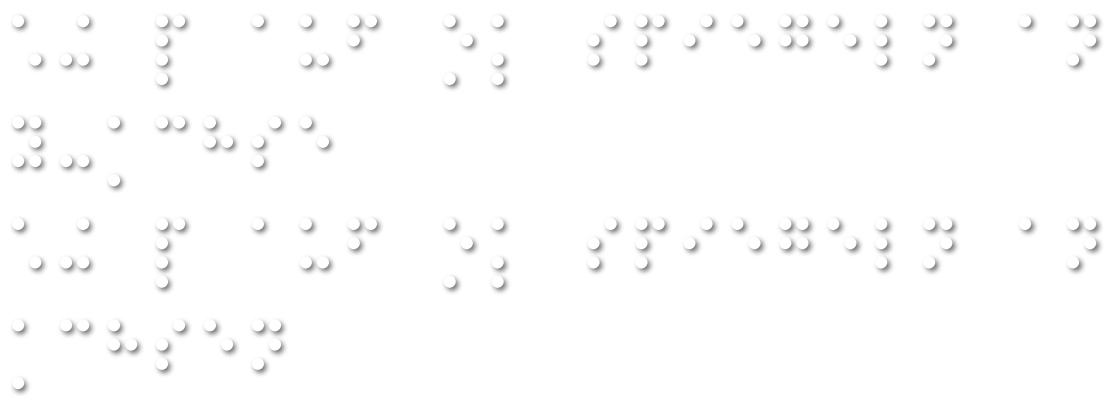


Trigonometrie

Schulstufe 10

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- 9** P auf EK spiegeln an x-Achse
- 10** P auf EK spiegeln an y-Achse
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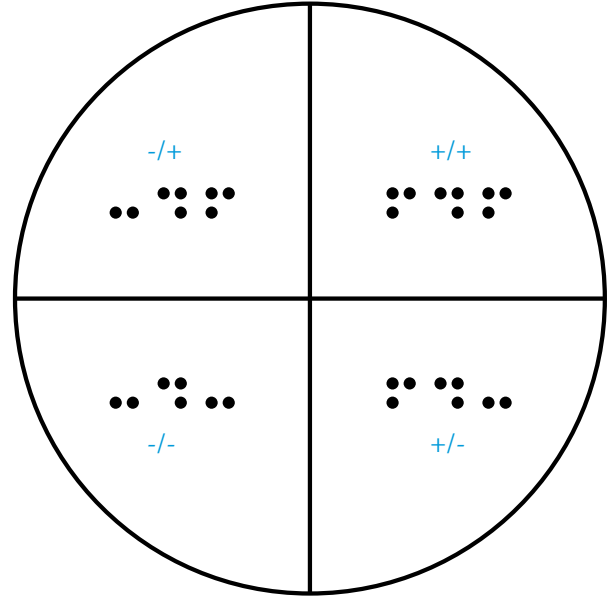
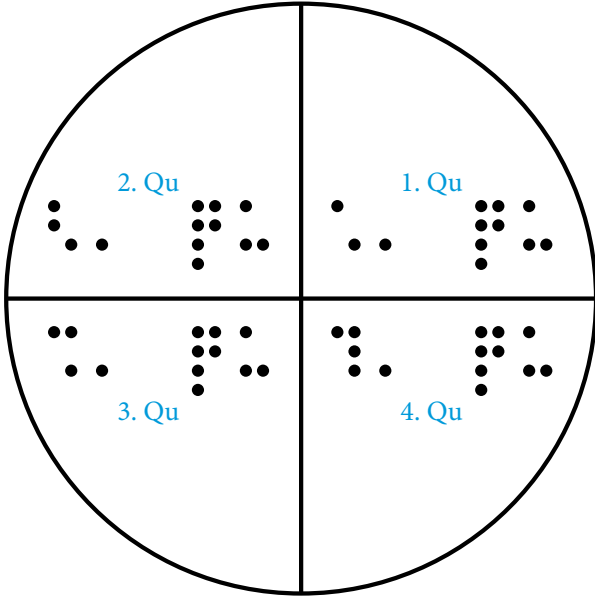


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Einheitskreis: $r = 1$; $M(0|0)$

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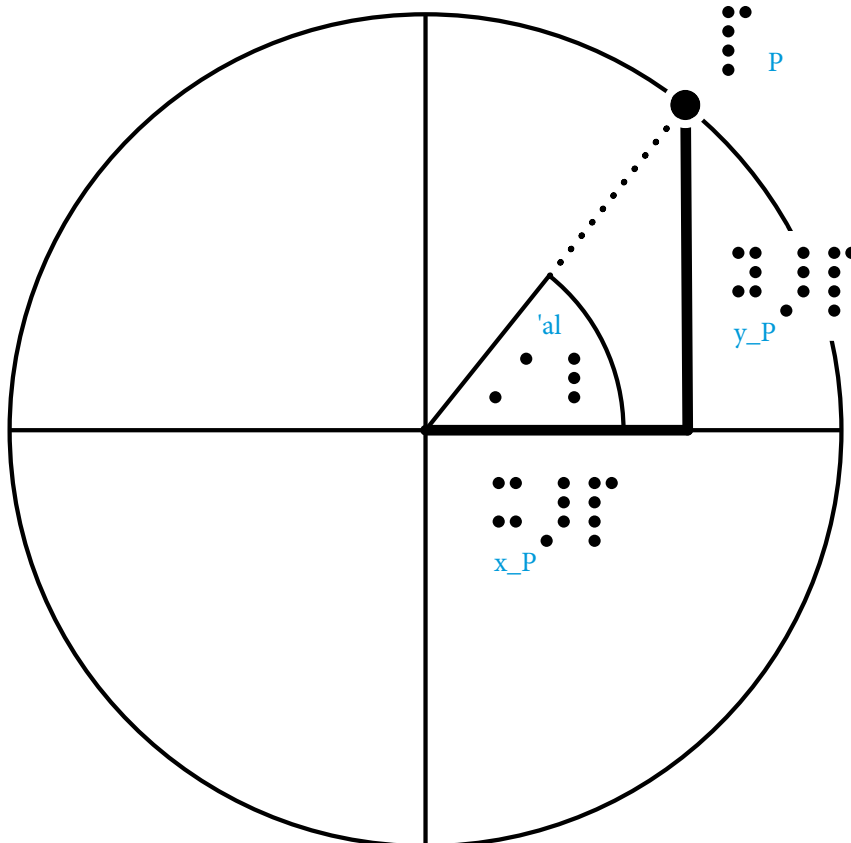


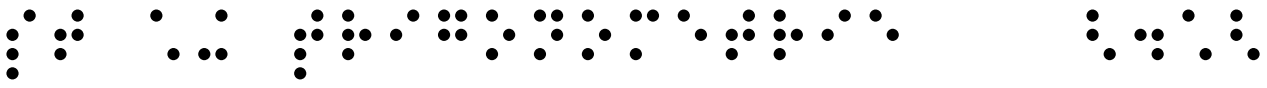
P auf dem Einheitskreis:

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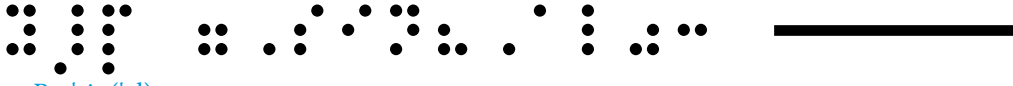
$x_P = \cos('al)$; $y_P = \sin('al)$

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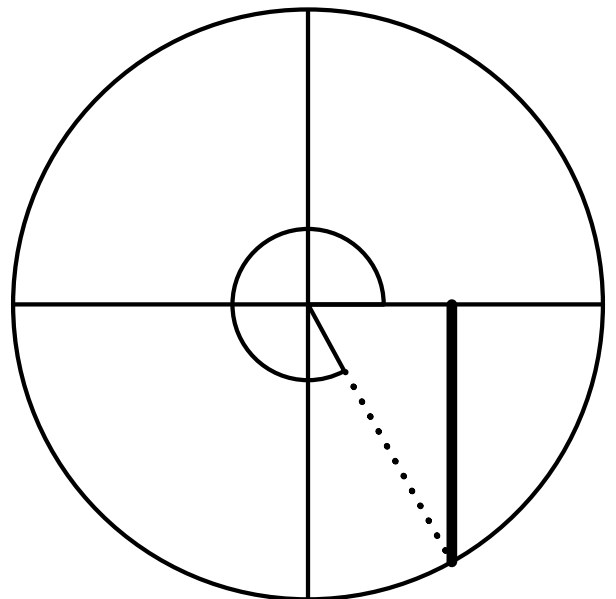
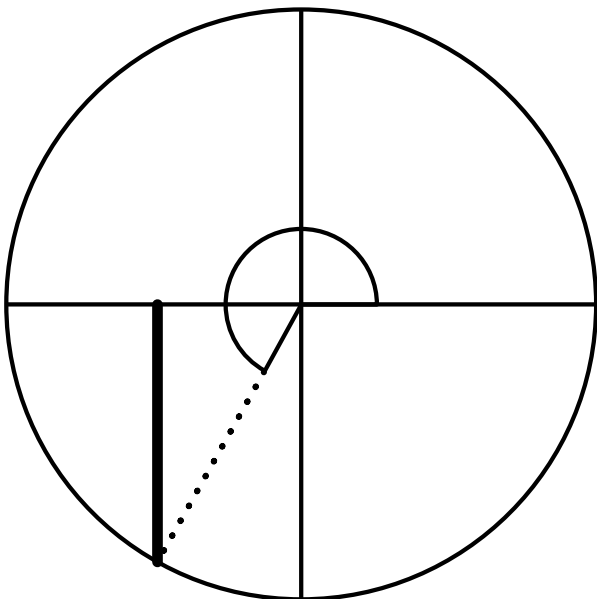
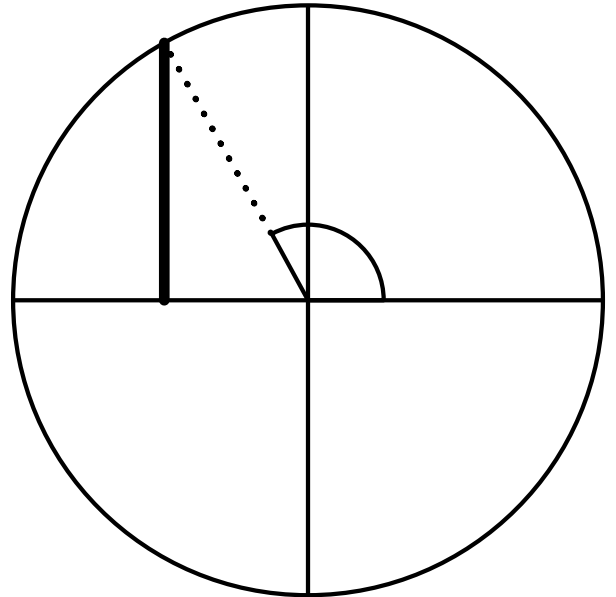
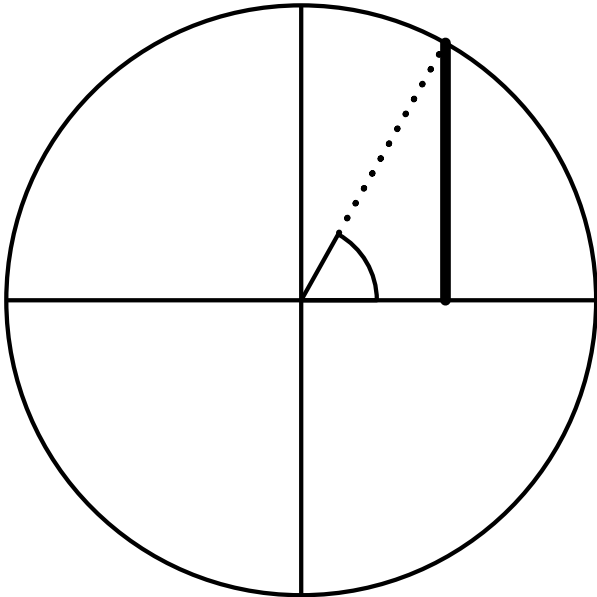
Sinus im Einheitskreis



$y_P = \sin(\alpha)$:



α :



St 10 Trigonometrie, 3/12

Gleiche Sinuswerte

St 10 Trigonometrie, 3/12

Gleiche Sinuswerte

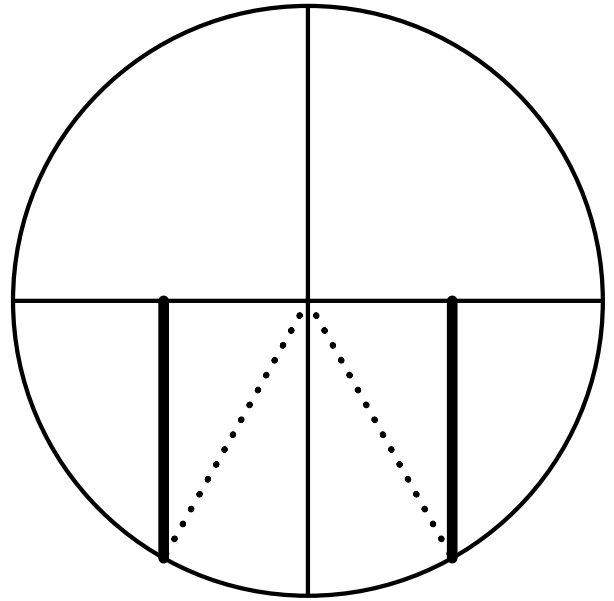
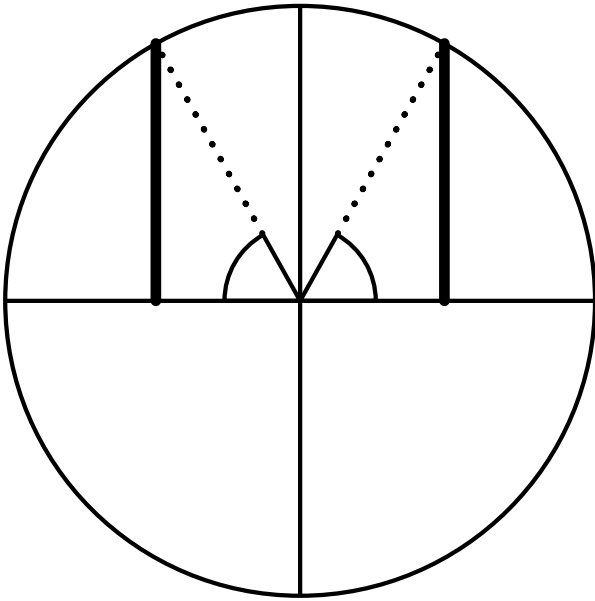
Gleiche Sinuswerte

$\sin(\alpha) = \sin(180^\circ - \alpha)$

$\sin(\alpha) = \sin(180^\circ - \alpha)$



α :



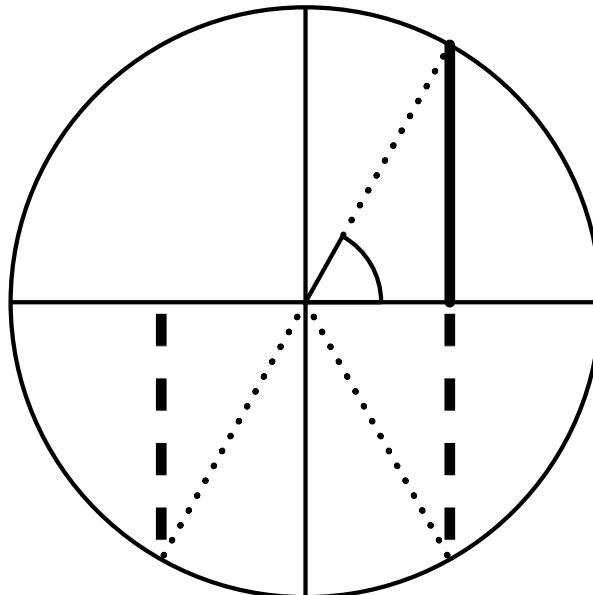
$\sin(\alpha) = -\sin(180^\circ + \alpha) = -\sin(-\alpha)$

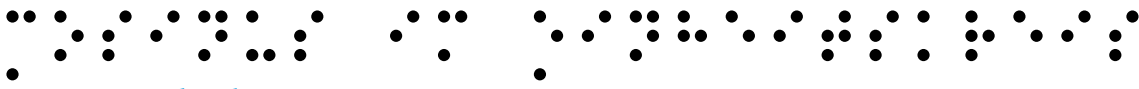
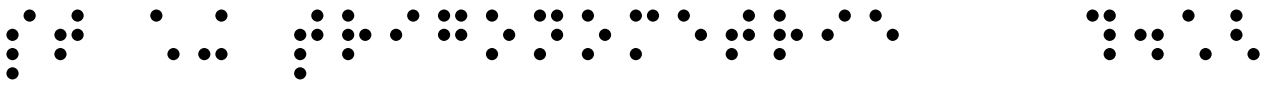
$\sin(\alpha) = -\sin(180^\circ + \alpha) = -\sin(-\alpha)$

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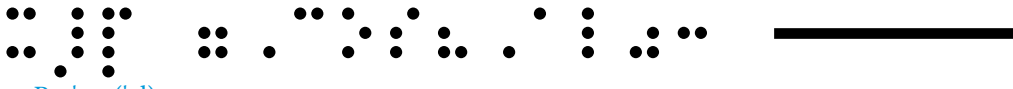


α :





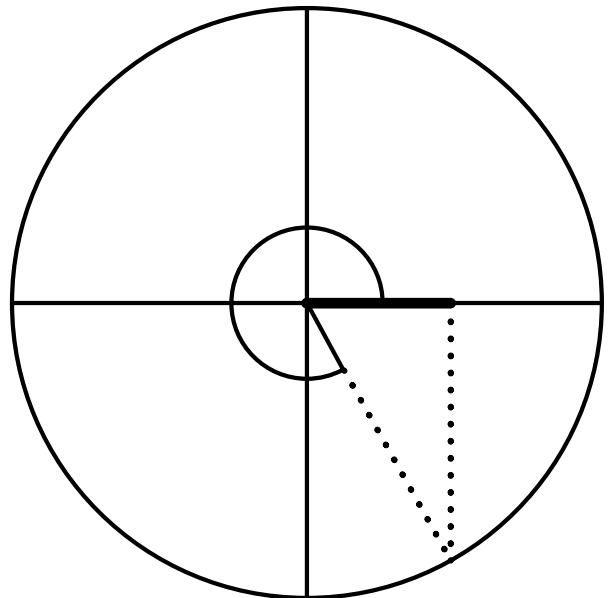
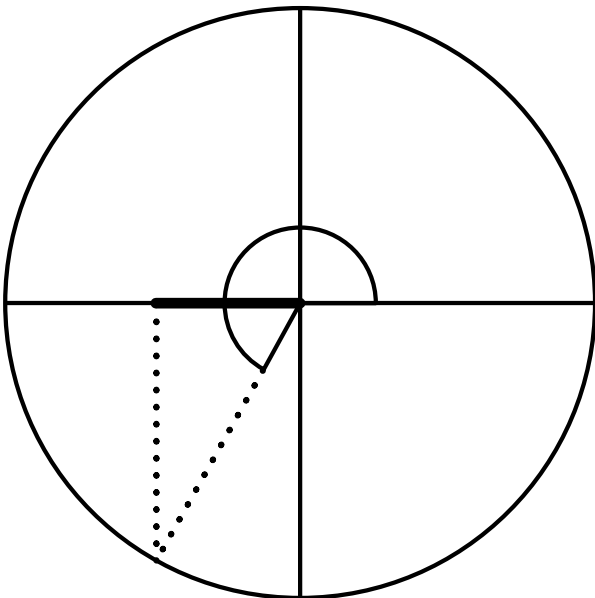
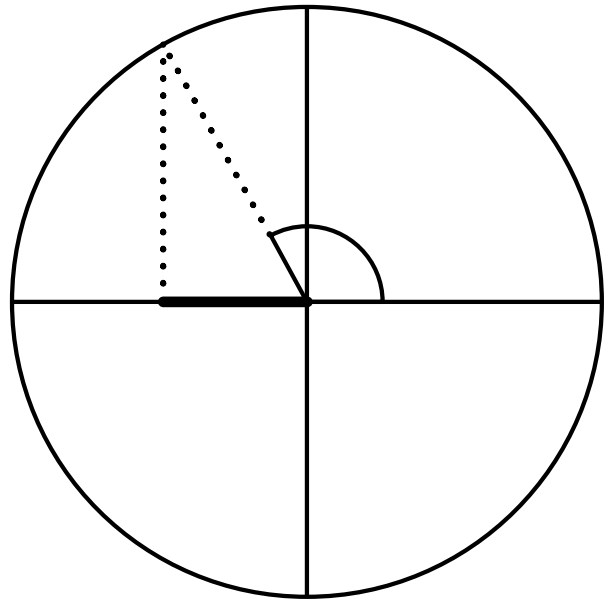
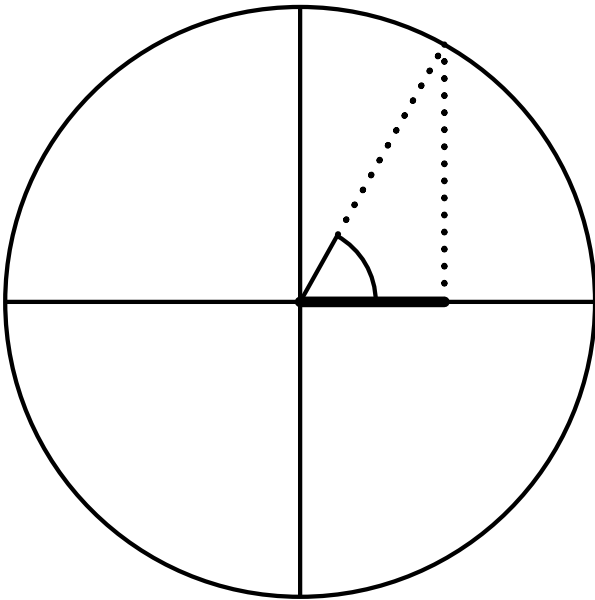
Cosinus im Einheitskreis

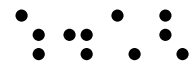
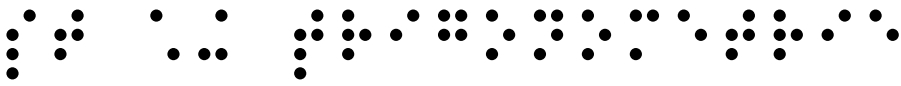


$x_P = \cos('al):$

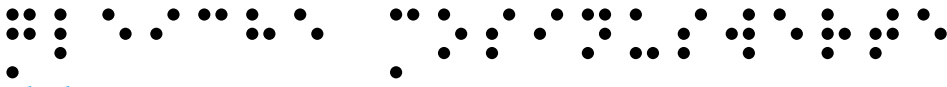


'al:





St 10 Trigonometrie, 5/12



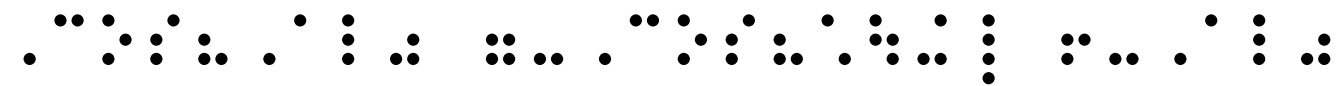
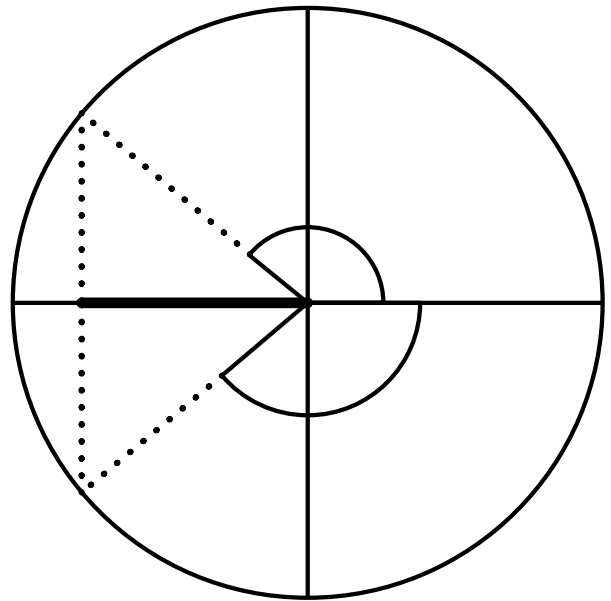
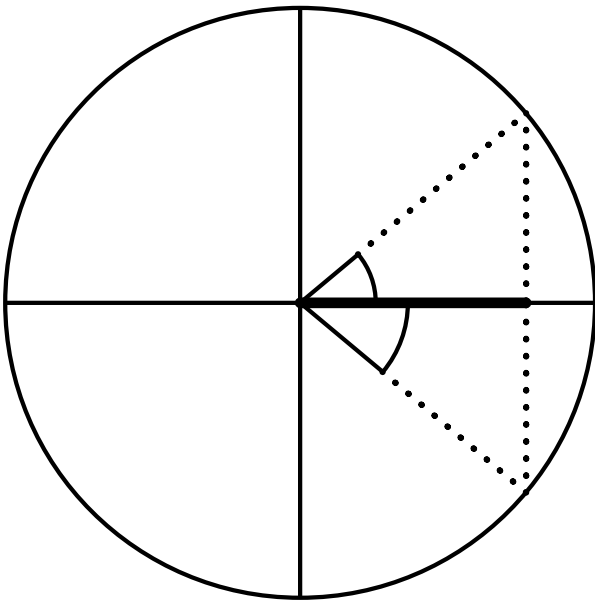
Gleiche Cosinuswerte



$\cos(\alpha) = \cos(-\alpha)$



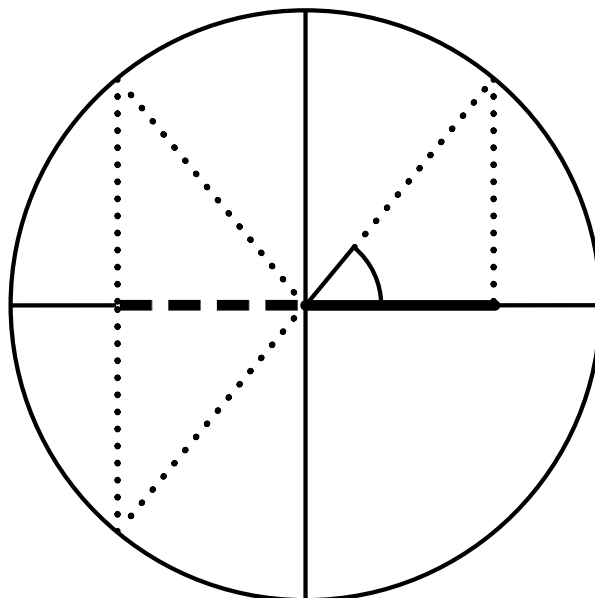
α:



$\cos(\alpha) = -\cos(180^\circ + \alpha)$



α:



St 10 Trigonometrie, 6/12

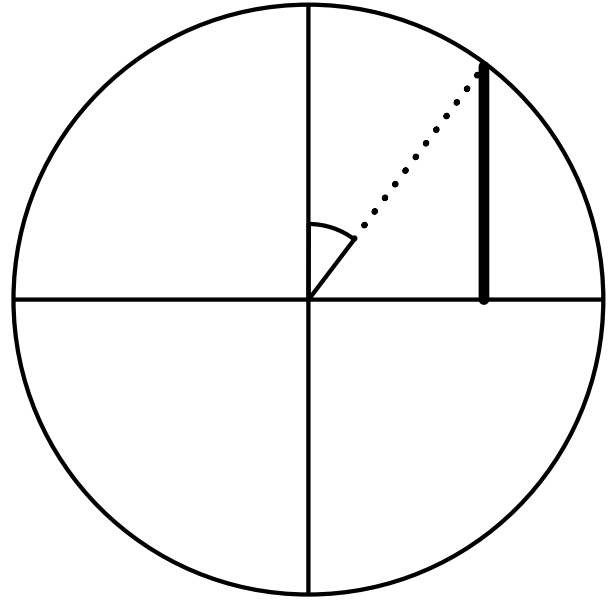
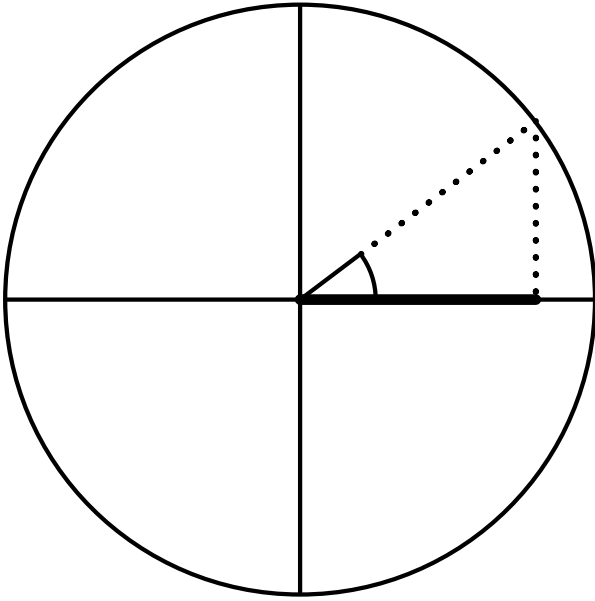
St 10 Trigonometrie, 6/12

Sinus und Cosinus

Sinus und Cosinus

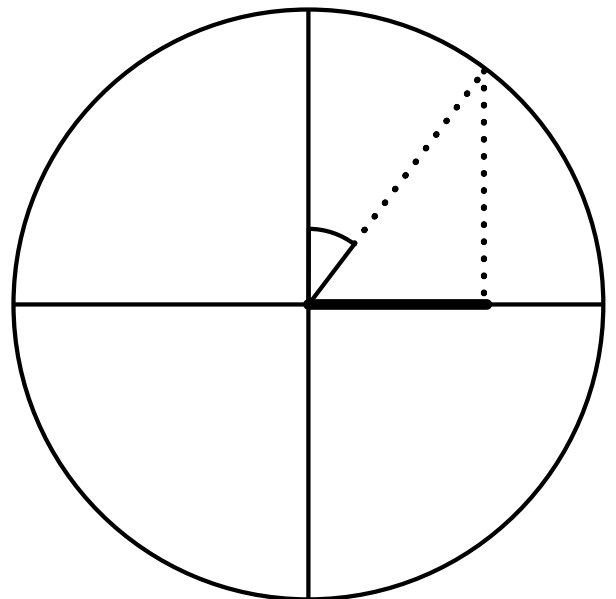
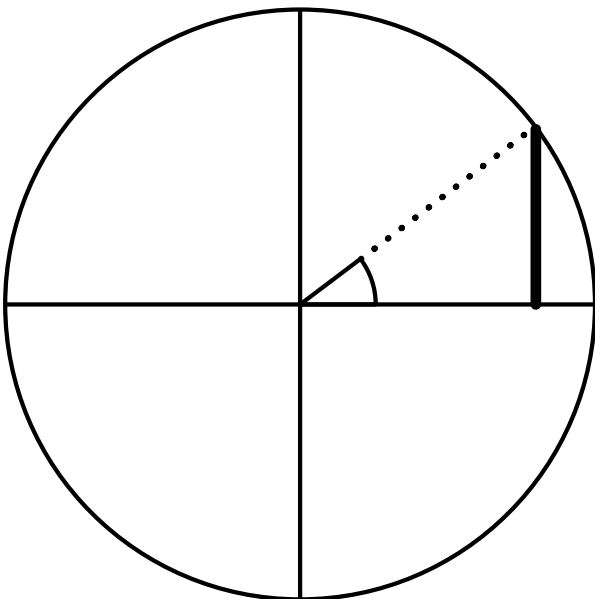
$\cos(\alpha) = \sin(90^\circ - \alpha)$

$\cos(\alpha) = \sin(90^\circ - \alpha)$



$\sin(\alpha) = \cos(90^\circ - \alpha)$

$\sin(\alpha) = \cos(90^\circ - \alpha)$



St 10 Trigonometrie, 7/12

St 10 Trigonometrie, 7/12

Tangens im Einheitskreis

Tangens im Einheitskreis

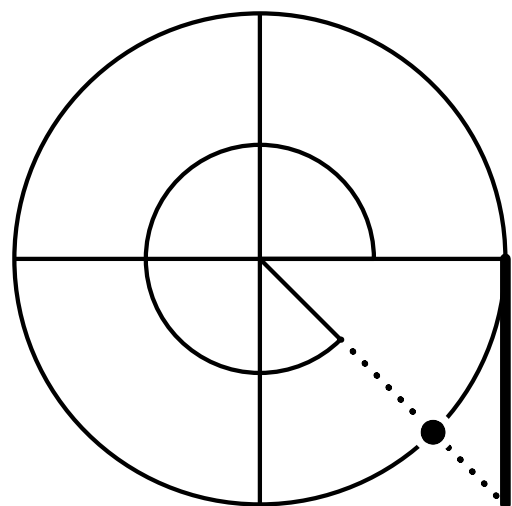
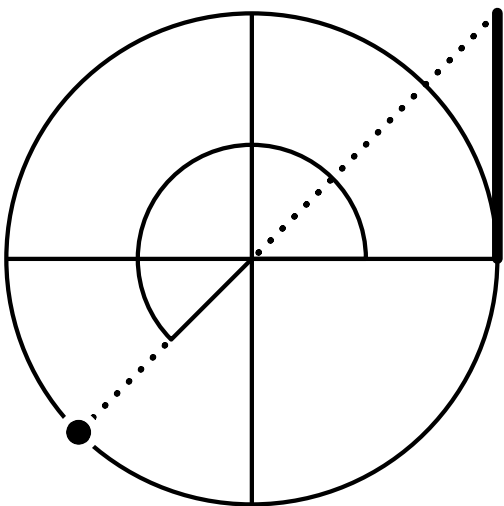
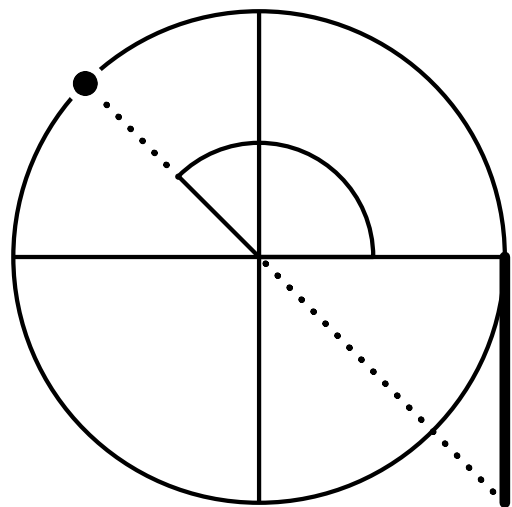
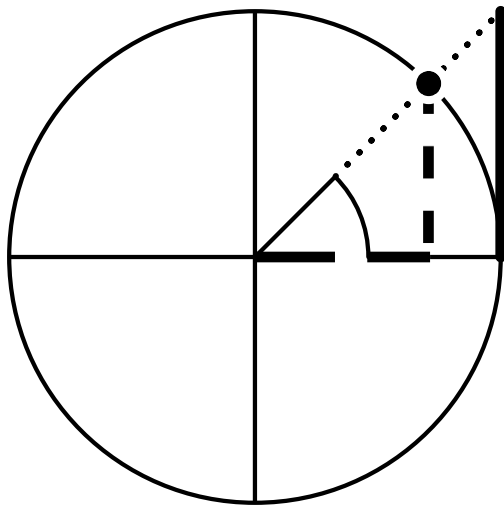
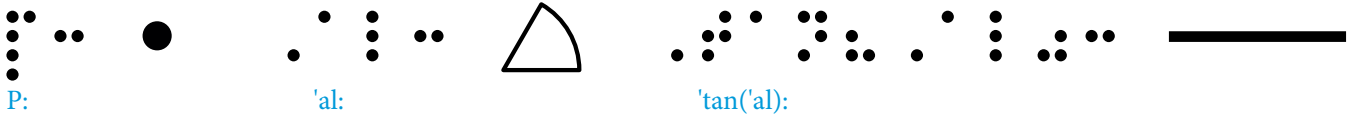
Strahlensatz:

Strahlensatz:

Strahlensatz:

Strahlensatz:

$$\tan(\alpha) / 1 = \sin(\alpha) / \cos(\alpha)$$



St 10 Trigonometrie, 8/12

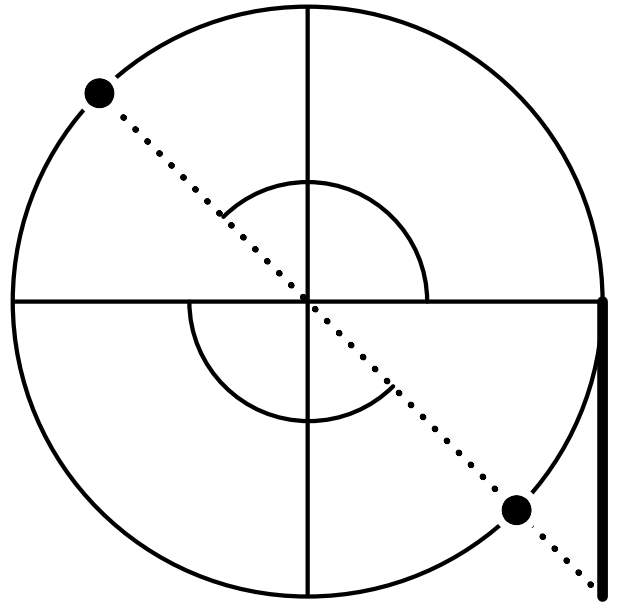
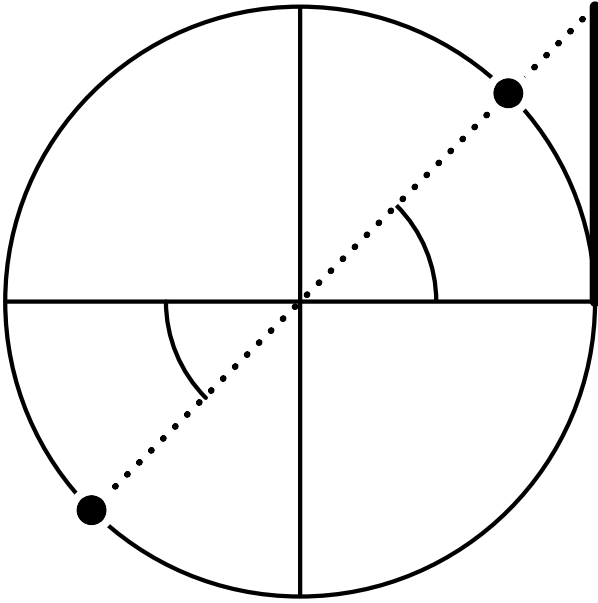
St 10 Trigonometrie, 8/12

Gleiche Tangenswerte

Gleiche Tangenswerte

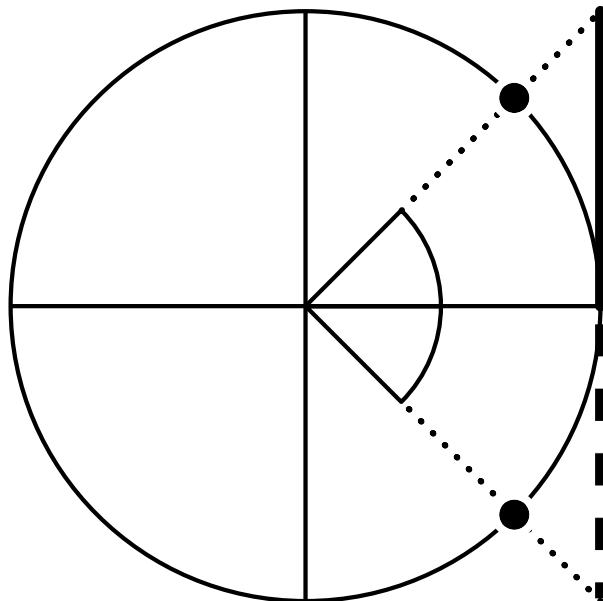
$\tan(\alpha) = \tan(180^\circ + \alpha)$

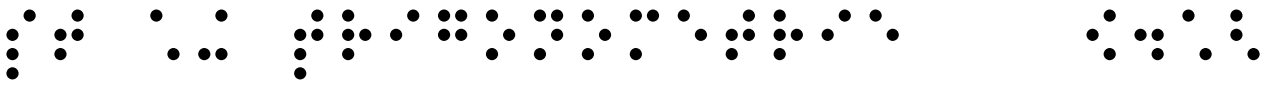
$\tan(\alpha) = \tan(180^\circ + \alpha)$



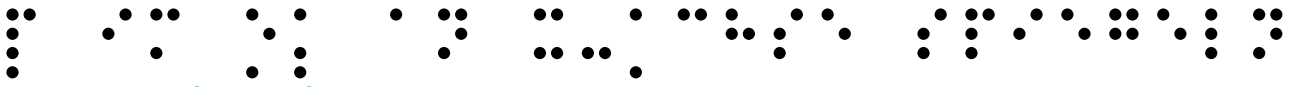
$\tan(\alpha) = -\tan(-\alpha)$

$\tan(\alpha) = -\tan(-\alpha)$





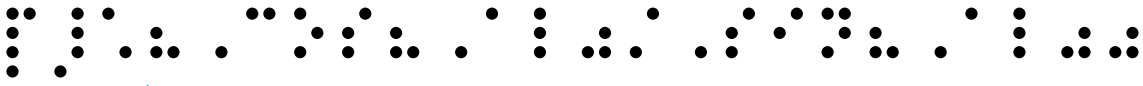
St 10 Trigonometrie, 9/12



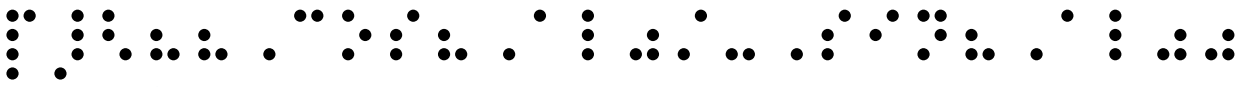
P im EK an x-Achse spiegeln



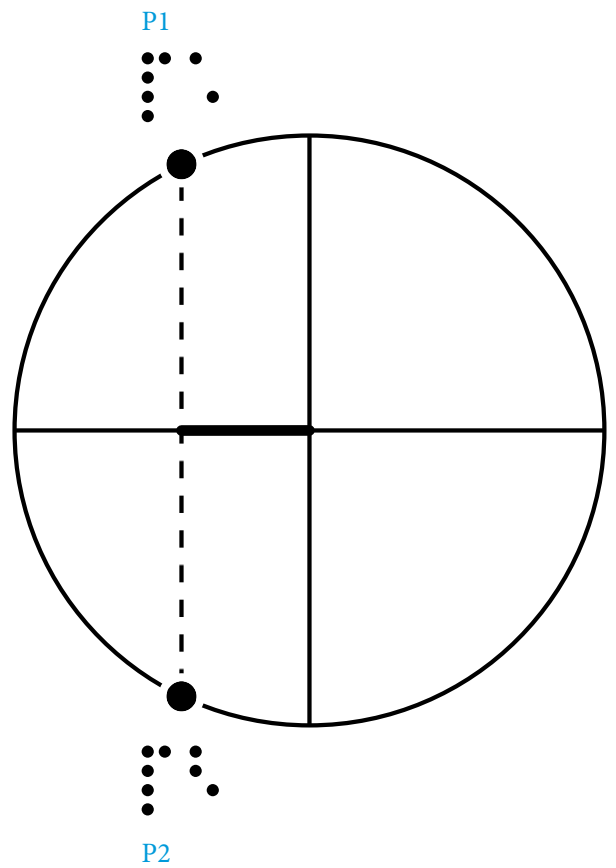
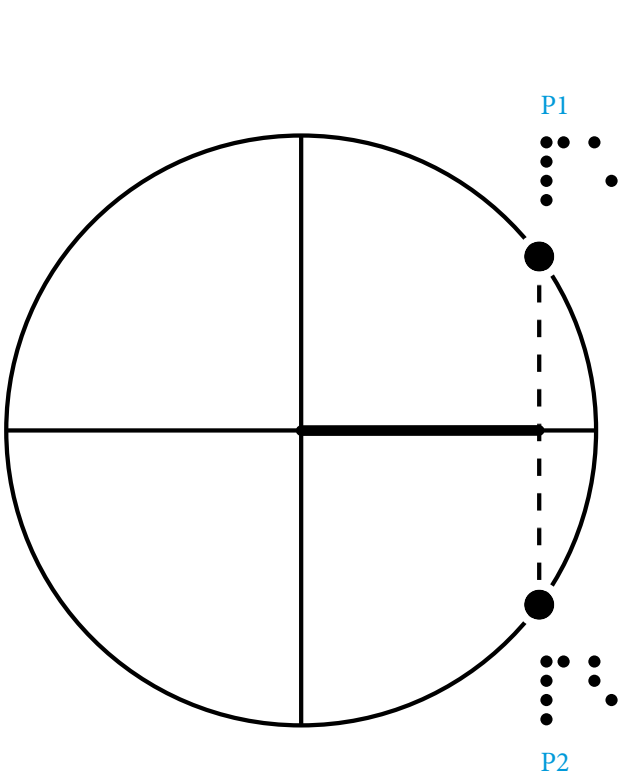
Cosinus bleibt gleich

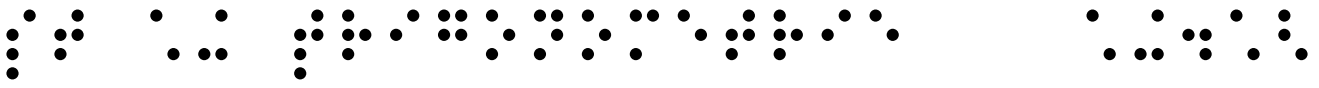


$P_1('cos('al)|'sin('al))$

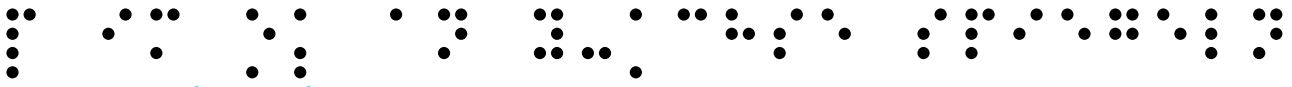


$P_2(('cos('al)|-'sin('al))$





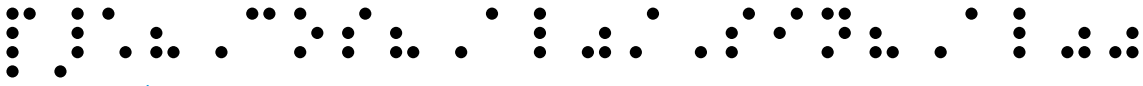
St 10 Trigonometrie, 10/12



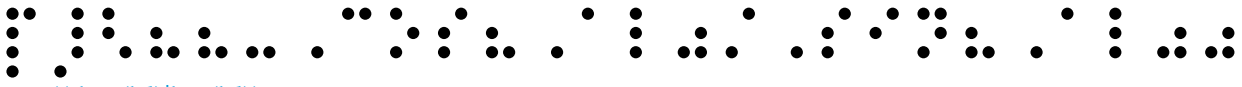
P im EK an y-Achse spiegeln



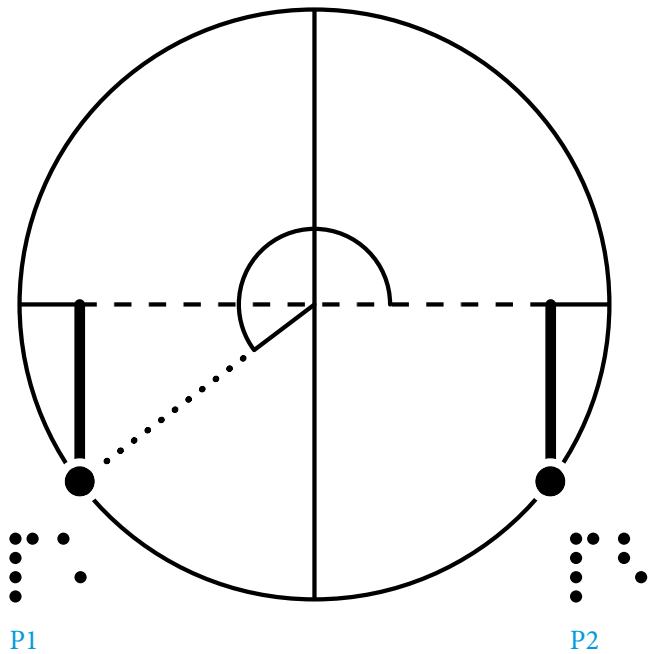
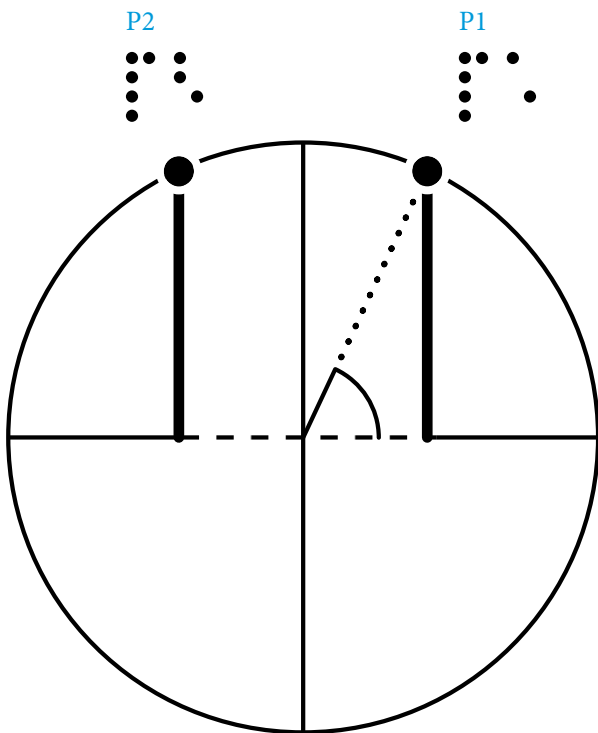
Sinus bleibt gleich

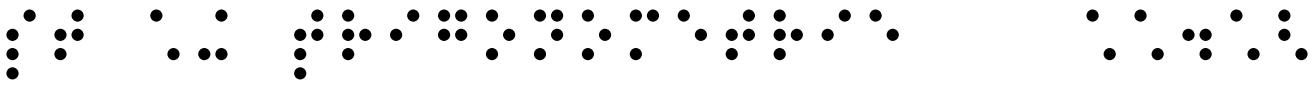


$P_1(\cos(\alpha) | \sin(\alpha))$



$P_2(-\cos(\alpha) | \sin(\alpha))$

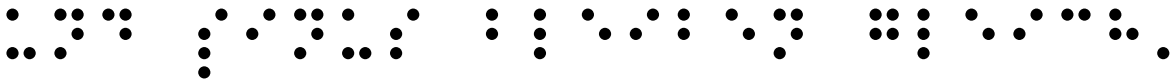
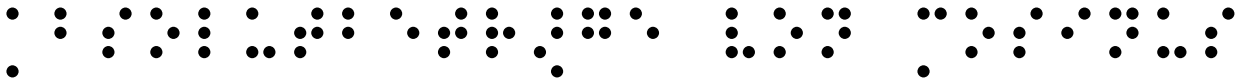




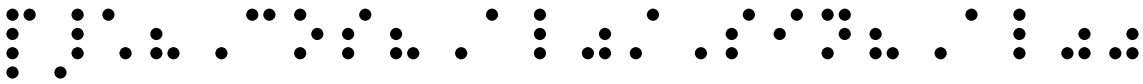
St 10 Trigonometrie, 11/12



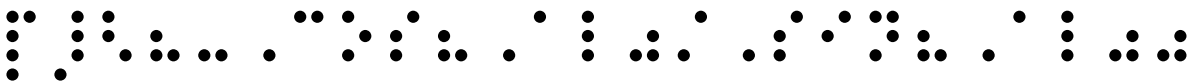
P im EK an Achsen spiegeln



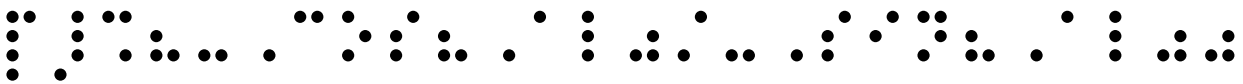
Absolutbeträge von Cosinus und Sinus bleiben gleich.



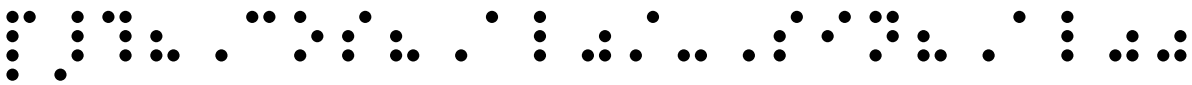
$P_1(|\cos(\alpha)|, |\sin(\alpha)|)$



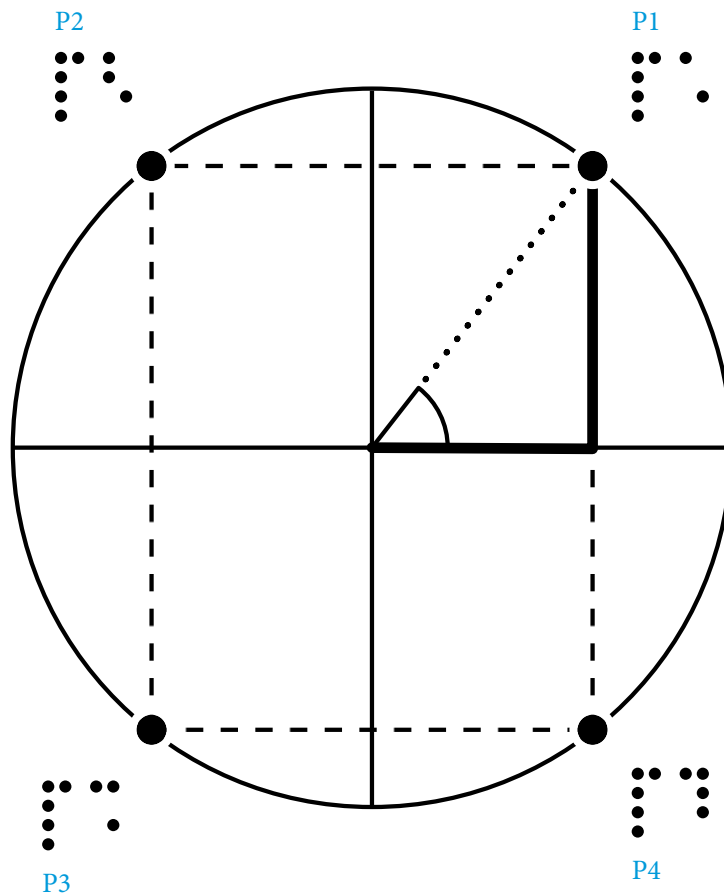
$P_2(-|\cos(\alpha)|, |\sin(\alpha)|)$



$P_3(-|\cos(\alpha)|, -|\sin(\alpha)|)$



$P_4(|\cos(\alpha)|, -|\sin(\alpha)|)$



St 10 Trigonometrie, 12/12

St 10 Trigonometrie, 12/12

Zusammenhänge

Zusammenhänge

EK: $r = 1$; $M(0|0)$; $P(x|y)$

EK: $r = 1$; $M(0|0)$; $P(x|y)$

$x = \cos(\alpha)$; $y = \sin(\alpha)$

$x = \cos(\alpha)$; $y = \sin(\alpha)$

Pythagoräischer Lehrsatz:

Pythagoräischer Lehrsatz:

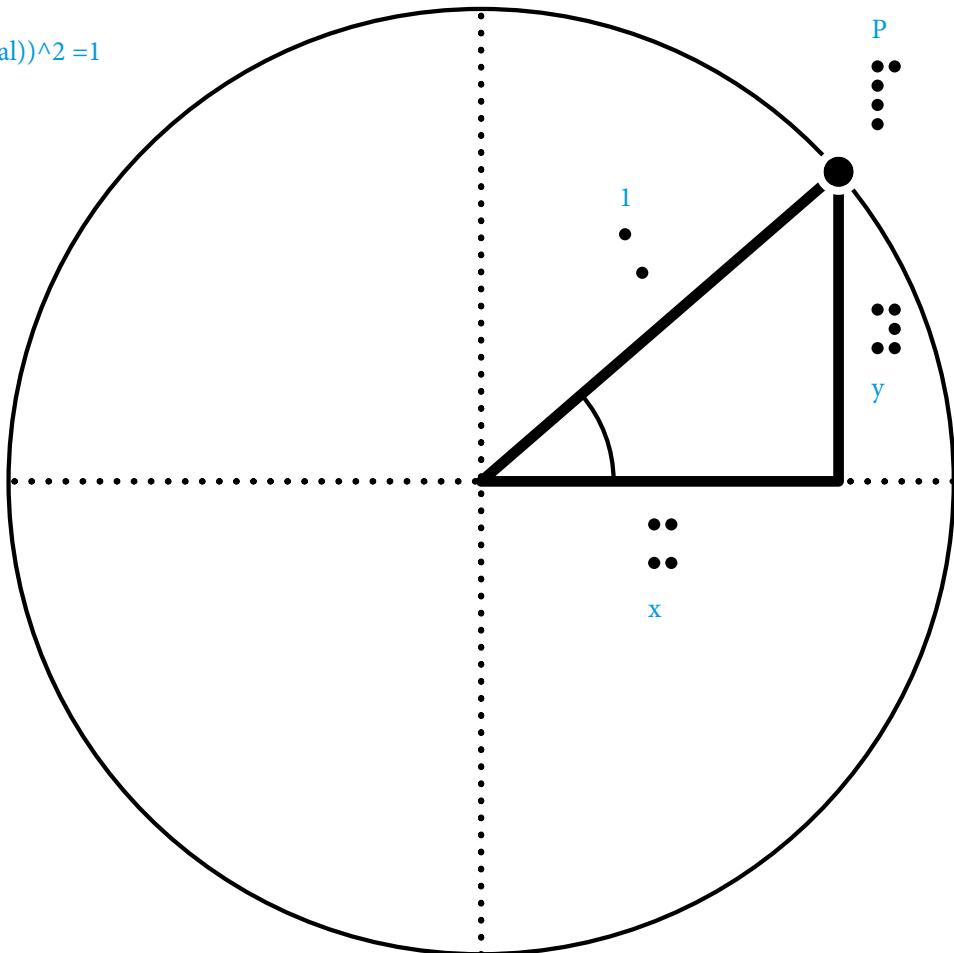
$x^2 + y^2 = r^2$

$x^2 + y^2 = r^2$

$(\cos(\alpha))^2 + (\sin(\alpha))^2 = 1$

$(\cos(\alpha))^2 + (\sin(\alpha))^2 = 1$

$(\cos(\alpha))^2 + (\sin(\alpha))^2 = 1$



Funktionen

Schulstufe 10

Inhalt

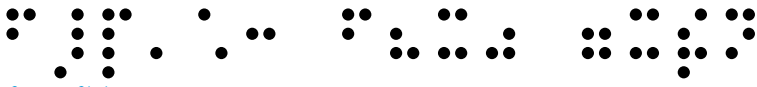
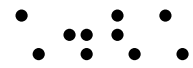
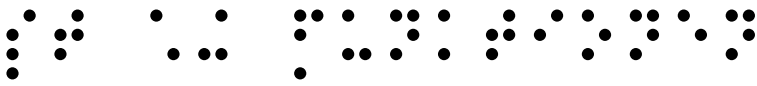
- 1 Potenzfunktion f_P.1: $f(x) = x^n$
- 2 Potenzfunktion f_P.2: $f(x) = 1/x^n = x^{-n}$
- 3 Potenzfunktion f_P.3: Umkehrfunktion
- 4 Potenzfunktion f_P.4: Wurzelfunktion
- 5 Exponentialfunktion f_Ex.1: $f(x) = a^x$
- 6 Exponentialfunktion f_Ex.2: $f(x) = a^x$ spiegeln
- 7 Exponentialfunktion f_Ex.3: $f(x) = e^x$ spiegeln
- 8 Exponentialfunktion f_Ex.4: --> Umkehrfkt (Basis a)
- 9 Exponentialfunktion f_Ex.5: Umkehrfkt (Basis e)
- 10 Logarithmusfunktion f_Log.1: $f(x) = \log_a(x)$
- 11 Logarithmusfunktion f_Log.2: $f(x) = \log_a(x)$
- 12 Logarithmusfunktion f_Log.3: $f(x) = \log_a(x)$
- 13 Winkelfunktion f_W.1: $f(x) = \sin(x)$
- 14 Winkelfunktion f_W.2: $f(x) = \cos(x)$
- 15 Winkelfunktion f_W.3: 'sin und 'cos
- 16 Winkelfunktion f_W.4: $f(x) = a \cdot \sin(x)$

17 Winkelfunktion f_W.5: $f(x) = \sin(b \cdot x)$

18 Winkelfunktion f_W.6: $f(x) = \sin(x) + c$

19 Winkelfunktion f_W.7: $f(x) = \sin(x+d)$

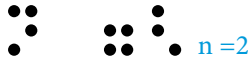
20 Winkelfunktion f_W.7: $f(x) = a \cdot \sin(b \cdot x + d) + c$



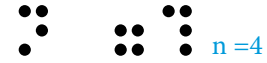
$f_{P.1}: f(x) = x^n$



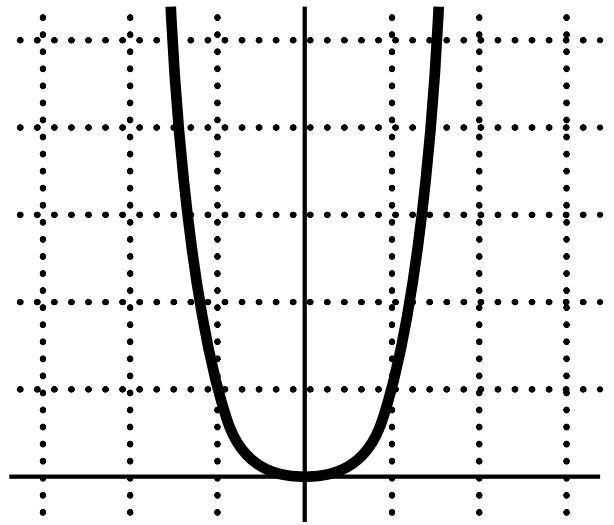
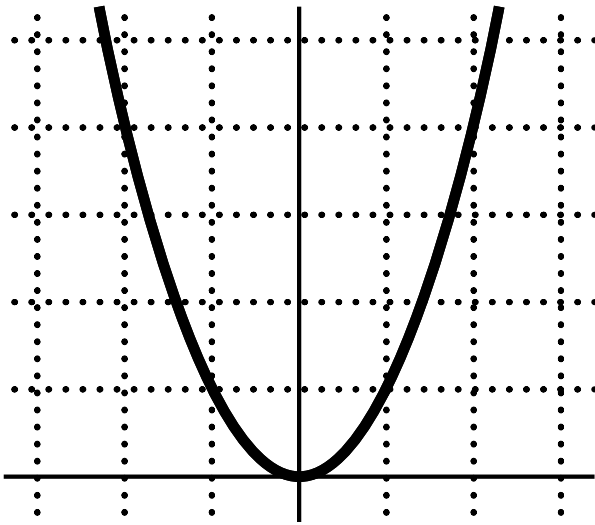
$n \in \mathbb{N}$, n gerade,



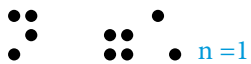
$n=2$



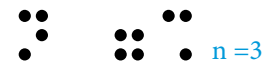
$n=4$



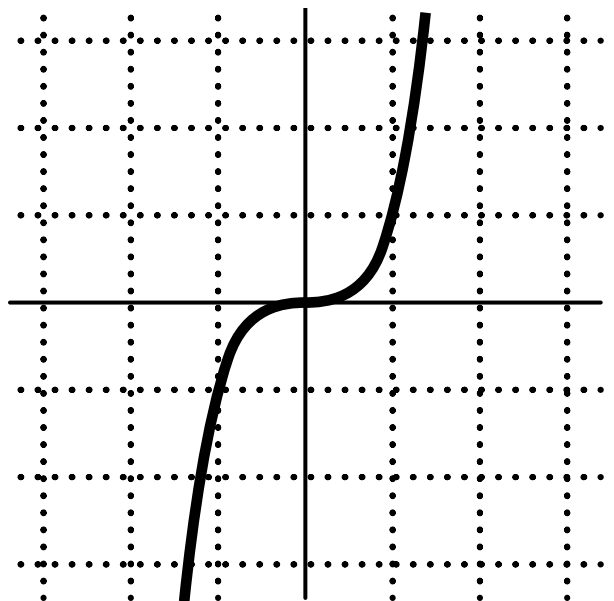
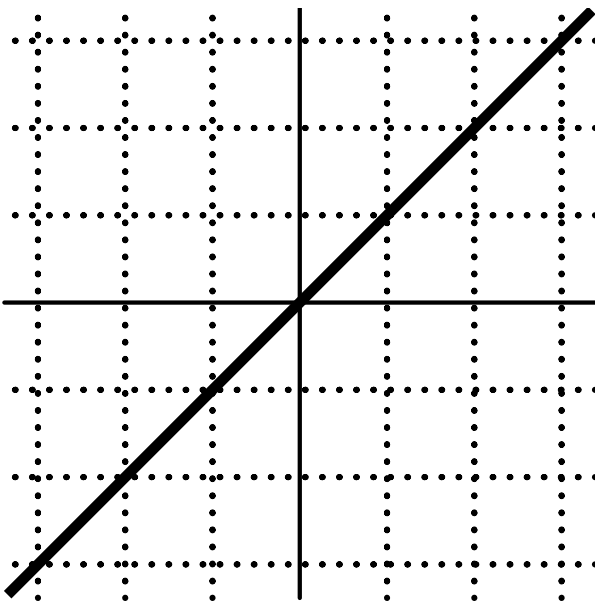
$n \in \mathbb{N}$, n ungerade,

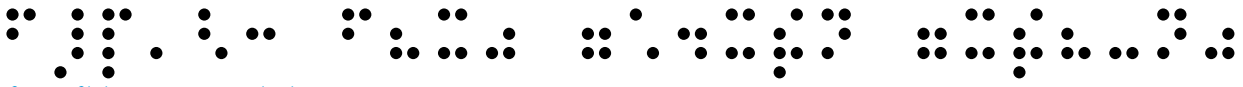
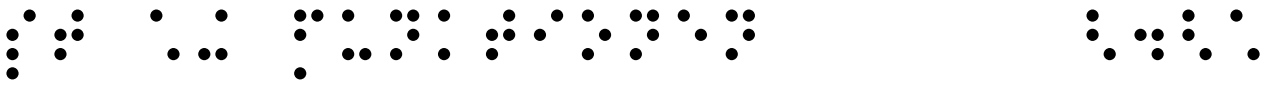


$n=1$

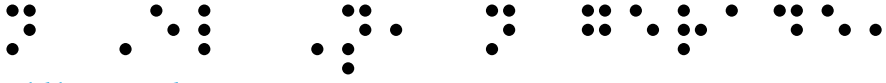


$n=3$



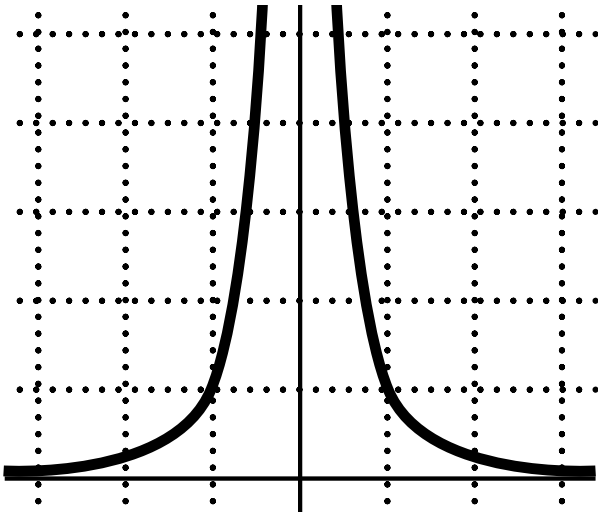


f_P.2: $f(x) = 1/x^n = x^{-n}$

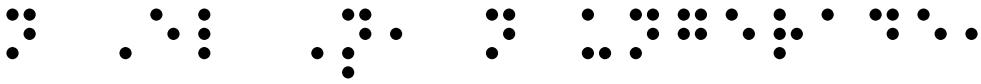
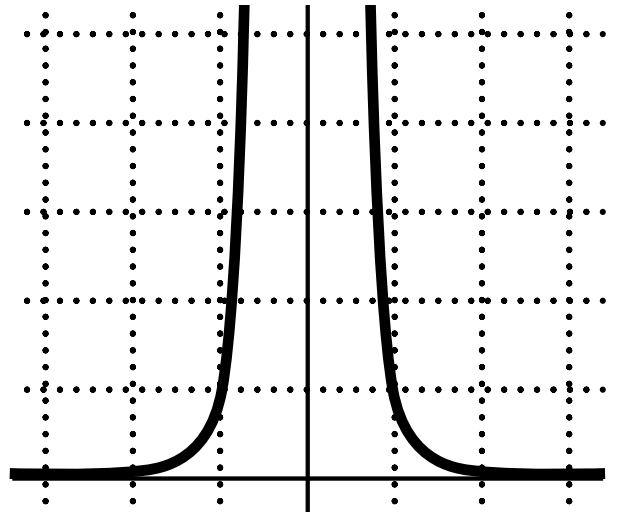


$n \in \mathbb{N}$, n gerade,

$n = -2$

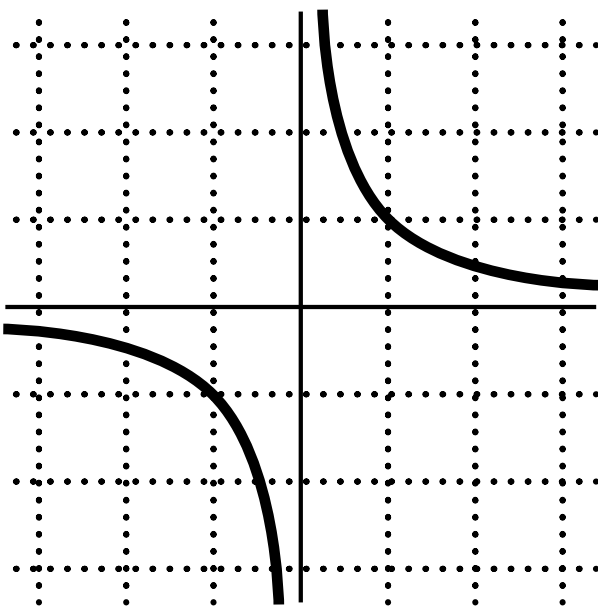


$n = -4$

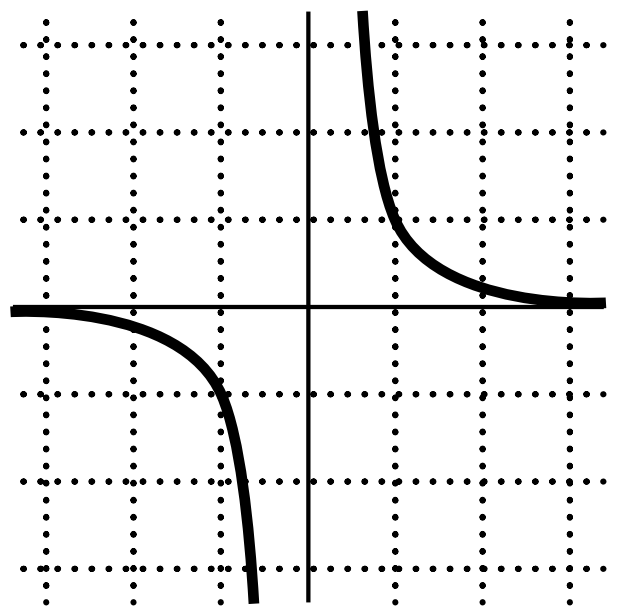


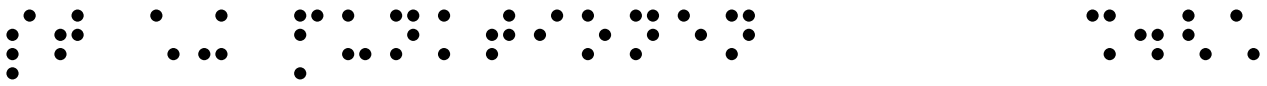
$n \in \mathbb{N}$, n ungerade,

$n = -1$



$n = -3$





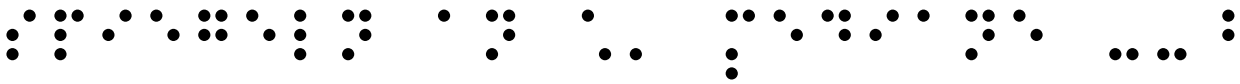
St 10 Funktionen, 3/21



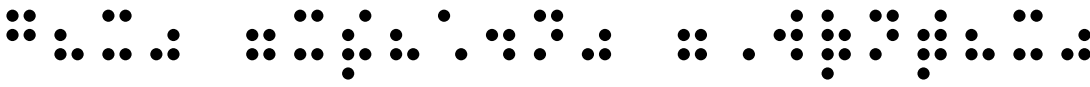
f_P.3: Umkehrfunktion



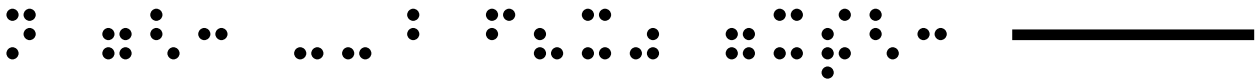
$f(x) = x^n$; $D_f: x \geq 0$



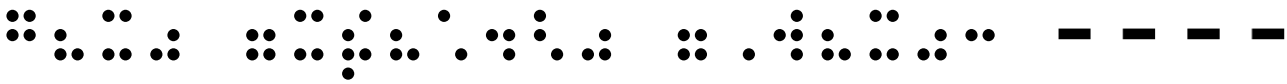
spiegeln an 1. Mediane -->



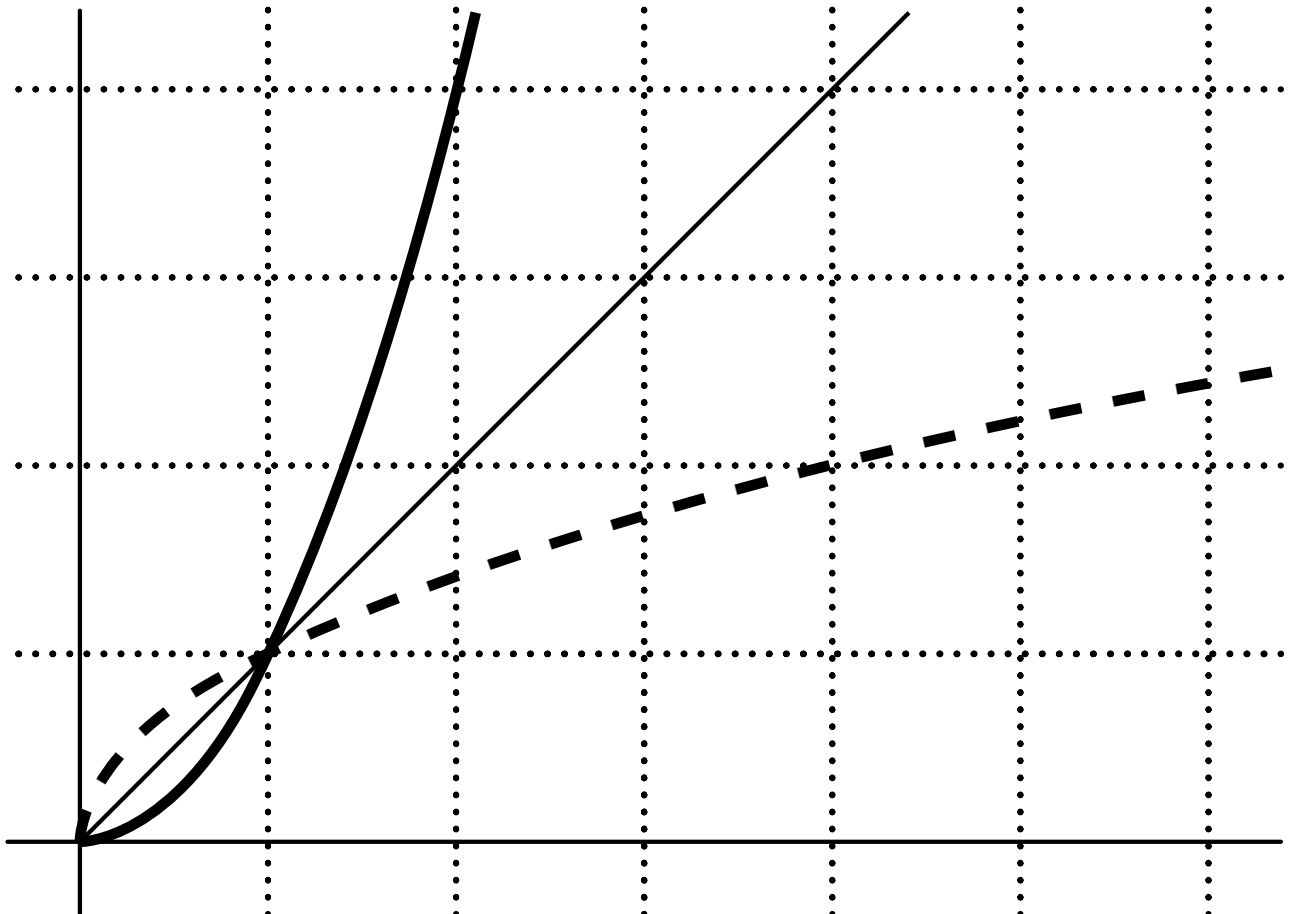
$g(x) = x^{1/n} = 'w[n](x)$

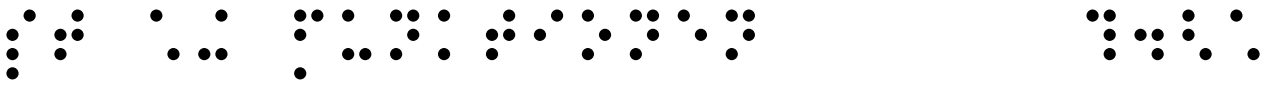


$n = 2: \rightarrow f(x) = x^2:$

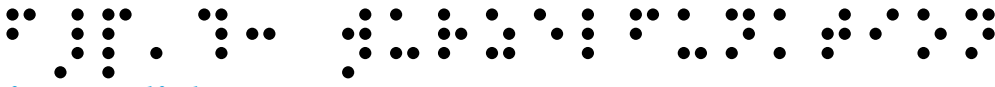


$g(x) = x^{1/2} = 'w(x):$

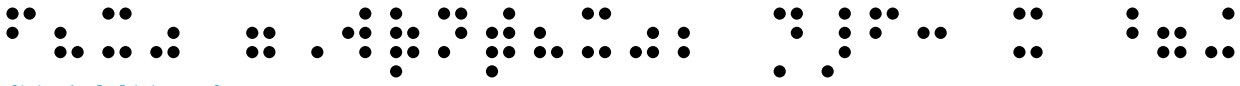




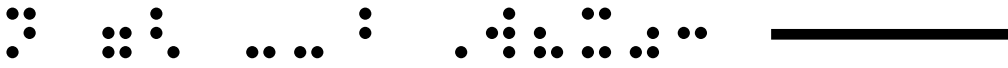
St 10 Funktionen, 4/21



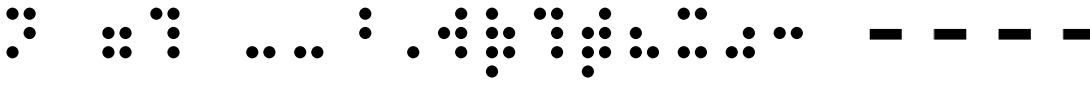
f_P.4: Wurzelfunktion



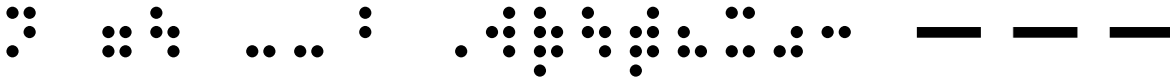
$f(x) = w[n](x); D_f: x \geq 0$



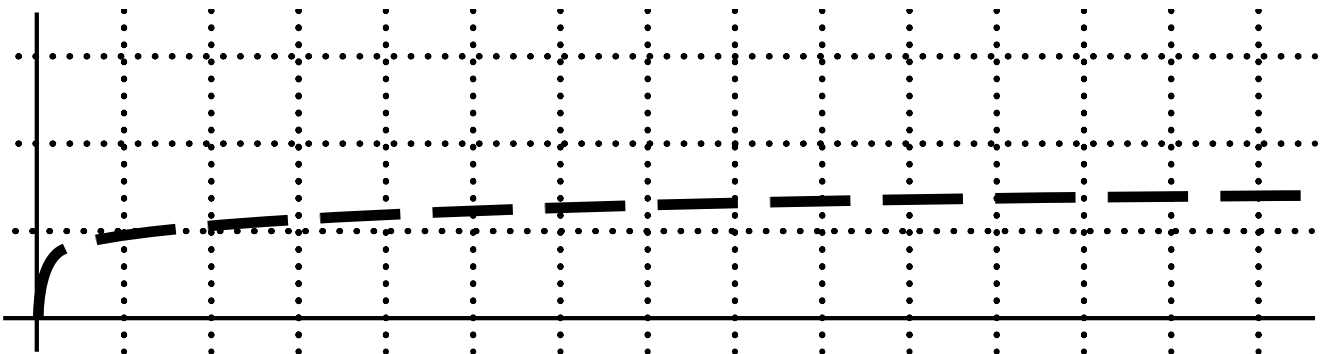
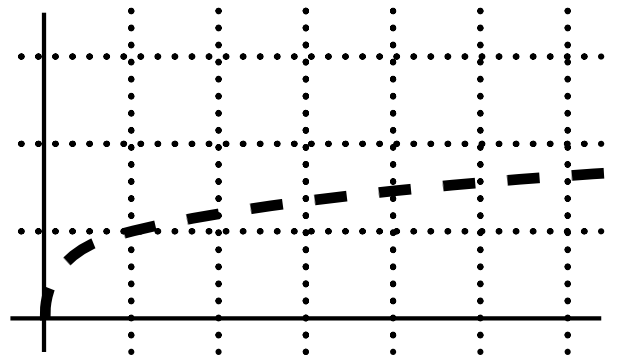
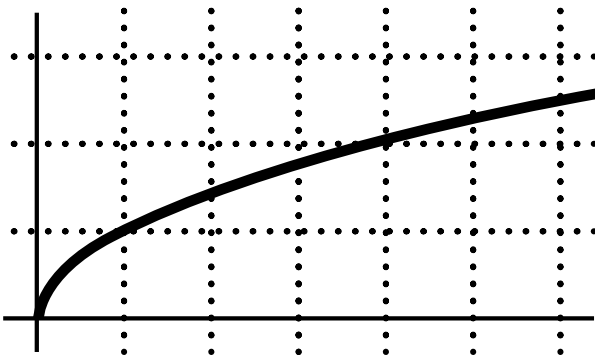
$n=2 \rightarrow w(x):$

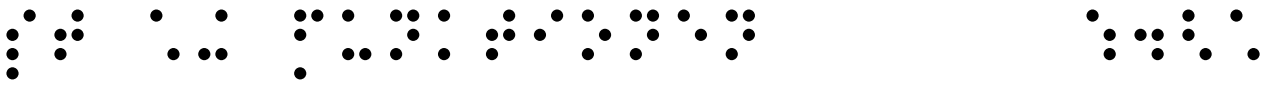


$n=4 \rightarrow w[4](x):$



$n=8 \rightarrow w[8](x):$





$f_{\text{Ex.1:}} f(x) = a^x$

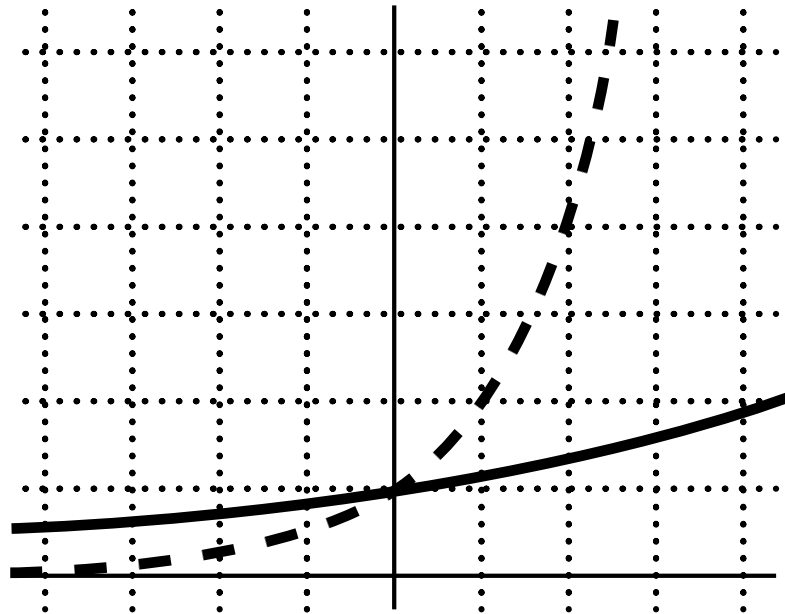


$a > 1$; steigend

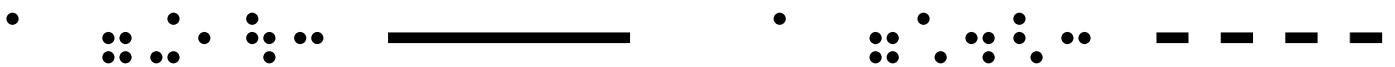


$a = 1, 2$:

$a = 2$:

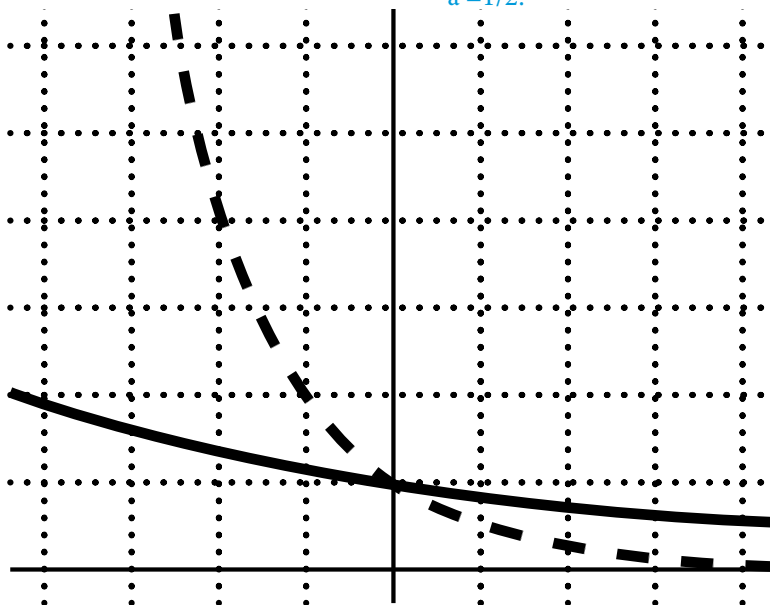


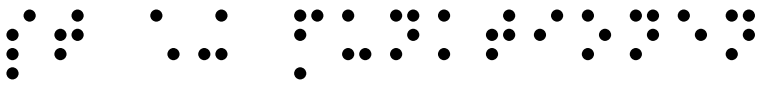
$0 < a < 1$; fallend



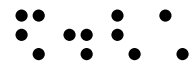
$a = 0,8$:

$a = 1/2$:

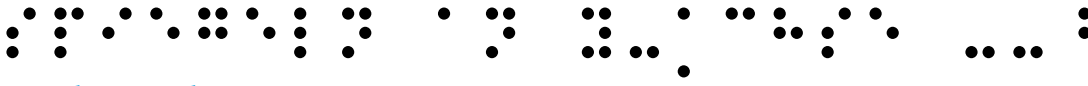




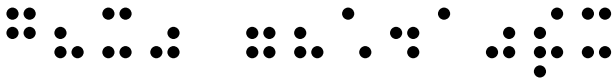
St 10 Funktionen, 6/21



f_Ex.2: $f(x) = a^x$



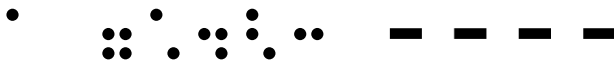
spiegeln an y-Achse -->



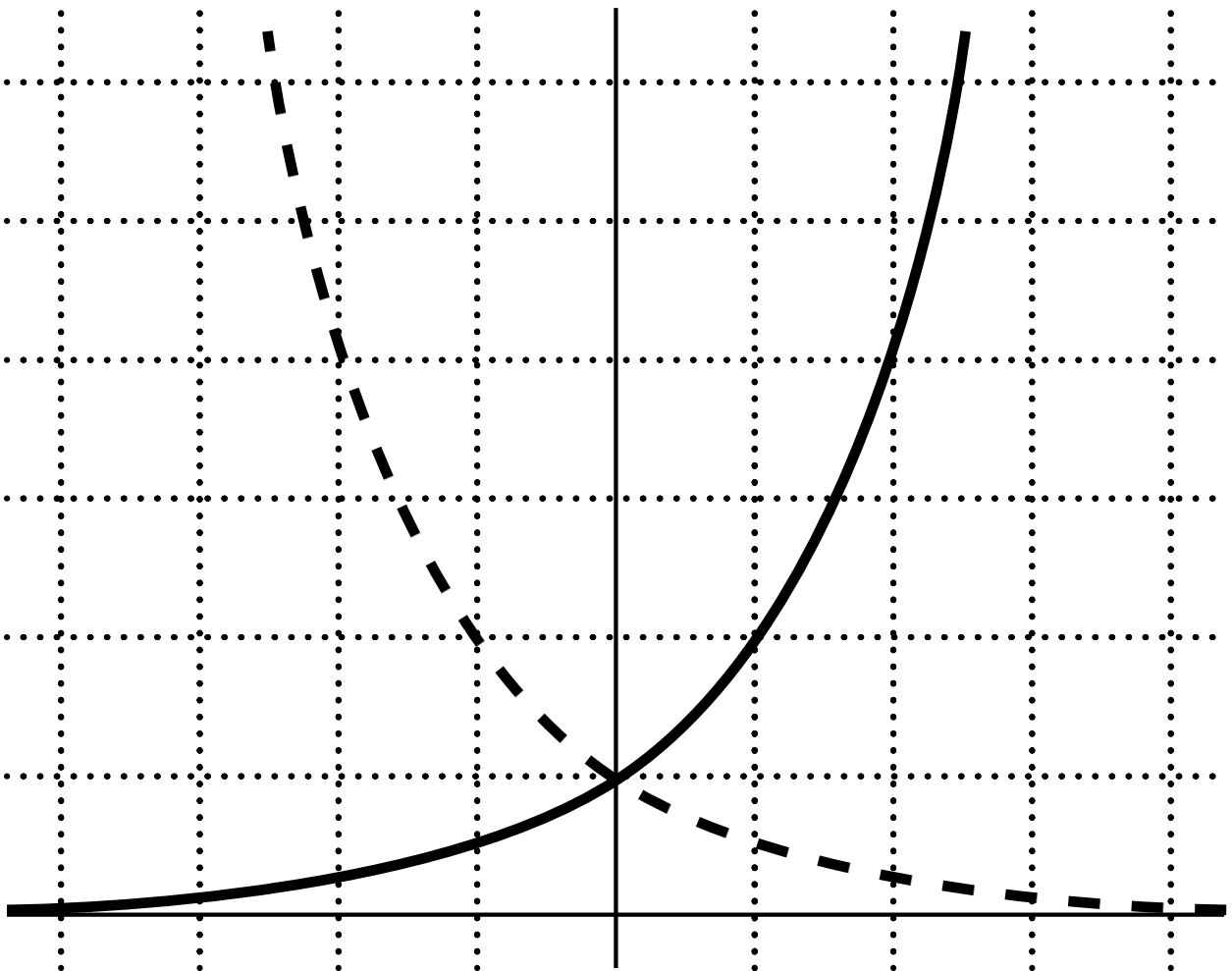
$g(x) = (1/a)^x$



$a = 2:$



$a = 1/2:$



St 10 Funktionen, 7/21

St 10 Funktionen, 7/21

St 10 Funktionen, 7/21

f_Ex.3: $f(x) = e^x$

f_Ex.3: $f(x) = e^x$

spiegeln an y-Achse -->

spiegeln an y-Achse -->

$g(x) = (1/e)^x$

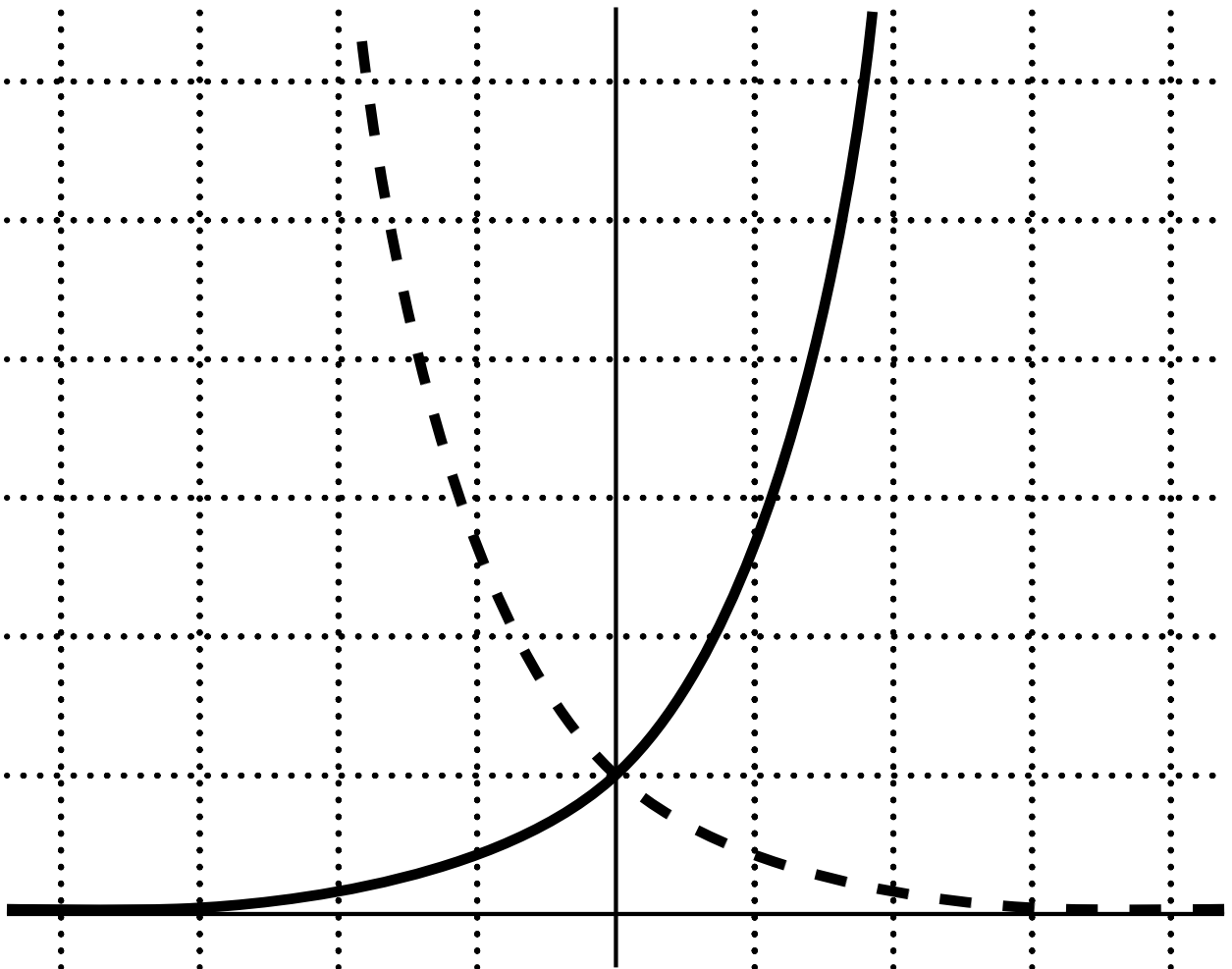
$g(x) = (1/e)^x$

$e \sim 2,7$

$e \sim 2,7$

$1/e \sim 0,37$

$1/e \sim 0,37$



St 10 Funktionen, 8/21

St 10 Funktionen, 8/21

f_Ex.4: Umkehrfktn (Basis a)

f_Ex.4: Umkehrfktn (Basis a)

$f(x) = a^x$

$f(x) = a^x$

spiegeln an 1. Mediane -->

spiegeln an 1. Mediane -->

$g(x) = \log_a(x)$

$g(x) = \log_a(x)$

$a = 10 \rightarrow$

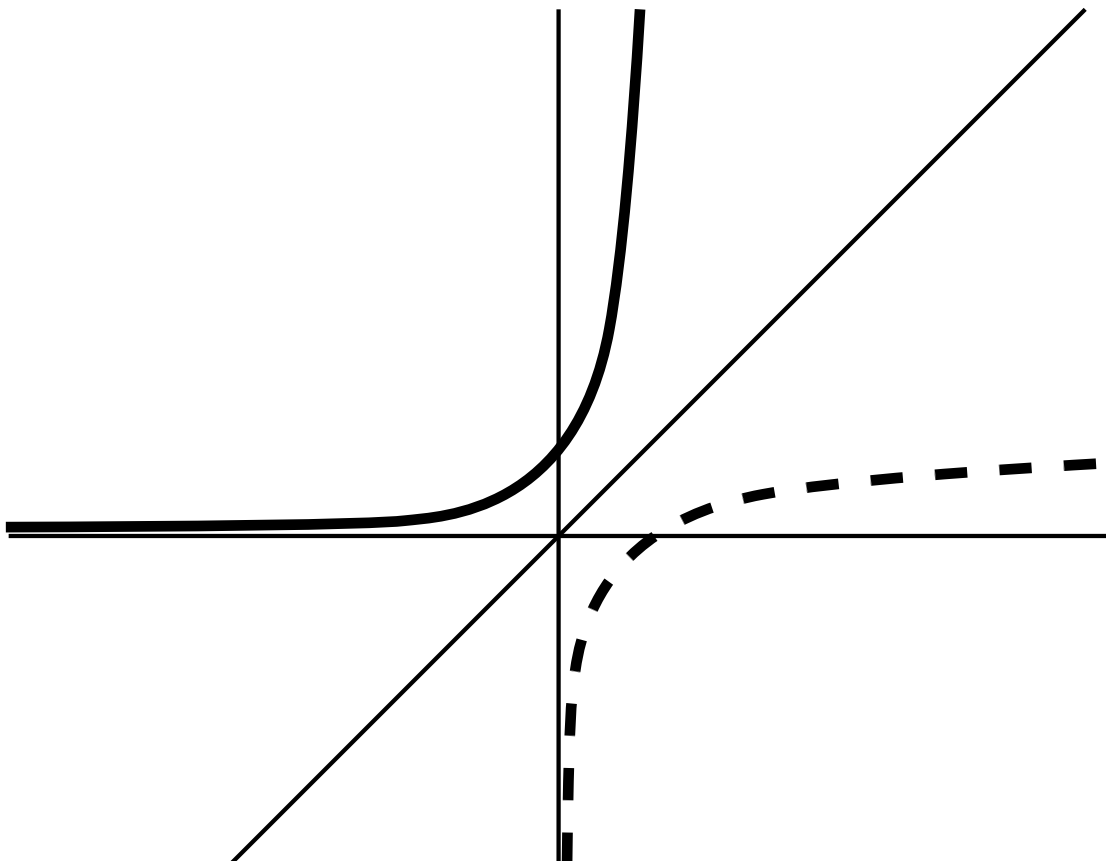
$a = 10 \rightarrow$

10^x

10^x

$\log_{10}(x) = \lg(x)$

$\log_{10}(x) = \lg(x)$



St 10 Funktionen, 9/21

St 10 Funktionen, 9/21

f_Ex.5: Umkehrfktn (Basis 'e)

$f(x) = e^x$

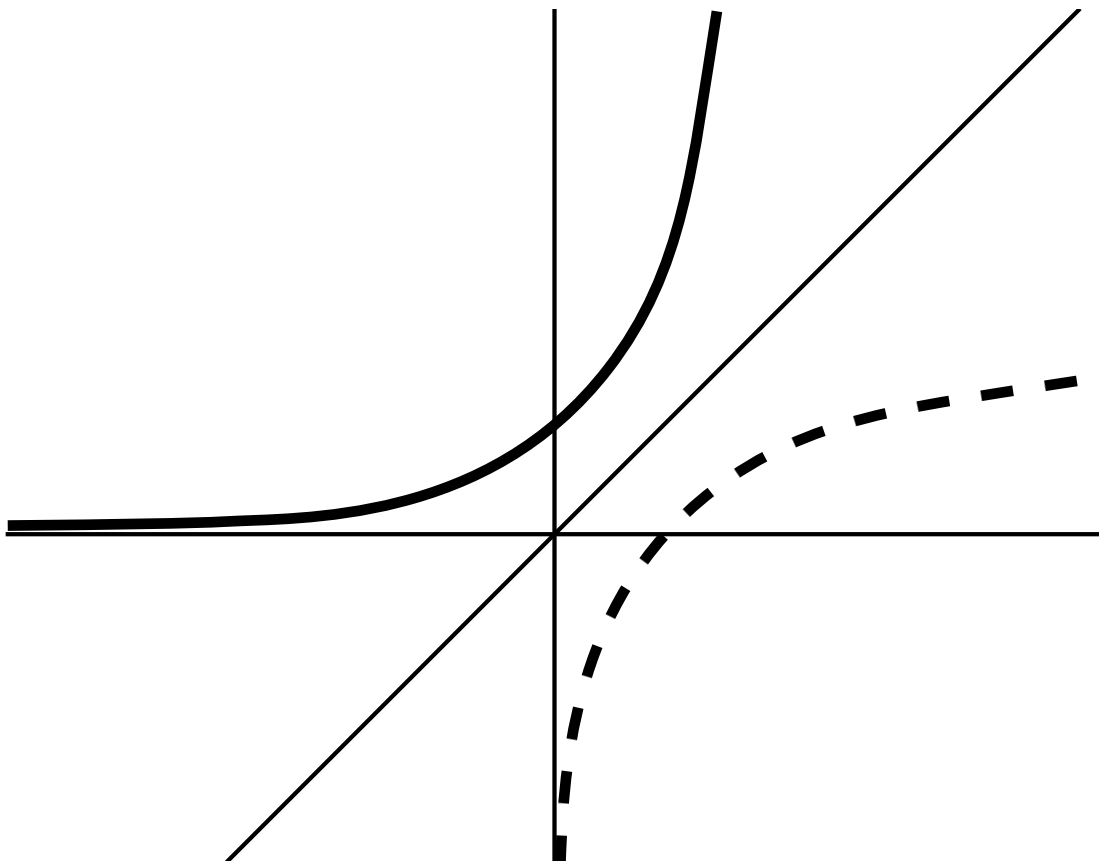
spiegeln an 1. Mediane -->

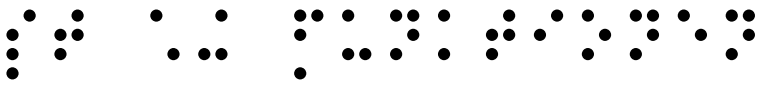
$g(x) = \ln(x) = \log_e(x)$

$e \sim 2,7 \rightarrow$

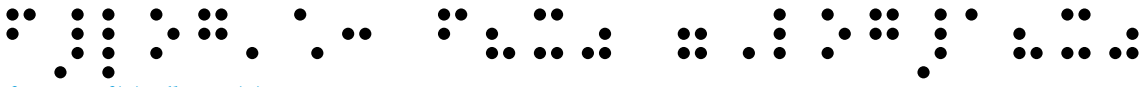
e^x :

$\ln(x)$:

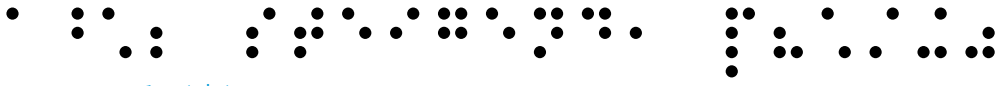




St 10 Funktionen, 10/21



f_Log.1: $f(x) = \log_a(x)$



$a > 1$; steigend, $P(1|0)$



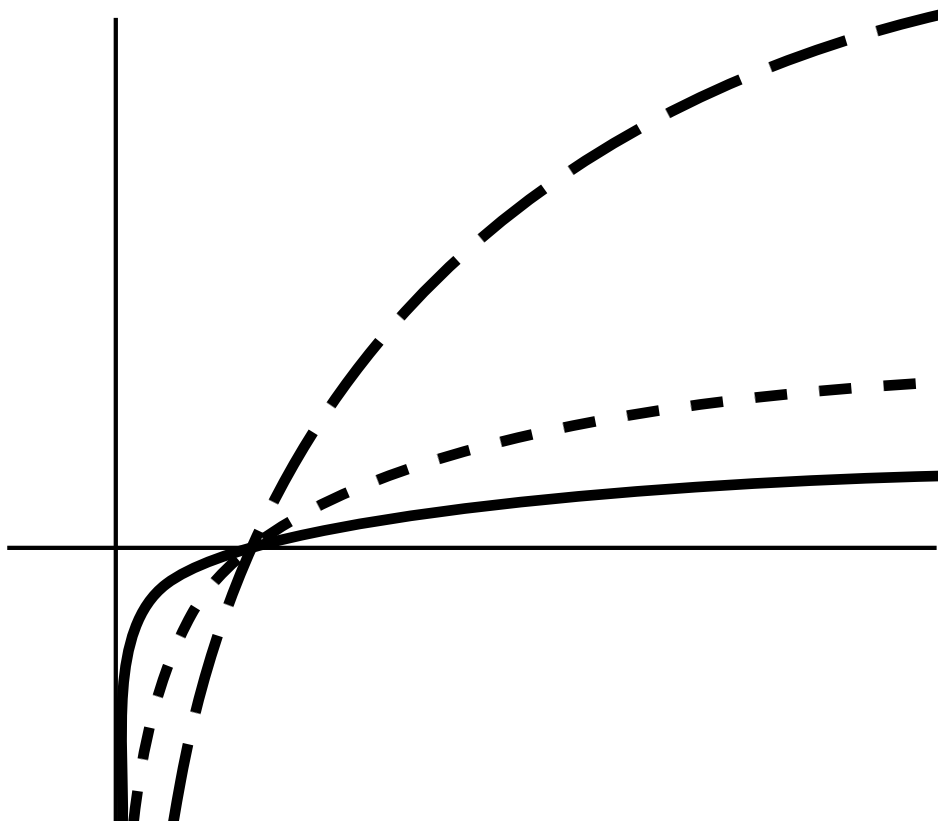
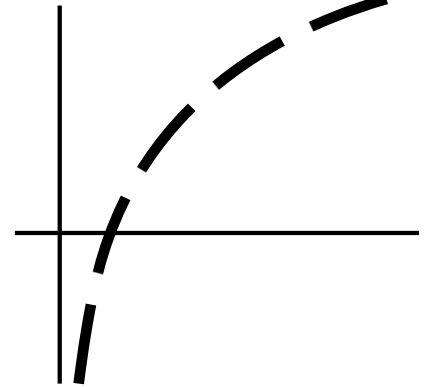
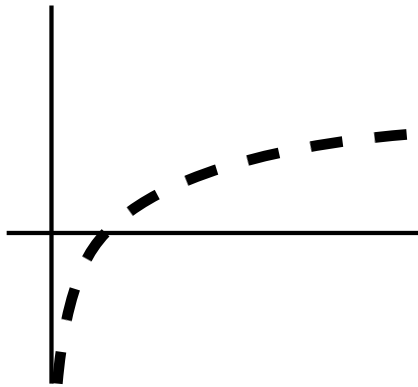
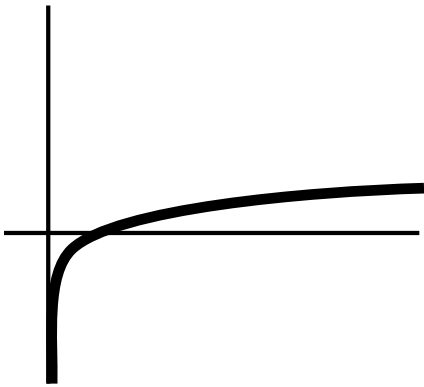
$a = 10$:

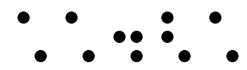
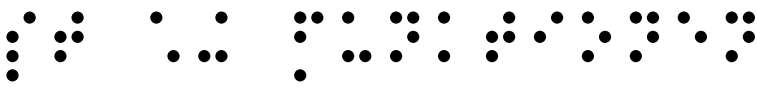


$a = e$:

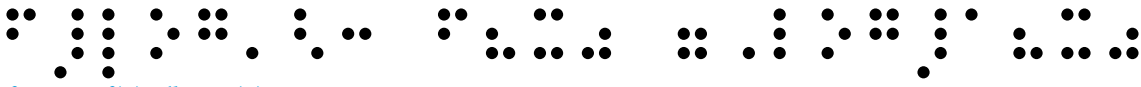


$a = 1,5$:

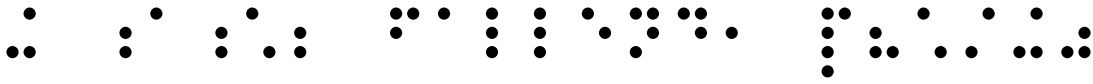




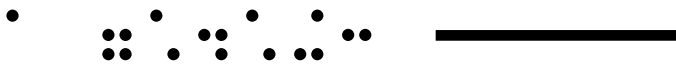
St 10 Funktionen, 11/21



f_Log.2: $f(x) = \log_a(x)$



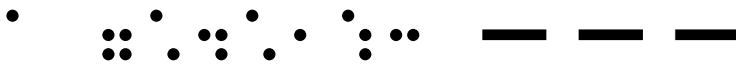
$0 < a < 1$; fallend, P(1|0)



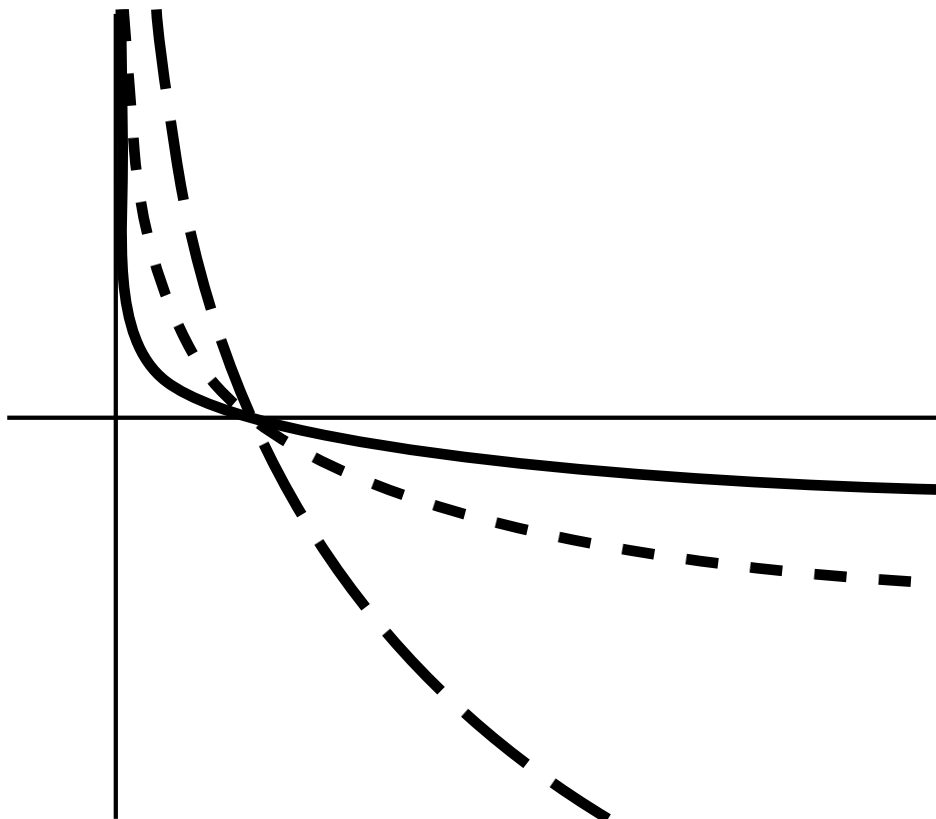
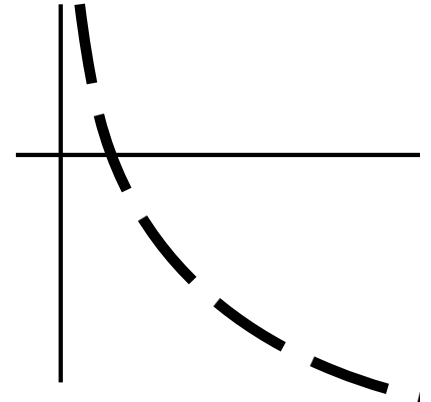
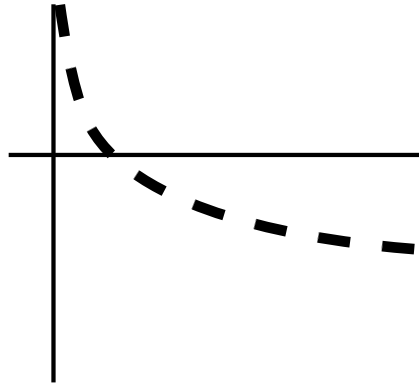
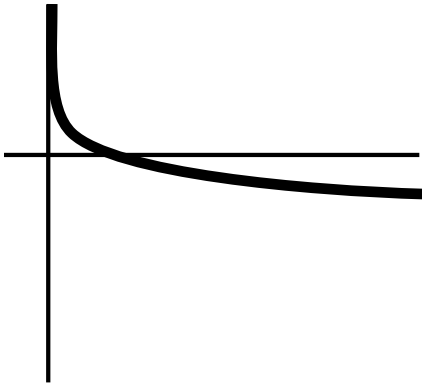
$a = 1/10$:

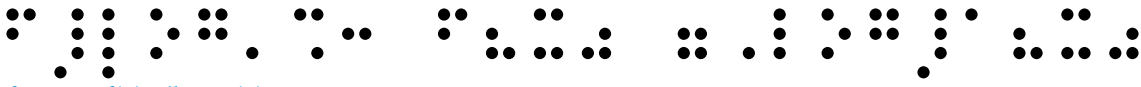
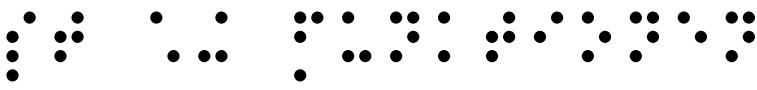


$a = 1/e$:



$a = 1/1,5$:





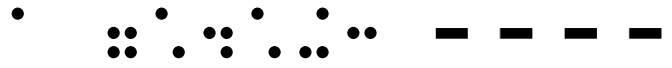
f_Log.3: $f(x) = \log_a(x)$



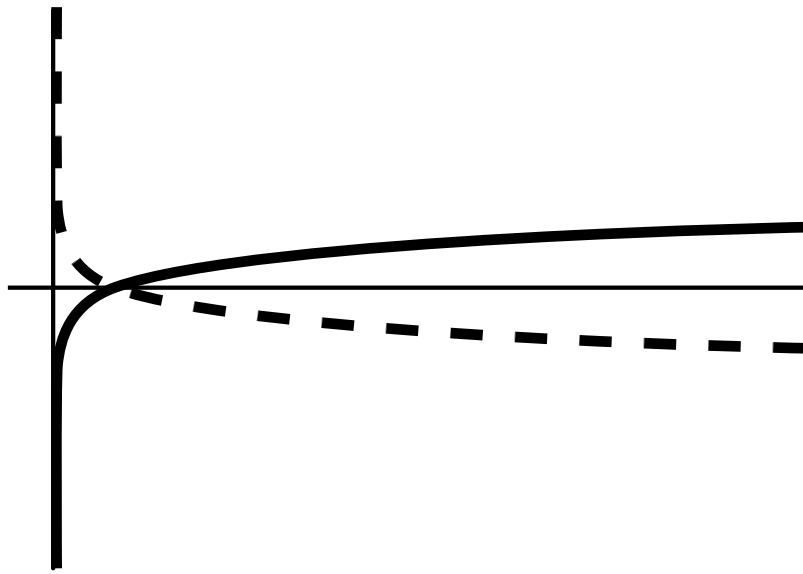
Spiegeln an x-Achse



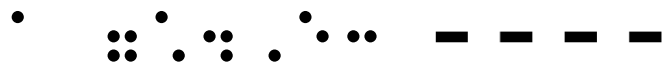
$a = 10:$



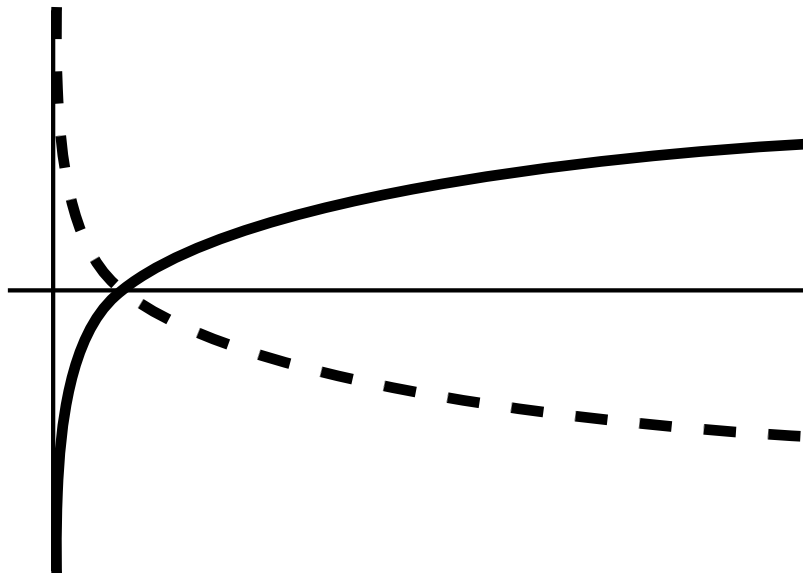
$a = 1/10:$



$a = e:$



$a = 1/e:$



St 10 Funktionen, 13/21

St 10 Funktionen, 13/21

$f_{W.1}: f(x) = \sin(x)$

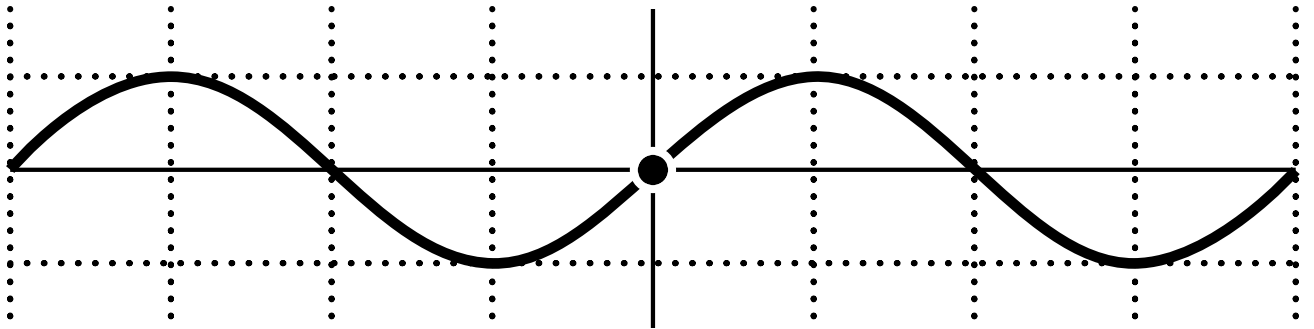
$f_{W.1}: f(x) = \sin(x)$

Skalierung: $x: \pi/2; y: 1$

Skalierung: $x: \pi/2; y: 1$

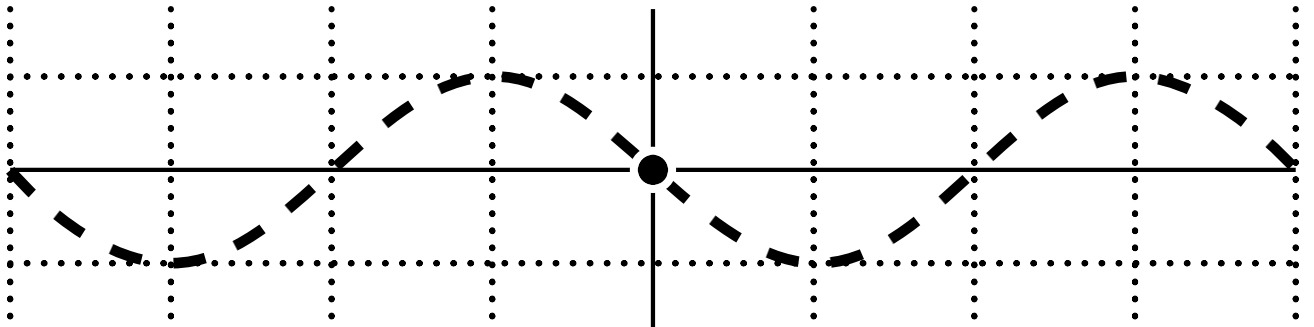
$\sin(x): [0; \pi/2]$ steigend

$\sin(x): [0; \pi/2]$ steigend



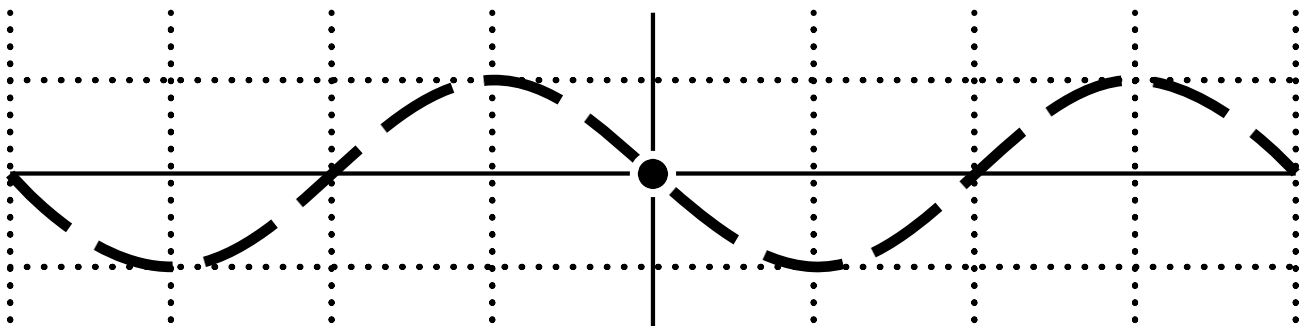
$\sin(-x): [0; \pi/2]$ fallend

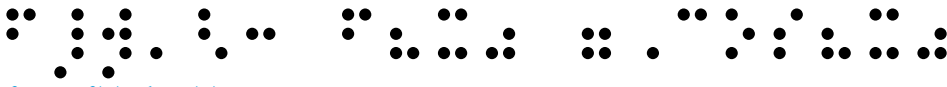
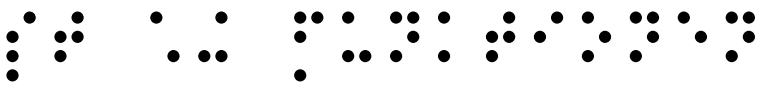
$\sin(-x): [0; \pi/2]$ fallend



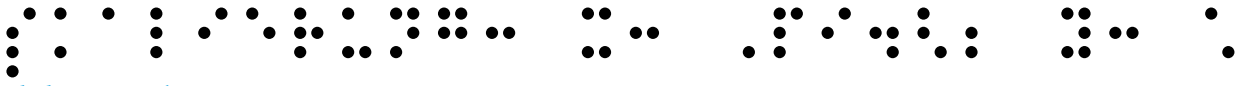
$-\sin(x): [0; \pi/2]$ fallend

$-\sin(x): [0; \pi/2]$ fallend

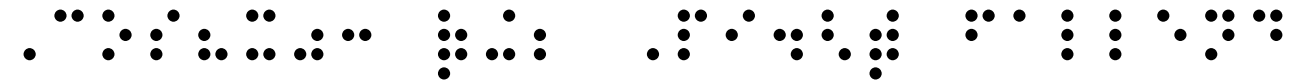




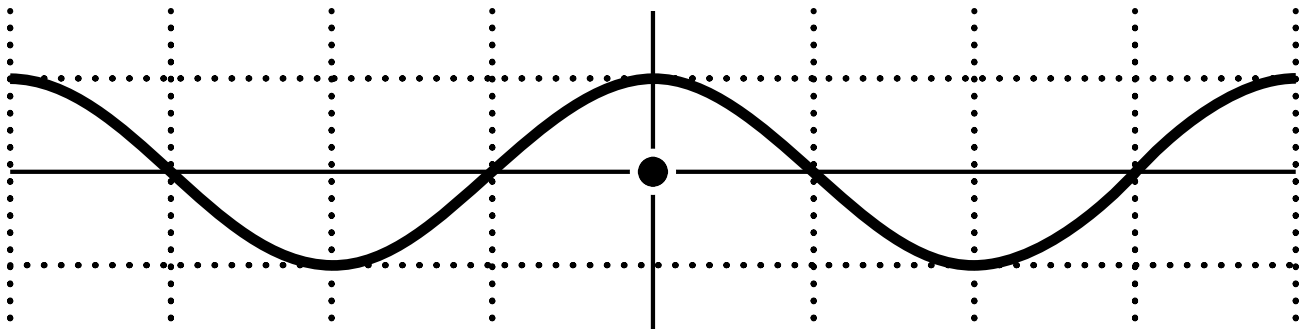
f_W.2: $f(x) = \cos(x)$



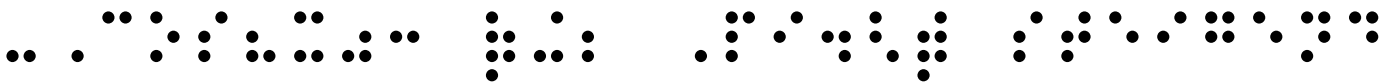
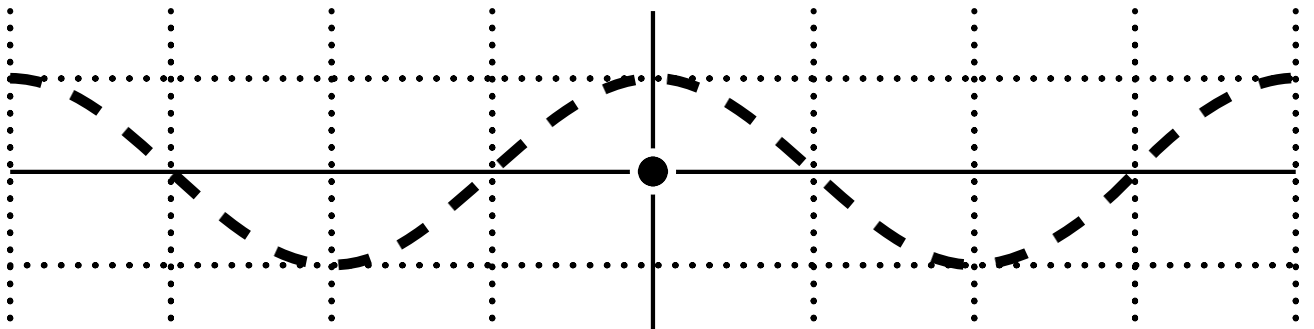
Skalierung: x: $\pi/2$; y: 1



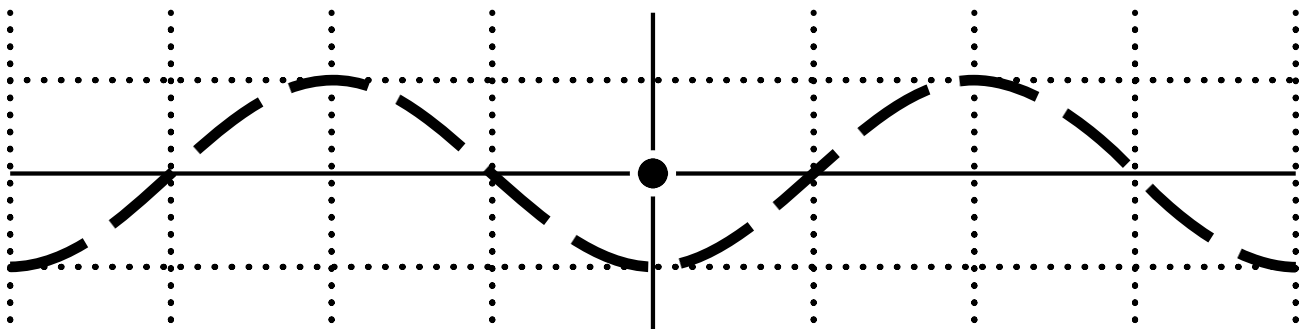
$\cos(x)$: $[0; \pi/2]$ fallend



$\cos(-x)$: $[0; \pi/2]$ fallend



$-\cos(x)$: $[0; \pi/2]$ steigend



St 10 Funktionen, 15/21

St 10 Funktionen, 15/21

f_W.3: 'sin und 'cos

f_W.3: 'sin und 'cos

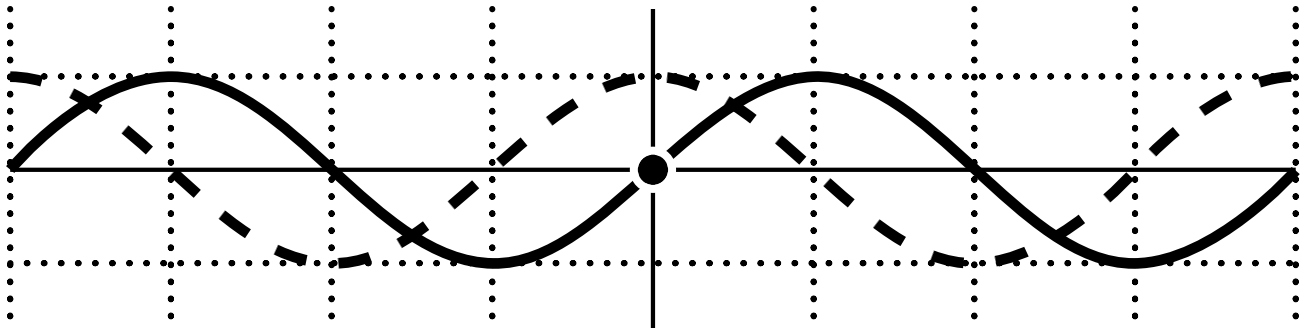
Skalierung: x: 'pi/2; y: 1

Skalierung: x: 'pi/2; y: 1

'sin(x): 'cos(x):

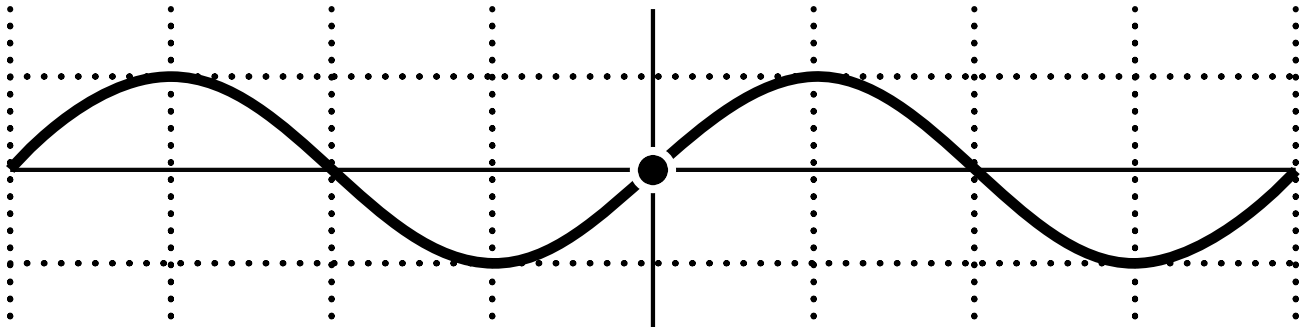
'sin(x):

'cos(x):



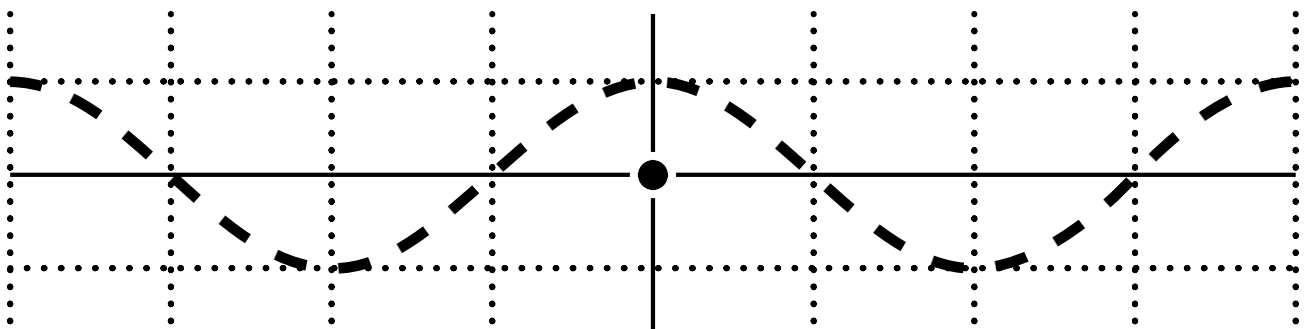
'sin(x) = 'cos(x - 'pi/2)

'sin(x) = 'cos(x - 'pi/2)



'cos(x) = 'sin(x + 'pi/2)

'cos(x) = 'sin(x + 'pi/2)



St 10 Funktionen, 16/21

St 10 Funktionen, 16/21

$f_{W.4}: f(x) = a \cdot \sin(x)$

$f_{W.4}: f(x) = a \cdot \sin(x)$

Skalierung: $x: \pi/2$; $y: 1$

Skalierung: $x: \pi/2$; $y: 1$

$N_1(0|0)$; $N_2(\pi|0)$

$N_1(0|0)$; $N_2(\pi|0)$

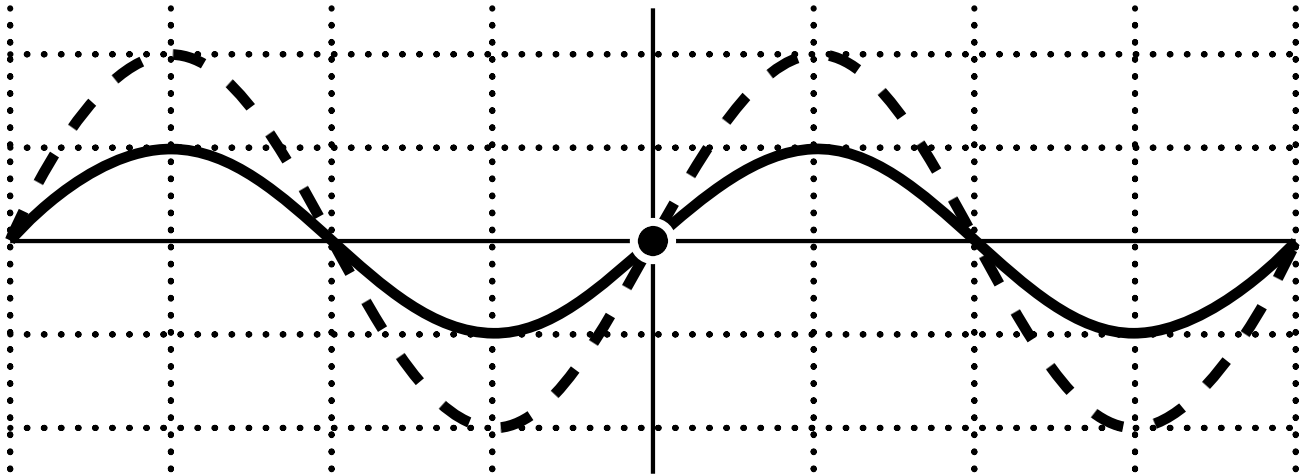
$H(\pi/2|a)$, $T(3 \cdot \pi/2|-a)$

$H(\pi/2|a)$, $T(3 \cdot \pi/2|-a)$

$a=1$: $a=2$:

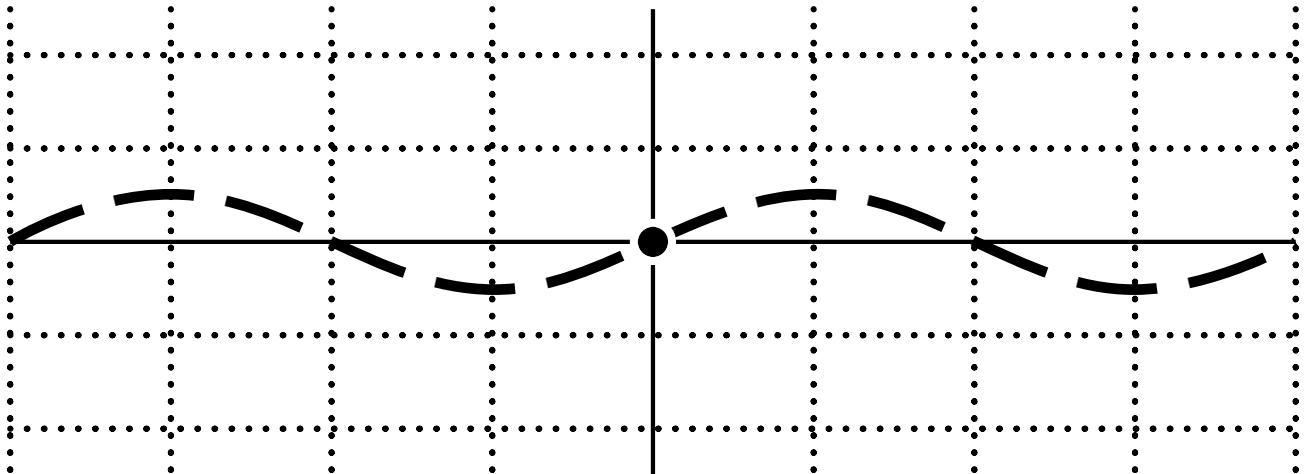
$a=1$:

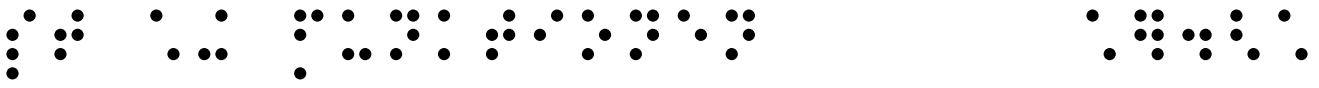
$a=2$:



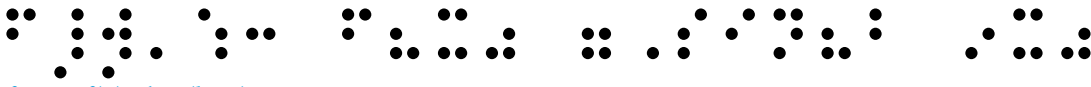
$a=1/2$:

$a=1/2$:

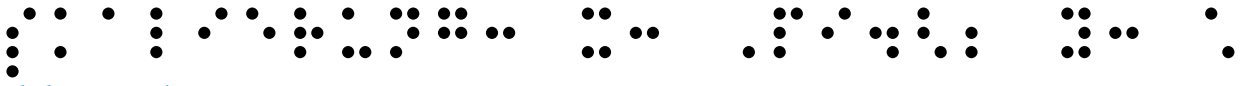




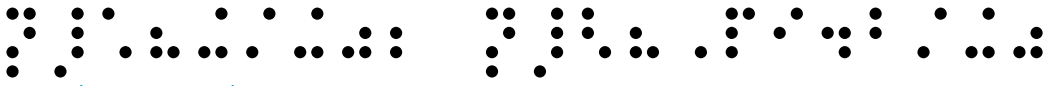
St 10 Funktionen, 17/21



f_W.5: $f(x) = \sin(b \cdot x)$



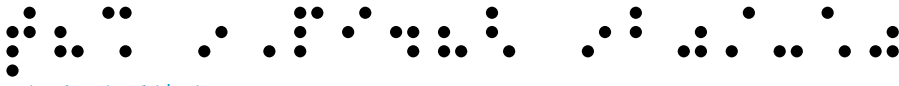
Skalierung: x: $\pi/2$; y: 1



$N_1(0|0)$; $N_2(\pi/b|0)$



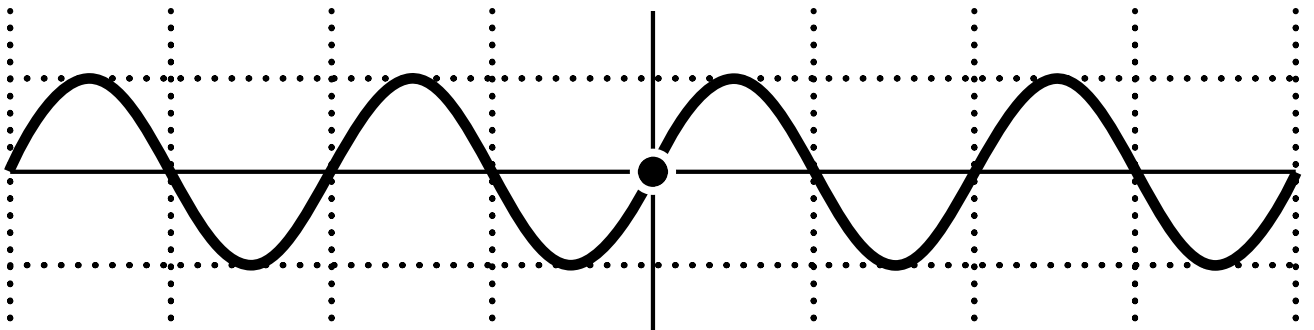
$H(\pi/(2 \cdot b)|1)$,



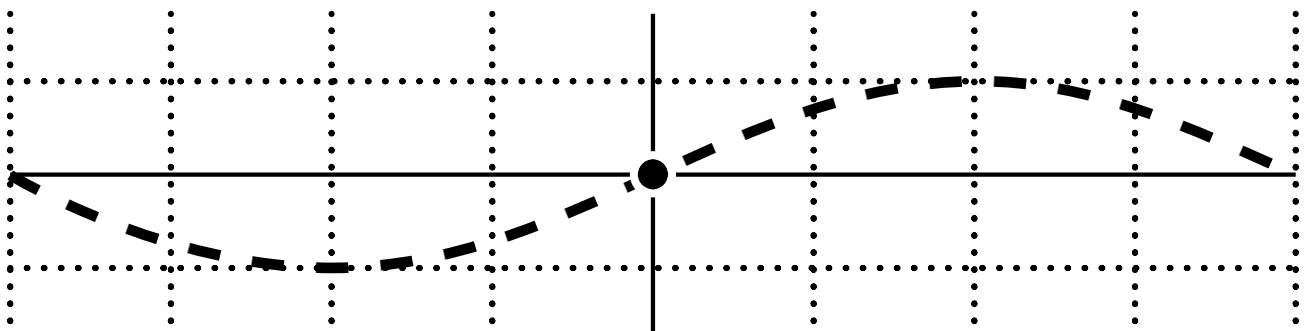
$T(3 \cdot \pi/(2 \cdot b)|-1)$



b = 2:



b = 1/2:



St 10 Funktionen, 18/21

St 10 Funktionen, 18/21

$f_{W.6}: f(x) = \sin(x) + c$

$f_{W.6}: f(x) = \sin(x) + c$

senkrechte Verschiebung

senkrechte Verschiebung

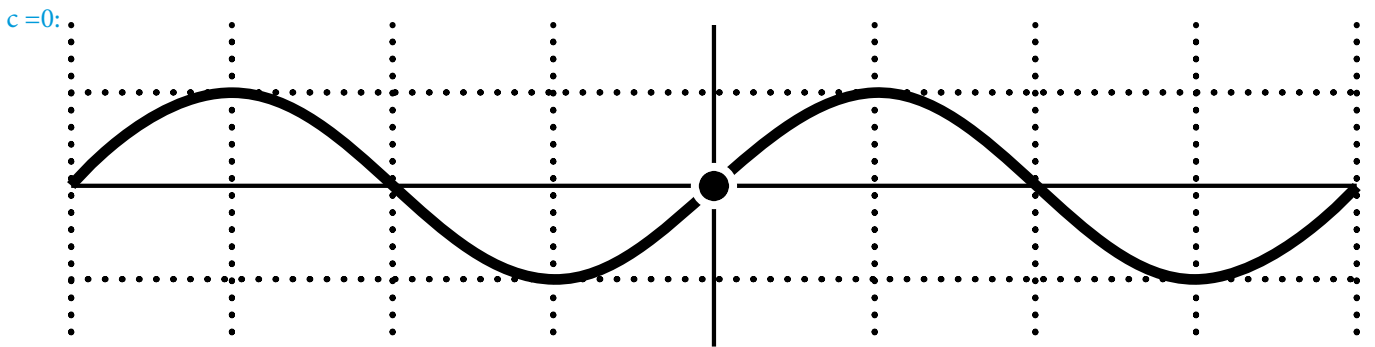
enthält P(0|c)

enthält P(0|c)

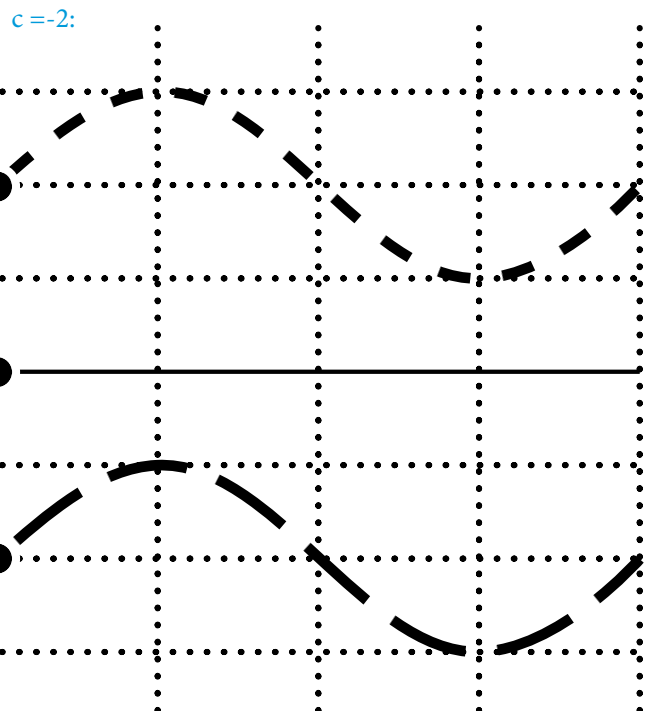
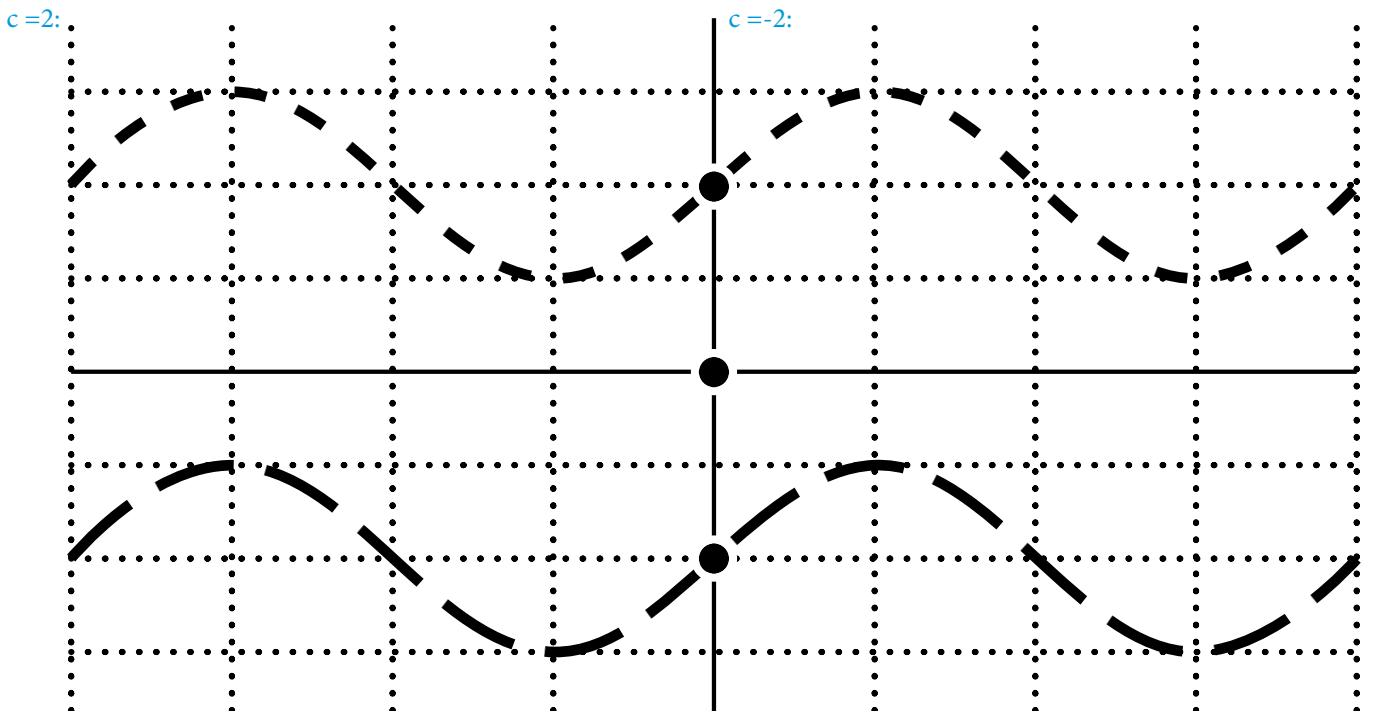
Skalierung: x: 'pi/2; y: 1

Skalierung: x: 'pi/2; y: 1

$c=0:$



$c=2:$



St 10 Funktionen, 19/21

St 10 Funktionen, 19/21

$f_{W.7}: f(x) = \sin(x+d)$

$f_{W.7}: f(x) = \sin(x+d)$

waagrechte Verschiebung

waagrechte Verschiebung

$N(-d|0)$

$N(-d|0)$

$d > 0$; nach links verschoben

$d > 0$; nach links verschoben

$d < 0$; nach rechts verschoben

$d < 0$; nach rechts verschoben

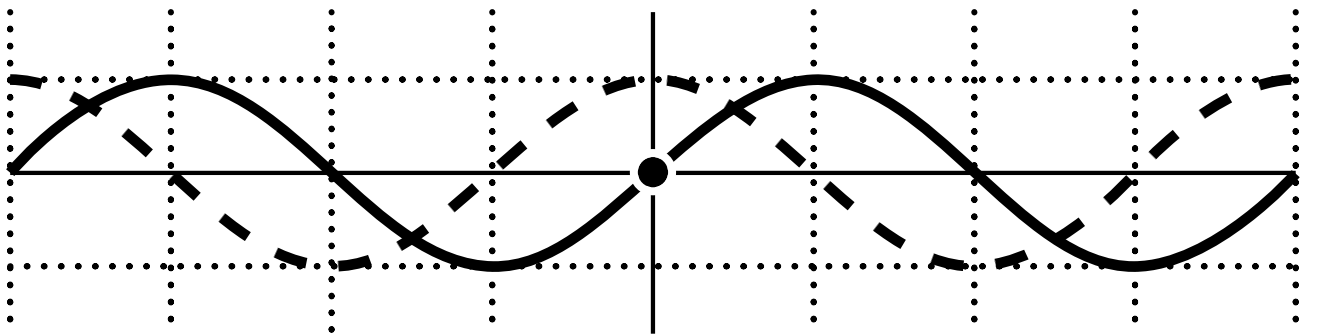
Skalierung: $x: \pi/2$; $y: 1$

Skalierung: $x: \pi/2$; $y: 1$

$d=0$: $d=+\pi/2$:

$d=0$:

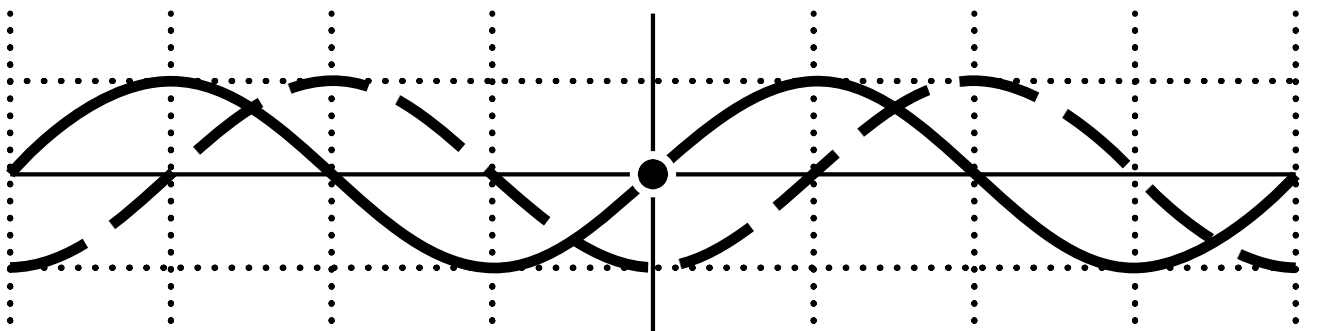
$d=+\pi/2$:

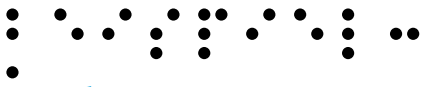
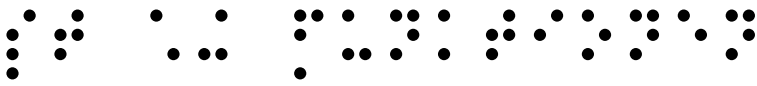


$d=0$: $d=-\pi/2$:

$d=0$:

$d=-\pi/2$:

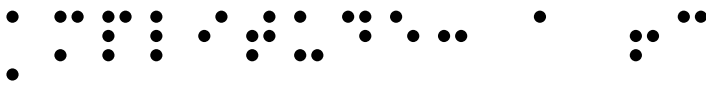




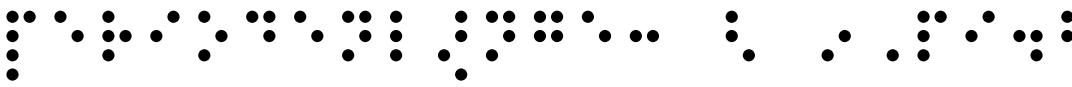
Beispiel:



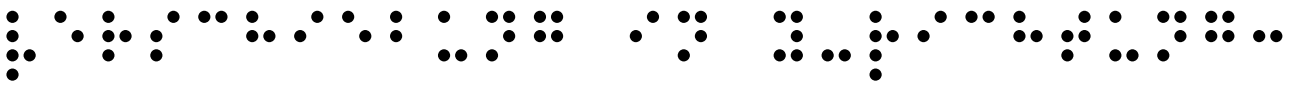
f_W.7: $f(x) = a \cdot \sin(b \cdot x + d) + c$



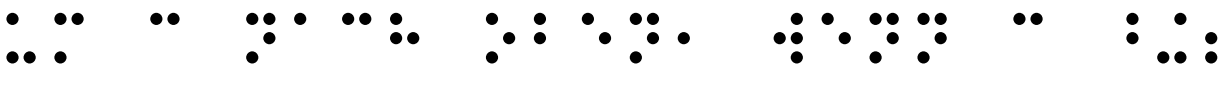
Amplitude: $a + c$



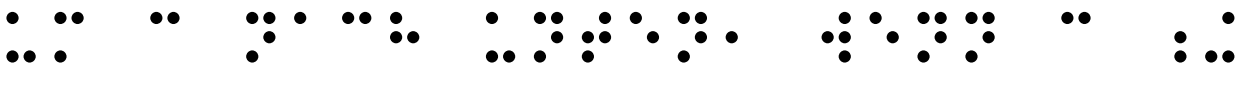
Periodenlänge: $2 \cdot \pi / b$



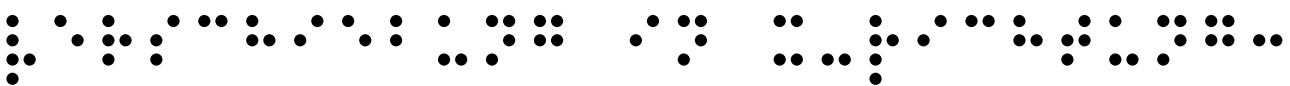
Verschiebung in y-Richtung:



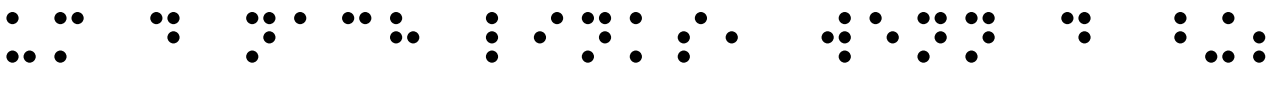
um c nach oben, wenn $c > 0$;



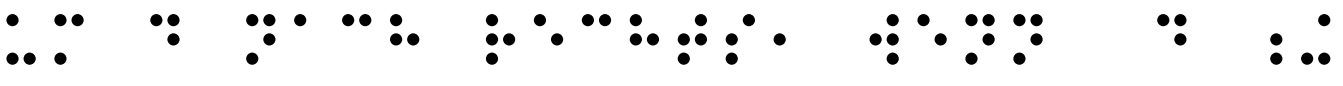
um c nach unten, wenn $c < 0$



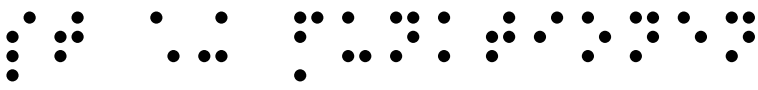
Verschiebung in x-Richtung:



um d nach links, wenn $d > 0$;



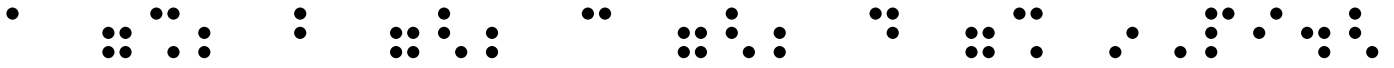
um d nach rechts, wenn $d < 0$



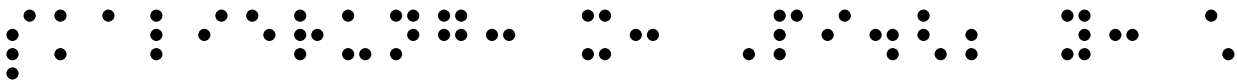
St 10 Funktionen, 21/21



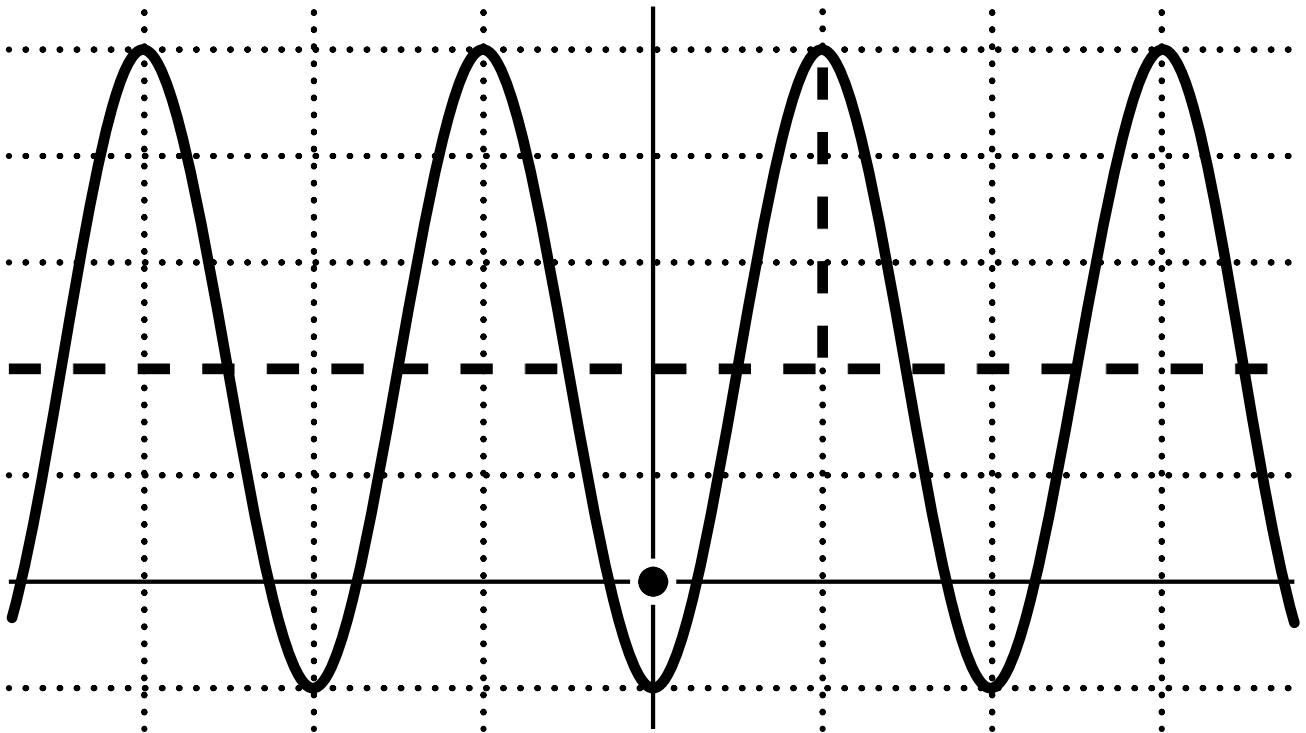
Beispiel:



$a = 3; b = 2; c = 2; d = 3 \cdot \pi/2$



Skalierung: $x: \pi/2; y: 1$



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1. Auflage