LEMA - Lehr- und Lernmaterialien
Hergestellt von der Abteilung für Inklusion und Lehrmittel (LMZ)
am Bundes-Blindeninstitut Wien

Grafikkatalog
12. Schulstufe (8. AHS)

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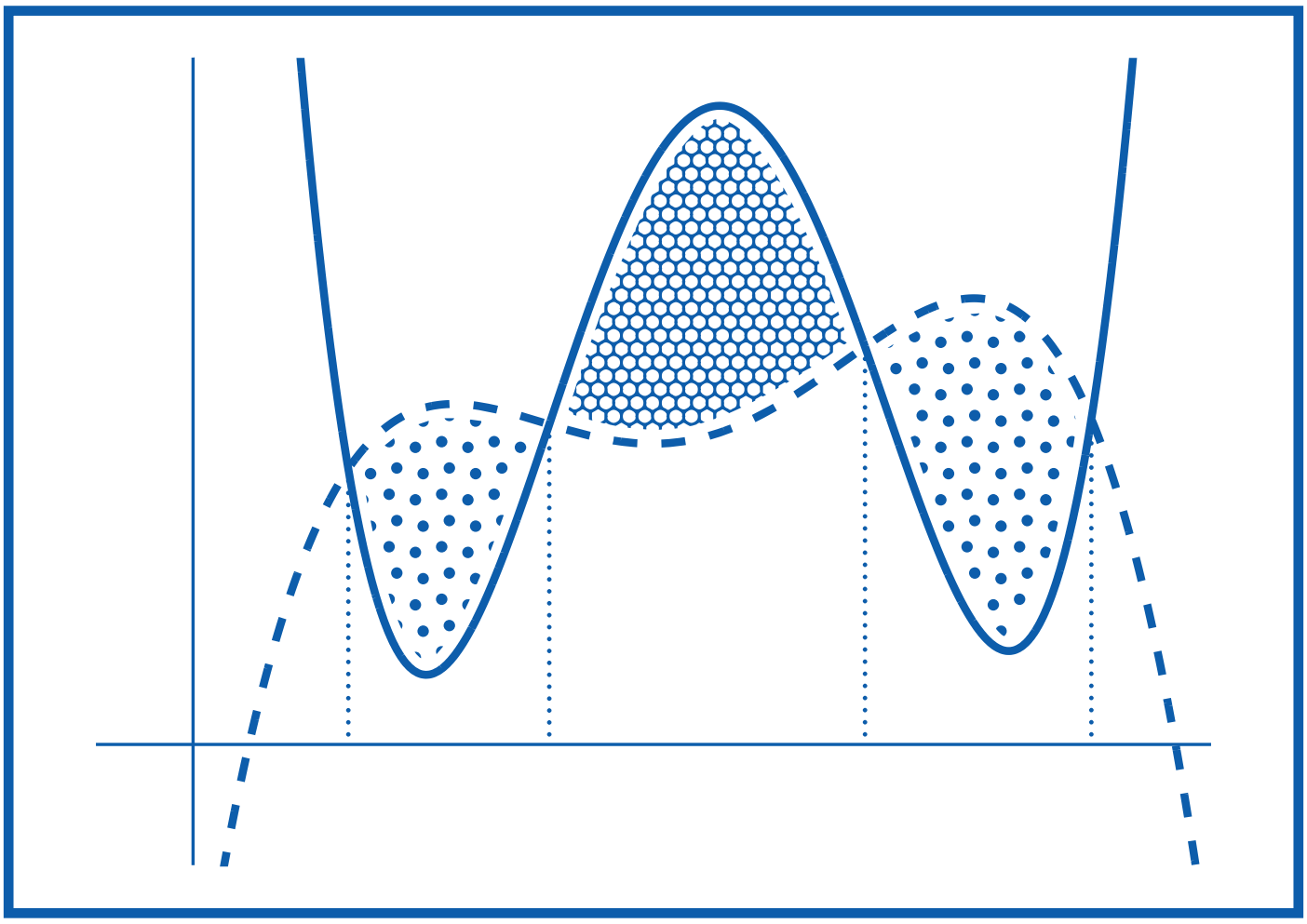

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Grafikkatalog
12. Schulstufe (8. AHS)

Inhalt

1 Stammfunktionen

2 Integrale

3 Wirtschaft

4 Normalverteilung

5 Reelle Funktionen

Stammfunktionen

Schulstufe 12

Mögliche Stammfunktionen (F)

diverser Funktionen (f)

[illegible]

[illegible]

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Inhalt

1 f(x)=d; d >0;

2 f(x)=d; d <0;
f(x) =x;

3 f(x) =x +d; d >0; d<0;

4 f(x) =-x;
f(x) =-x +d; d >0;

5 f(x) =-x +d; d <0;
f(x) =x^2;

6 f(x) =x^2 +d; d >0; d <0;

7 f(x) =-x^2 +d; d >0; d <0;

8 f(x) =x^3;
f(x) ='sin(x)

9 f(x) ='cos(x)
f(x) =2^x

10 f(x) =1/x
f(x) ='ln(x)

⡊⠝⠓⠁⠇⠞

⠡ ⠋⠦⠭⠴⠶⠙⠆ ⠙ ⠘⠬⠆

⠣ ⠋⠦⠭⠴⠶⠙⠆ ⠙ ⠰⠬⠆

⠋⠦⠭⠴ ⠶⠭⠆

⠩ ⠋⠦⠭⠴ ⠶⠭ ⠖⠙⠆ ⠙ ⠘⠬⠆ ⠙ ⠰⠬⠆

⠹ ⠋⠦⠭⠴ ⠶⠤⠭⠆

⠋⠦⠭⠴ ⠶⠤⠭ ⠖⠙⠆ ⠙ ⠘⠬⠆

⠱ ⠋⠦⠭⠴ ⠶⠤⠭ ⠖⠙⠆ ⠙ ⠰⠬⠆

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⠫ ⠋⠦⠭⠴ ⠶⠭⡦⠣ ⠖⠙⠆ ⠙ ⠘⠬⠆ ⠙ ⠰⠬⠆

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⠋⠦⠭⠴ ⠶⡶⠎⠊⠝⠦⠭⠴

⠪ ⠋⠦⠭⠴ ⠶⡶⠉⠕⠎⠦⠭⠴

⠋⠦⠭⠴ ⠶⠣⡦⠭

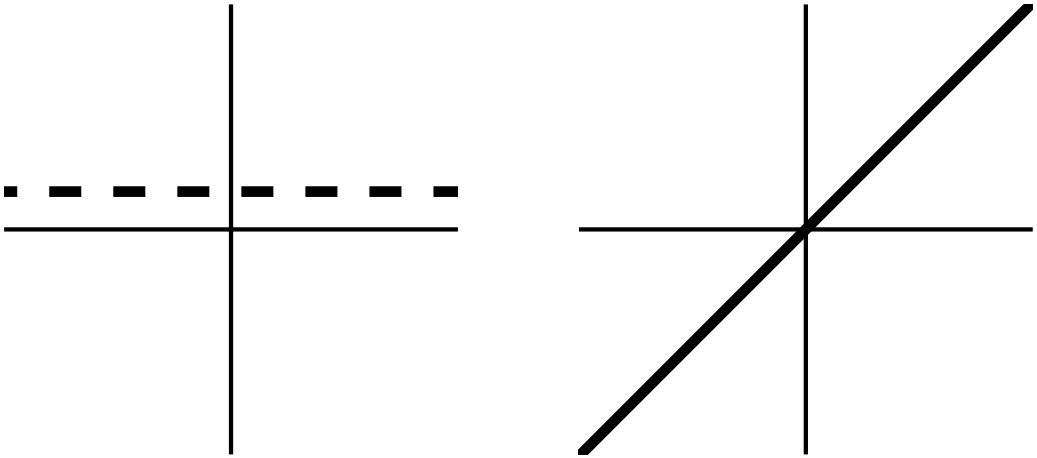
⠡⠬ ⠋⠦⠭⠴ ⠶⠡⠲⠭

⠋⠦⠭⠴ ⠶⡶⠇⠝⠦⠭⠴

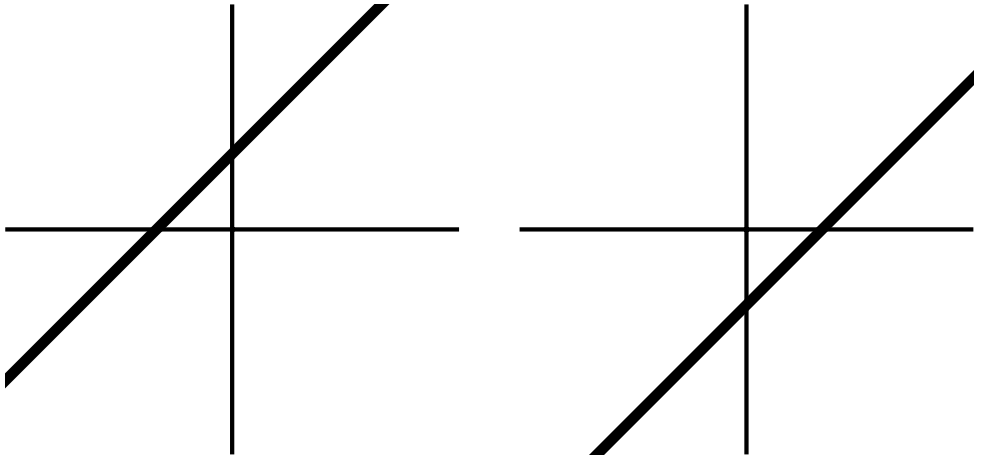
Stammfunktionen, 1/10

f … Funktion F … eine mögliche Stammfunktion

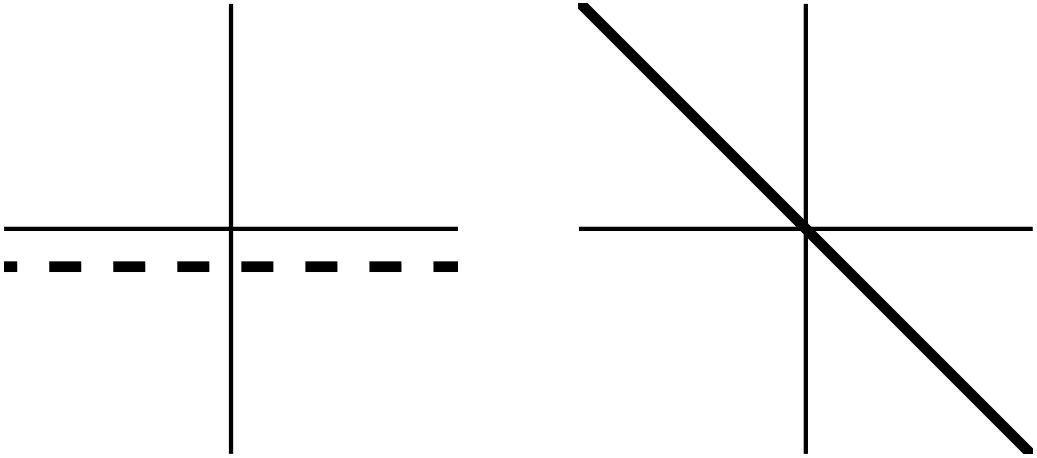
f(x) = 1



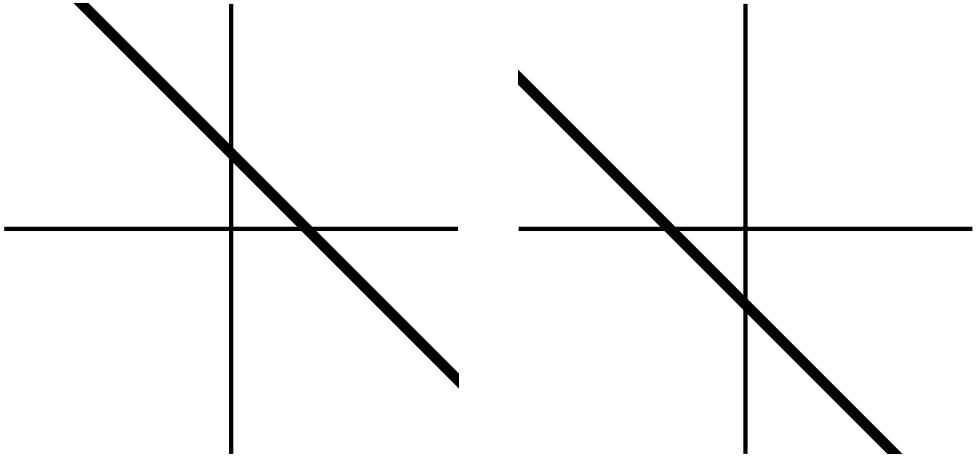
F(x) = x + C



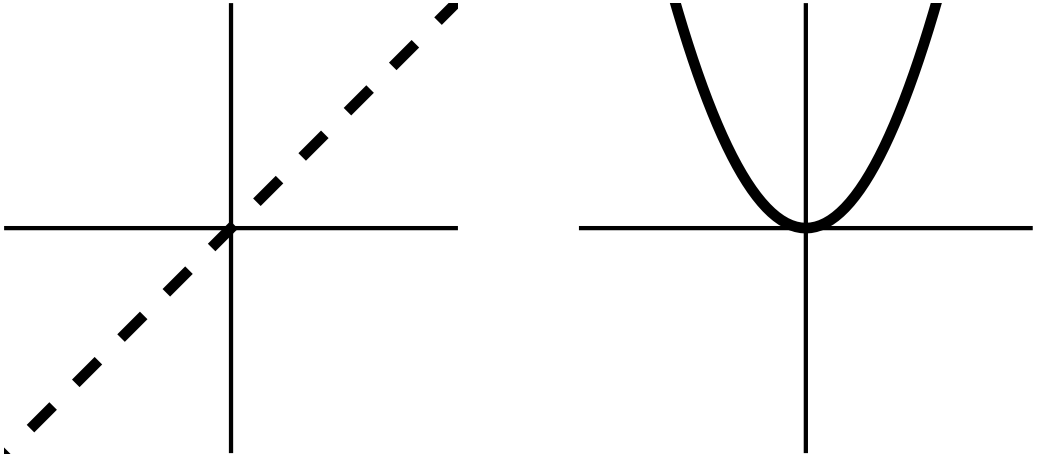
f(x) = -1



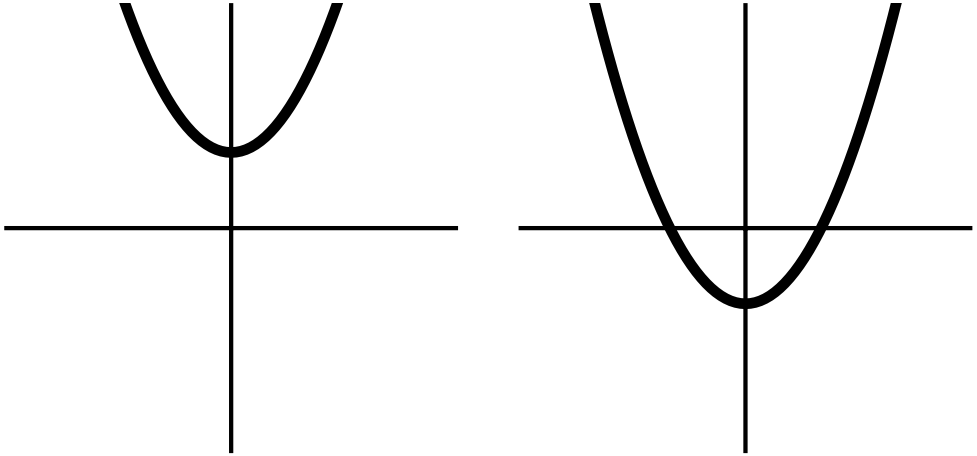
F(x) = -x + C



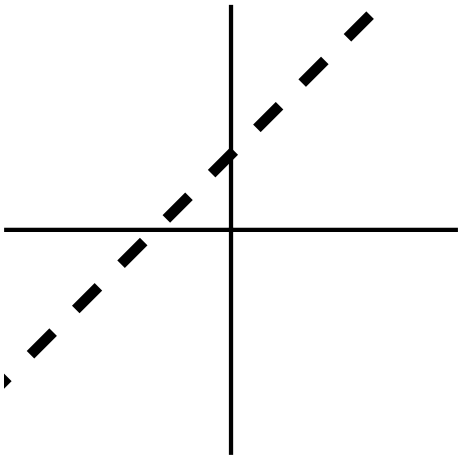
f(x) = x

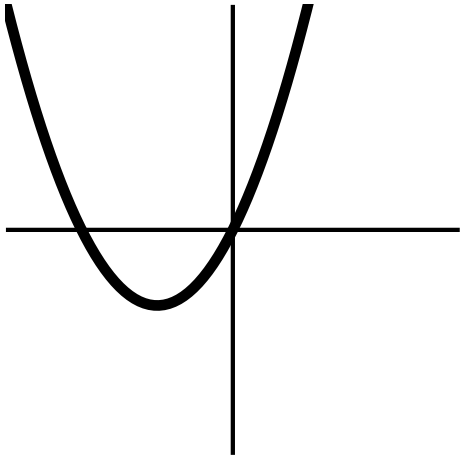


F(x) = x²/2 + C

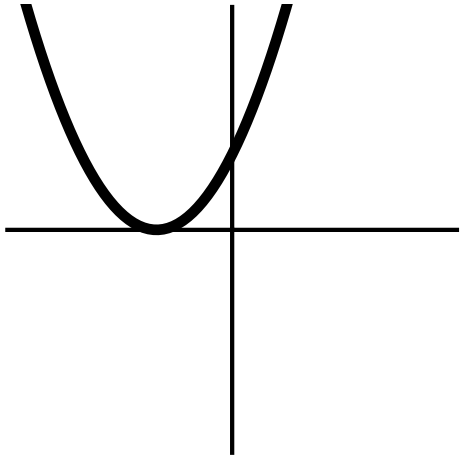


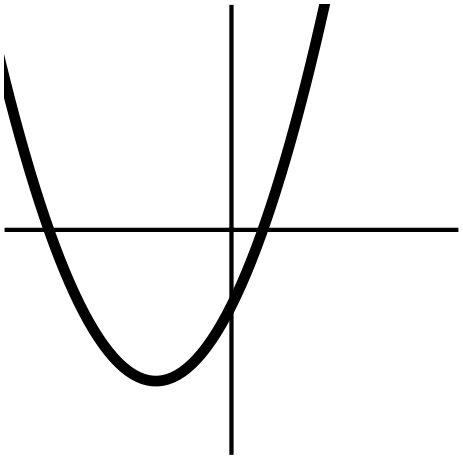
f(x) = x + 2



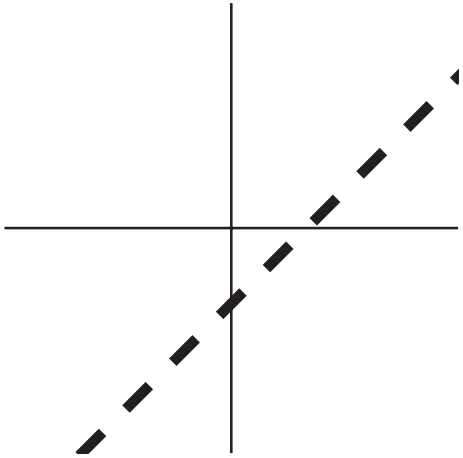


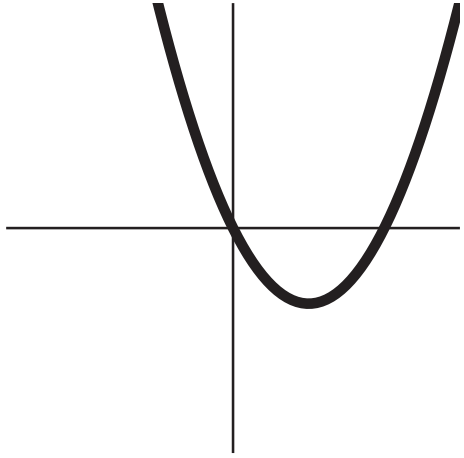
F(x) = x²/2 + 2x + C



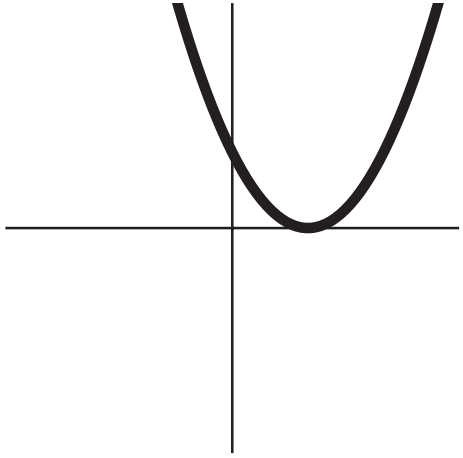


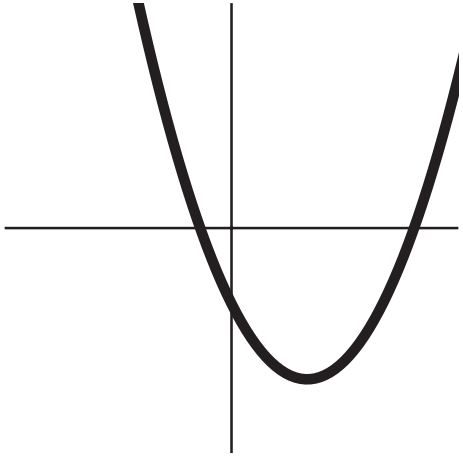
f(x) = x - 2



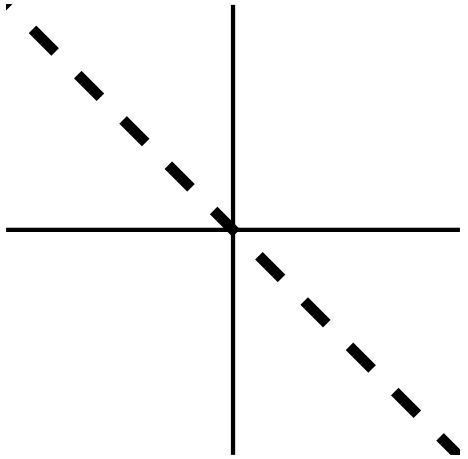


F(x) = x²/2 - 2x + C

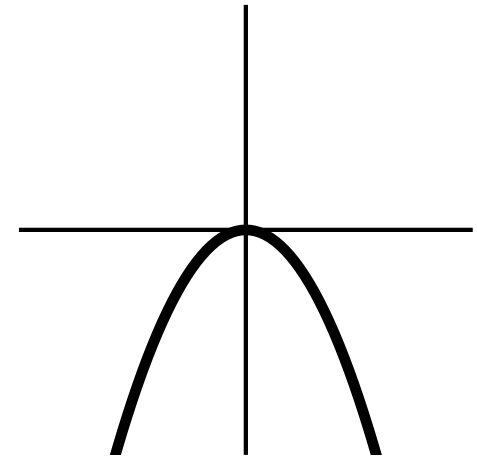


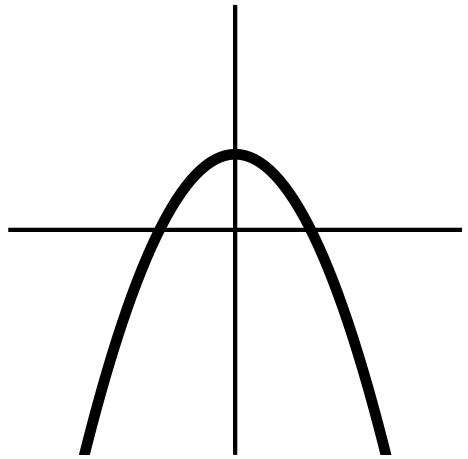


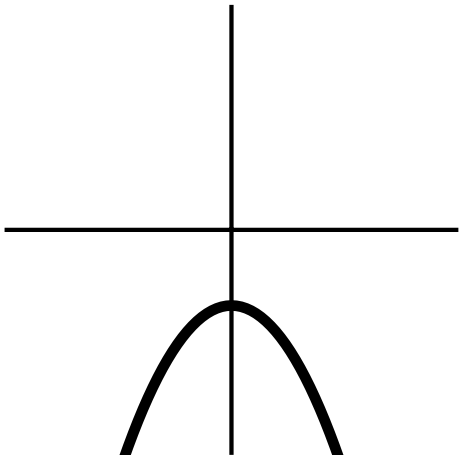
f(x) = -x



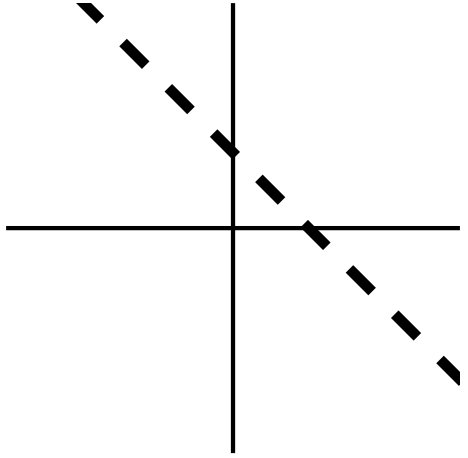
F(x) = $-x^2/2 + C$



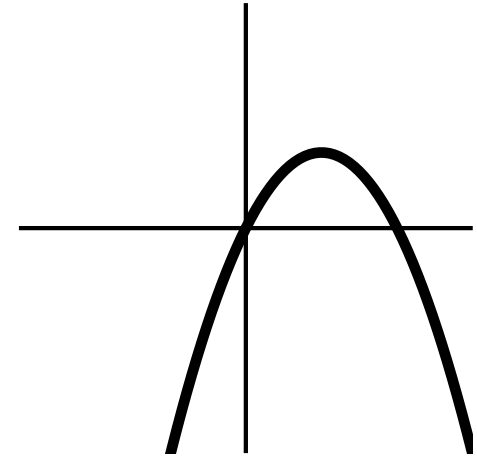


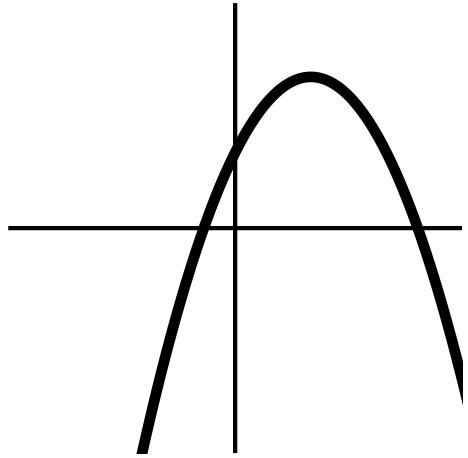


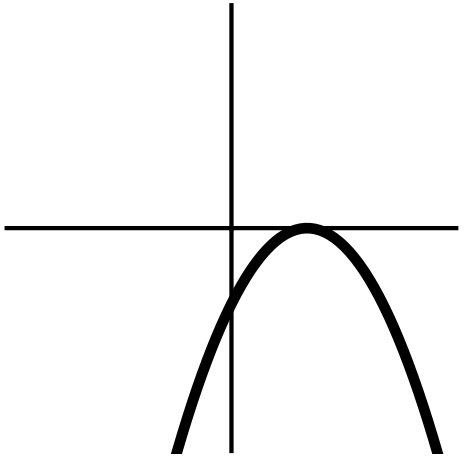
f(x) = -x + 2



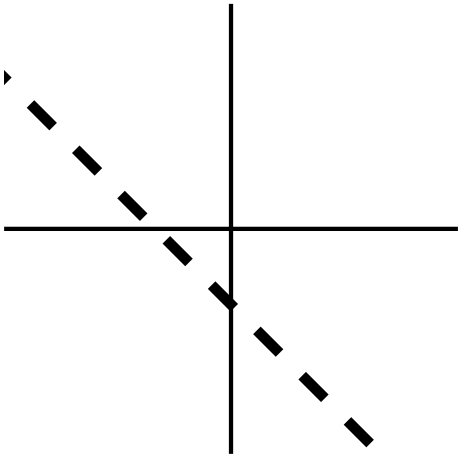
F(x) = $-x^2/2 + 2x + C$



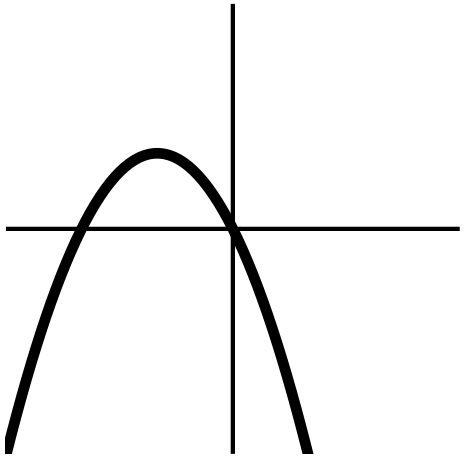


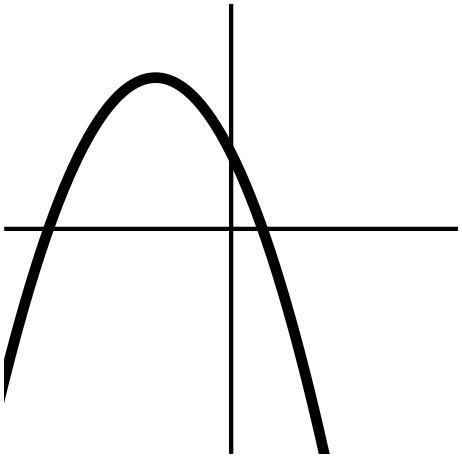


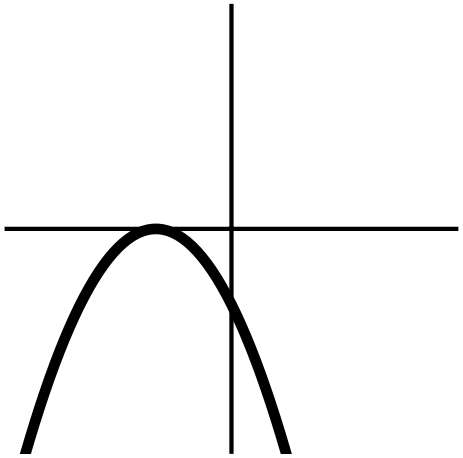
f(x) = -x - 2



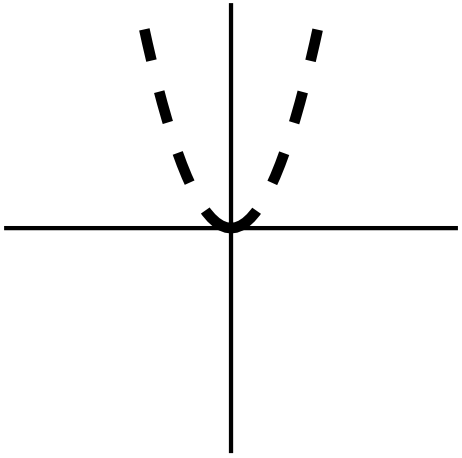
F(x) = $-x^2/2 - 2x + C$



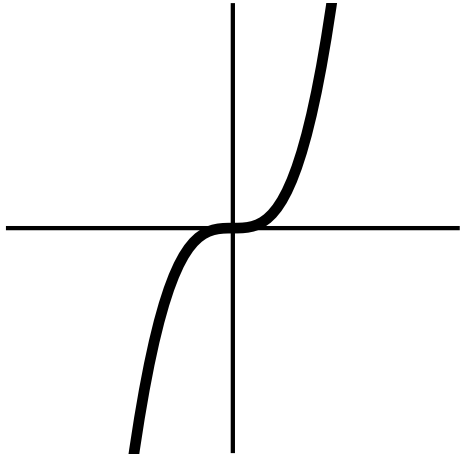


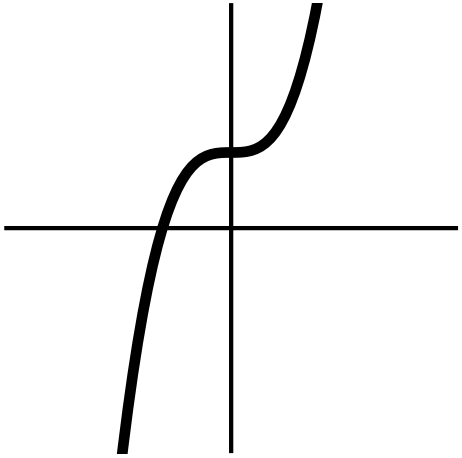


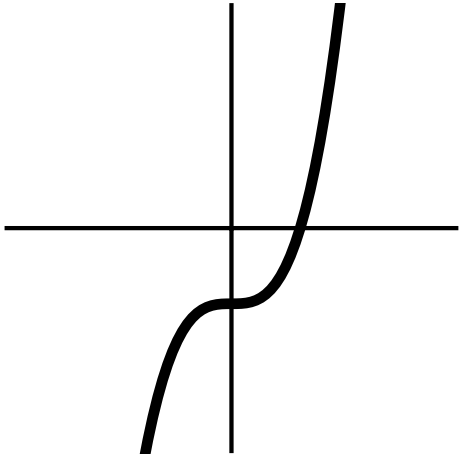
f(x) = x^2



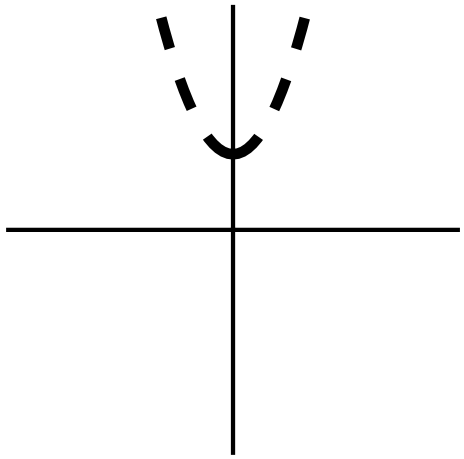
F(x) = $x^3/3 + C$

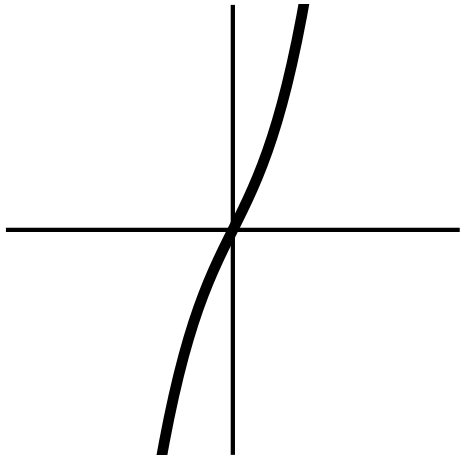




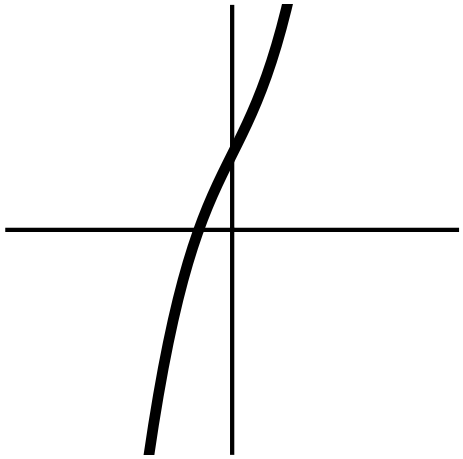


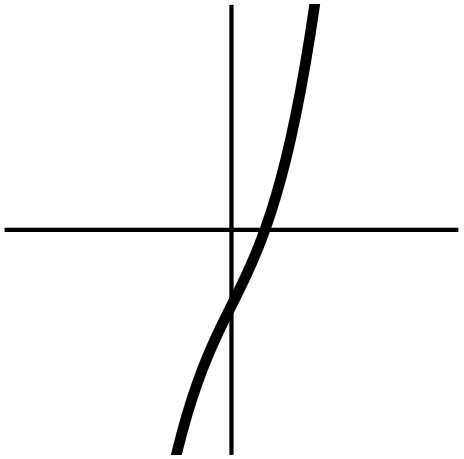
f(x) = $x^2 + 2$



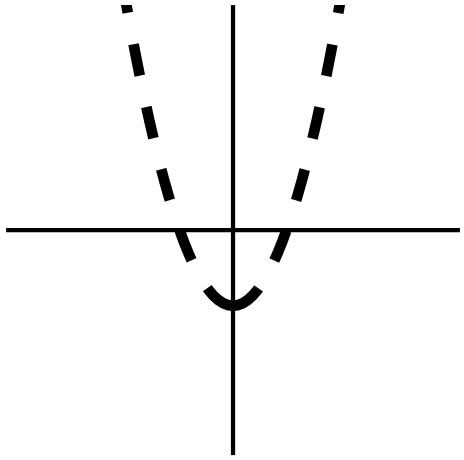


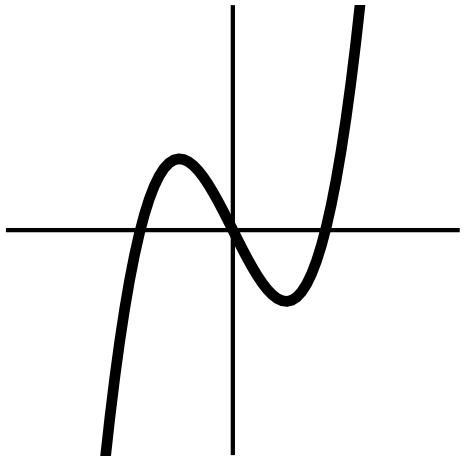
F(x) = $x^3/3 + 2x + C$



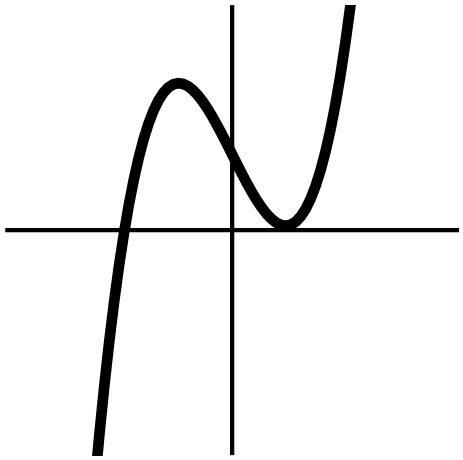


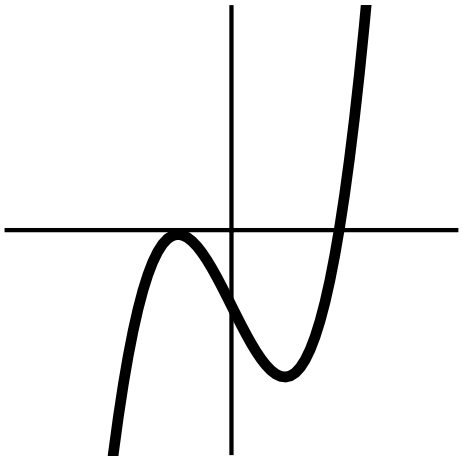
f(x) = $x^2 - 2$



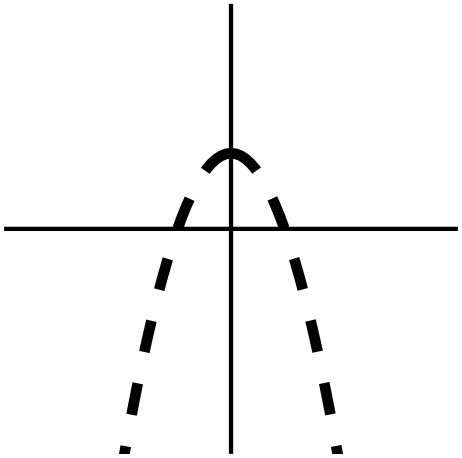


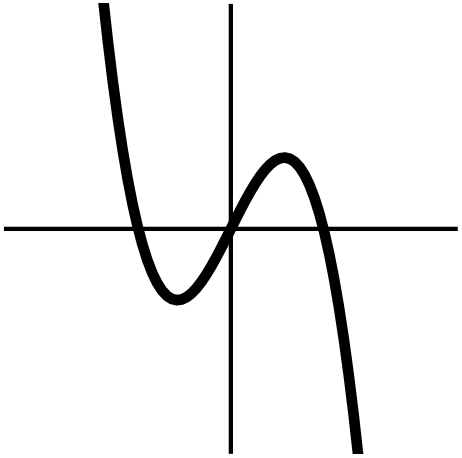
F(x) = $x^3/3 -2x + C$



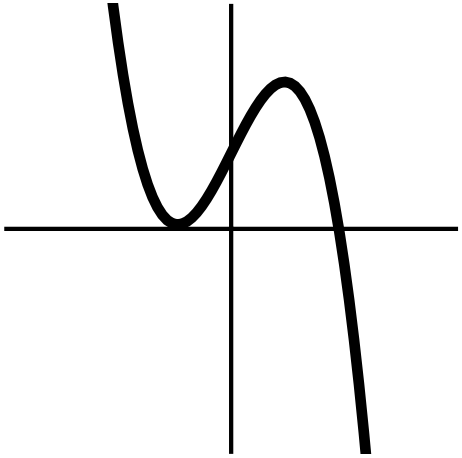


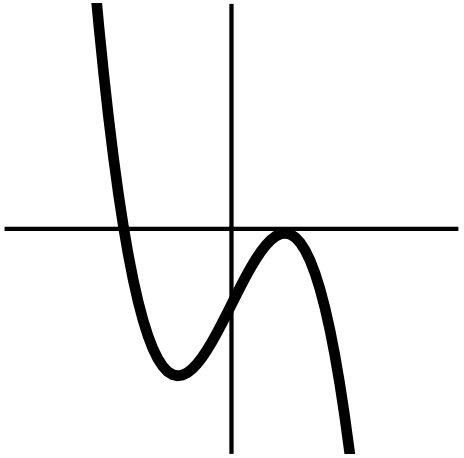
$f(x) = -x^2 + 2$



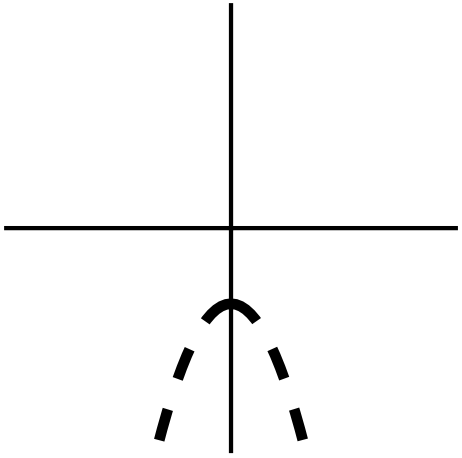


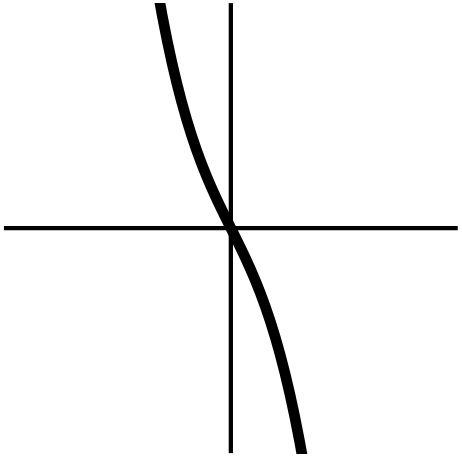
$F(x) = -x^3/3 + 2x + C$



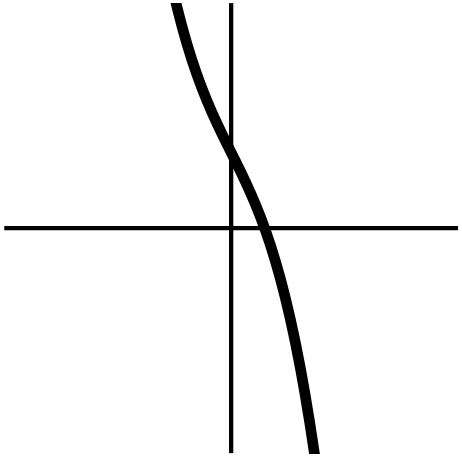


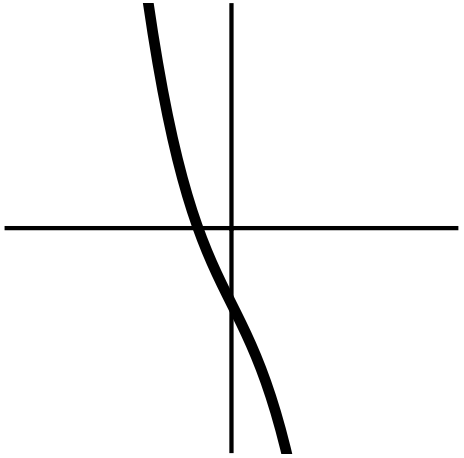
$f(x) = -x^2 - 2$



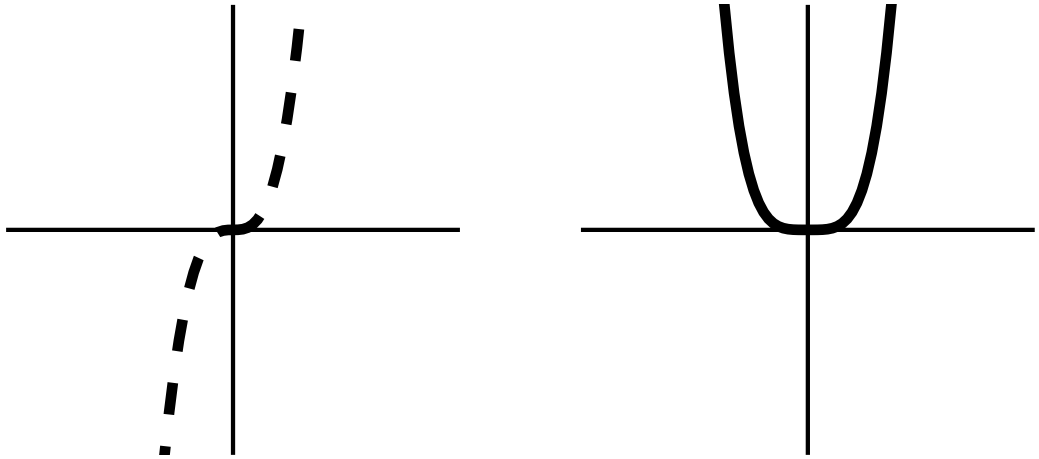


$F(x) = -x^3/3 - 2x + C$

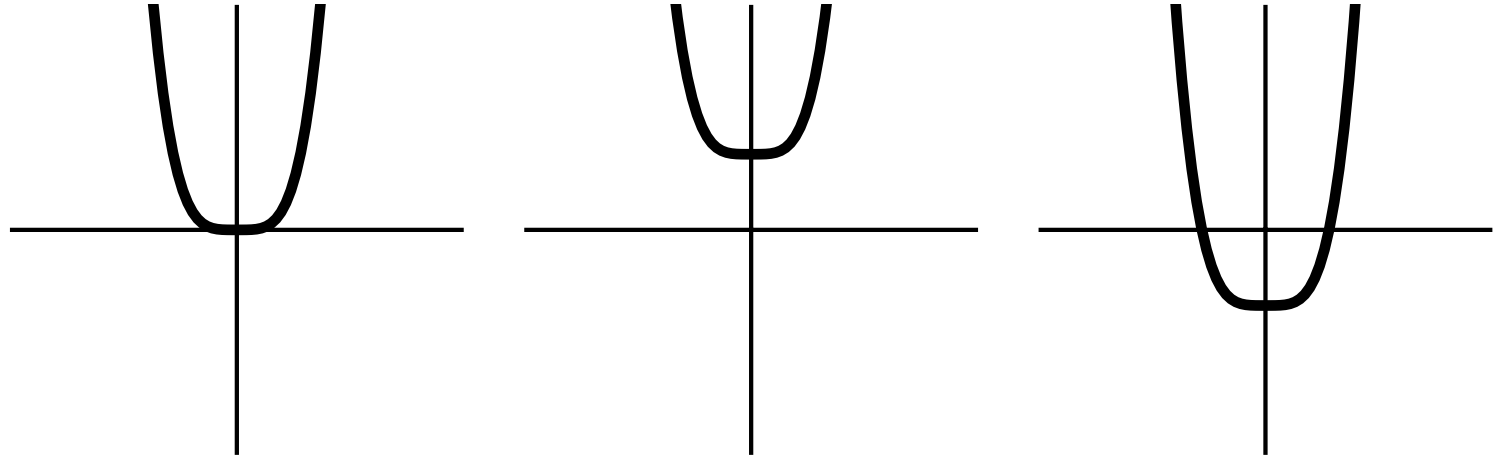




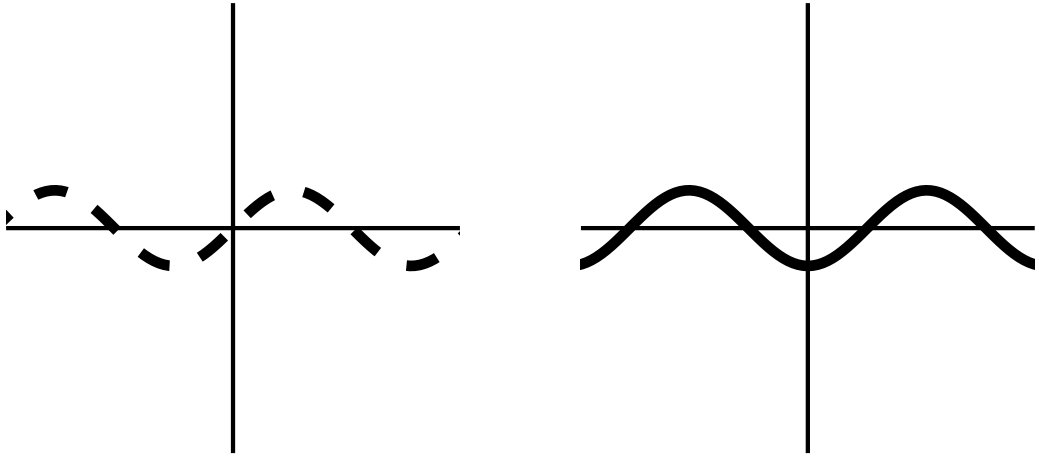
f(x) = x^3



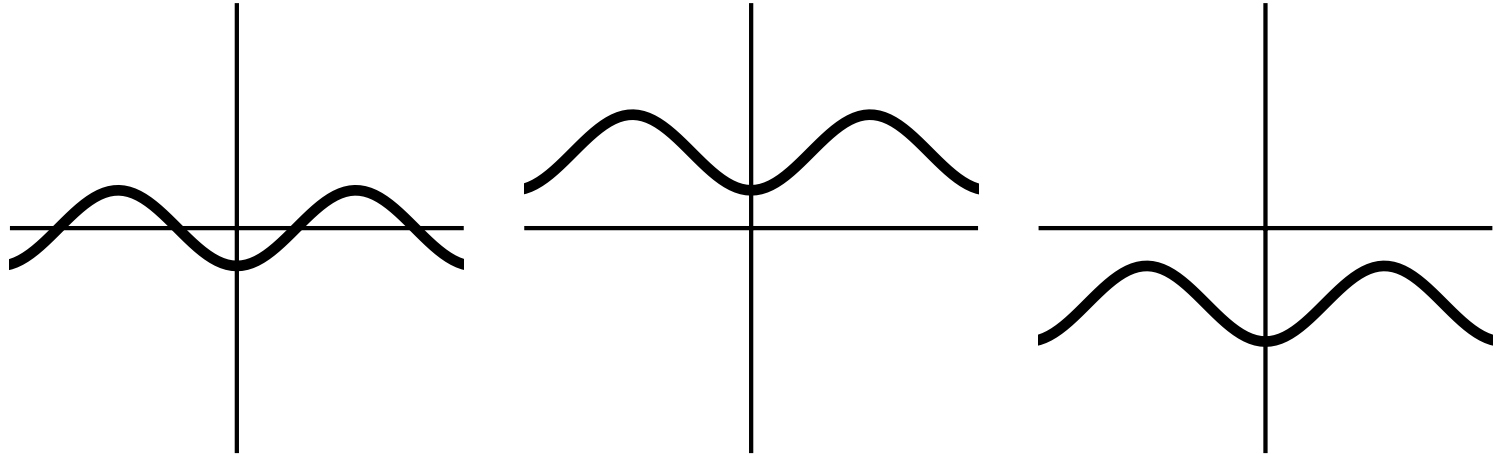
F(x) = $x^4/4 + C$



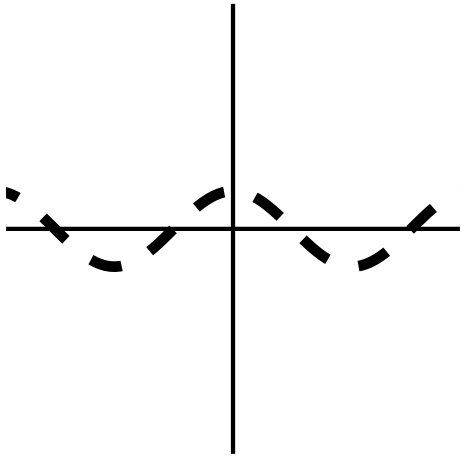
f(x) = sin(x)



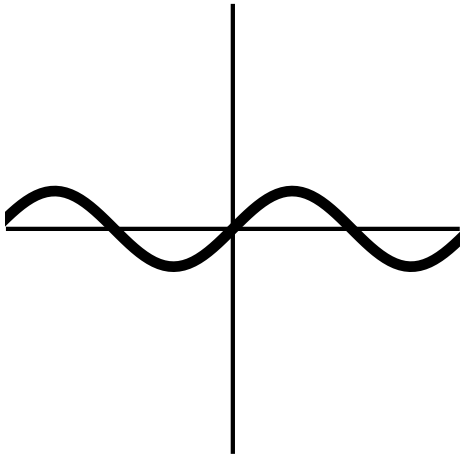
F(x) = -cos(x) + C

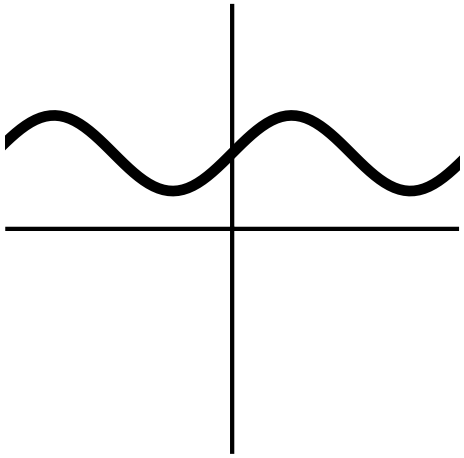


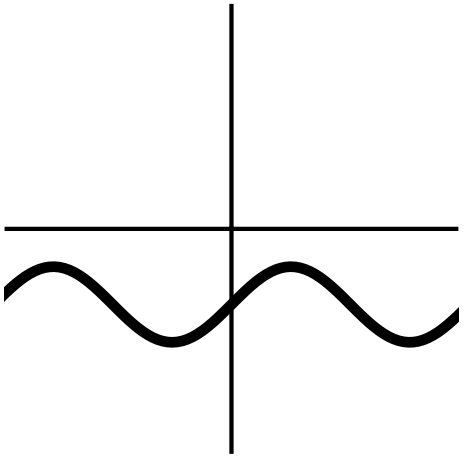
f(x) = cos(x)



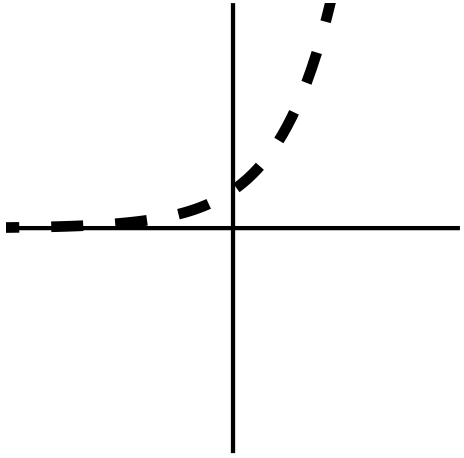
F(x) = sin(x) + C



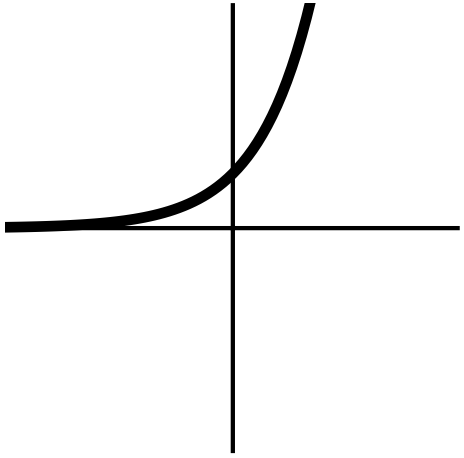


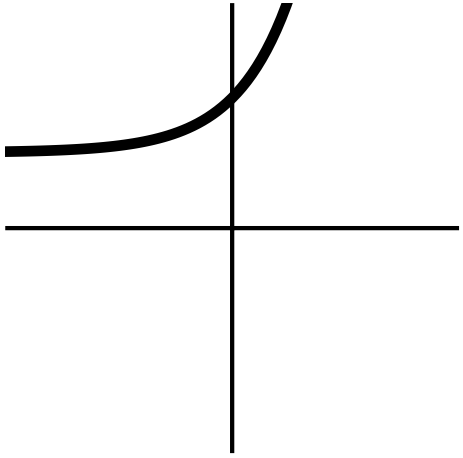


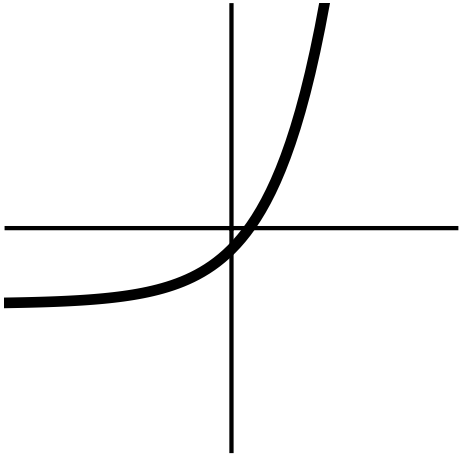
f(x) = 2^x



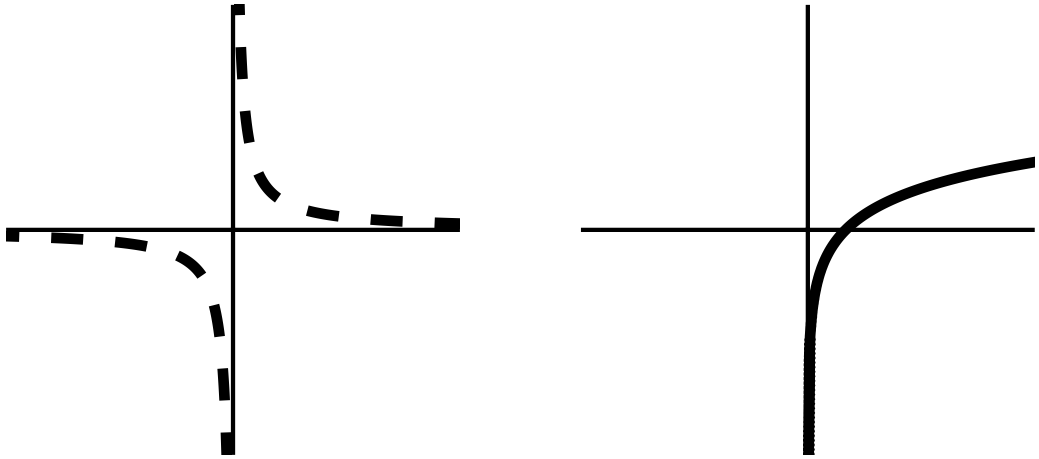
F(x) = $2^x/\ln(2) + C$



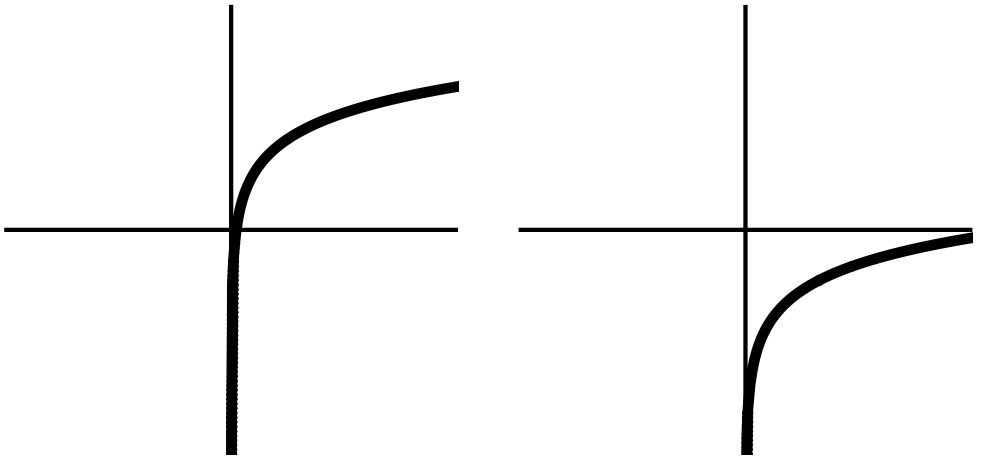




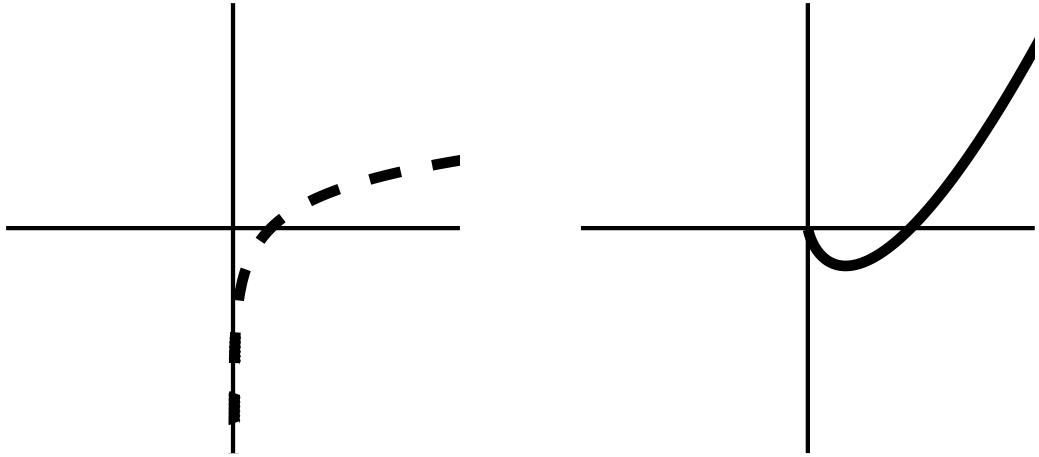
f(x) = 1/x



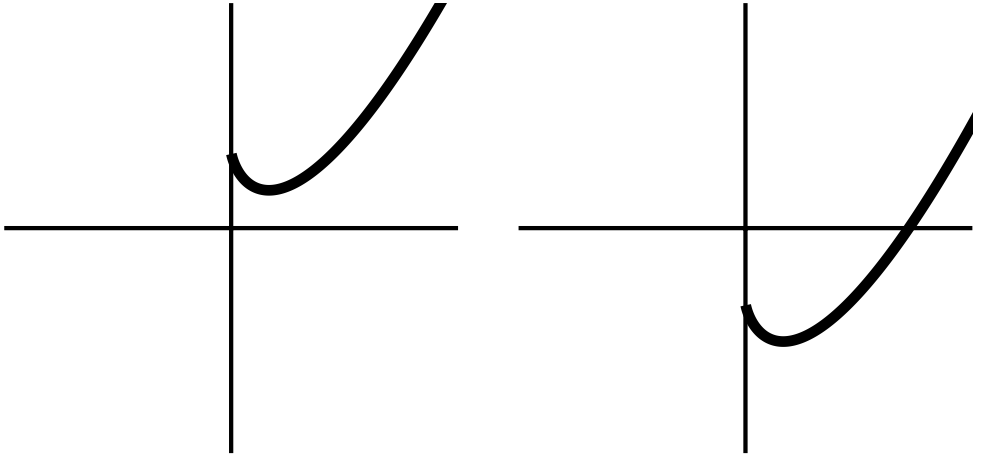
F(x) = ln(x) + C



f(x) = ln(x)



F(x) = x*ln(x) - x + C



Integrale

Schulstufe 12

Darstellung und Berechnung von Flächen

Weglängen

Volumina

Integrale

Schulstufe 12

Darstellung und Berechnung

von Flächen

Weglängen

Volumina

Inhalt

1 - 9 Flächenberechnungen

1 Fläche zwischen Kurve und x- Achse

2 Unter- und Obersumme n =3

3 Unter- und Obersumme n =5

4 A ='int[a; b], f(x) >0

5 A ='int[a; b], f(x) <0

6 A =|'int[a; b]| +|'int[b; c]|

7 A ='int[a; b](f(x) -g(x))'dx)

8 A ='int[a; b](g(x) -f(x))'dx)

9 Abschnittweises Integrieren

10-12 Streckenberechnungen

10 s ='int[t_1; t_2](v(t) 'dt) mit v konstant

11 s ='int[t_1; t_2](v_1(t)'dt)
+'int[t_2; t_3](v_2(t'dt)
mit v_1, v_2 konstant

12 s ='int[t_1; t_2](v(t) 'dt)
mit v nicht konstant

13 - 24 Volumsberechnungen

13 Kreis x^2 +y^2 =r^2

14 Kreisrotation um x-Achse

15 Kreisrotation um y-Achse

16 Ellipse b^2x^2 +a^2y^2 =a^2b^2

17 Ellipsenrotation um x-Achse

18 Ellipsenrotation um y-Achse

19 Hyperbel b^2x^2 -a^2y^2 =a^2b^2

20 Hyperbelrotation um x-Achse

21 Hyperbelrotation um y-Achse

22 Parabel y^2 =2px

23 Parabelrotation um x-Achse

24 Parabelrotation um y-Achse

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⠡⠱ ⡅⠗⠑⠊⠎⠗⠕⠞⠁⠞⠊⠕⠝ ⠥⠍ ⠽⠤⡁⠉⠓⠎⠑
⠡⠫ ⡑⠇⠇⠊⠏⠎⠑ ⠃⡮⠣⠭⡮⠣ ⠖⠁⡮⠣⠽⡮⠣
⠶⠁⡮⠣⠃⡮⠣

17 Ellipsenrotation um x-
Achse

18 Ellipsenrotation um y-
Achse

19 Hyperbel b^2x^2 -a^2y^2
=a^2b^2

20 Hyperbelrotation um x-
Achse

21 Hyperbelrotation um y-
Achse

22 Parabel y^2 =2px

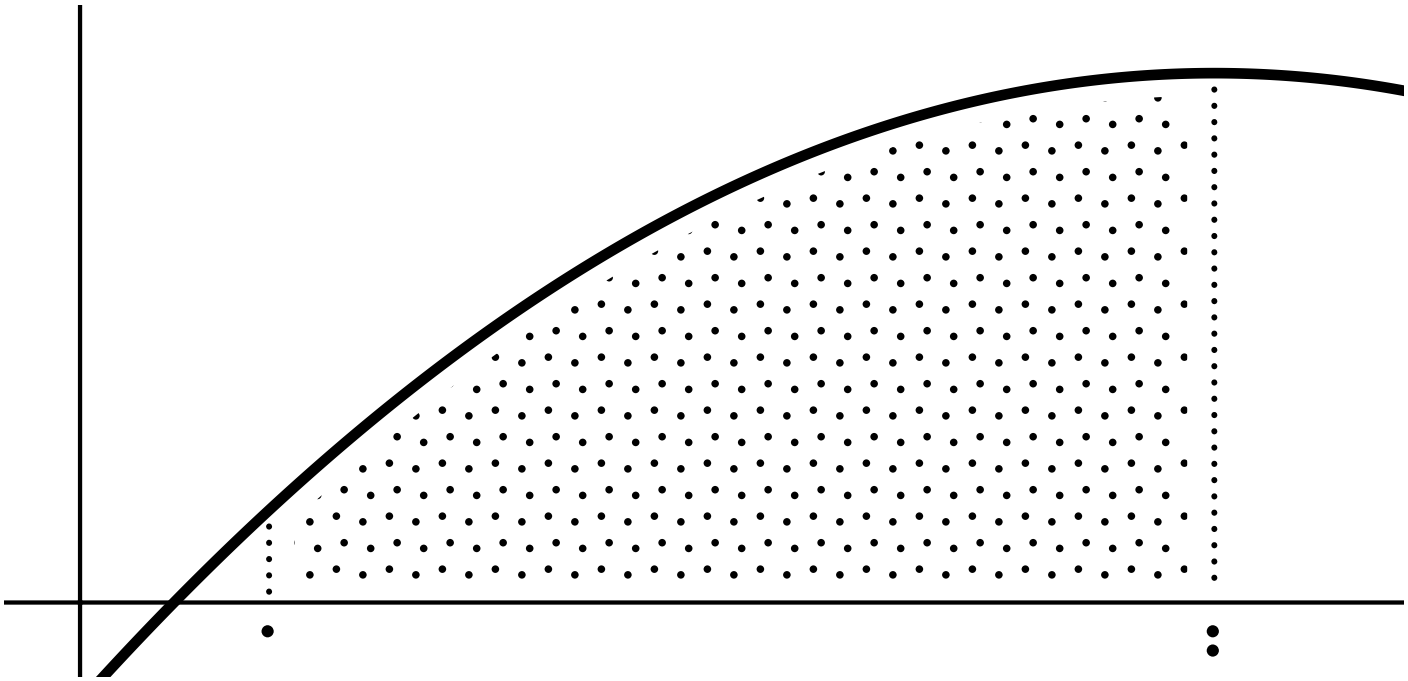
23 Parabelrotation um x-Ach-
se

24 Parabelrotation um y-Ach-
se

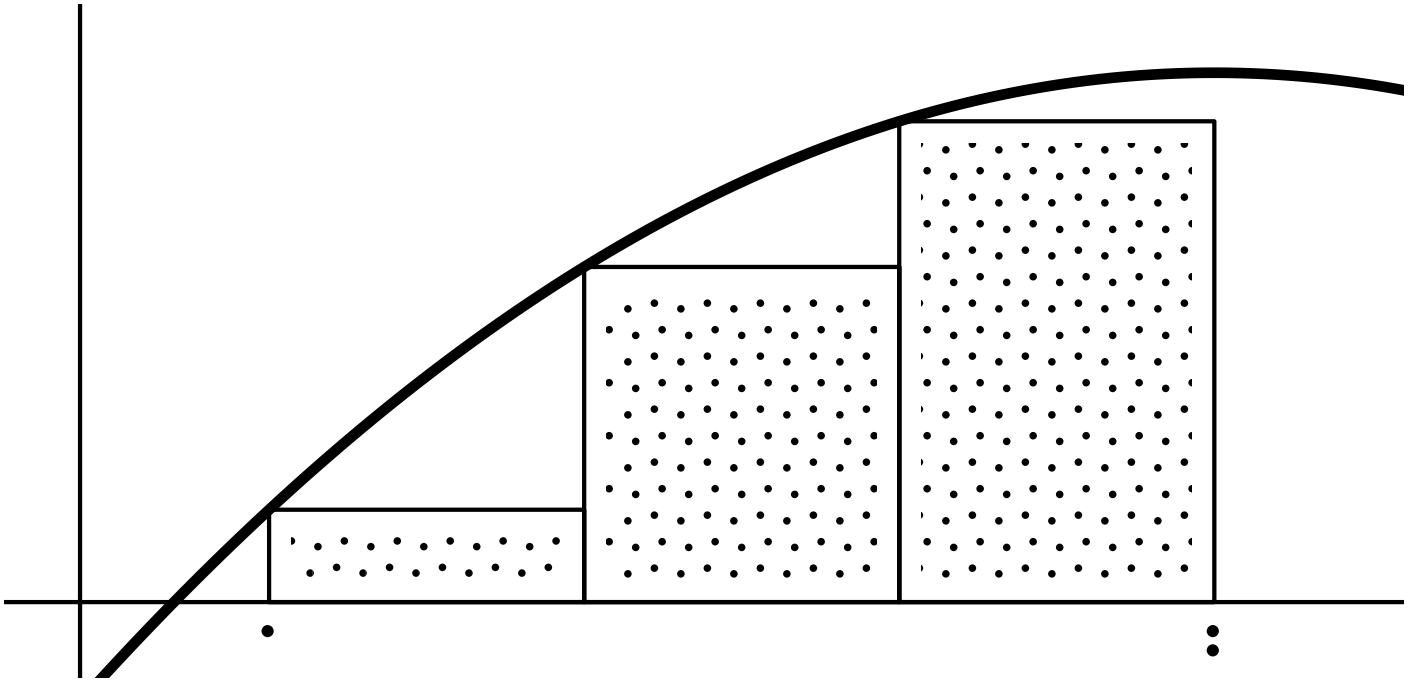
Integral Flächen, 1/24

f … Funktion a, b … Grenzen

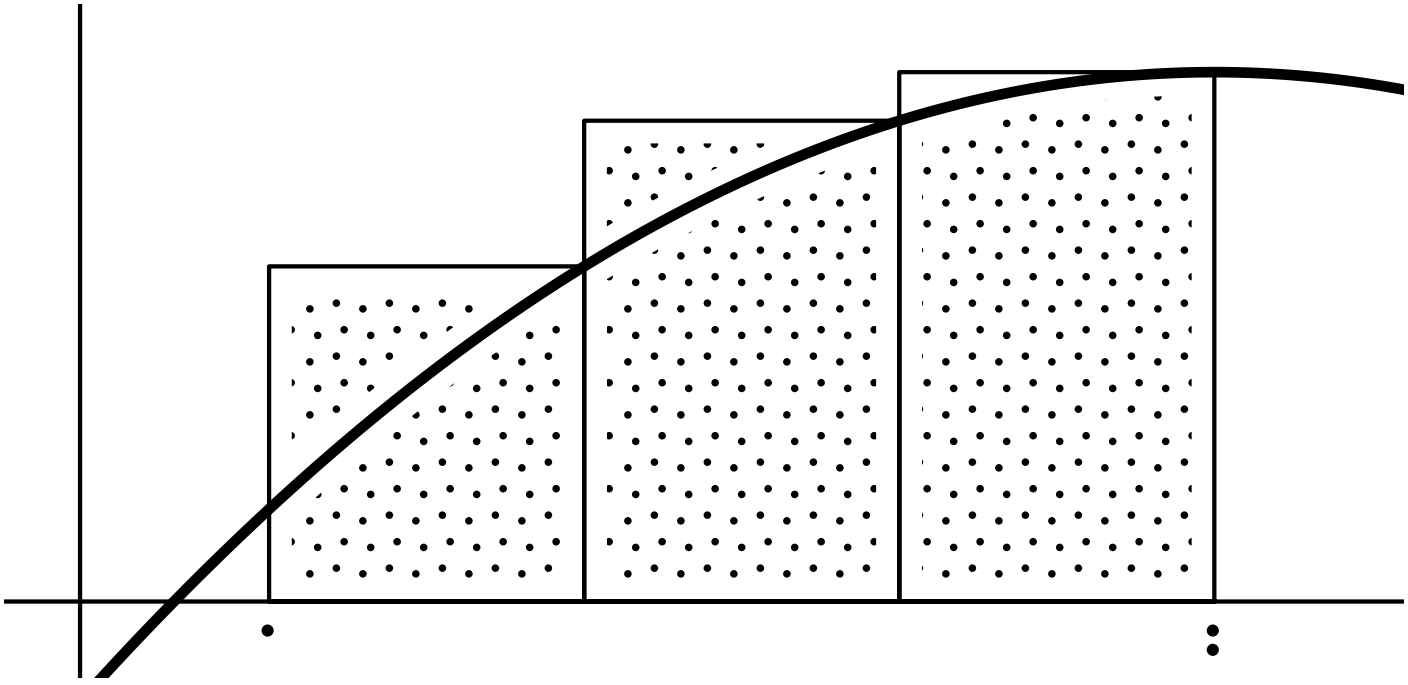
Integral



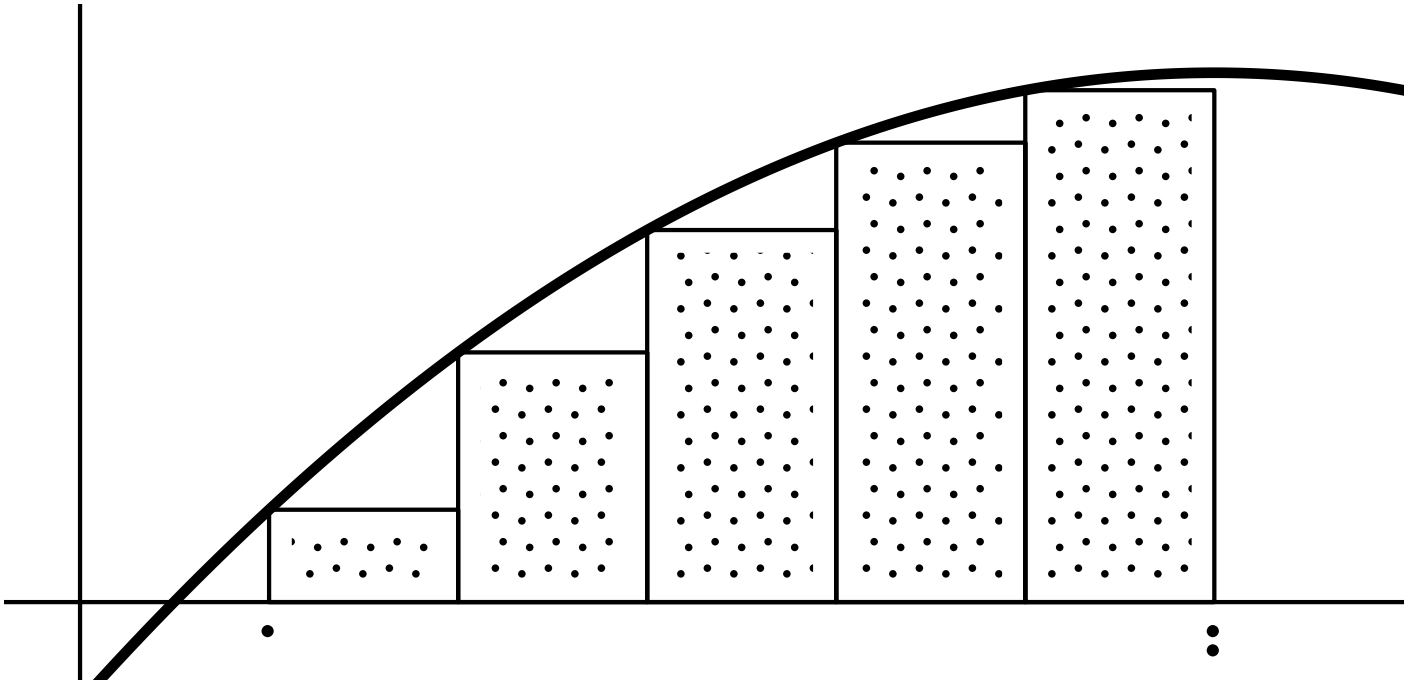
Untersumme, n = 3



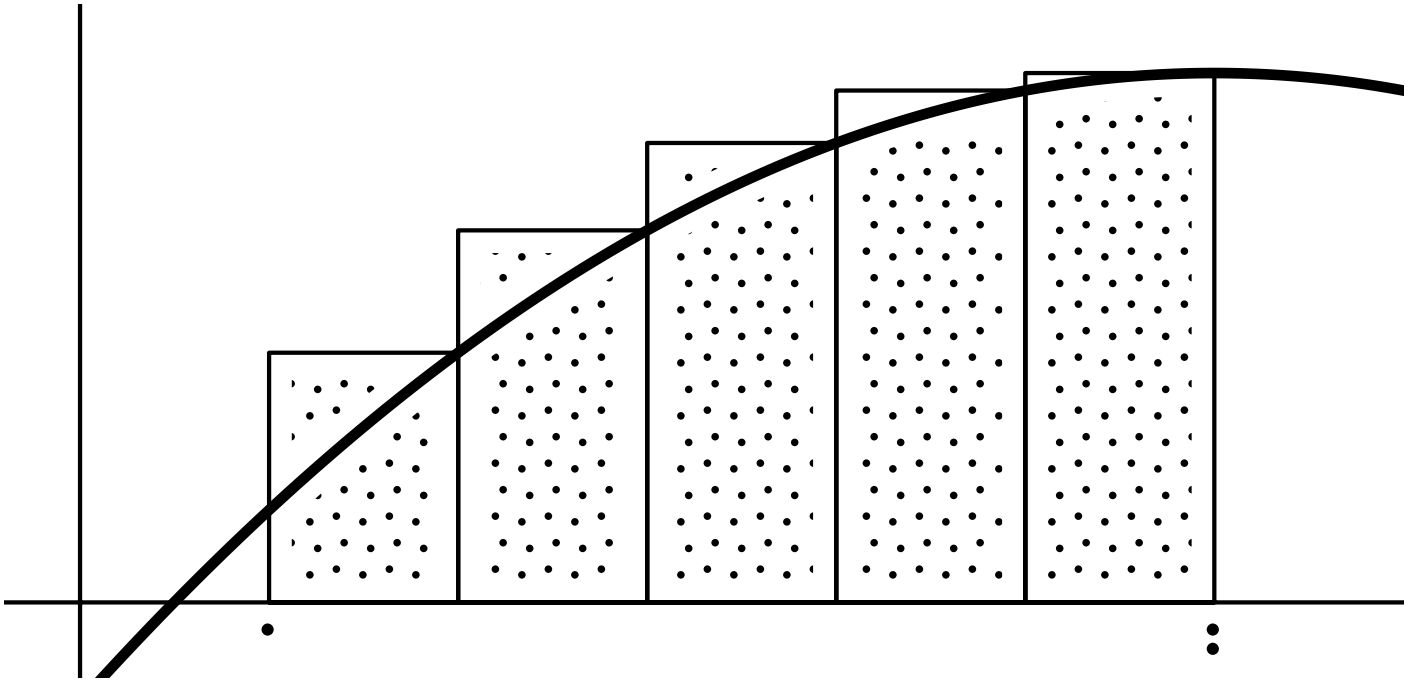
Obersumme, n = 3



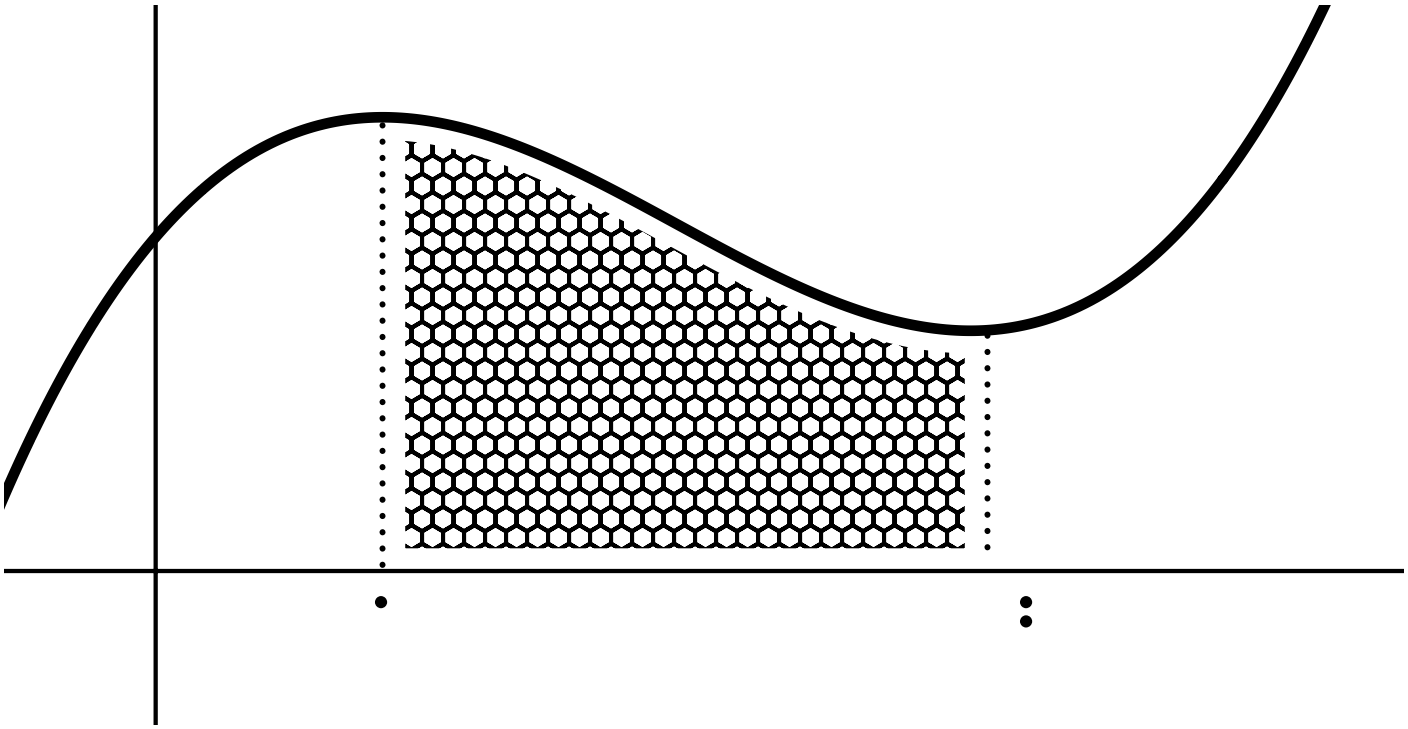
Untersumme, n = 5

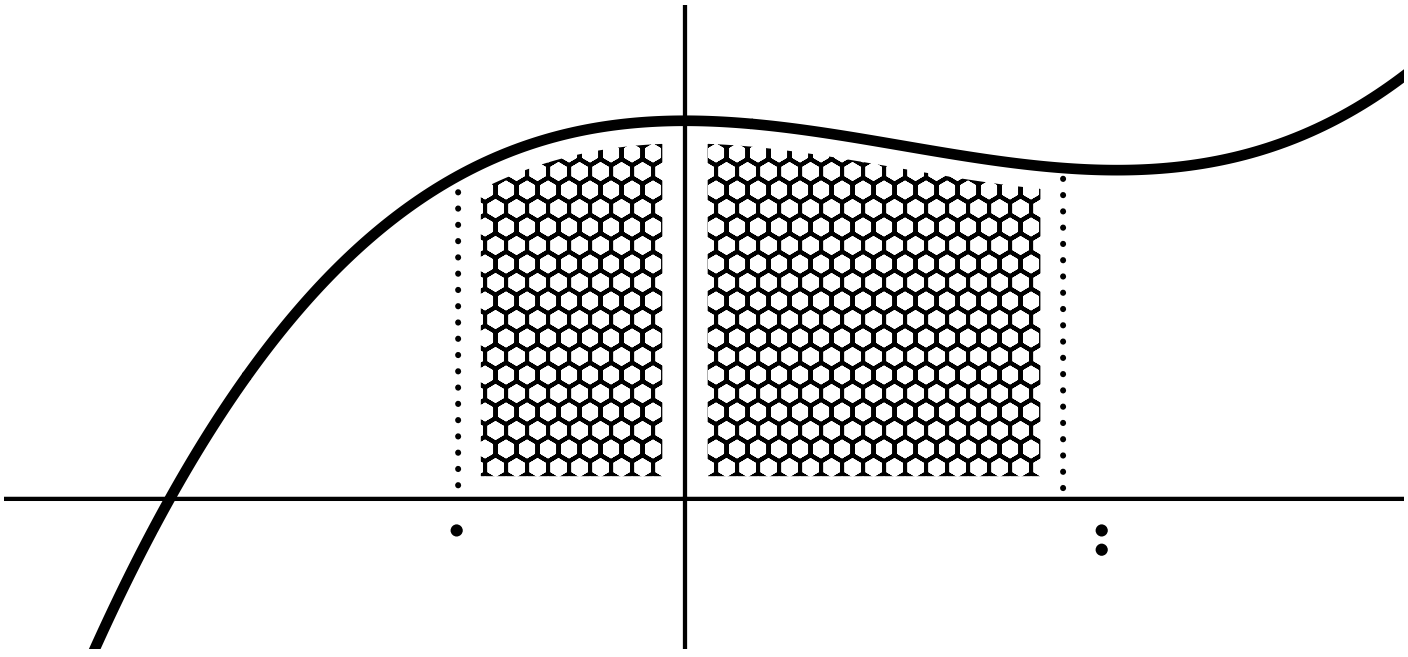


Obersumme, n = 5

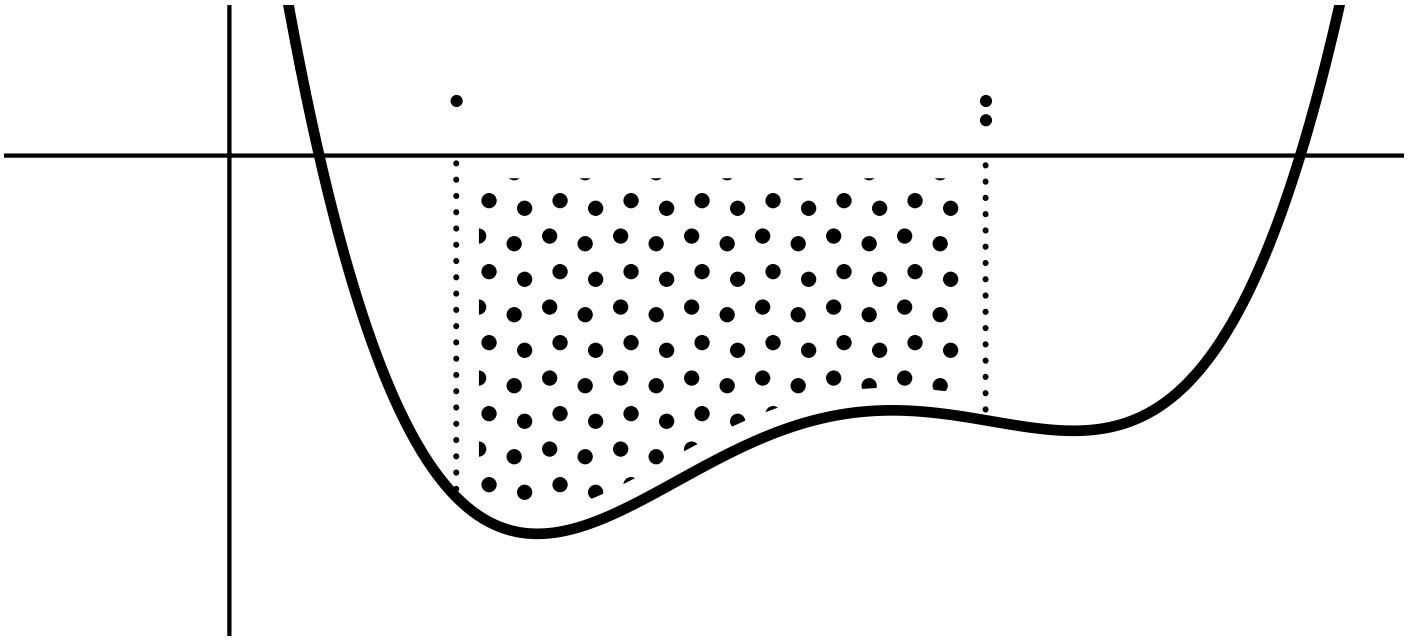


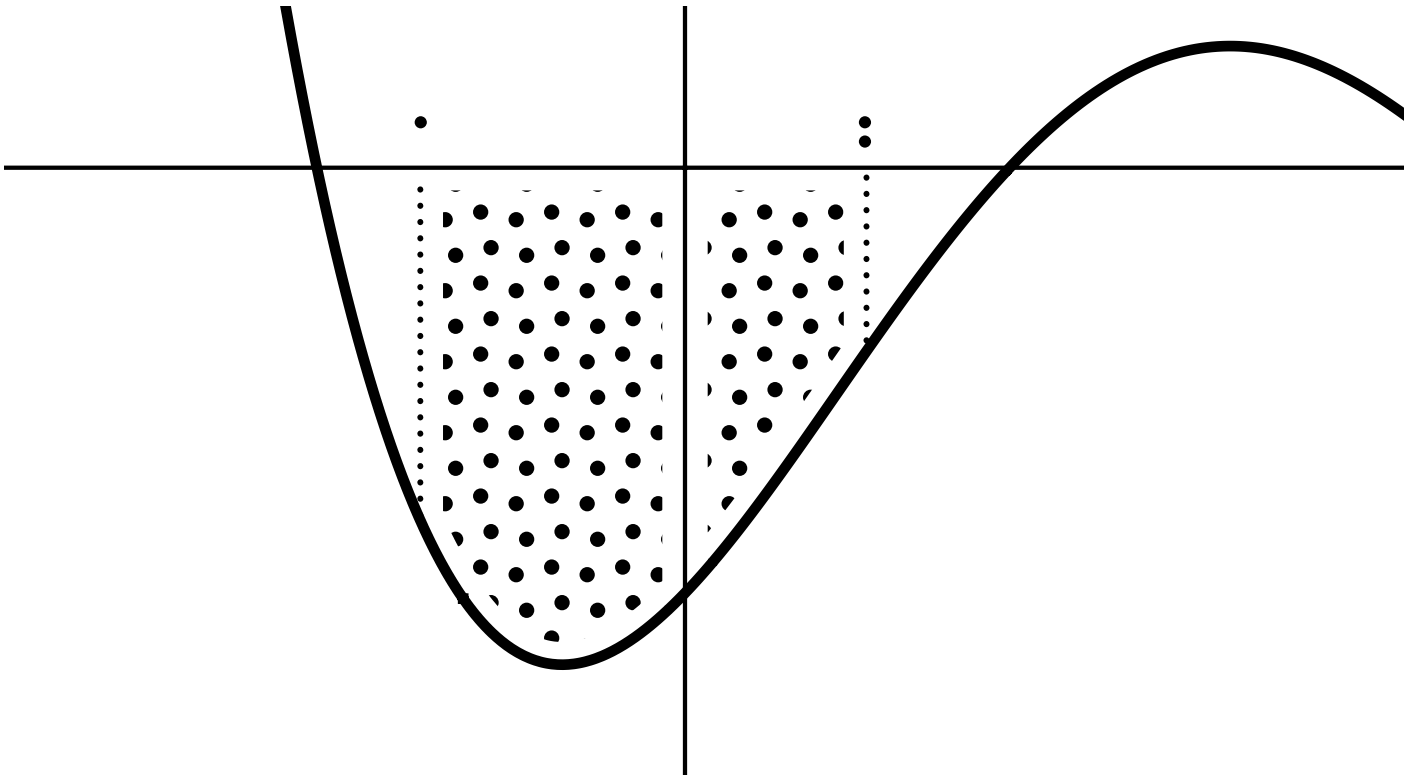
int [a; b] (f(x) dx) = A



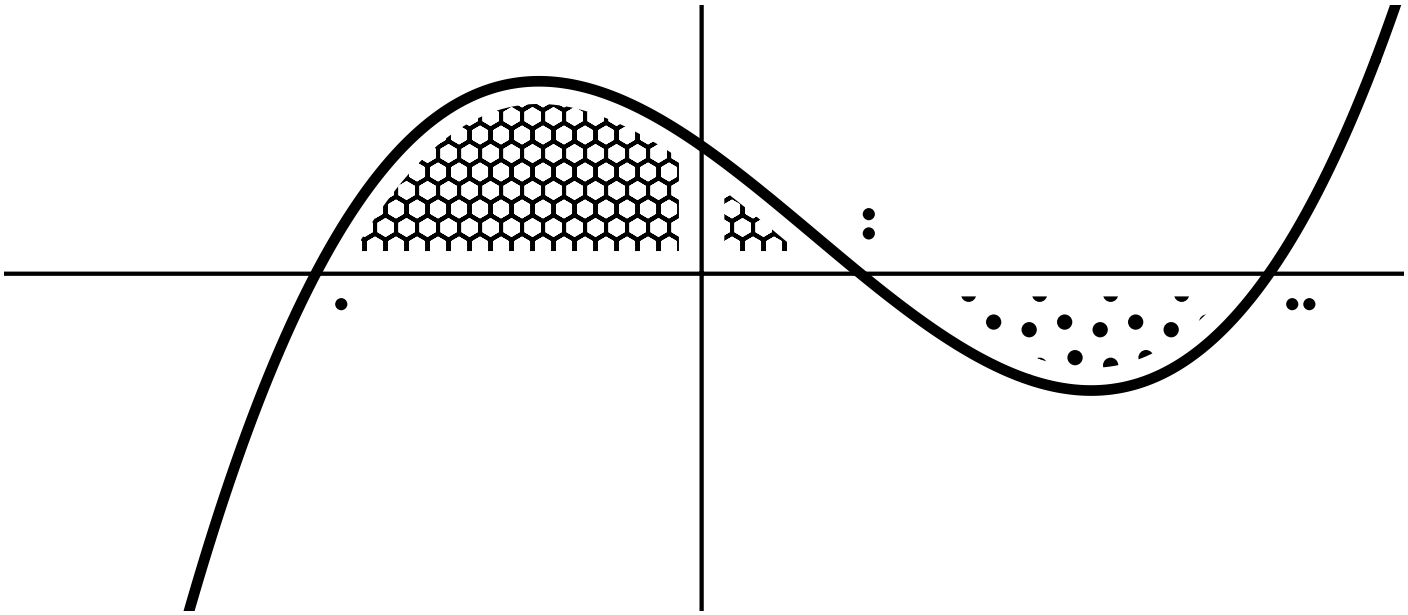


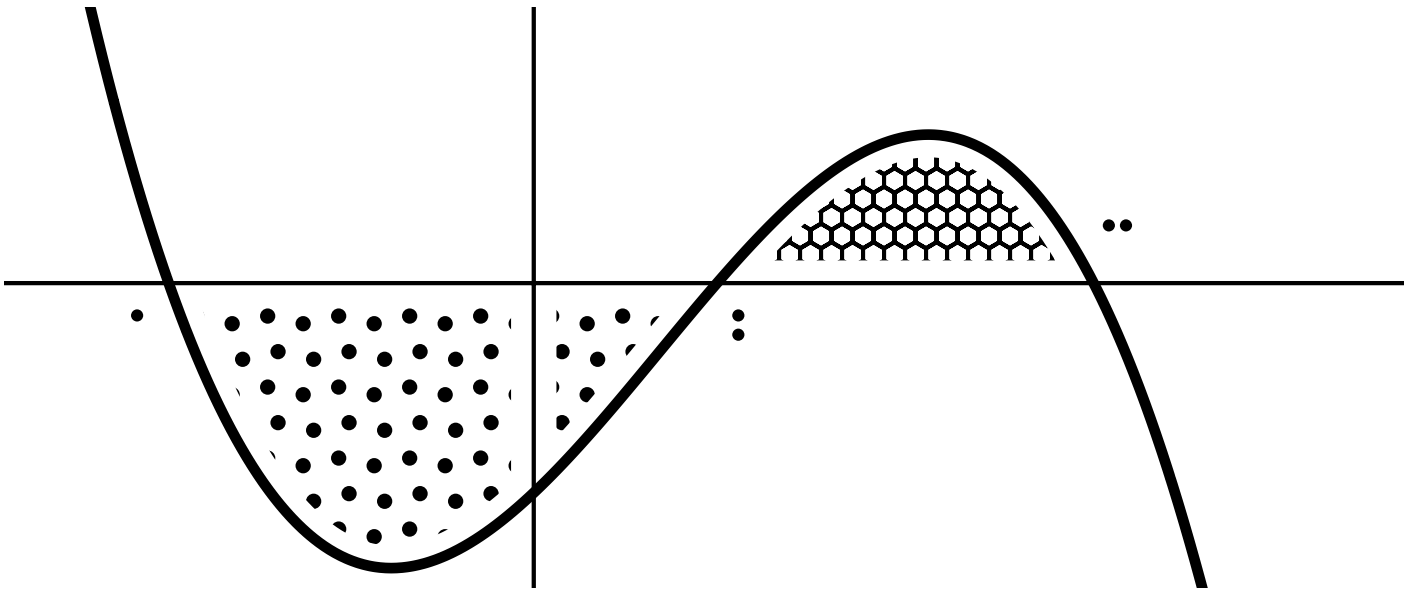
int [a; b] (f(x) dx) = -A



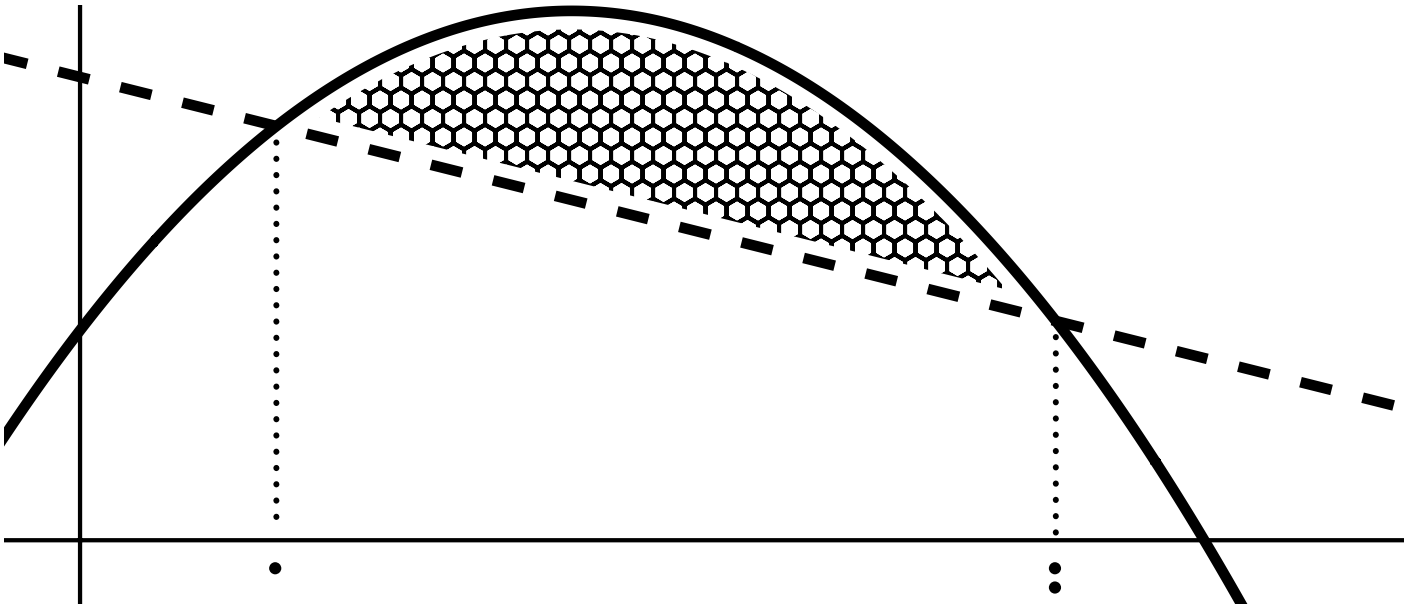


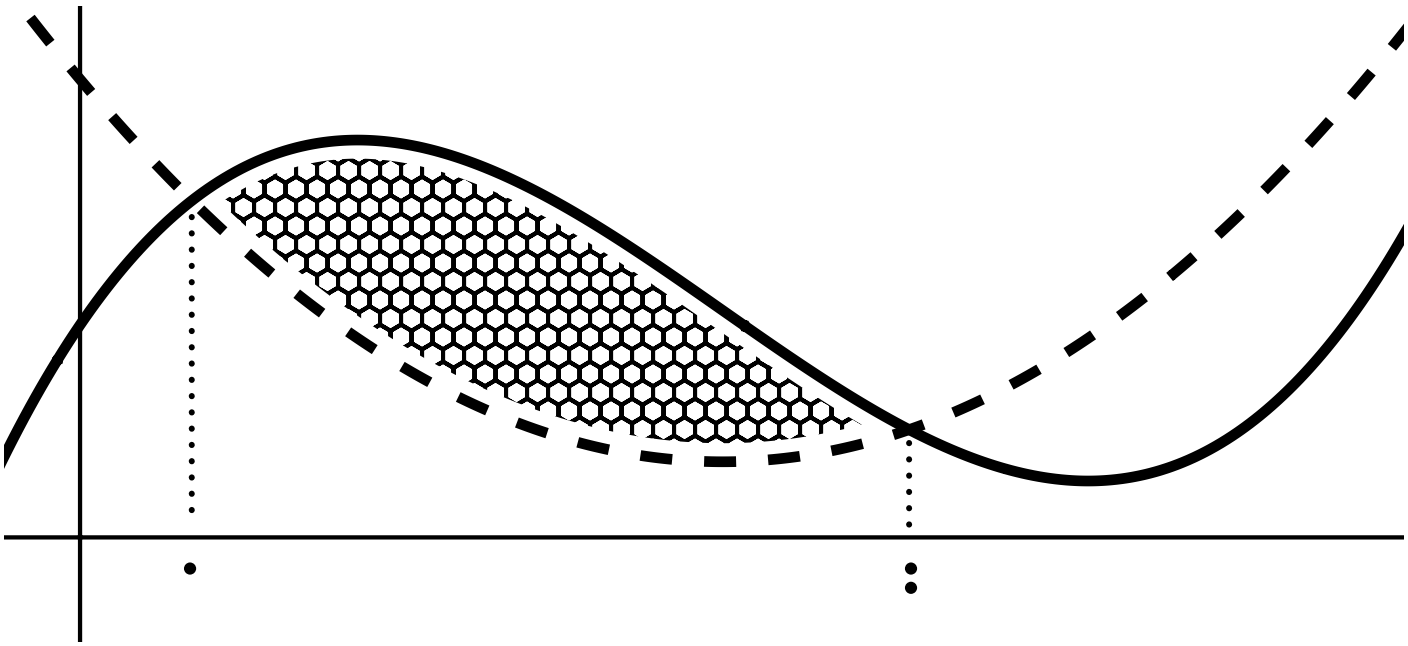
$A_1 = |\text{int}\,[a; b]\,(f(x)\,dx)|$ $A_2 = |\text{int}\,[b; c]\,(f(x)\,dx)|$ $A = A_1 + A_2$



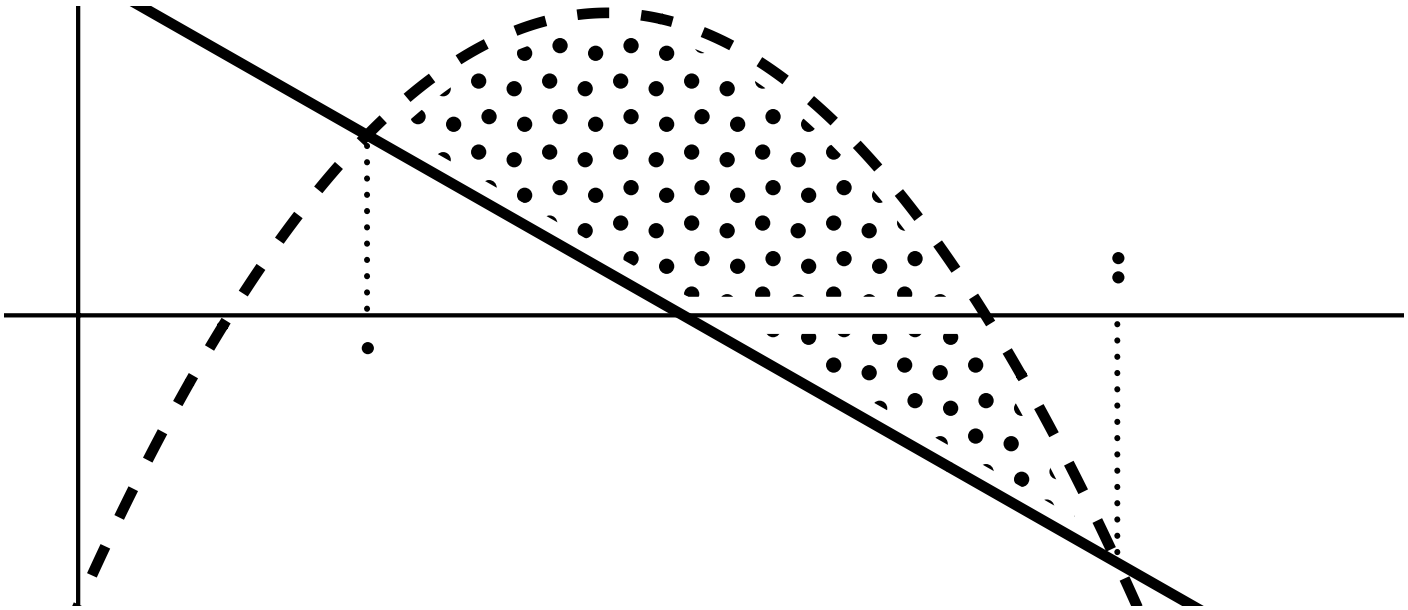


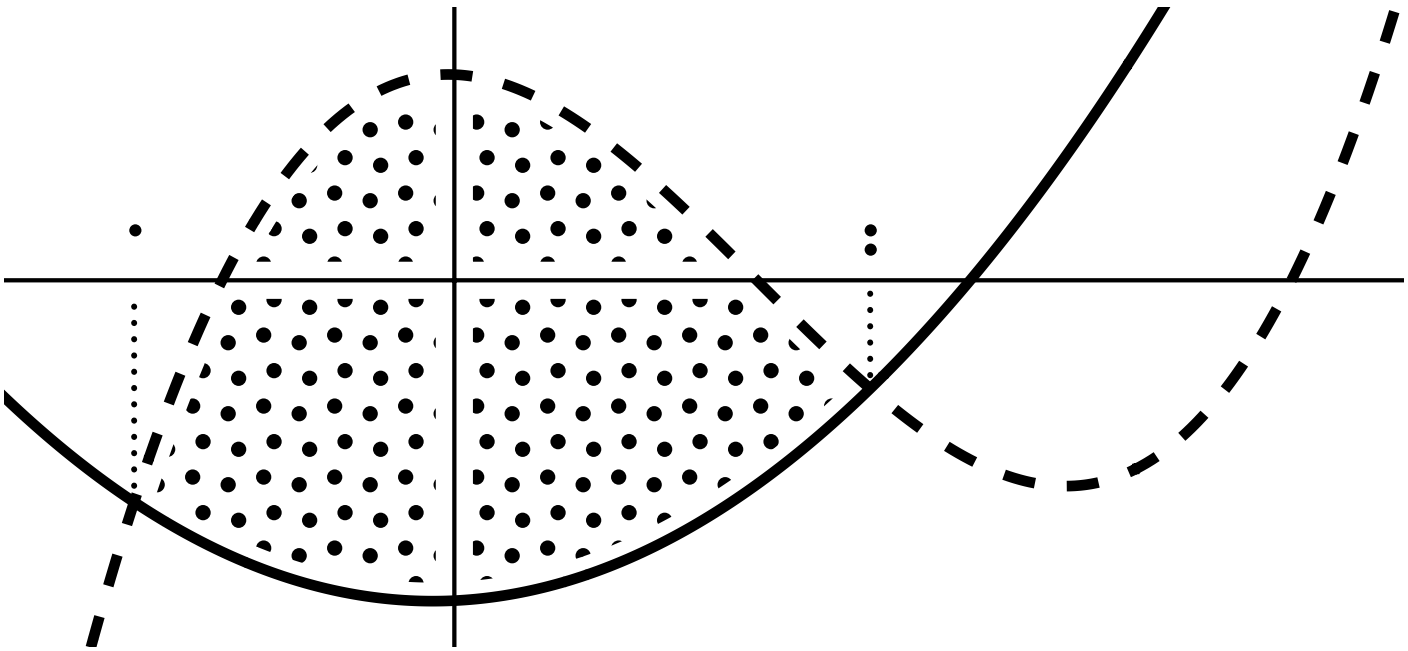
A = int [a; b] ((f(x) - g(x)) dx)



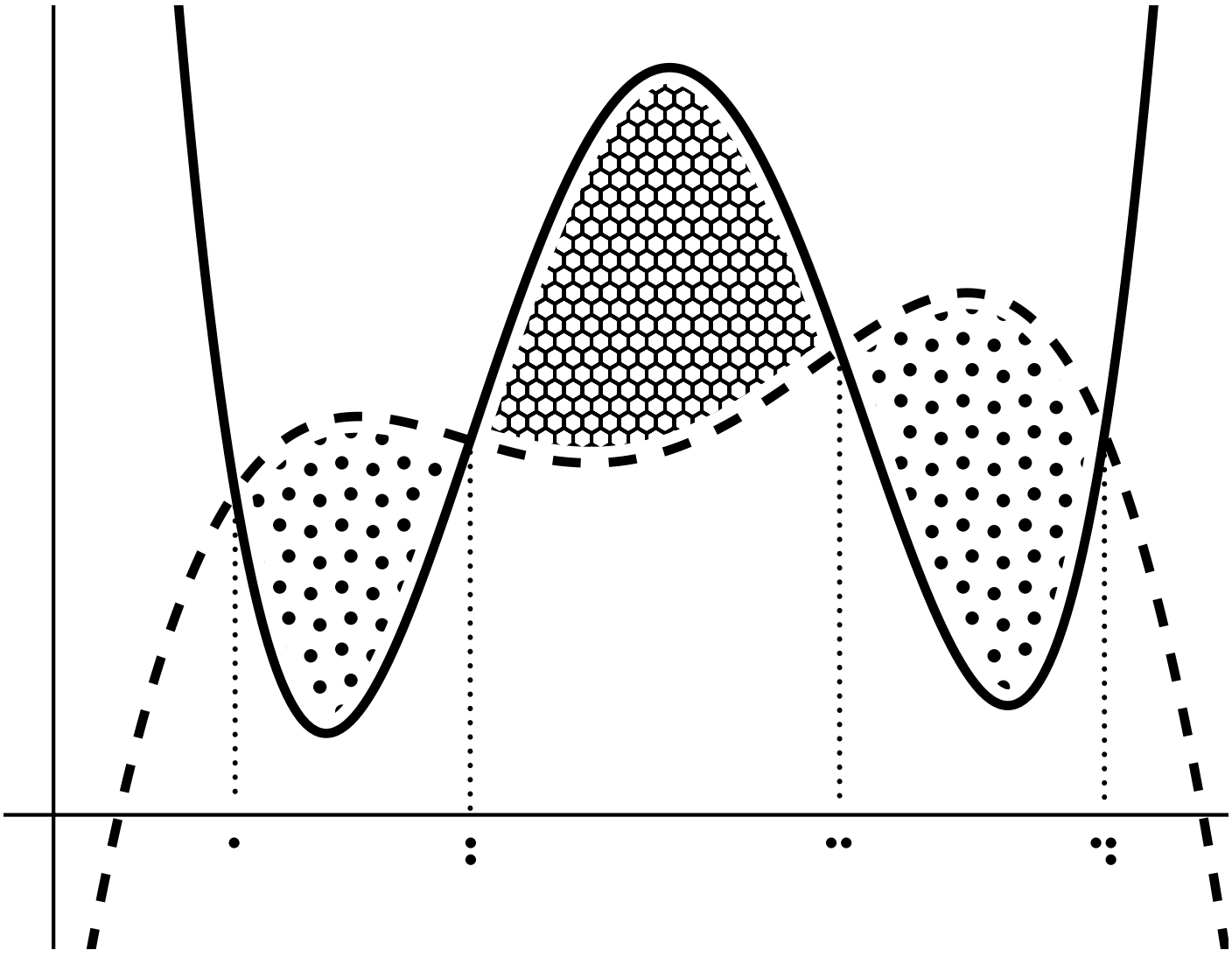


A = int [a; b] ((g (x) - f (x)) dx)

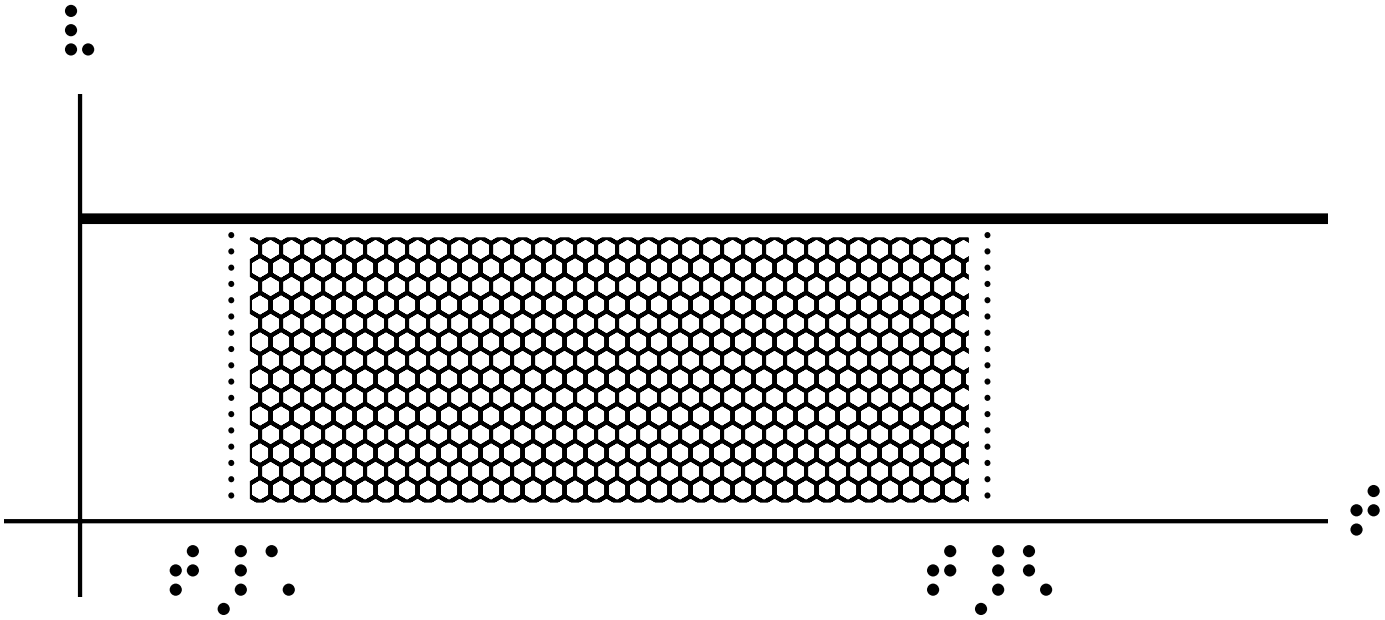


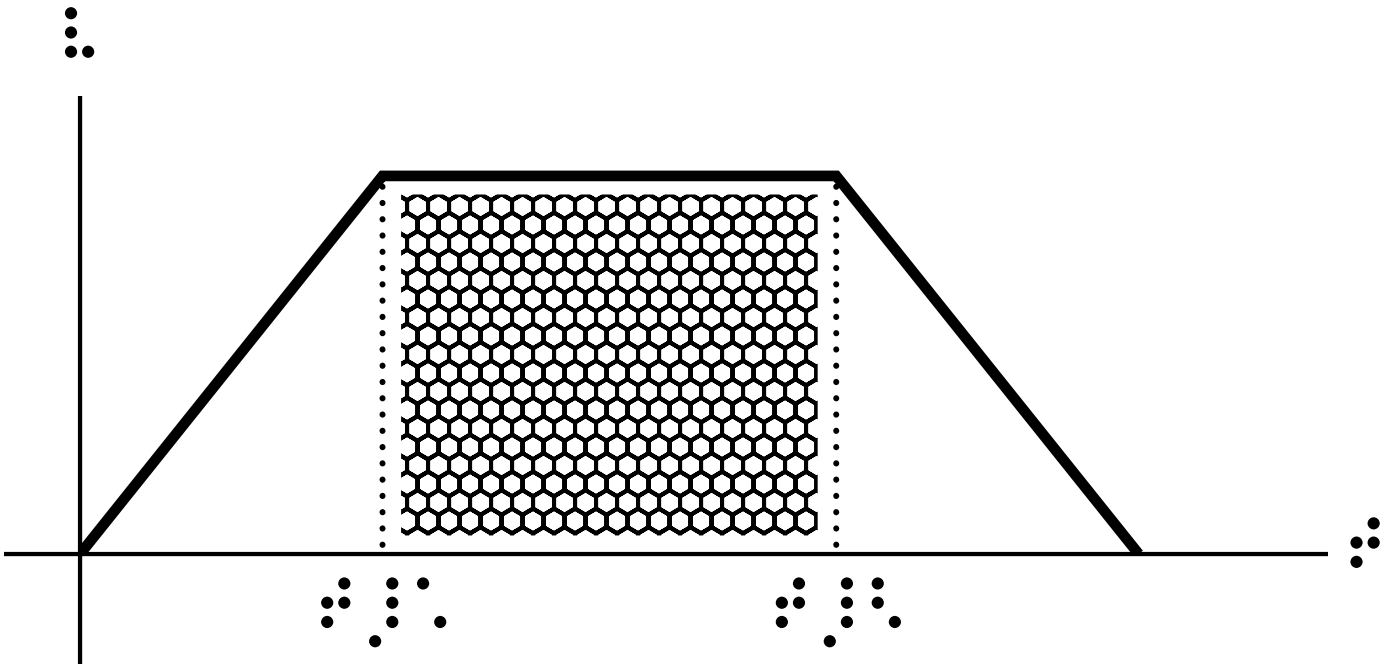


A = int [a; b] ((g (x) - f(x)) dx) + int [b; c] ((f(x) - g (x)) dx) + int [c; d] ((g (x) - f(x)) dx)

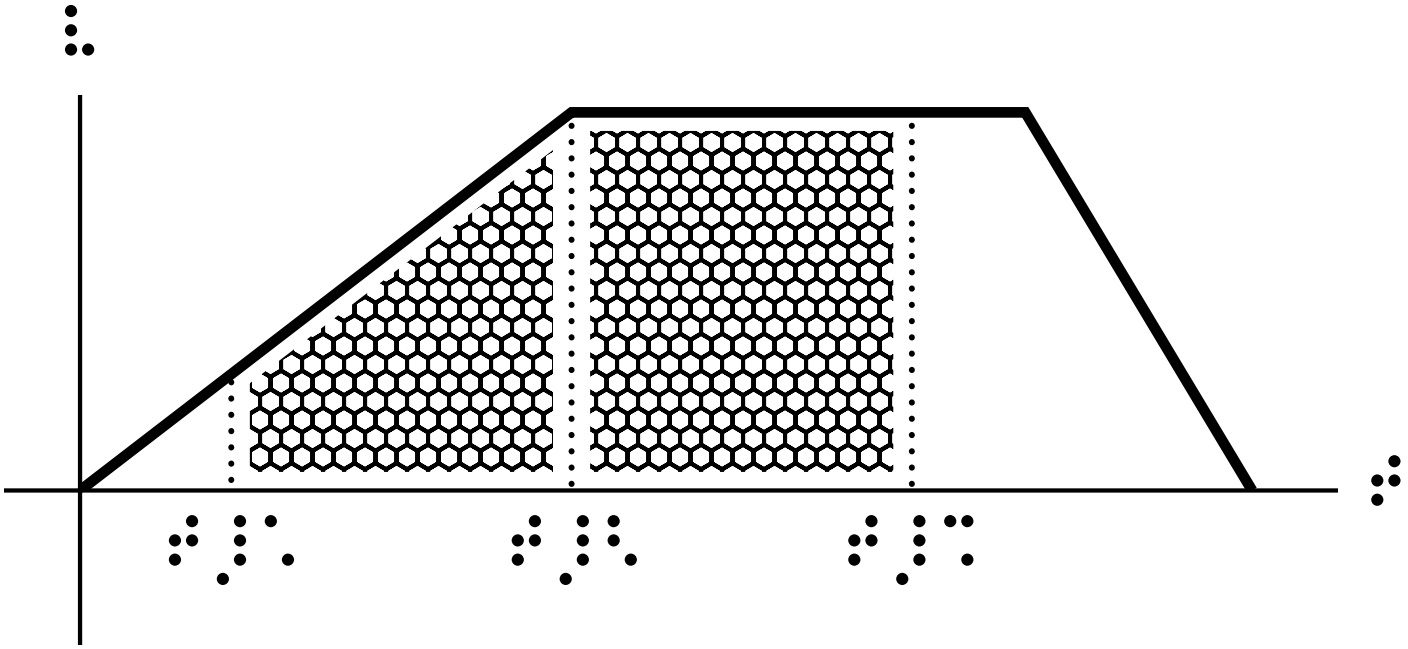


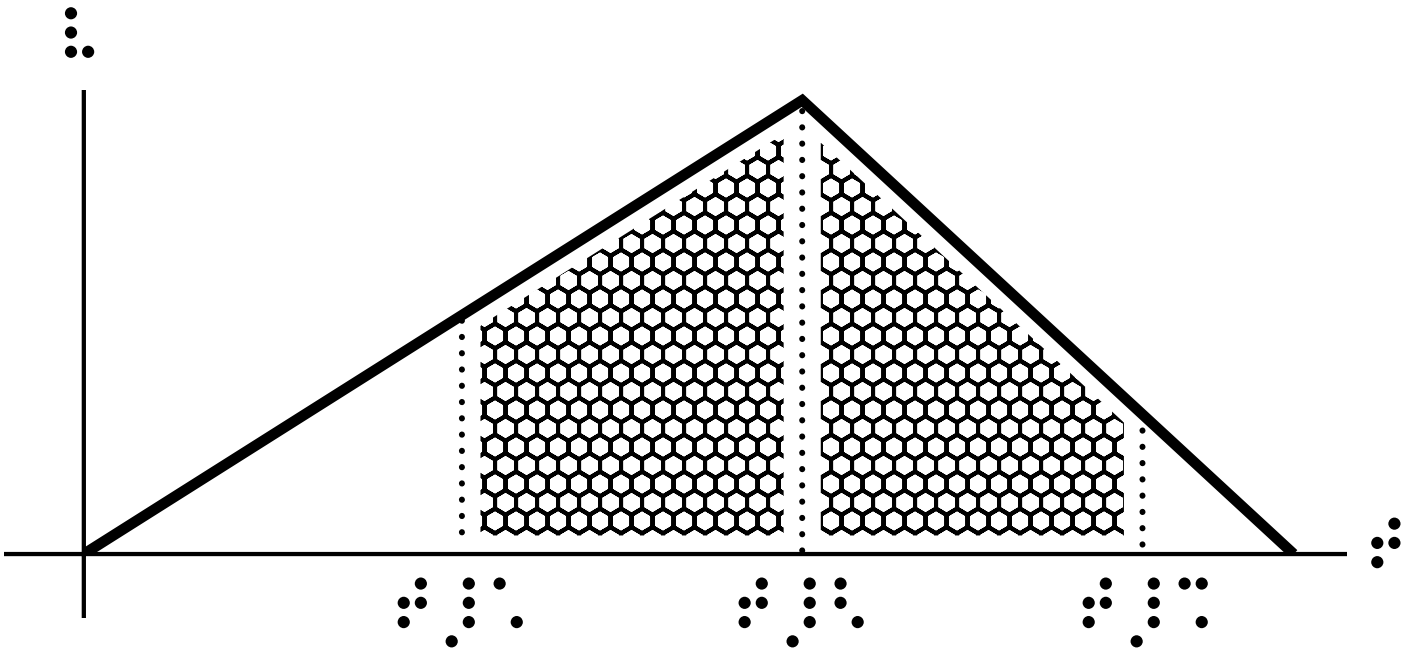
$s = \int_{t_1}^{t_2} (v(t)\, dt)$



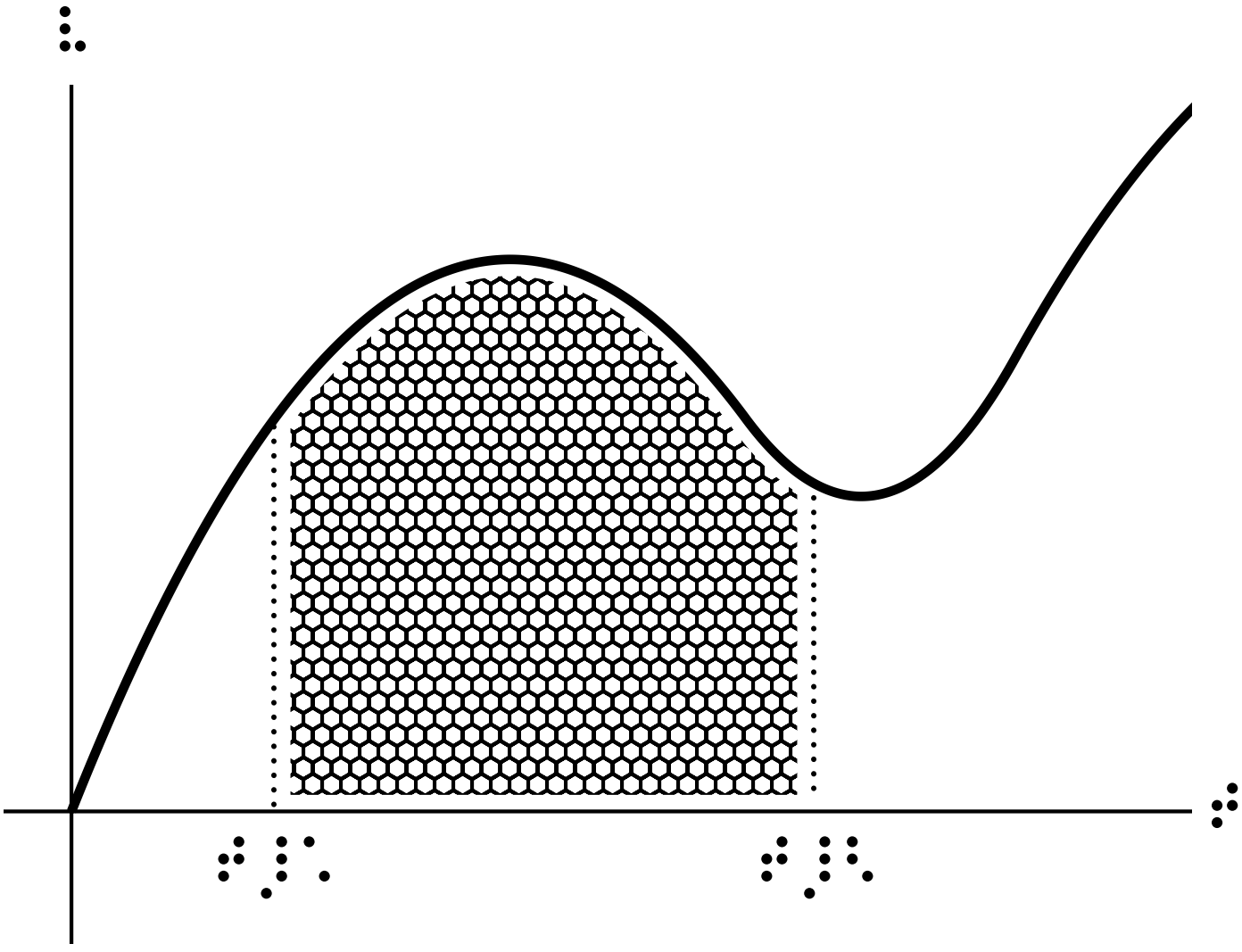


$$s = \int_{t_1}^{t_2} v(t)\,dt + \int_{t_2}^{t_3} v(t)\,dt$$

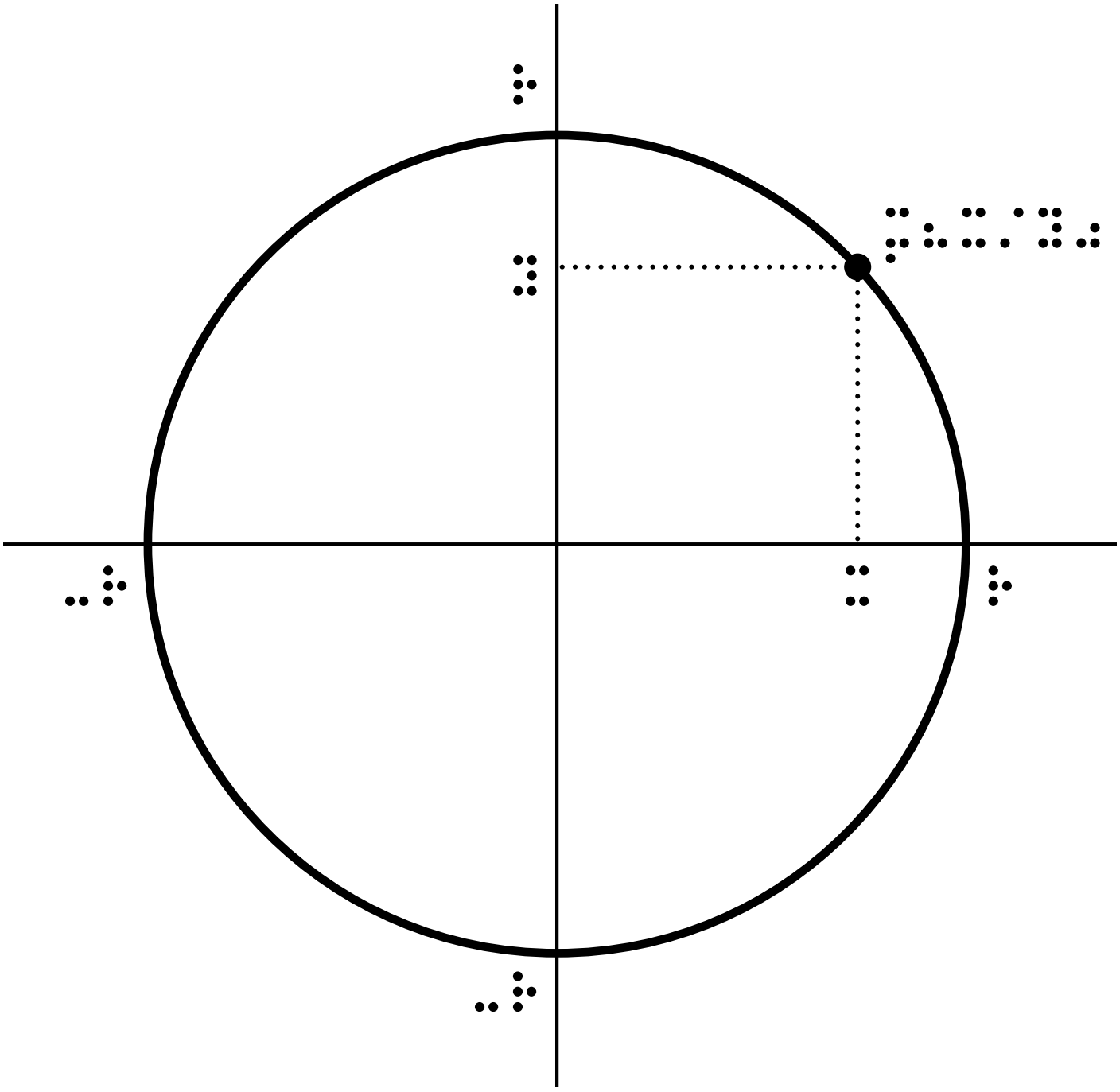




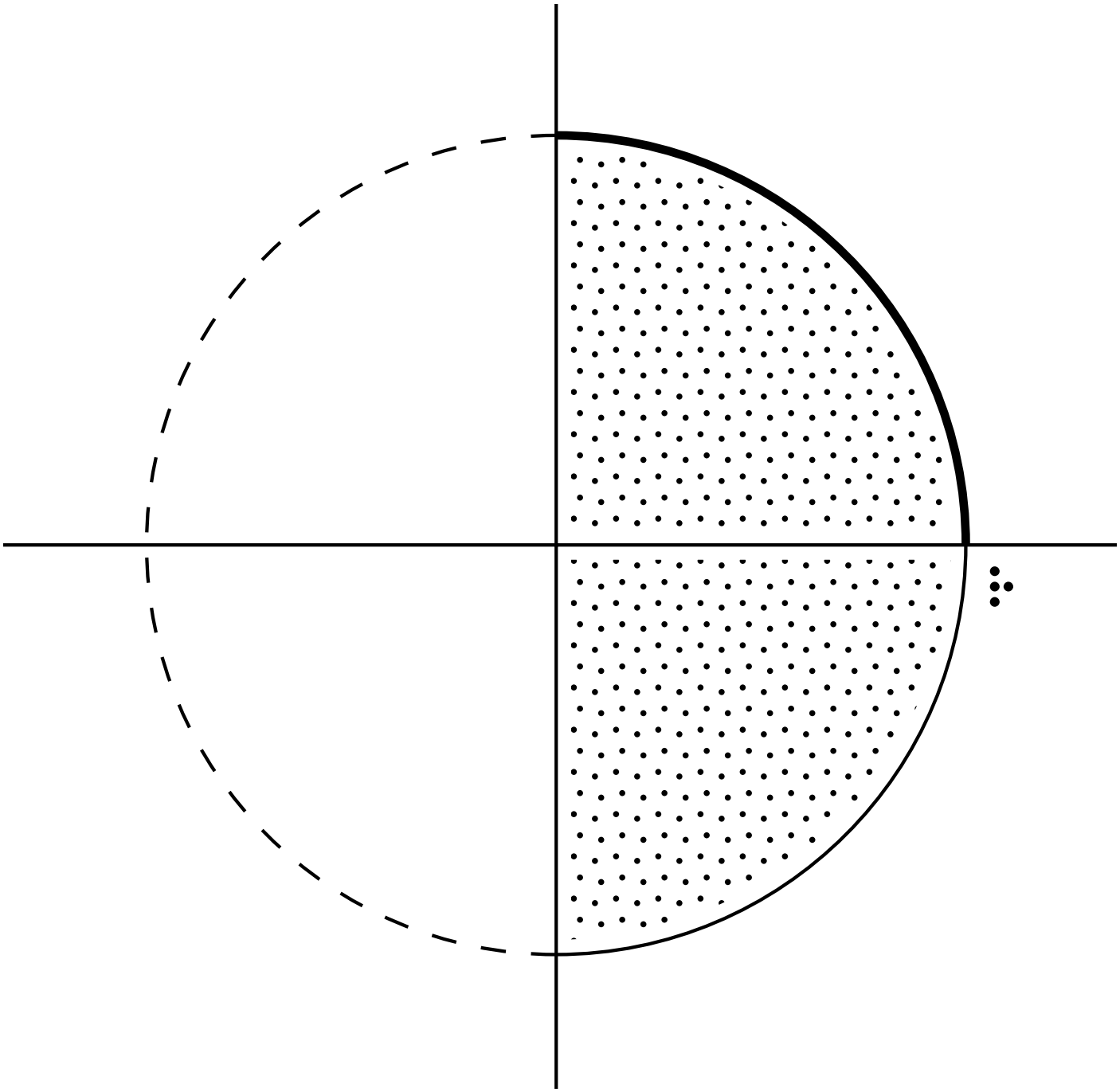
$s = \int_{t_1}^{t_2} v(t)\,dt$



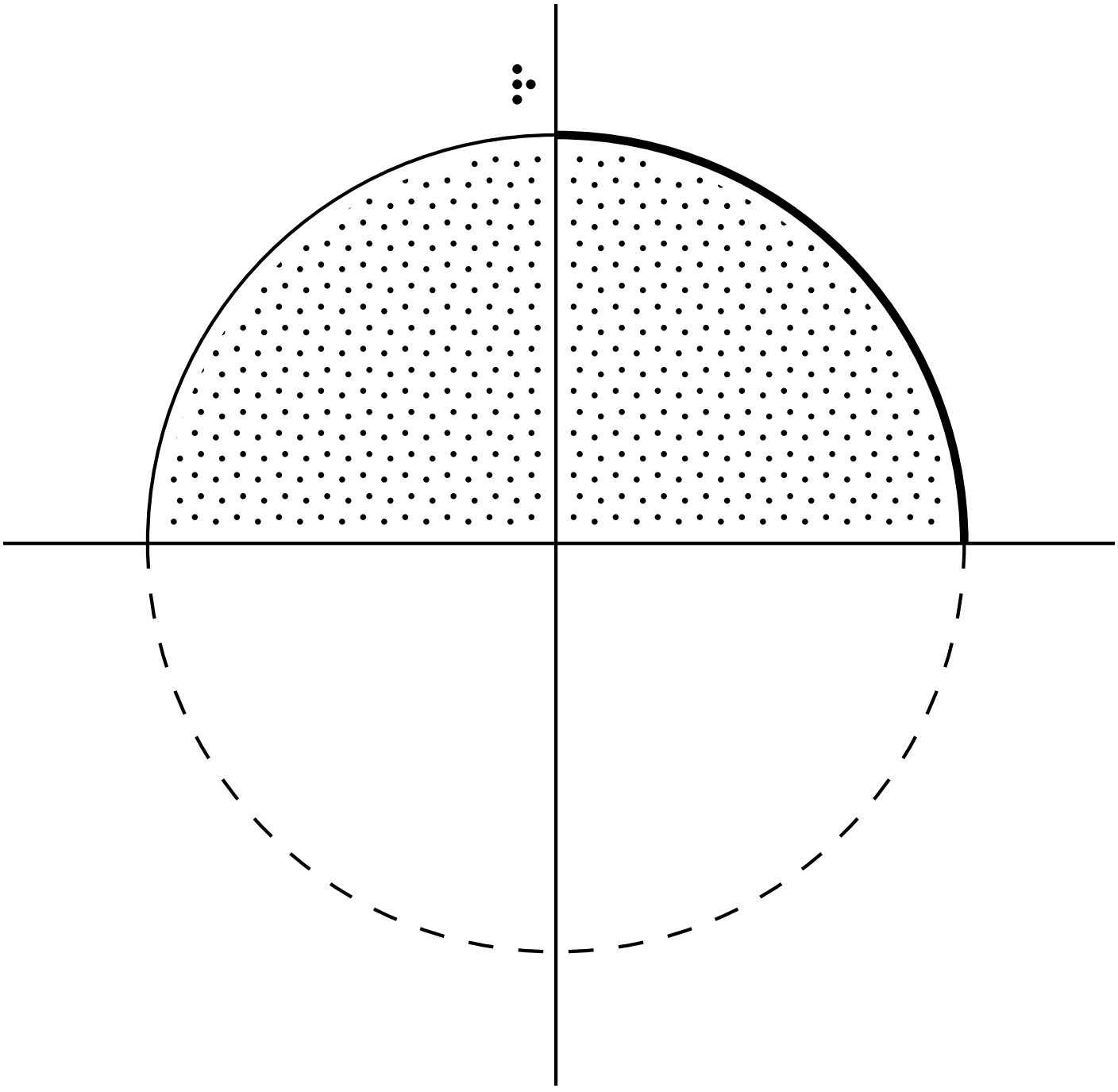
Kreis: $x^2 + y^2 = r^2$



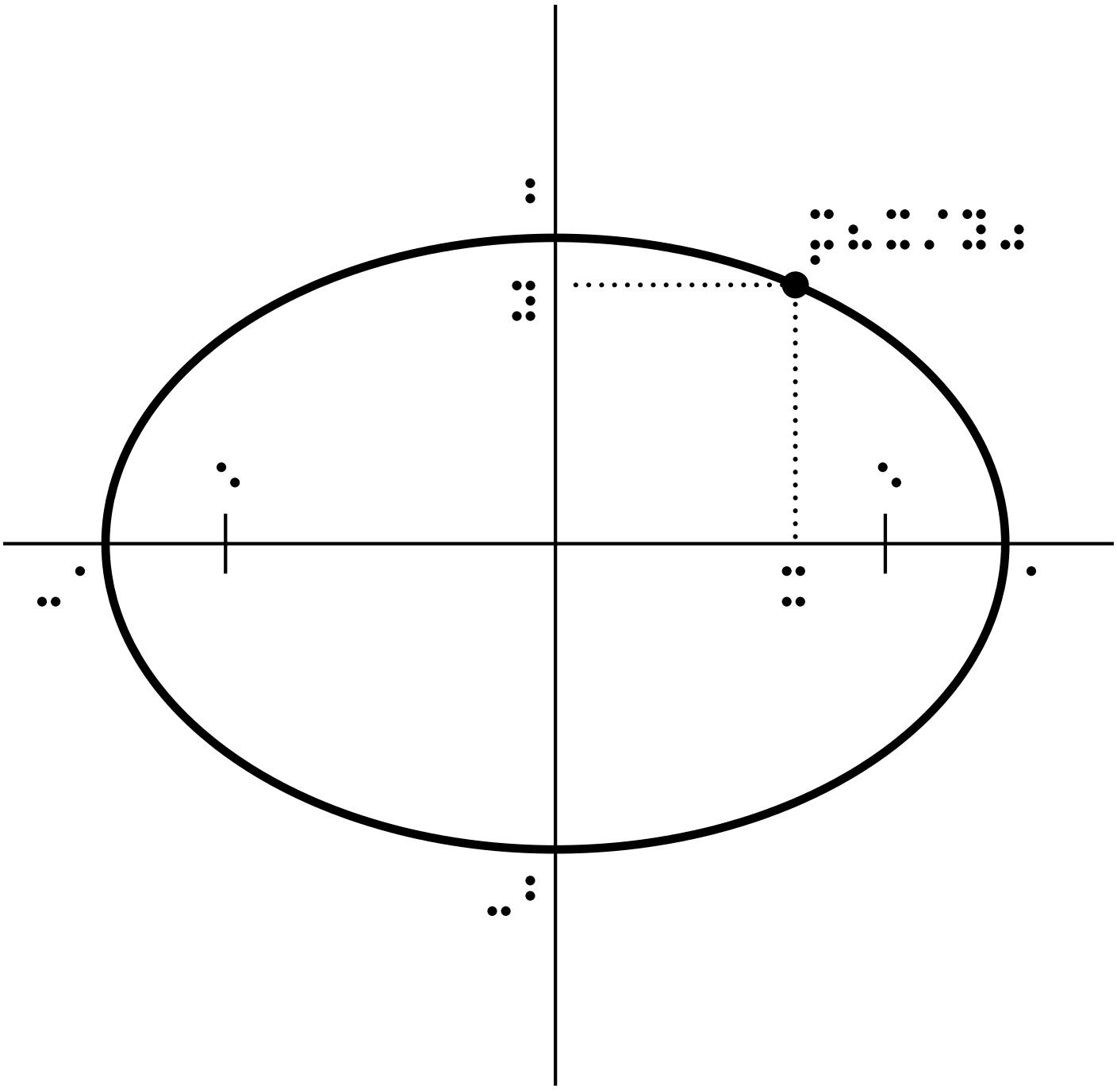
$y^2 = r^2 - x^2$ $V/2 = \pi * \text{int}[0; r] (y^2 dx)$



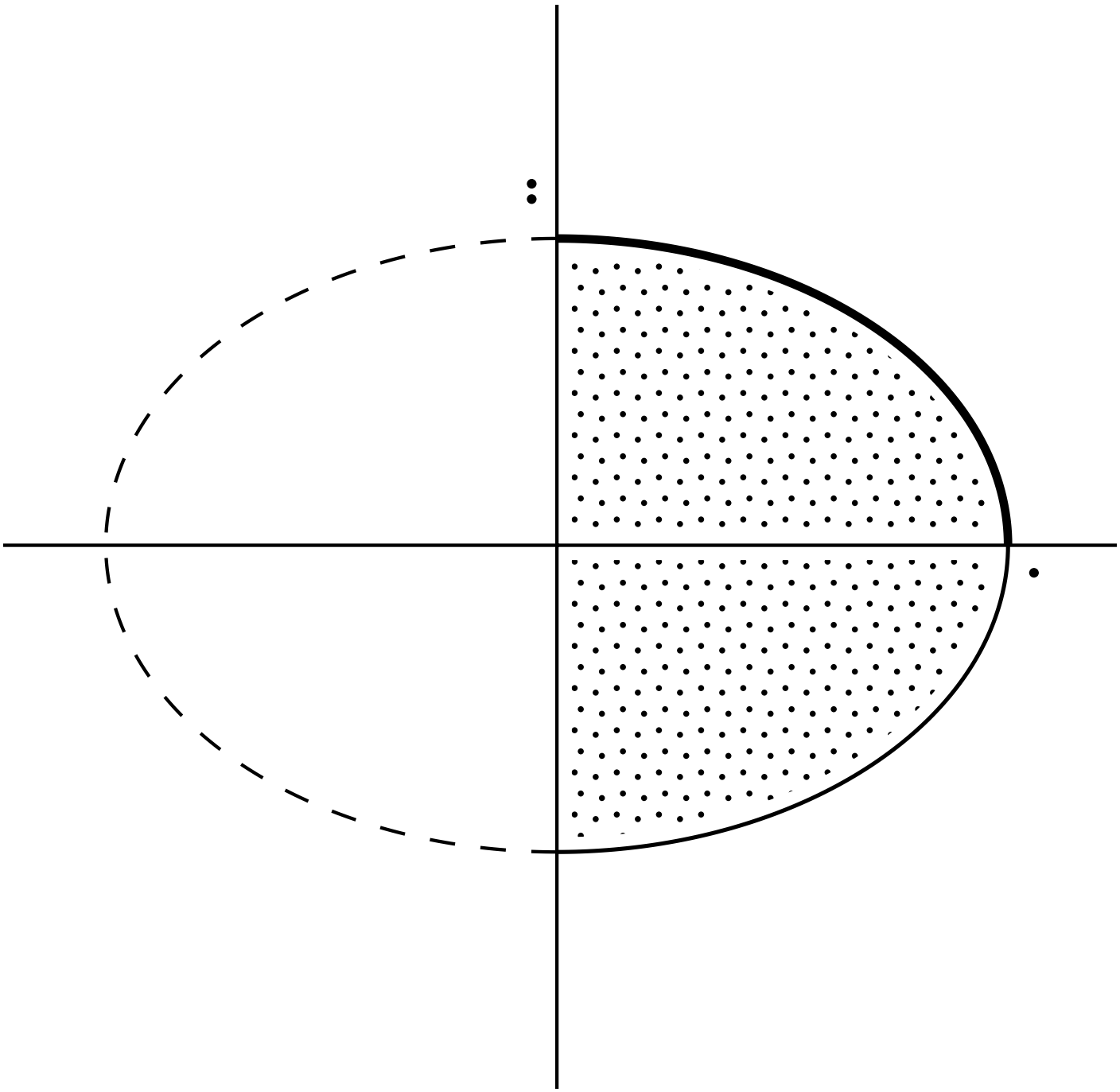
$x^2 = r^2 - y^2$ $V/2 = \pi * \text{int}[0; r] (x^2 dy)$



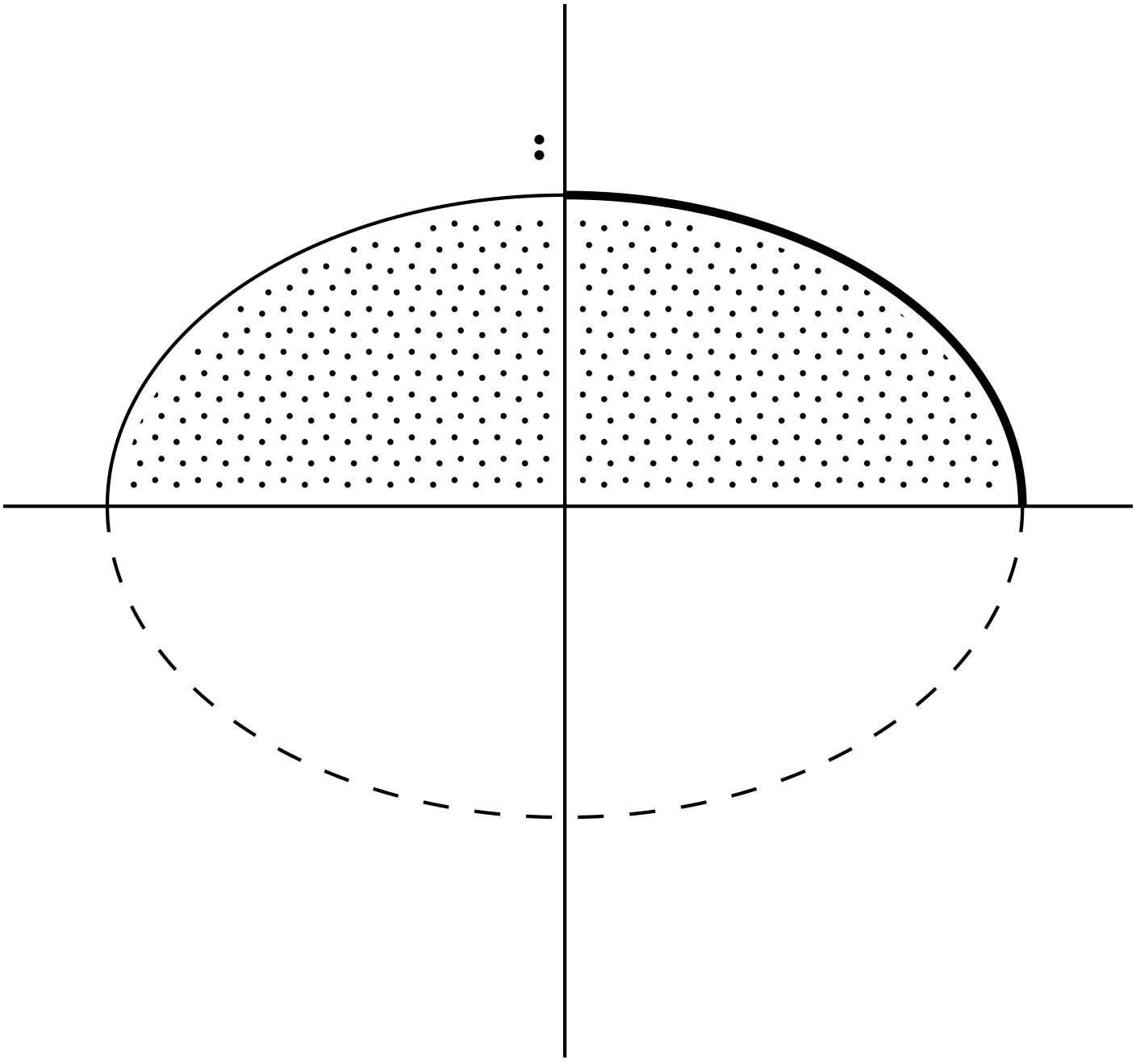
Ellipse: $b^2 * x^2 + a^2 * y^2 = b^2 * a^2$ $e^2 = a^2 - b^2$



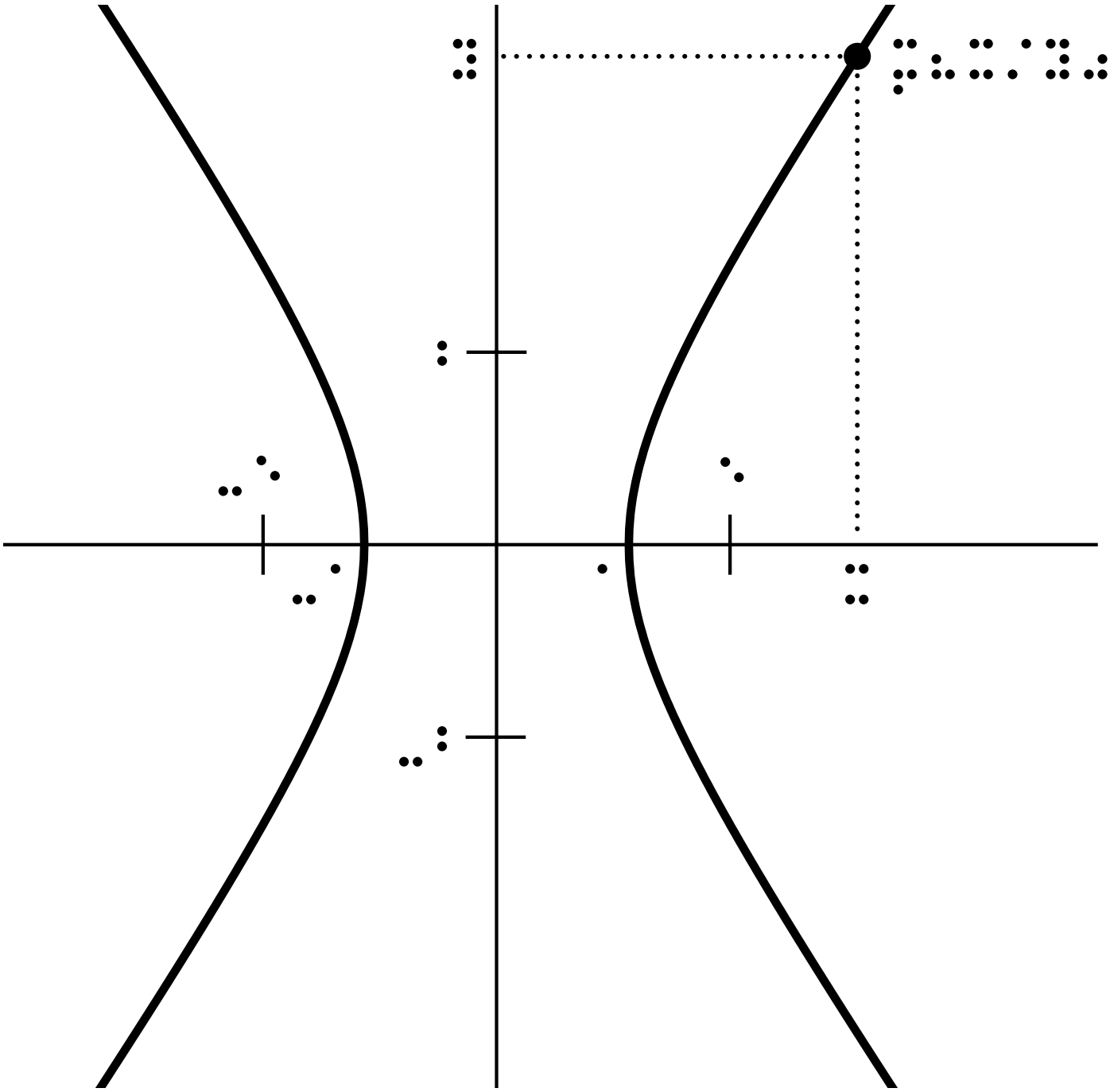
$y^2 = b^2 - b^2/a^2 * x^2$ $V/2 = \pi * \text{int}[0; a] (y^2 dx)$



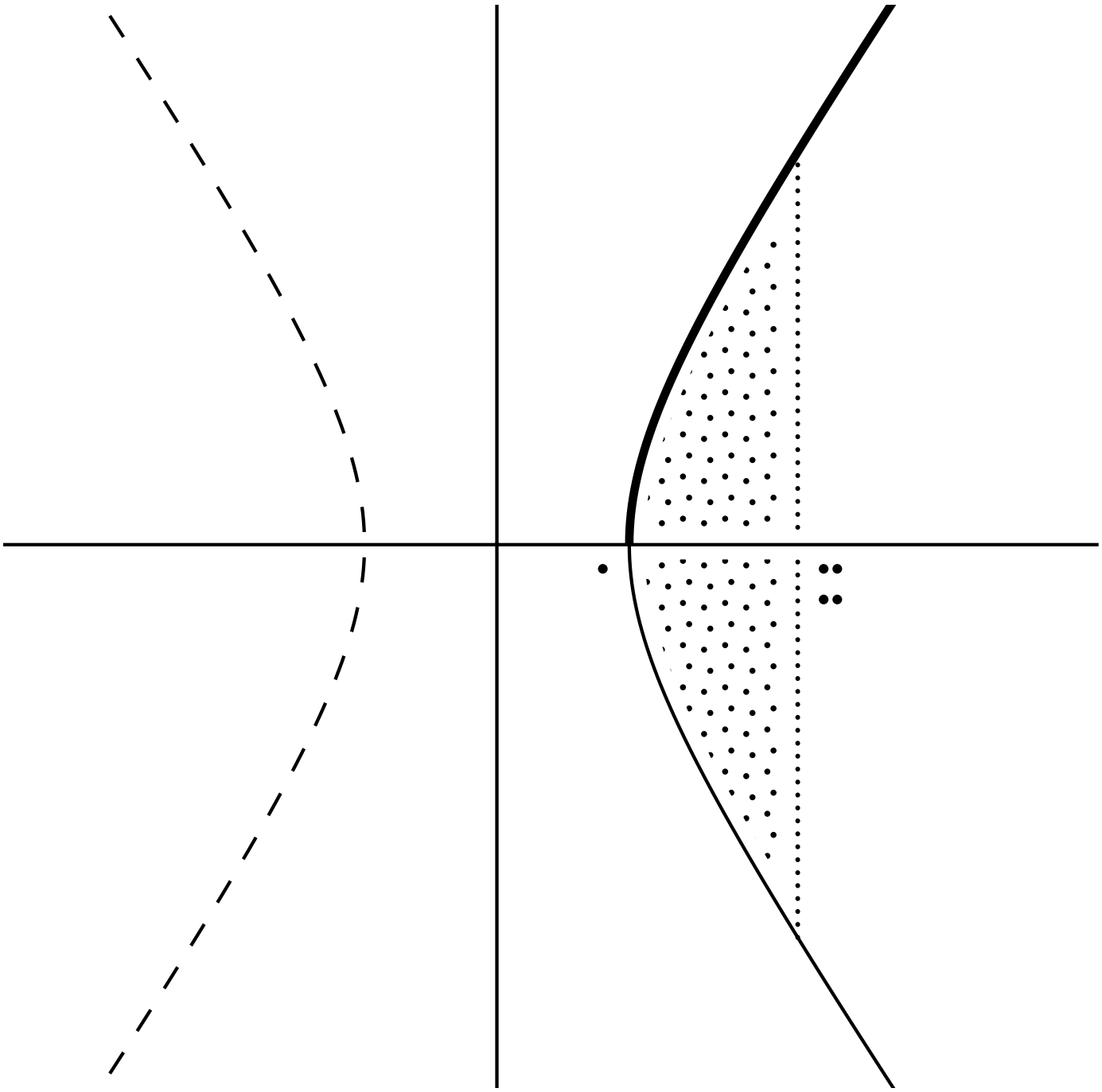
$x^2 = a^2 - a^2 / b^2 * y^2$ $V/2 = \pi * \text{int}[0; b] (x^2 dy)$



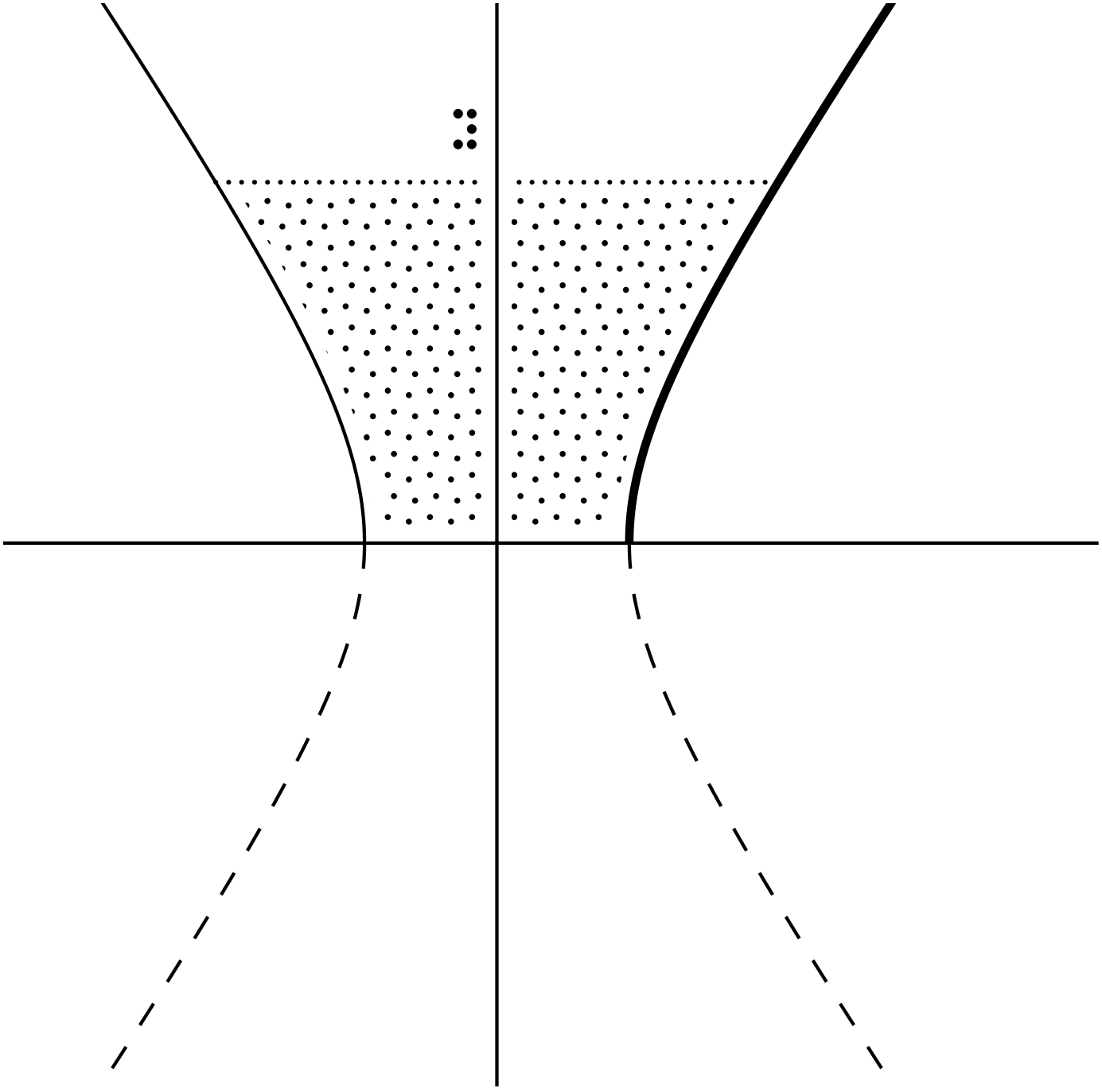
Hyperbel: $b^2 * x^2 - a^2 * y^2 = b^2 * a^2$ $e^2 = a^2 + b^2$



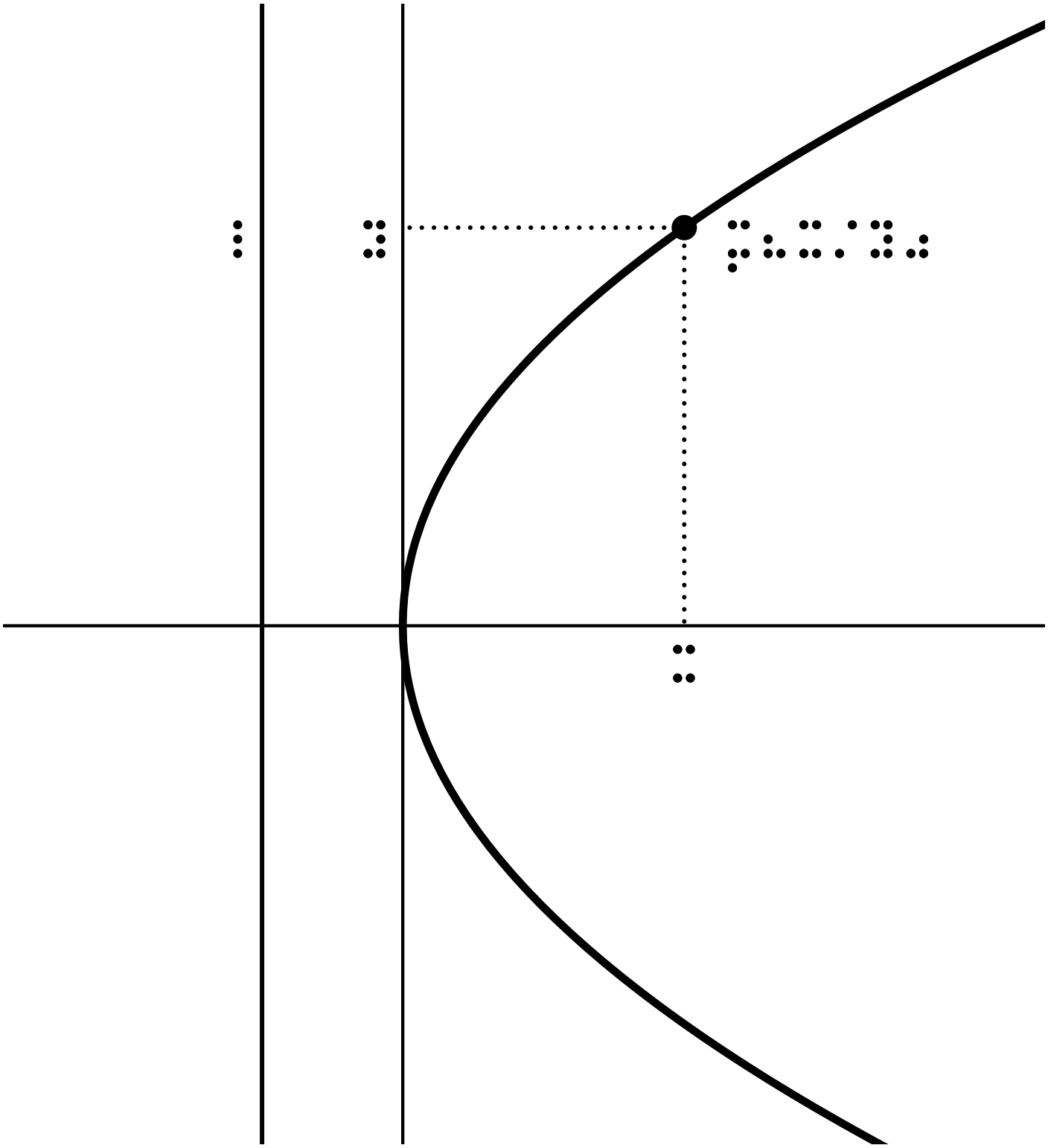
$y^2 = b^2 + b^2 / a^2 * x^2$ $V/2 = \pi * \text{int}[a; x] (y^2 dx)$



$x^2 = a^2 + a^2 / b^2 * y^2$ $V/2 = \pi * \text{int}[0; y] (x^2 dy)$

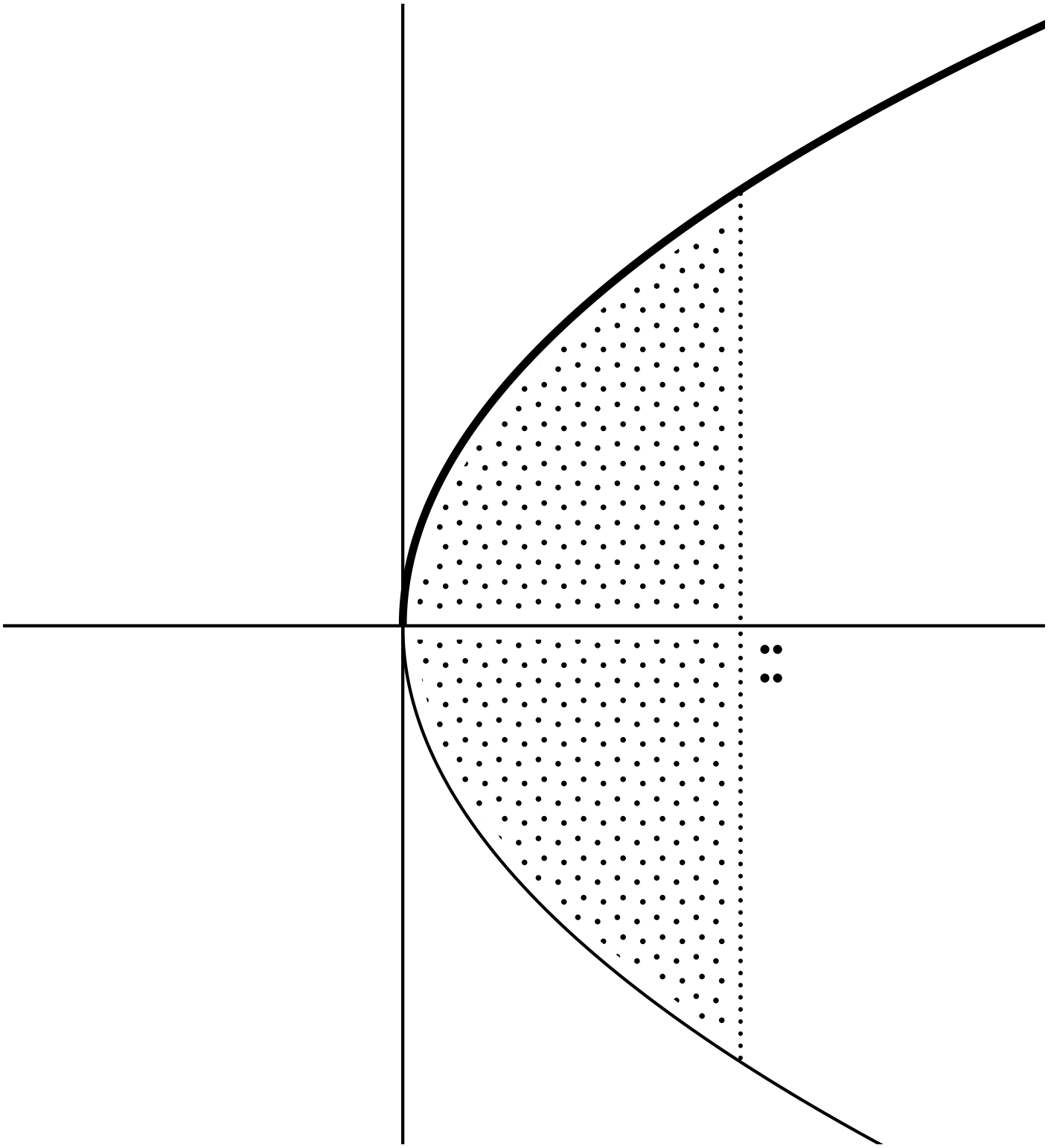


Parabel: $y^2 = 2 * p * x$ $e = p/2; l = -p/2$



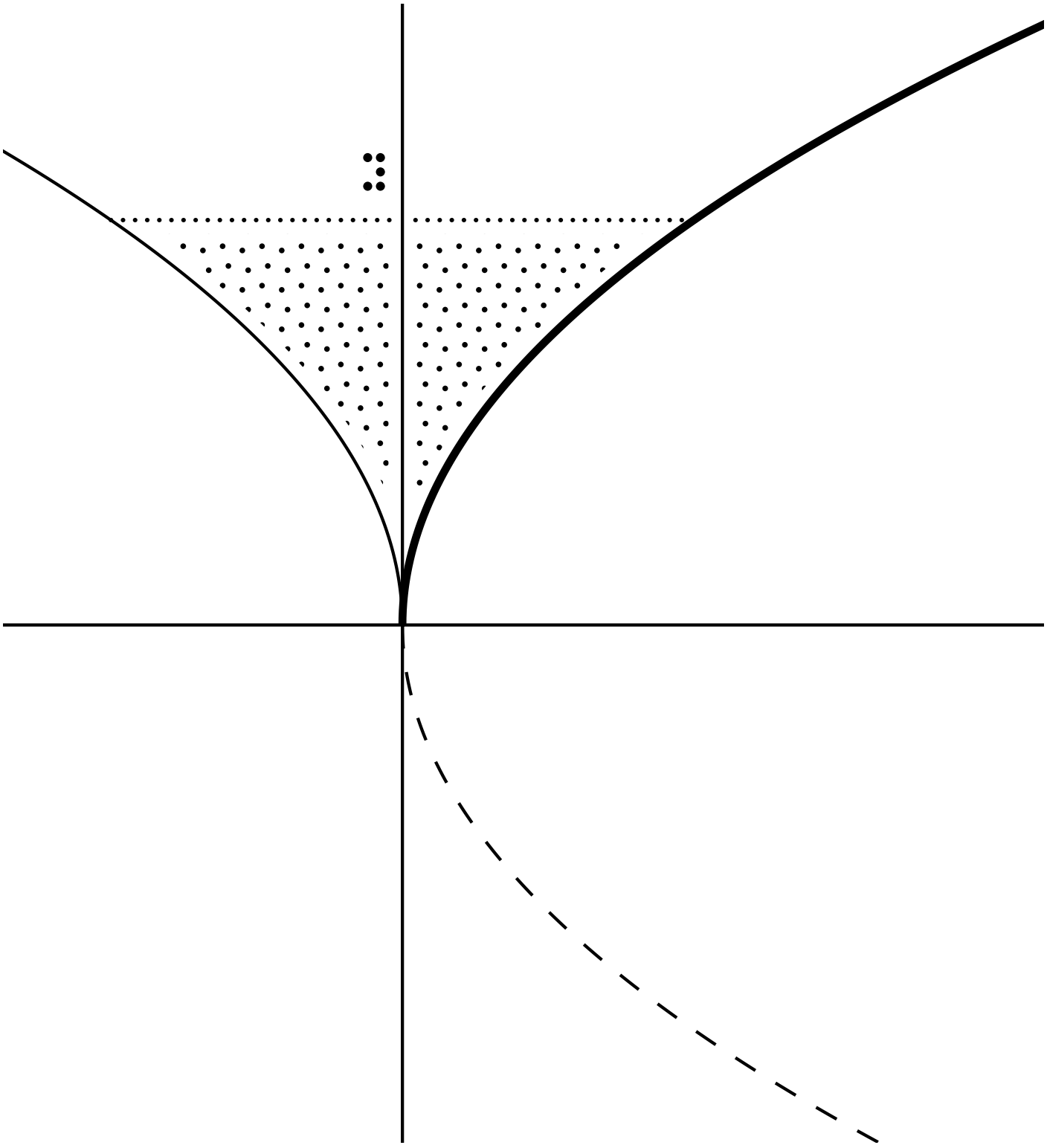
Int Volumina Parabel, 23/24

$y^2 = 2 * p * x$ $V = \pi * \text{int}[0; x] (y^2 dx)$



Int Volumina Parabel, 24/24

$x^2 = (y^2 / 2p)^2$ $V = \pi * \text{int}[0; y]\ (x^2\ dy)$



Wirtschaft

Schulstufe 12

Kostenfunktionen

Grenzkostenfunktionen

Stückkostenfunktionen

Betriebsoptimum

Preis- und Erlösfunktionen

Gewinngrenzen

Gewinnmaximierung

Gewinnfunktionen

Cournot‘sche Punkt

Wirtschaft

Schulstufe 12

Kostenfunktionen

Grenzkostenfunktionen

Stückkostenfunktionen

Betriebsoptimum

Preis- und Erlösfunktionen

Gewinngrenzen

Gewinnmaximierung

Gewinnfunktionen

Cournot'sche Punkt

Inhalt

0 Liste der Abkürzungen

1 Kostenfunktionen K
Variable Kosten KV
K, KV (4 Arten: lin,
prog, degr, s-förm)

2 Grenzkostenfunktionen K'
K und K' (4 Arten)

3 Stückkostenfunktionen K/x
K und K/x (4 Arten)

4 Betriebsoptimum x_(opt)
K'(x) =K(x)/x
(Ks, Ks', Ks/x)

5 Preisfunktionen (p, p(x))
(p_konst, p_lin.fall)
Erlösfunktionen (E, Eq)
(E_lin, E_quadr)

6 - 9 Gewinngrenzen K =E

6 Gewinngrenzen
(K_lin, K_prog, E_lin)

7 Gewinngrenzen
(K_deg, K_s-förm, E_lin)

8 Gewinngrenzen
(K_lin, K_prog, E_quad)

9 Gewinngrenzen
(K_deg, K_s-förm, E_quad)

10 - 11 Gewinnmaximierung

10 Gewinnmaximierung x_m
(K_prog, E_lin)

11 Gewinnmaximierung x_m
(K_s-förm, E_lin)

12 - 15 Gewinnfunktionen

12 Gewinnfunktionen G
G =K -E
(K_lin, K_prog, E_lin)

13 Gewinnfunktionen G
(K_deg, K_s-förm, E_lin)

14 Gewinnfunktionen
(K_lin, K_prog, E_quad)

15 Gewinnfunktionen
(K_deg, K_s-förm, E_quad)

16 Cournot‘sche Punkt C
C (x_m/p(x_m))
(p(x), G, x_C =x_m)

⠦⡅⡸⠇⠊⠝⠂ ⡅⡸⠏⠗⠕⠛⠂ ⡑⡸⠇⠊⠝⠴
⠻ ⡛⠑⠺⠊⠝⠝⠛⠗⠑⠝⠵⠑⠝
⠦⡅⡸⠙⠑⠛⠂ ⡅⡸⠎⠤⠋⢪⠗⠍⠂ ⡑⡸⠇⠊⠝⠴
⠳ ⡛⠑⠺⠊⠝⠝⠛⠗⠑⠝⠵⠑⠝
⠦⡅⡸⠇⠊⠝⠂ ⡅⡸⠏⠗⠕⠛⠂ ⡑⡸⠟⠥⠁⠙⠴
⠪ ⡛⠑⠺⠊⠝⠝⠛⠗⠑⠝⠵⠑⠝
⠦⡅⡸⠙⠑⠛⠂ ⡅⡸⠎⠤⠋⢪⠗⠍⠂ ⡑⡸⠟⠥⠁⠙⠴

⠡⠬ ⠤ ⠡⠡ ⡛⠑⠺⠊⠝⠝⠍⠁⠭⠊⠍⠊⠑⠗⠥⠝⠛
⠡⠬ ⡛⠑⠺⠊⠝⠝⠍⠁⠭⠊⠍⠊⠑⠗⠥⠝⠛ ⠭⡸⠍
⠦⡅⡸⠏⠗⠕⠛⠂ ⡑⡸⠇⠊⠝⠴
⠡⠡ ⡛⠑⠺⠊⠝⠝⠍⠁⠭⠊⠍⠊⠑⠗⠥⠝⠛ ⠭⡸⠍
⠦⡅⡸⠎⠤⠋⢪⠗⠍⠂ ⡑⡸⠇⠊⠝⠴

⠡⠣ ⠤ ⠡⠱ ⡛⠑⠺⠊⠝⠝⠋⠥⠝⠅⠞⠊⠕⠝⠑⠝
⠡⠣ ⡛⠑⠺⠊⠝⠝⠋⠥⠝⠅⠞⠊⠕⠝⠑⠝ ⡛
⡛ ⠶⡅ ⠤⡑
⠦⡅⡸⠇⠊⠝⠂ ⡅⡸⠏⠗⠕⠛⠂ ⡑⡸⠇⠊⠝⠴
⠡⠩ ⡛⠑⠺⠊⠝⠝⠋⠥⠝⠅⠞⠊⠕⠝⠑⠝ ⡛
⠦⡅⡸⠙⠑⠛⠂ ⡅⡸⠎⠤⠋⢪⠗⠍⠂ ⡑⡸⠇⠊⠝⠴

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Wirtschaft - Abkürzungen

Index

C: Cournot‘sche Punkt (liegt auf der Preisfunktion mit dem x-Wert: x_m

E: Erlösfunktion (p *x)

Eq: E_quad, quadrat. E

F: Fixkosten

G: Gewinnfunktion (Erlös minus Kosten)

GE: Geldeinheiten

H: Höchstpreis

K: Kostenfunktion:

Kl: K_lin, lineare K,

Kp: K_prog, progressive K

Kd: K_deg, degressive K

Ks: K_s-förm, S-förmige K

K': Grenzkostenfunktion
Preis pro zusätzlich erzeugter ME

K^-: Stückkostenfunktion
K/x

ME: Mengeneinheiten

p: Preis (fix)

p(x): von Nachfrage abhängiger Preis (Nachfragefunktion)

p_C: Preis bei Verkauf von Cournot‘scher Menge

W: Wirtschaft

x_C: Cournot‘sche Menge,
gewinnmaximale Produktionsmenge
eines Monopolisten
x_C =x_m

x_m: gewinnmaximale Produktionsmenge
(G'(x_(m) =0)

x_o: obere Gewinngrenze
(2. Break-Even-Point)

x_S: Sättigungsmenge

x_u: untere Gewinngrenze
(1. Break-Even-Point)

x_W Kostenkehre
Wendestelle von K

x_(opt): Produktionsmenge
mit minimalen Stückkosten
K'(x) =K^-

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⡅⠲⠭
⡍⡑⠒ ⡍⠑⠝⠛⠑⠝⠑⠊⠝⠓⠑⠊⠞⠑⠝
⠏⠒ ⡏⠗⠑⠊⠎ ⠦⠋⠊⠭⠴
⠏⠦⠭⠴⠒ ⠧⠕⠝ ⡝⠁⠉⠓⠋⠗⠁⠛⠑ ⠁⠃⠓⢜⠝⠤
⠛⠊⠛⠑⠗ ⡏⠗⠑⠊⠎ ⠦⡝⠁⠉⠓⠋⠗⠁⠛⠑⠋⠥⠝⠅⠤
⠞⠊⠕⠝⠴
⠏⡸⡉⠒ ⡏⠗⠑⠊⠎ ⠃⠑⠊ ⡧⠑⠗⠅⠁⠥⠋ ⠧⠕⠝
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⡺⠒ ⡺⠊⠗⠞⠎⠉⠓⠁⠋⠞
⠭⡸⡉⠒ ⡉⠕⠥⠗⠝⠕⠞⡶⠎⠉⠓⠑ ⡍⠑⠝⠛⠑⠂
⠛⠑⠺⠊⠝⠝⠍⠁⠭⠊⠍⠁⠇⠑ ⡏⠗⠕⠙⠥⠅⠤
⠞⠊⠕⠝⠎⠍⠑⠝⠛⠑
⠑⠊⠝⠑⠎ ⡍⠕⠝⠕⠏⠕⠇⠊⠎⠞⠑⠝
⠭⡸⡉ ⠶⠭⡸⠍
⠭⡸⠍⠒ ⠛⠑⠺⠊⠝⠝⠍⠁⠭⠊⠍⠁⠇⠑ ⡏⠗⠕⠙⠥⠅⠤
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⠦⡛⡶⠦⠭⡸⠦⠍⠴ ⠶⠬⠴

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Ergänzungen zu Seite 1

Kl: linearer Kostenverlauf
F: Fixkosten
KVl: linearer Kostenverlauf ohne Fixkosten

Kp: progressiver Kostenverlauf
linksgekrümmt (pos. gekrümmt), strg mon st,
überproportional
$K' > 0$; $K'' > 0$

Kd: degressiver Kostenverlauf
rechtsgekrümmt (neg. gekrümmt), strg mon st,
unterproportional
$K' > 0$; $K'' < 0$

Ks: S-förmiger Kostenverlauf
erst rechtsgekrümmt (neg. gekrümmt),
dann linksgekrümmt (pos. gekrümmt)
x_W =Kostenkehre (Wendestelle)
$K''(x_W) = 0$

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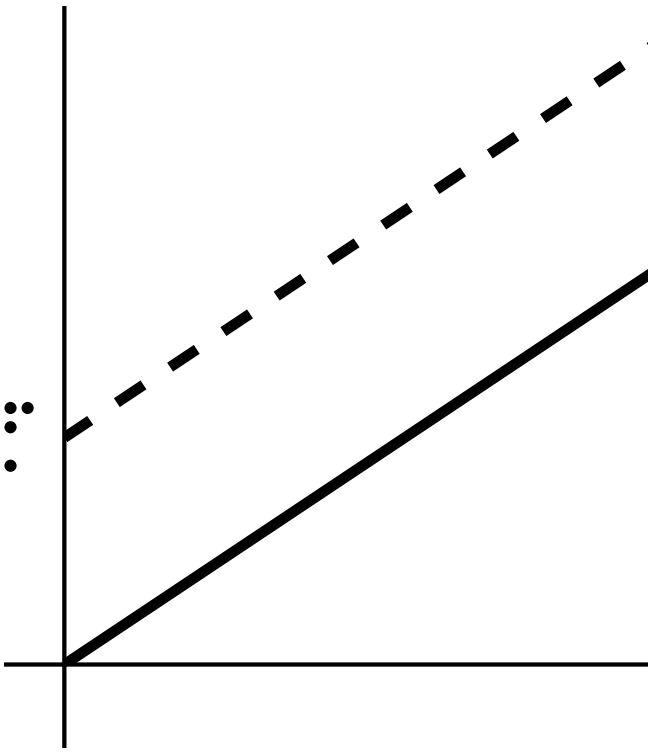
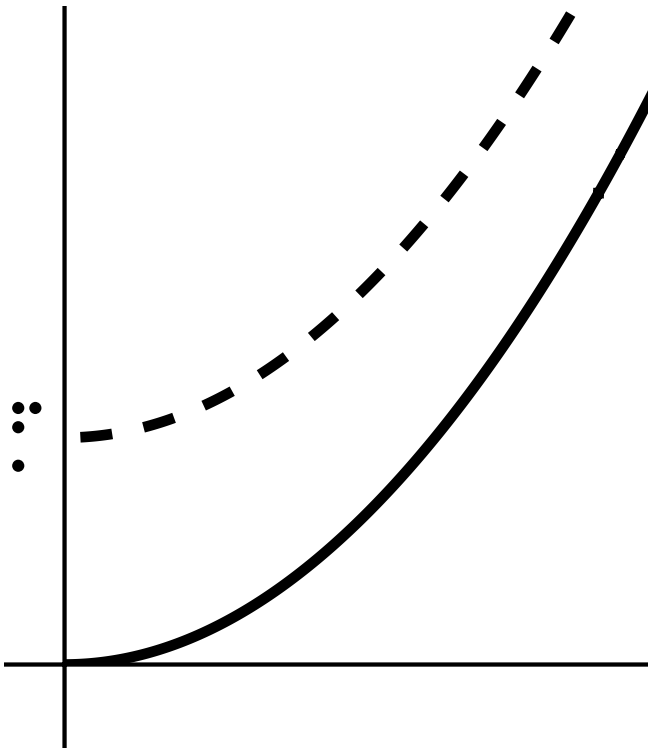
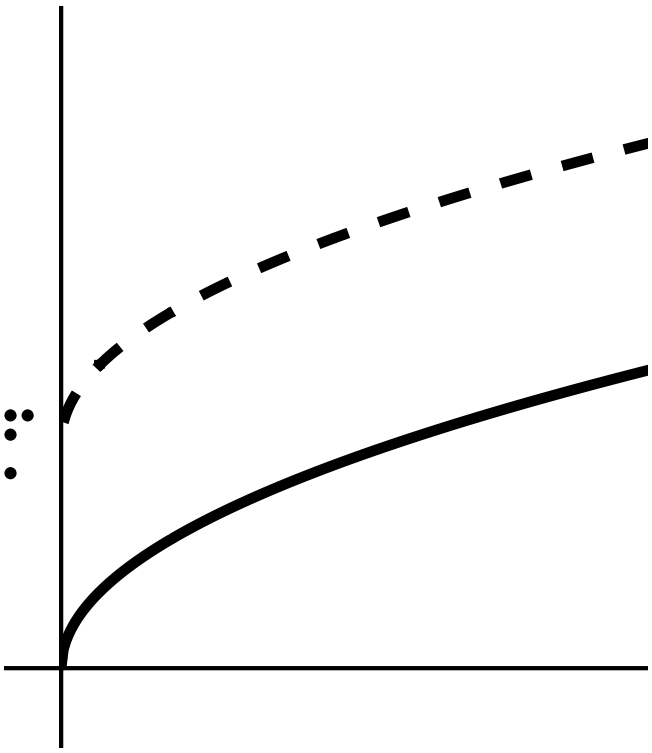
K_l; KV_l



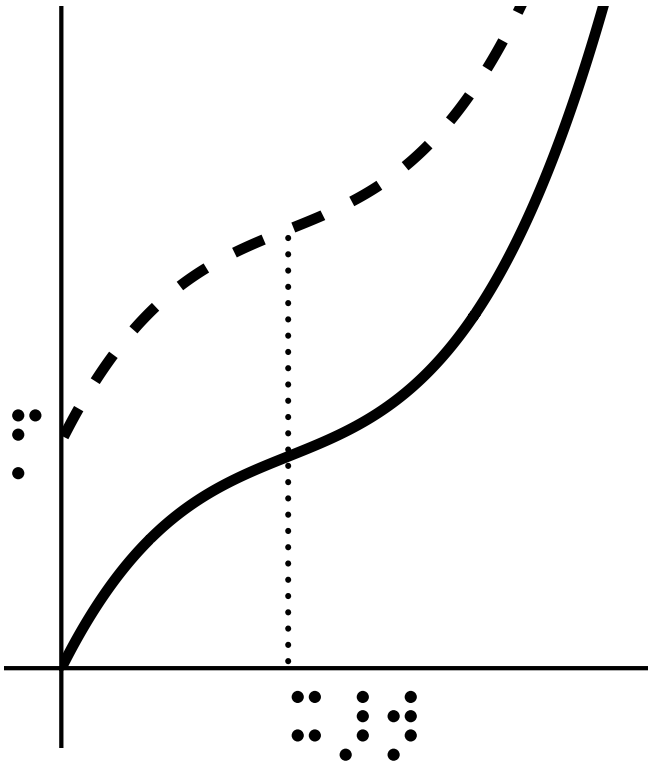
K_p; KV_p



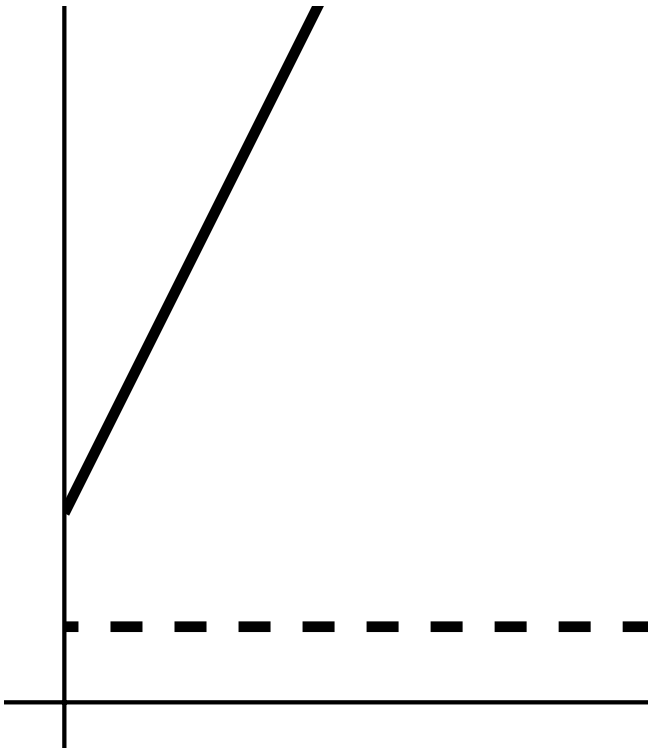
K_d; KV_d



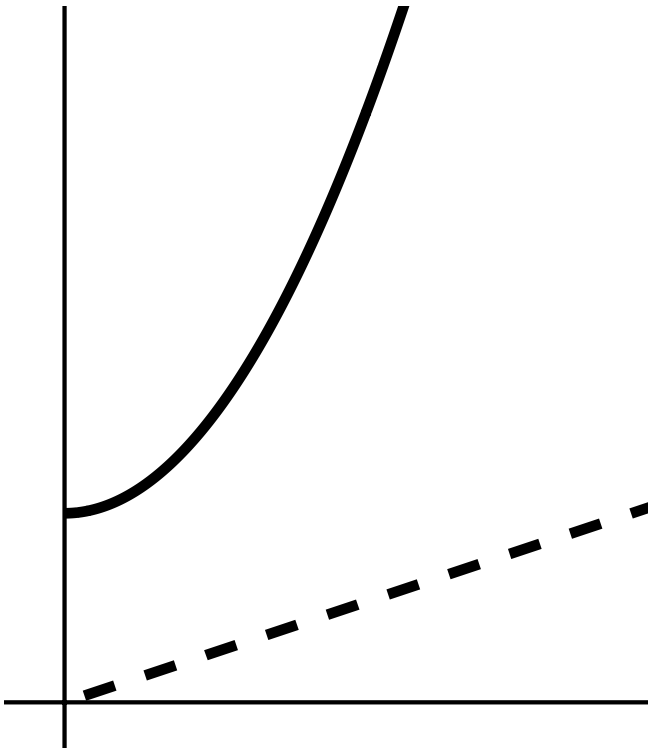
K_s; KV_s



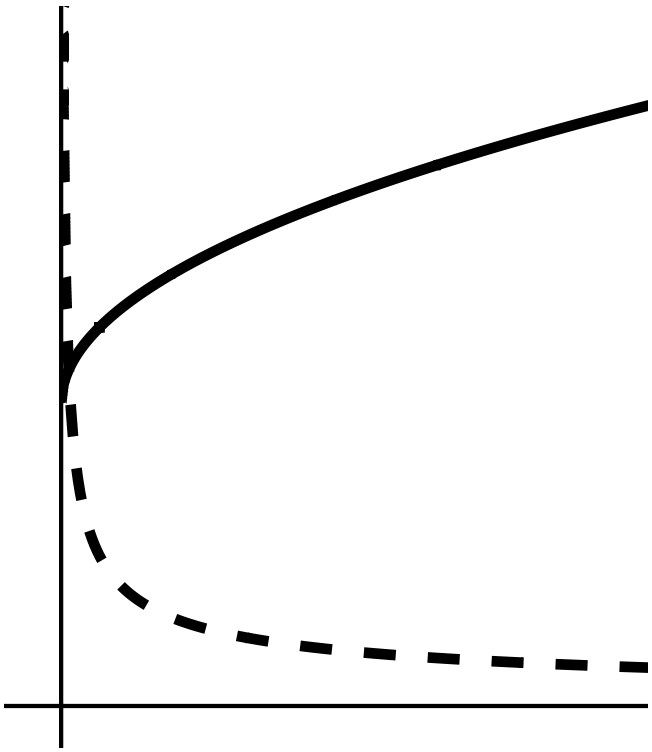
K_l; K_l'



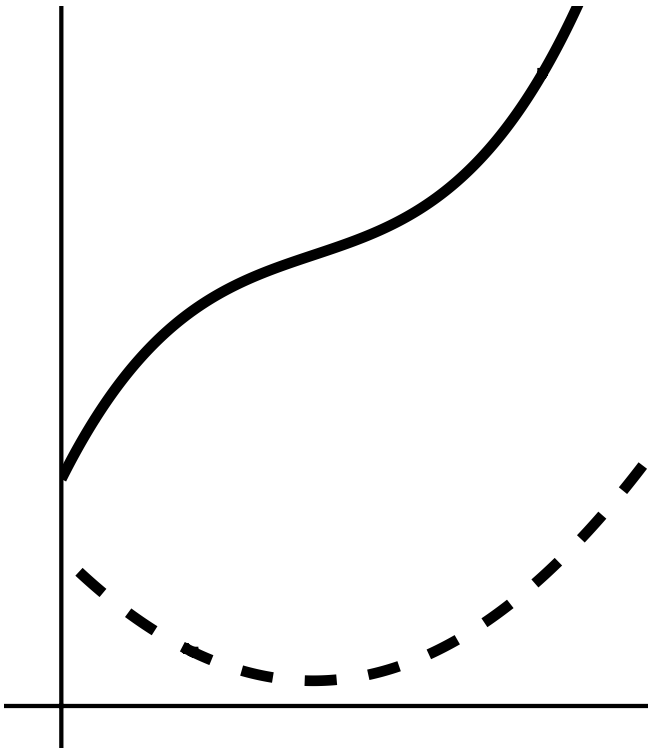
K_p; K_p'



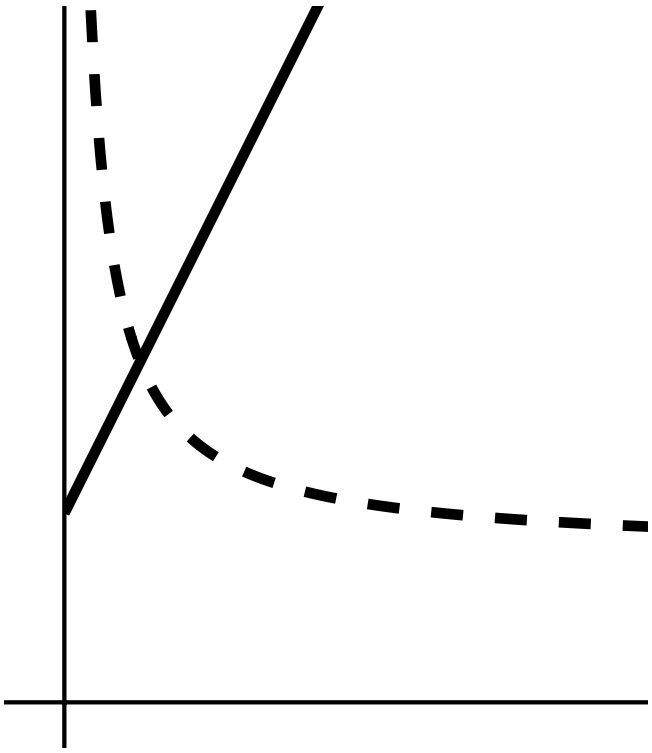
K_d; K_d'



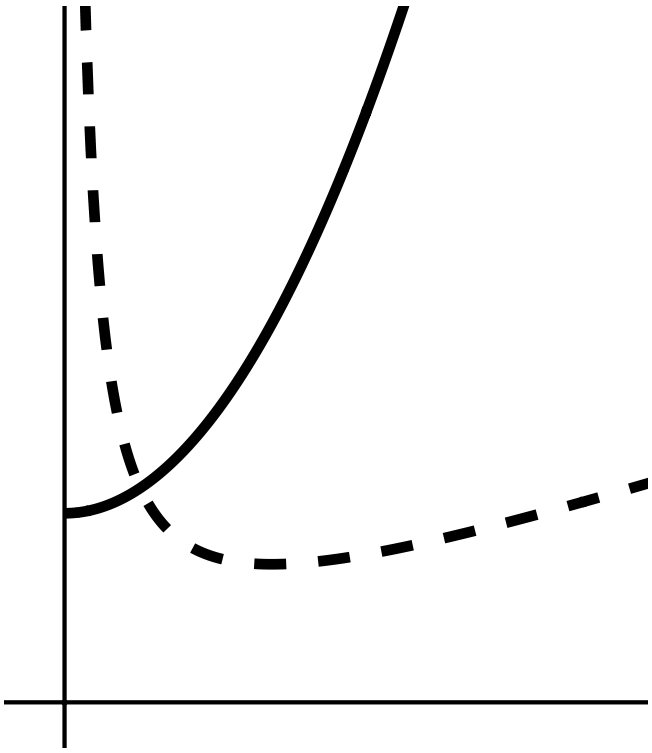
K_s; K_s'



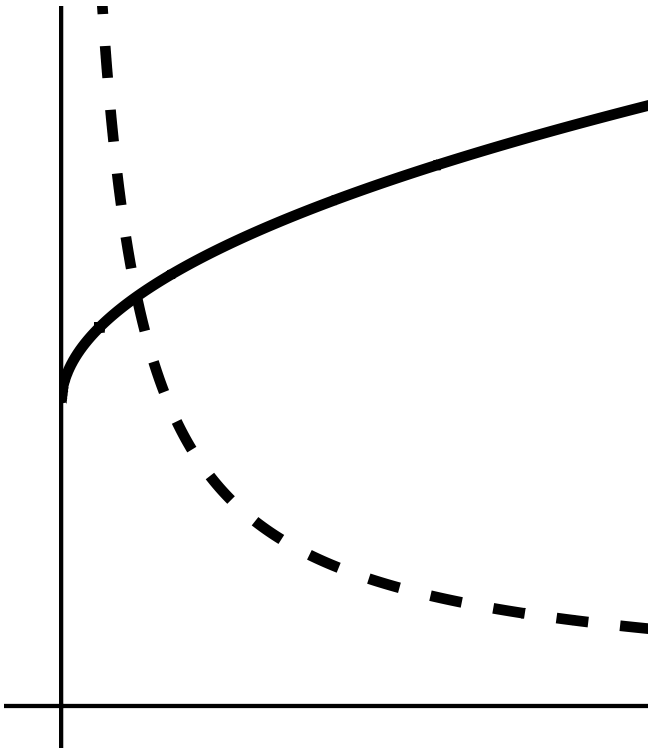
K_I; K_I/x



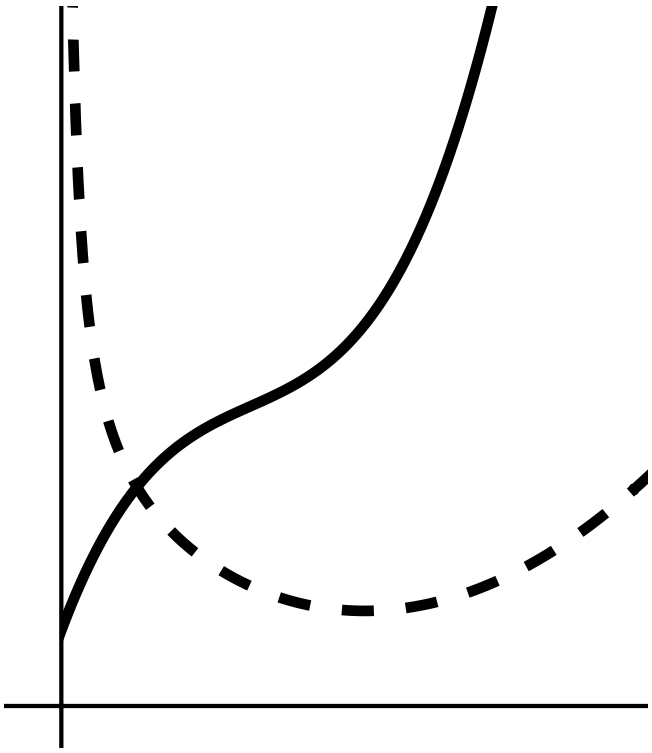
K_p; K_p/x



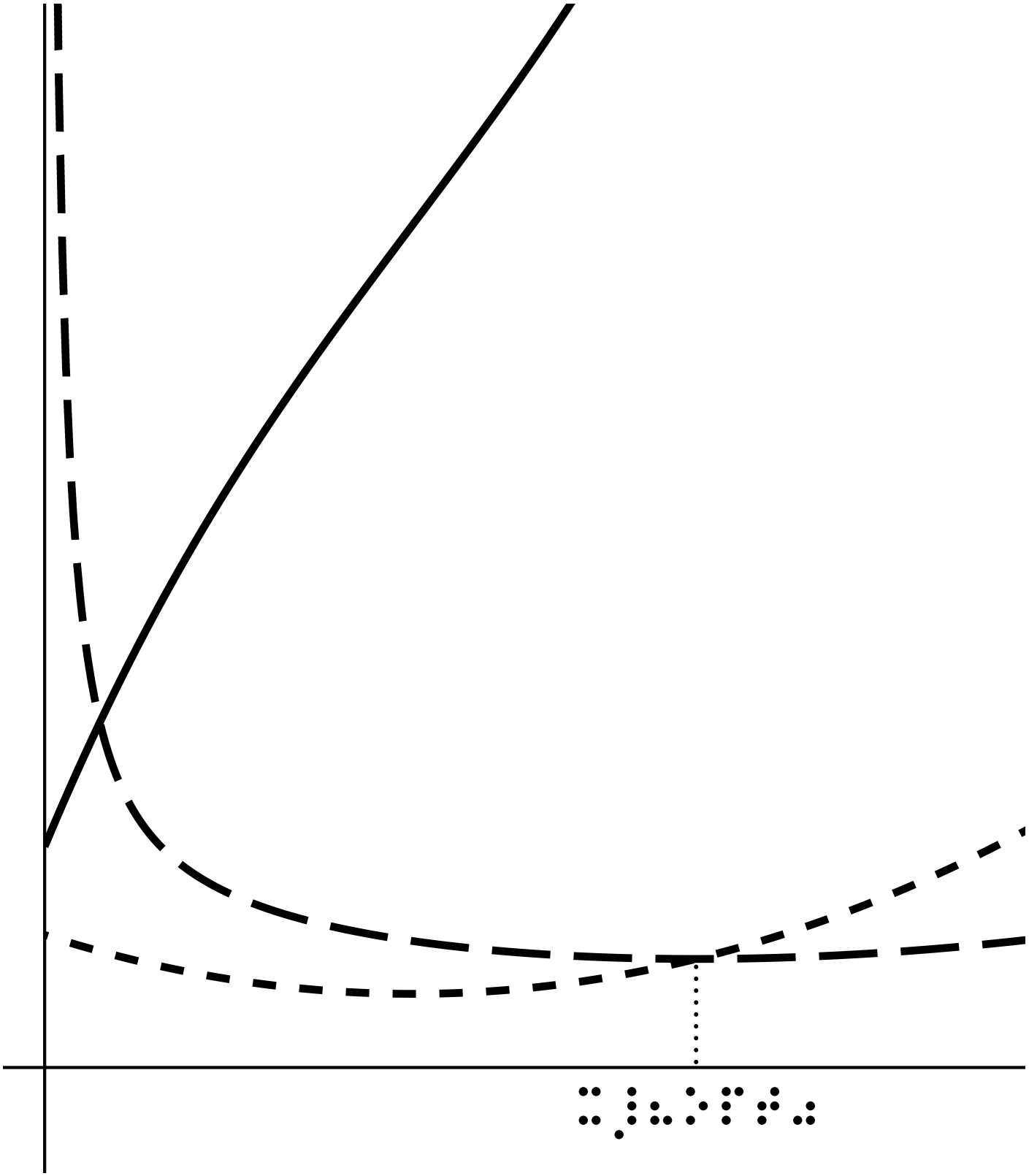
K_d; K_d/x



K_s; K_s/x



K_s; K_s'; K_s/x



Ergänzungen zu Seite 5

Fixpreis: p

Erlös linear steigend

Nachfragefunktion: p(x)

H(0|p(0))

Höchstpreis H =Schnittpunkt mit der y-Achse

S(x_S|0)

Sättigungsmenge x_S =Schnittstelle mit der x-Achse

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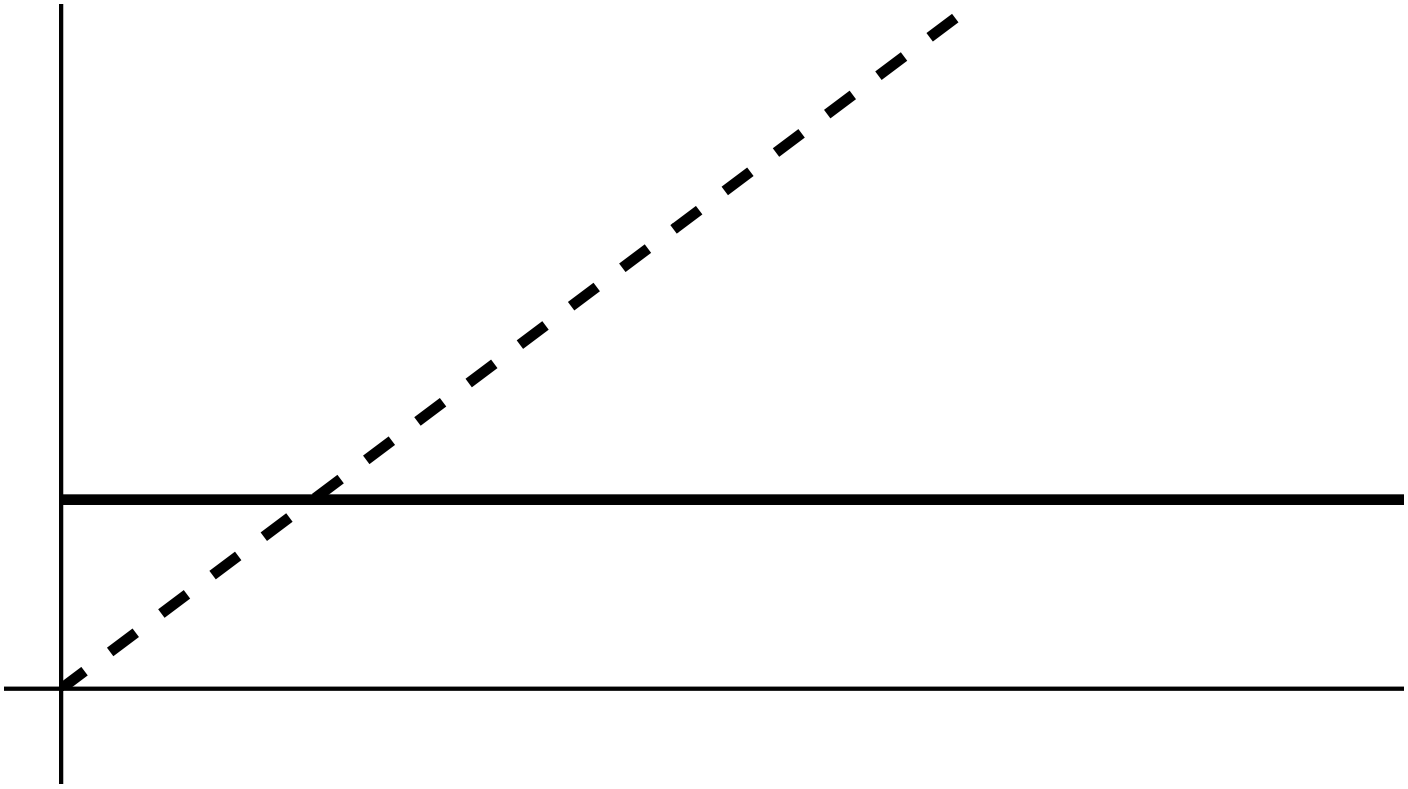
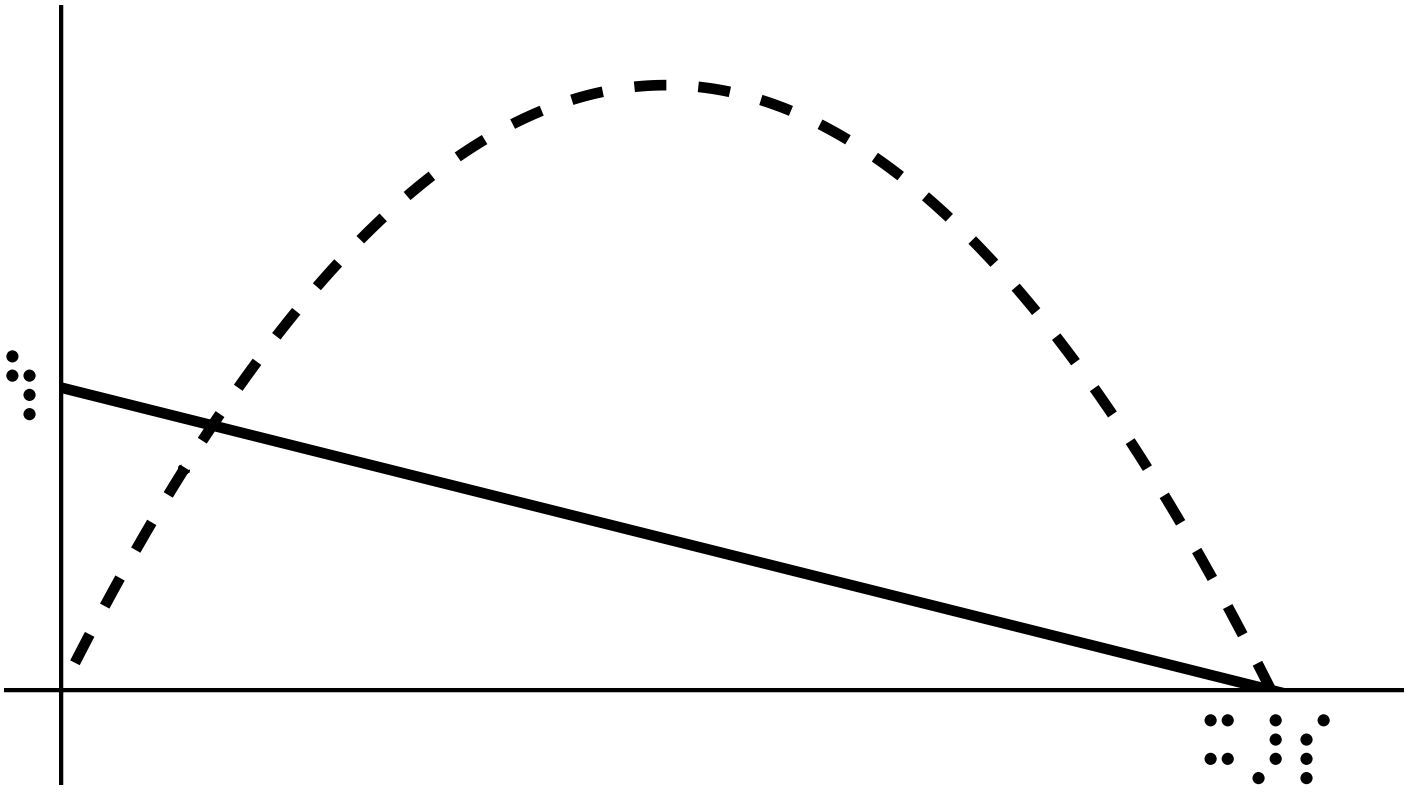
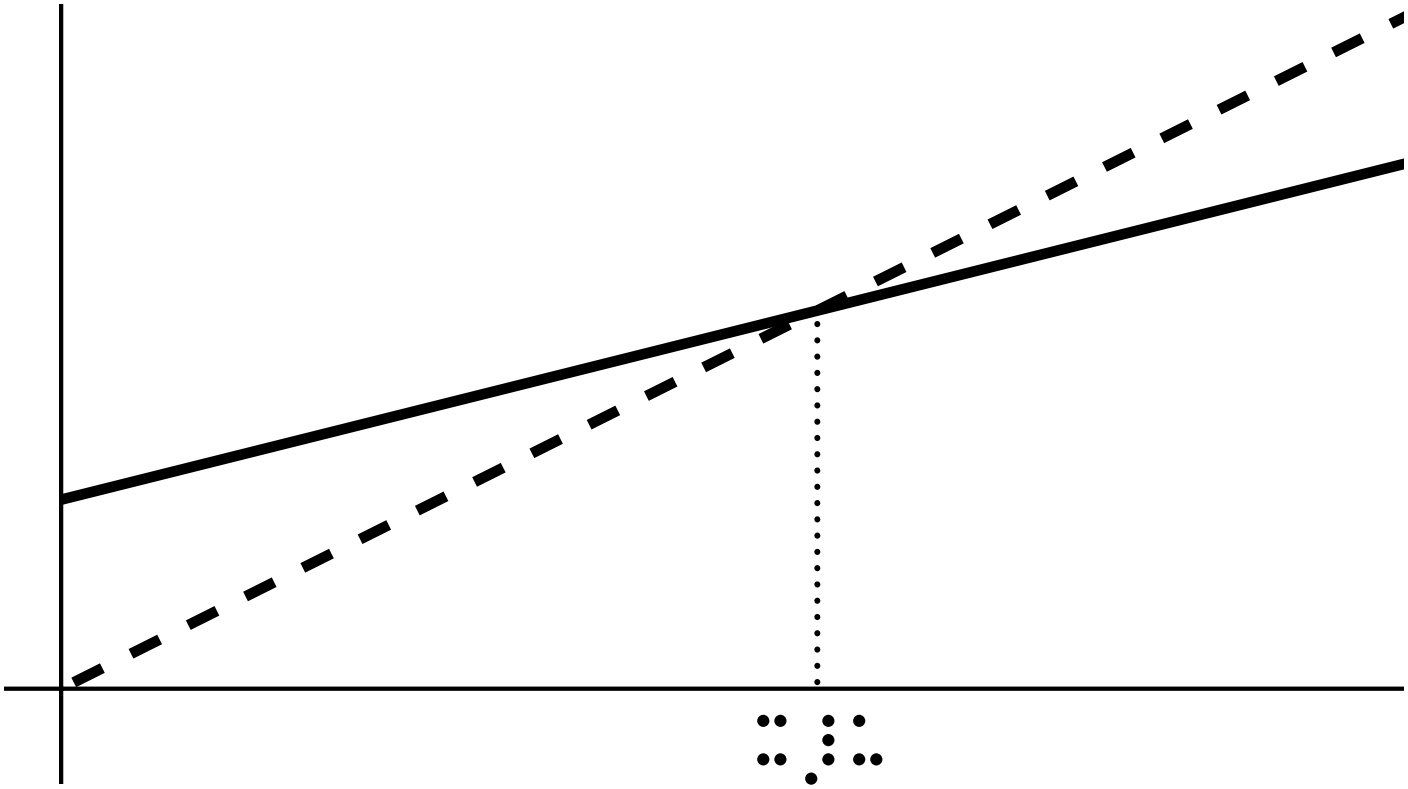
p; E



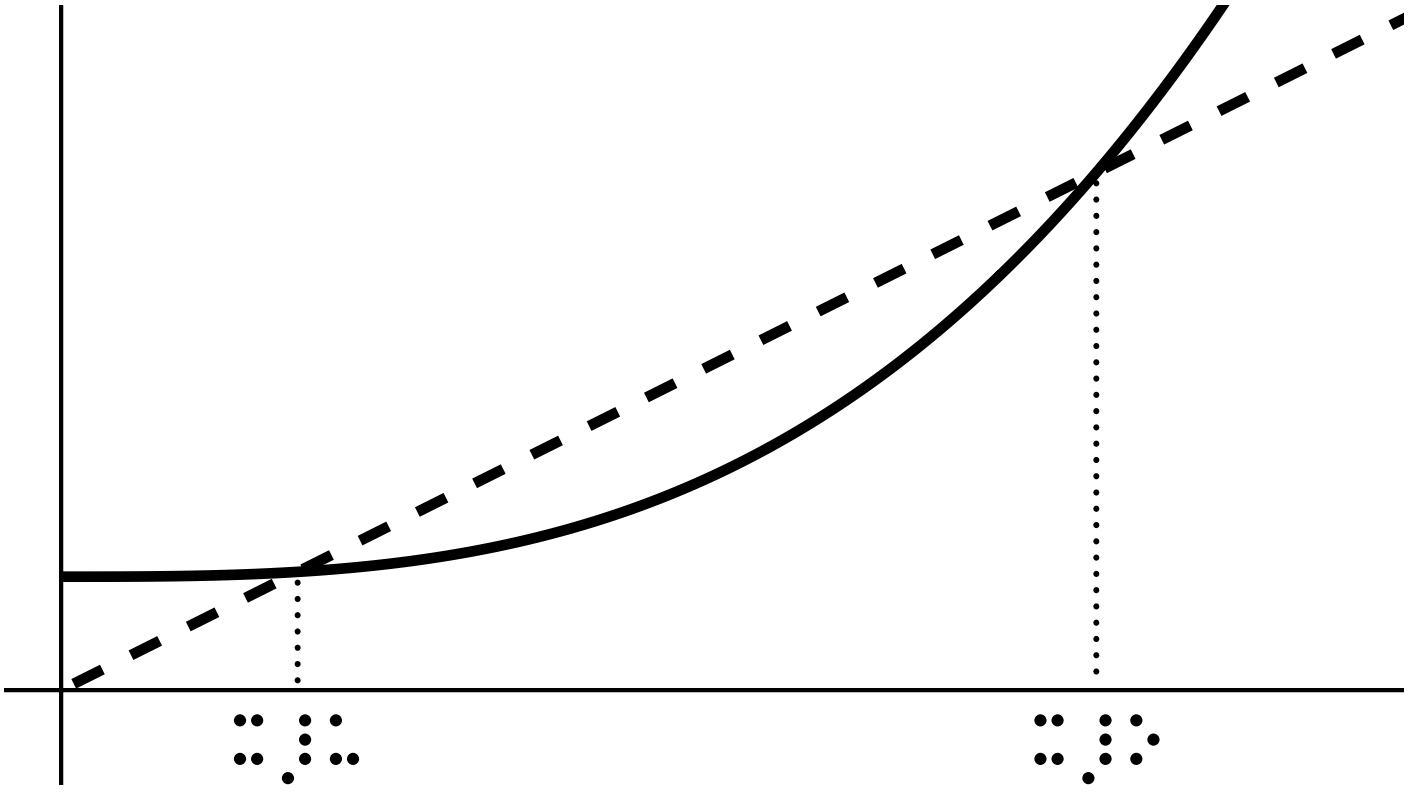
p(x); E_q



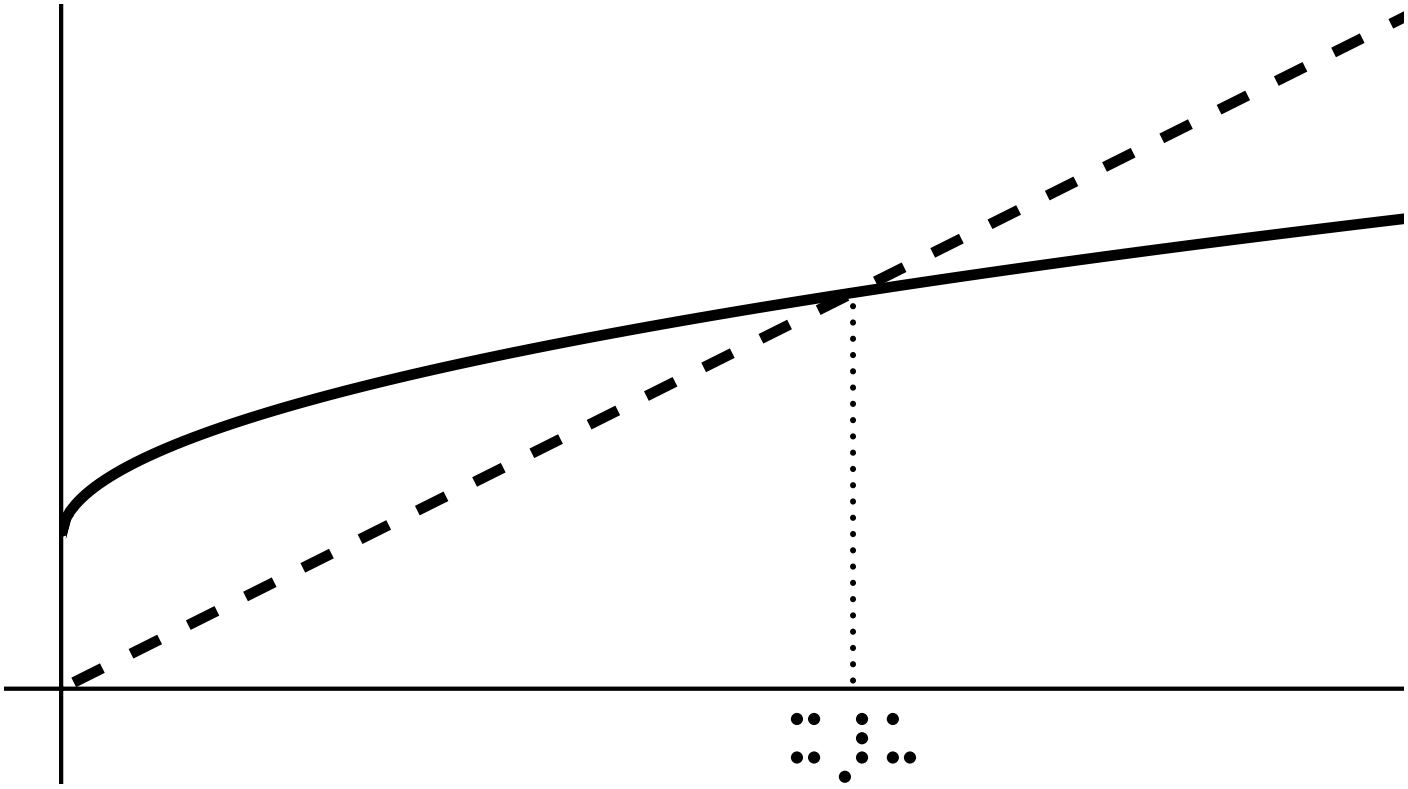
K_I; E



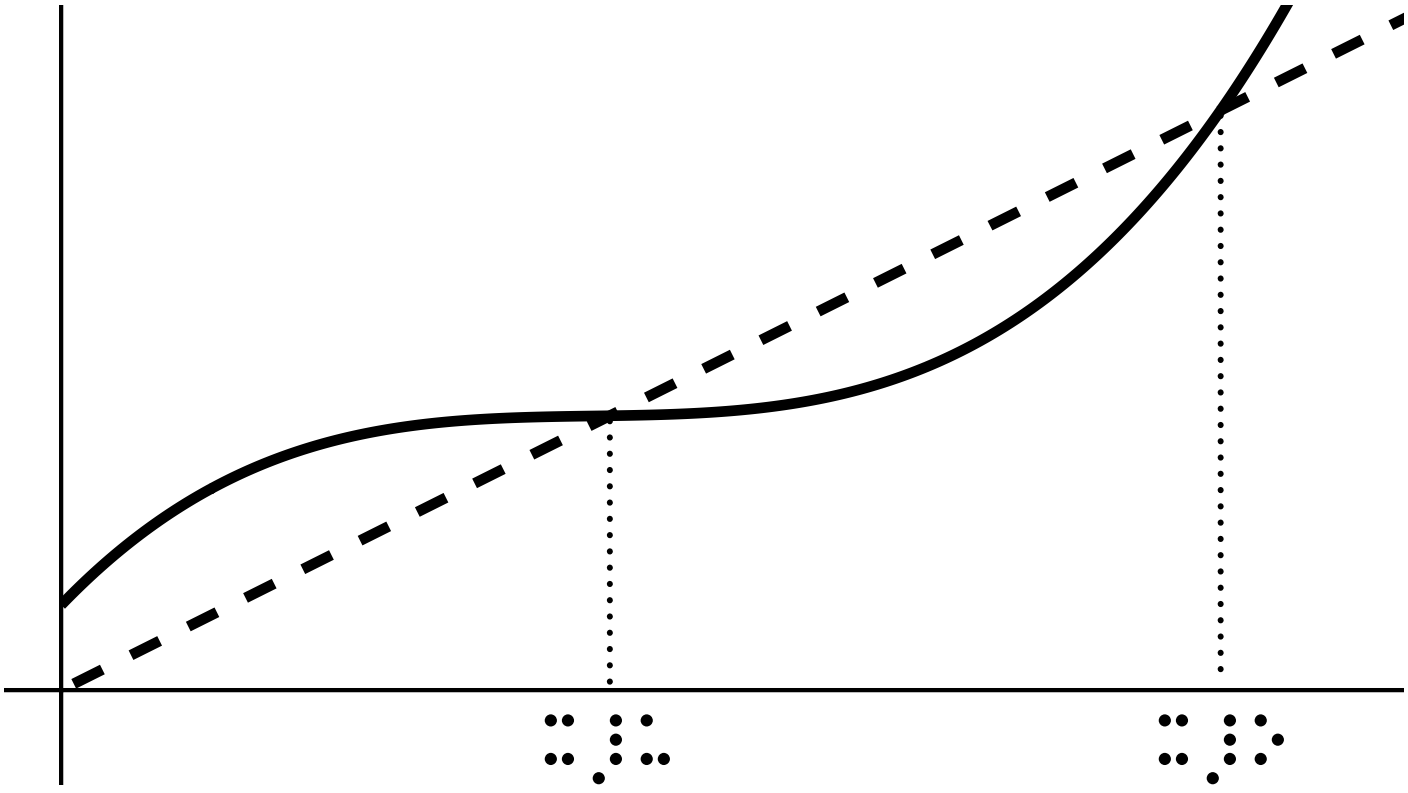
K_p; E



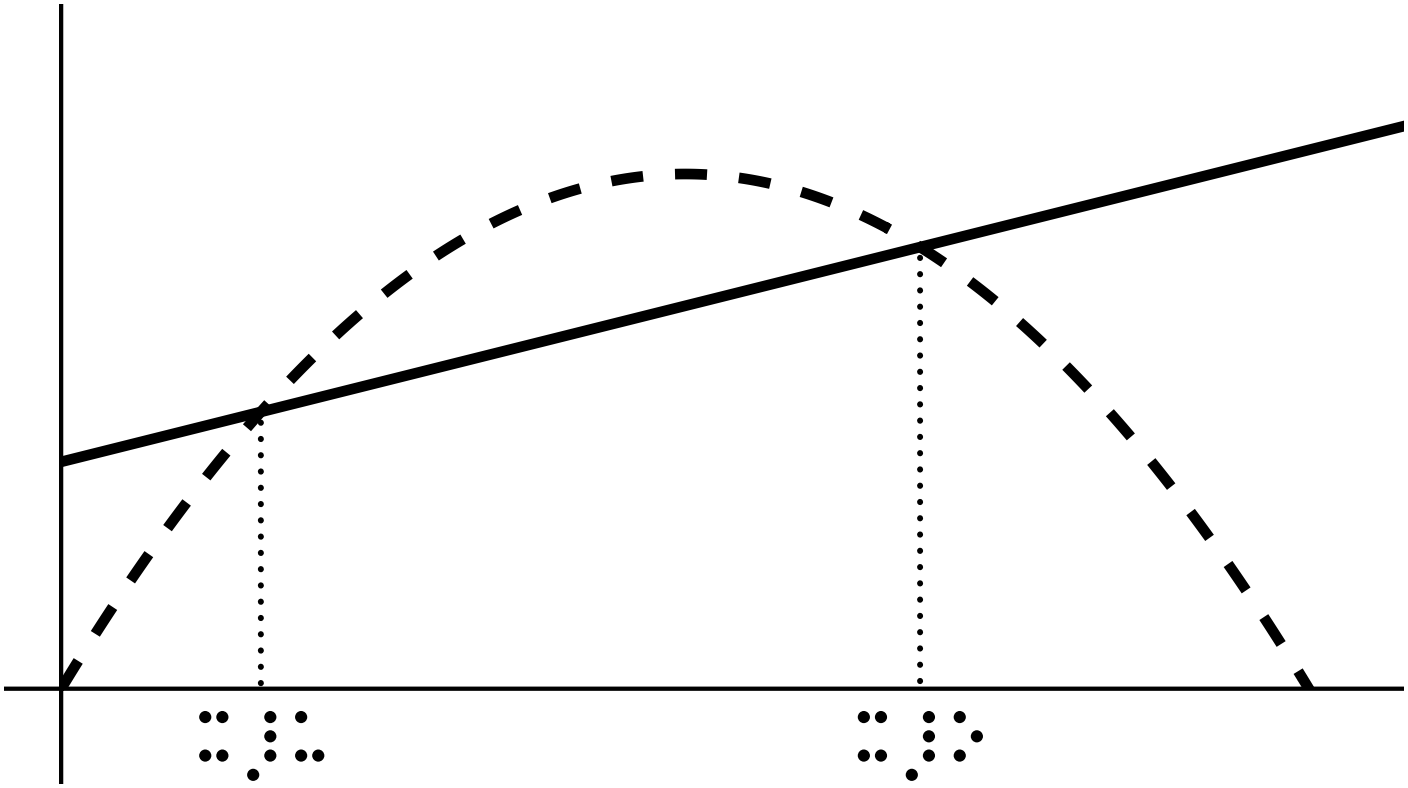
K_d; E



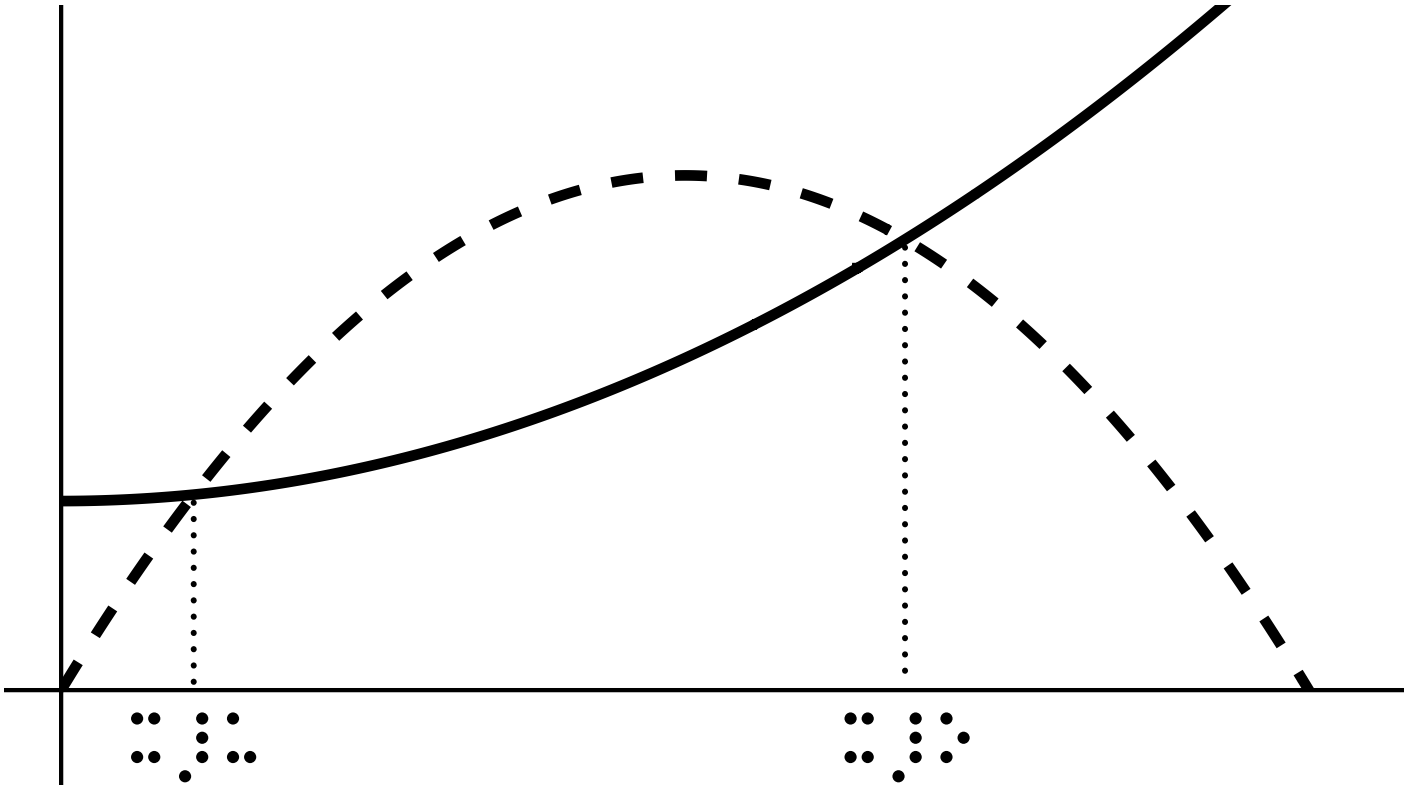
K_s; E



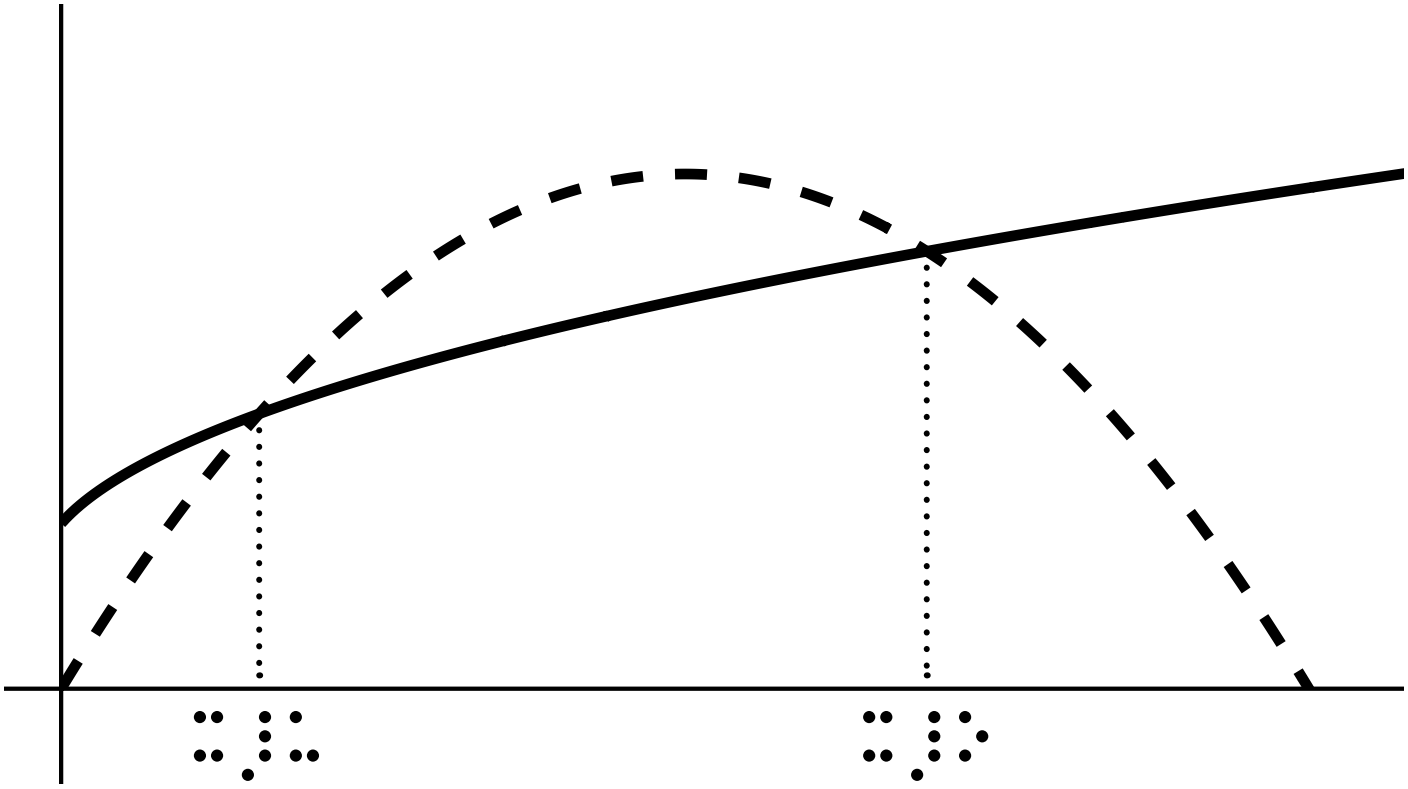
K_I; E_q



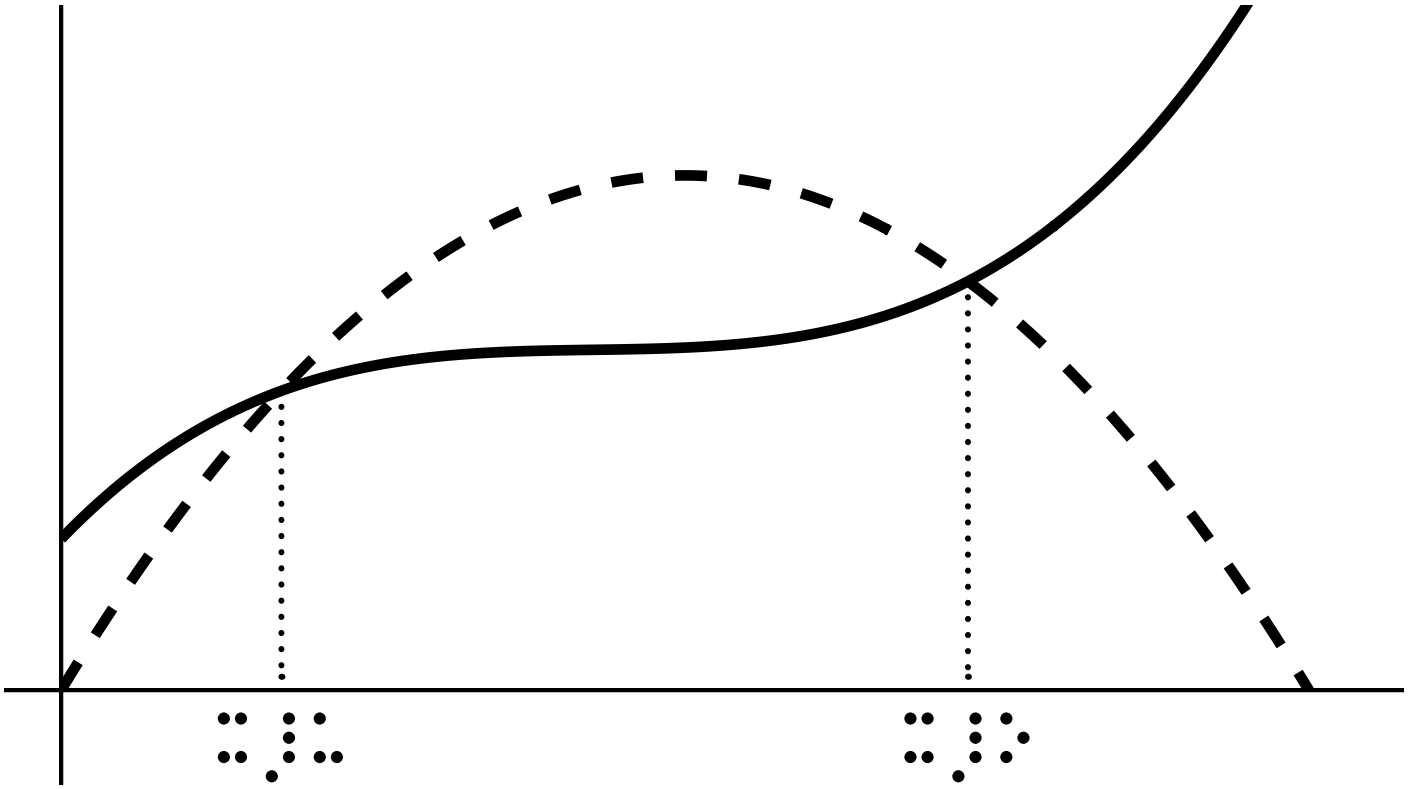
K_p; E_q



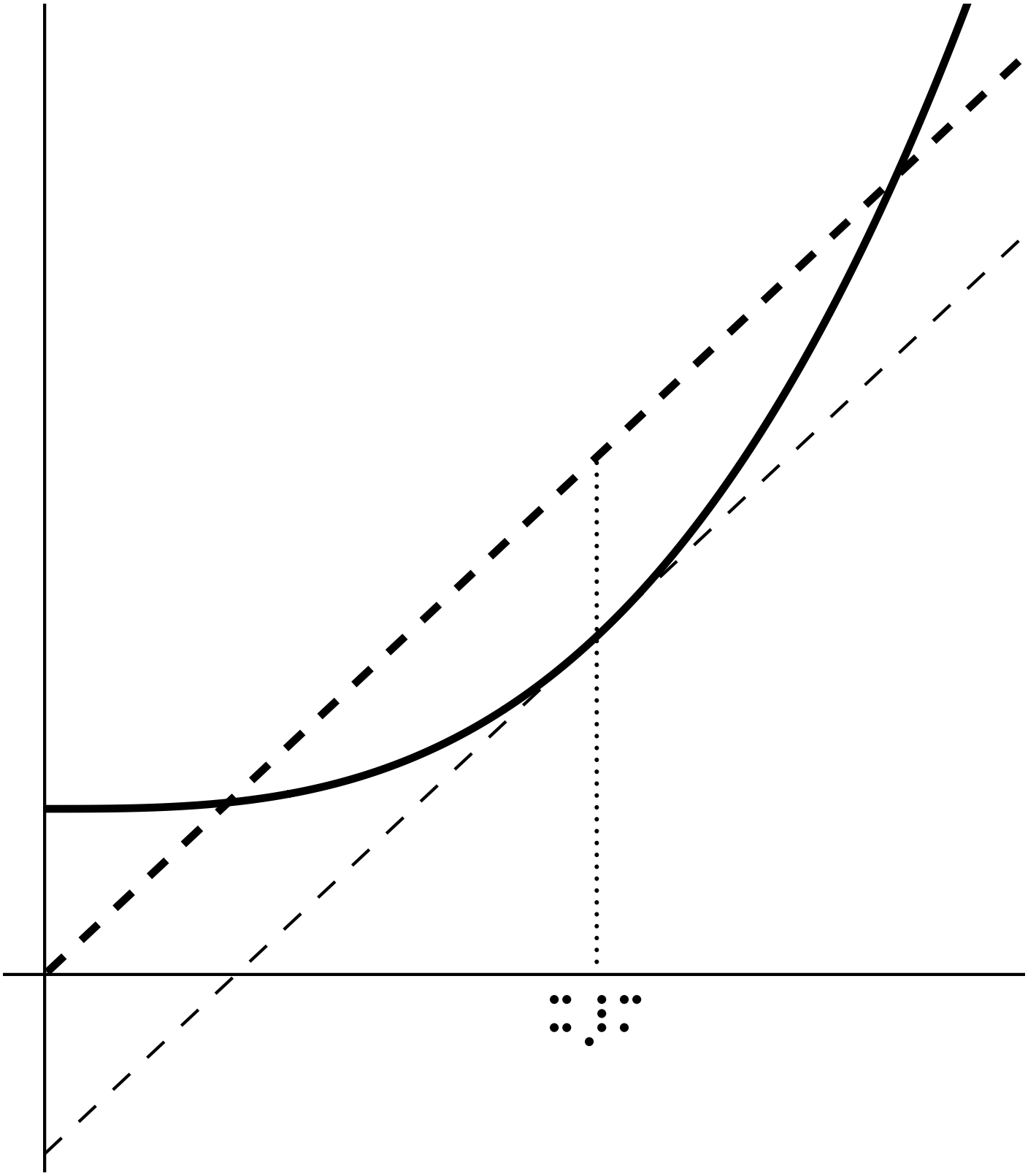
K_d; E_q



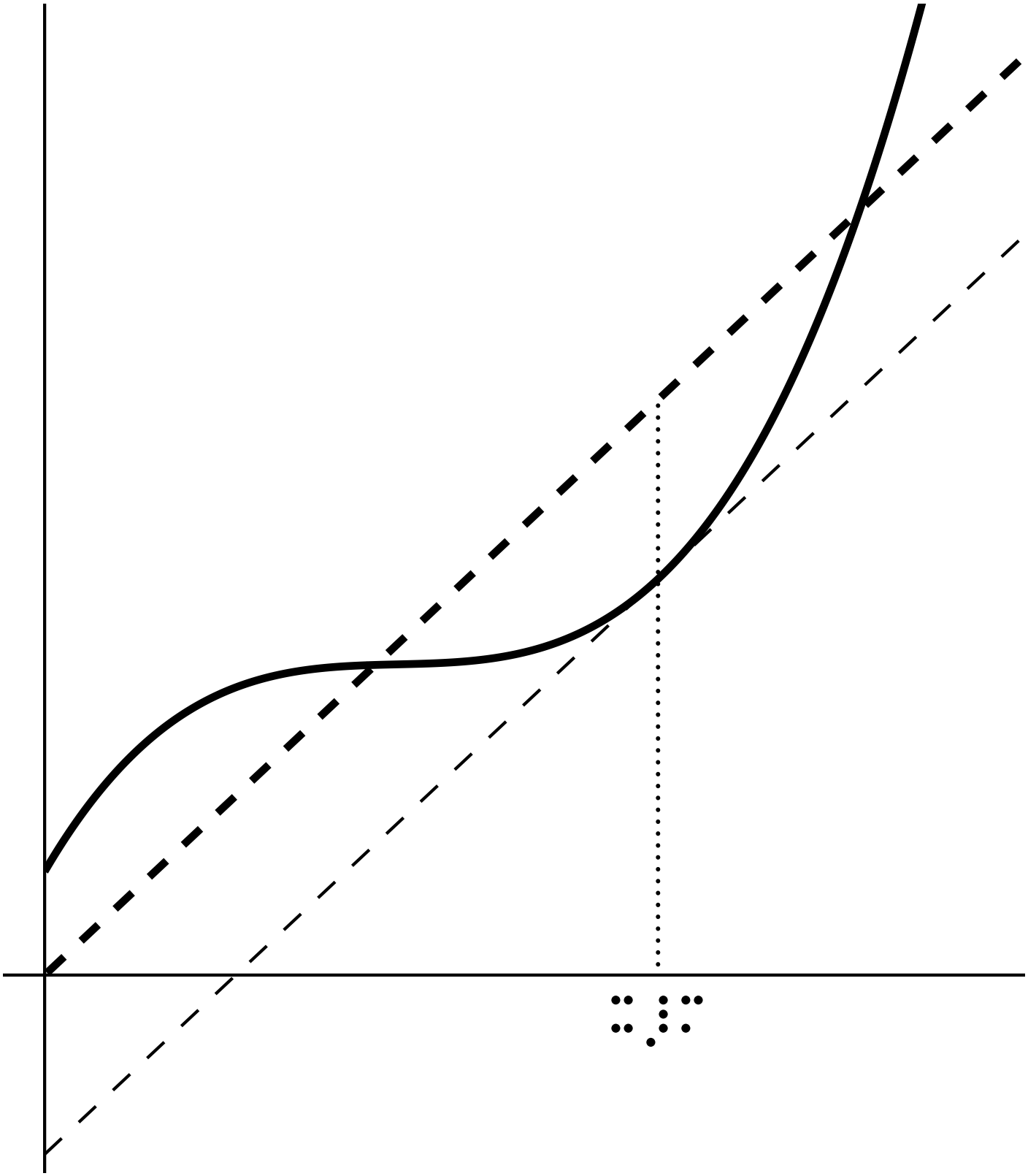
K_s; E_q



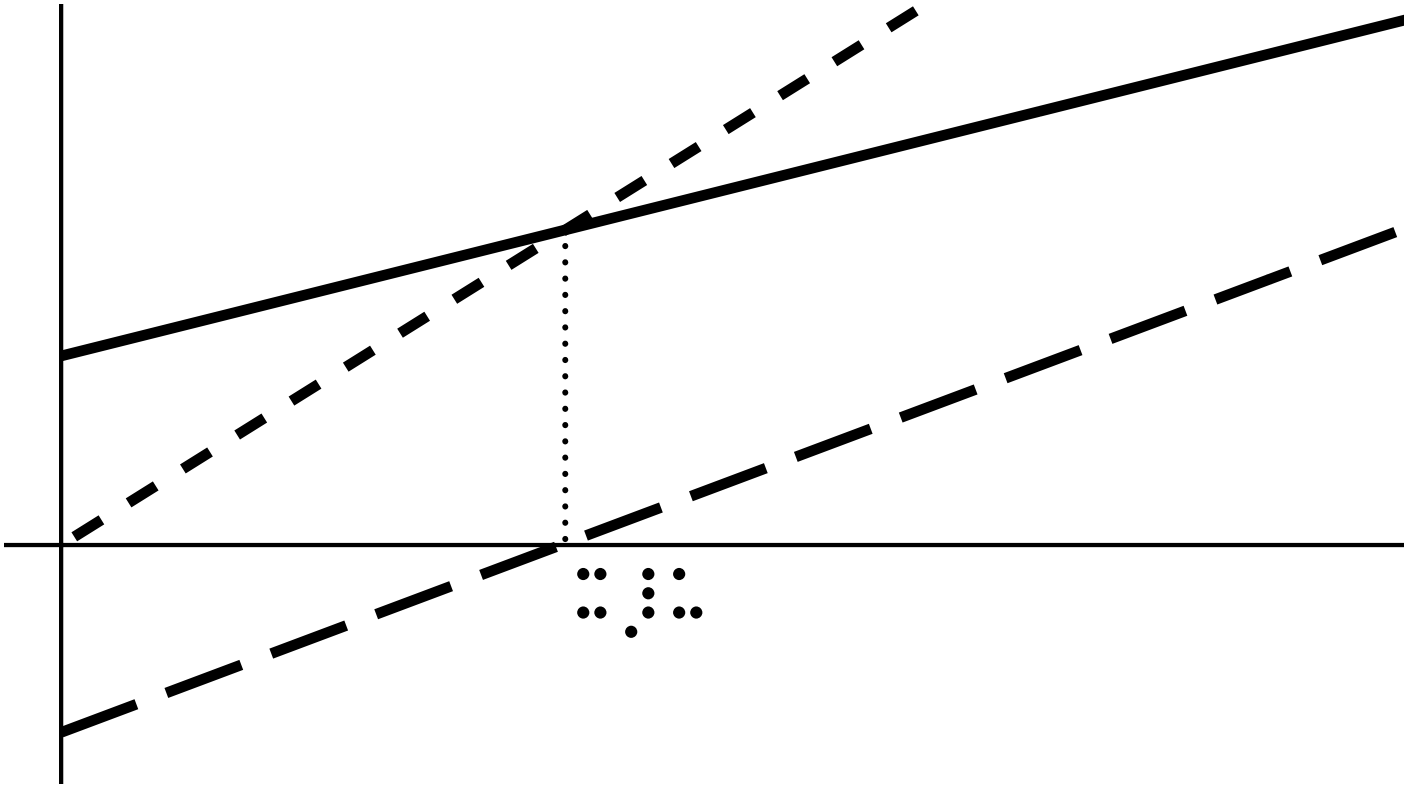
K_p; $E = p * x$; $E_{||}$



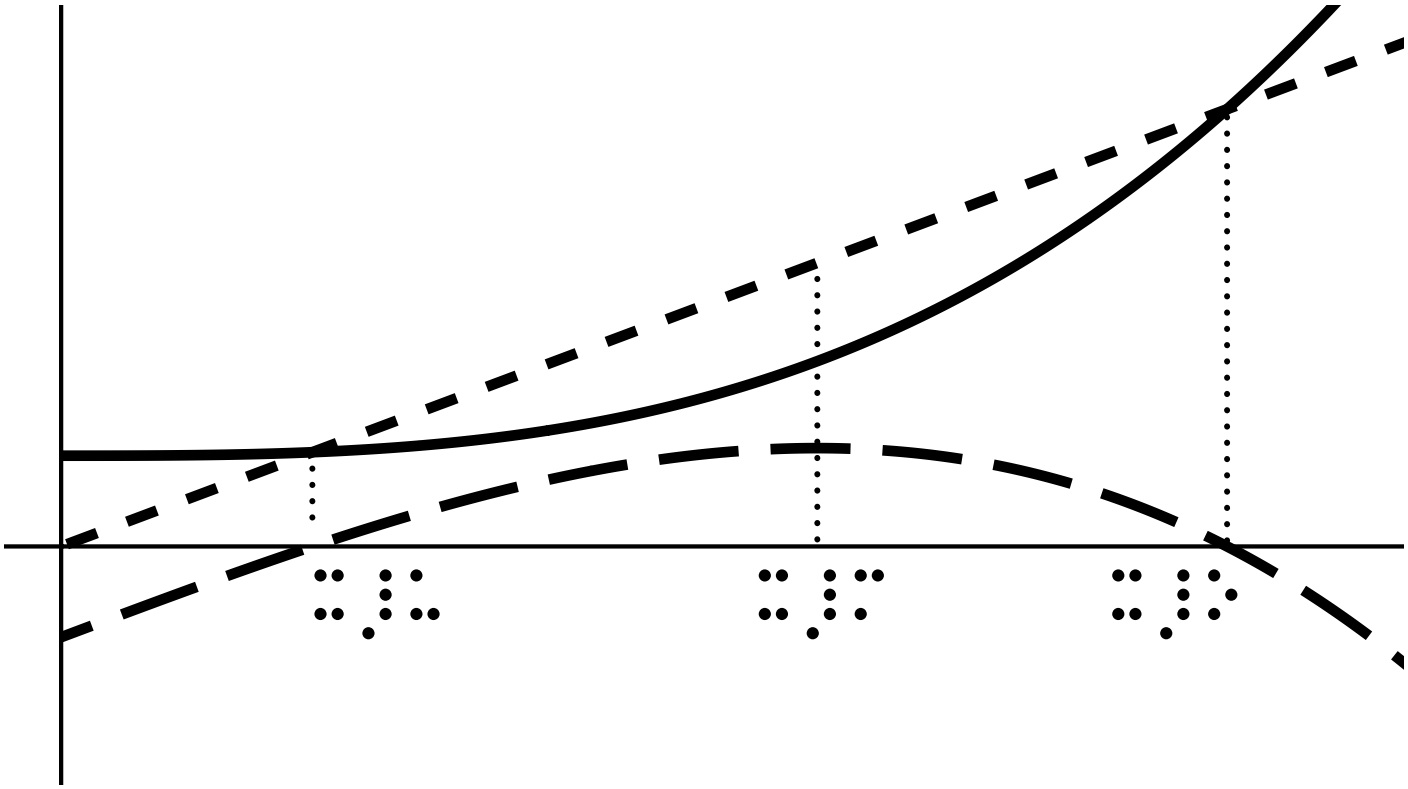
K_s; $E = p * x$; $E_{||}$



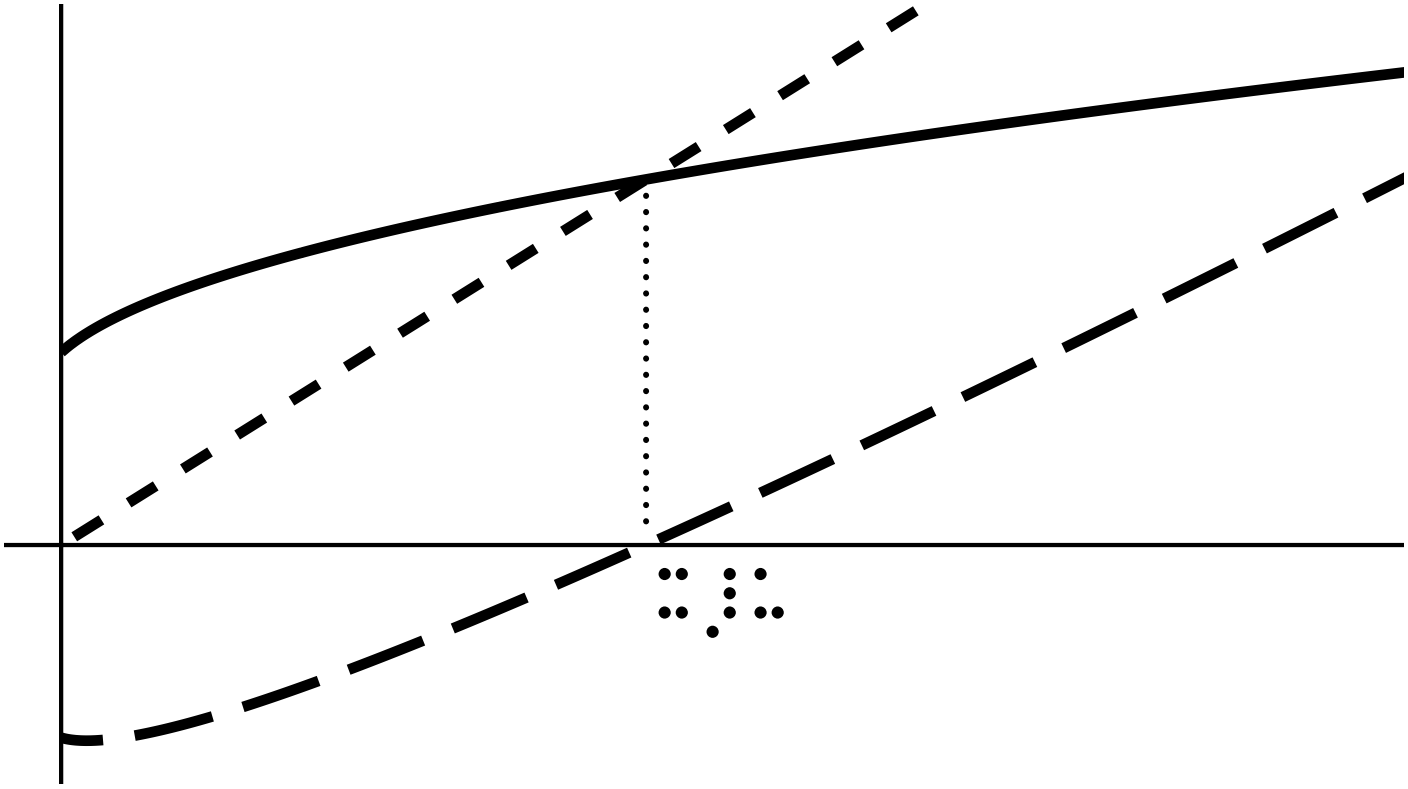
K_I; E; G



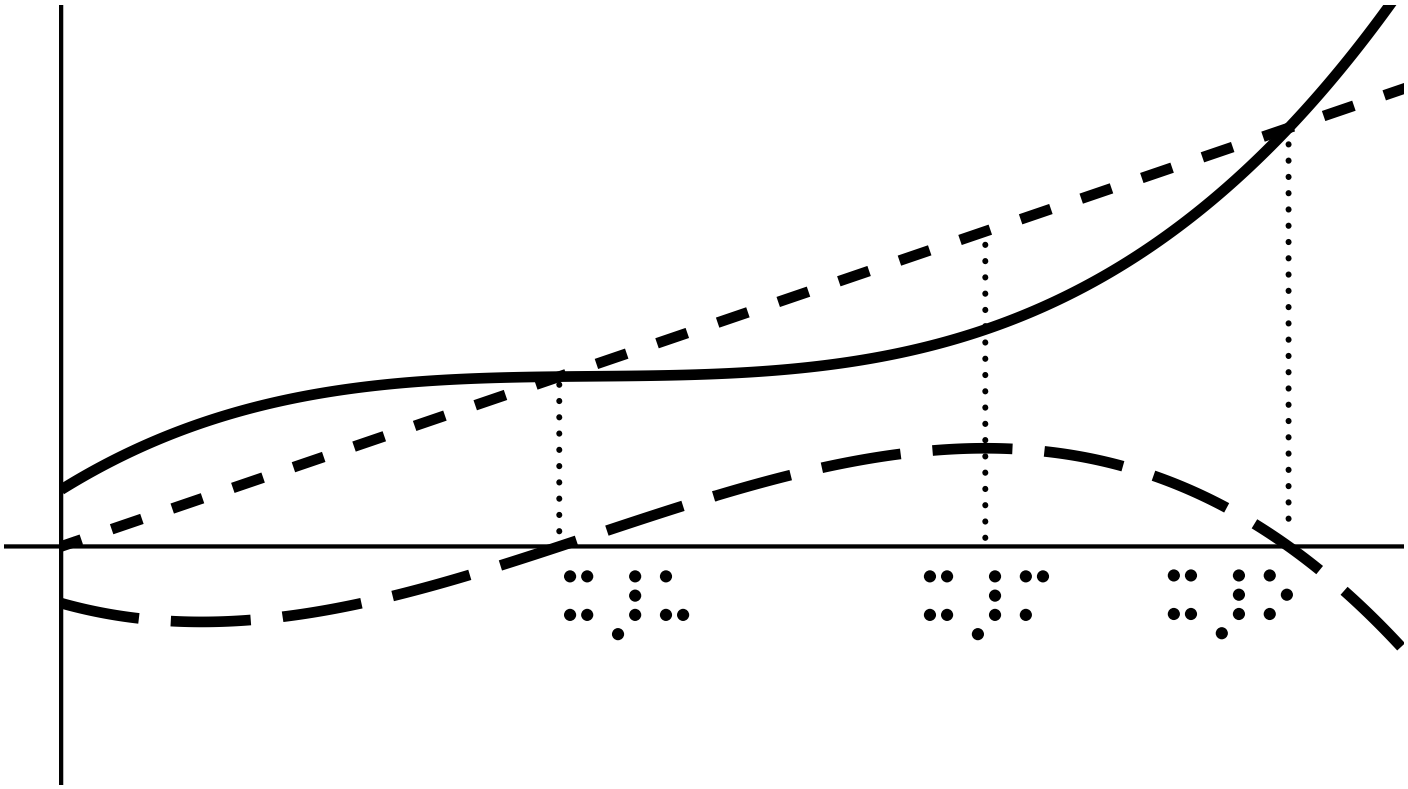
K_p; E; G



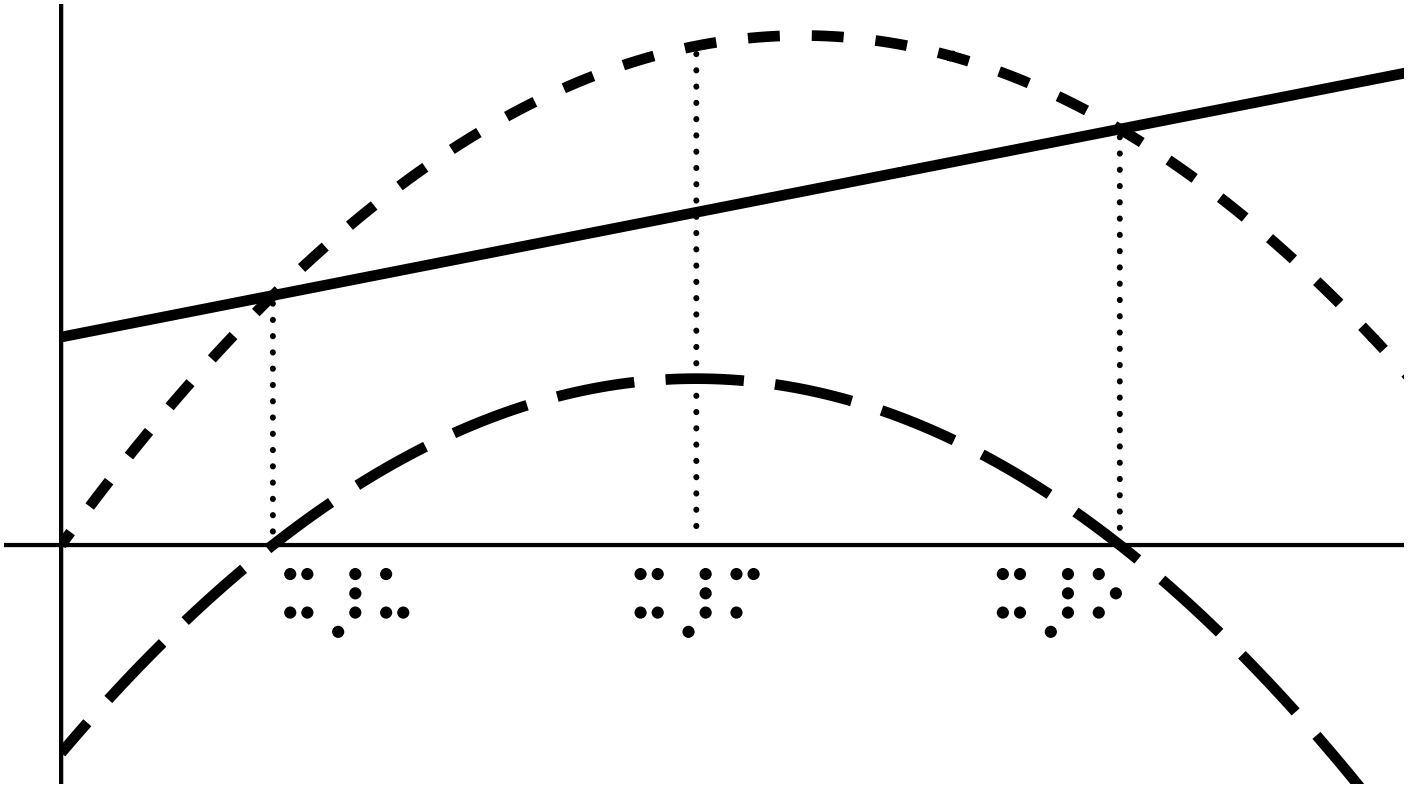
K_d; E; G



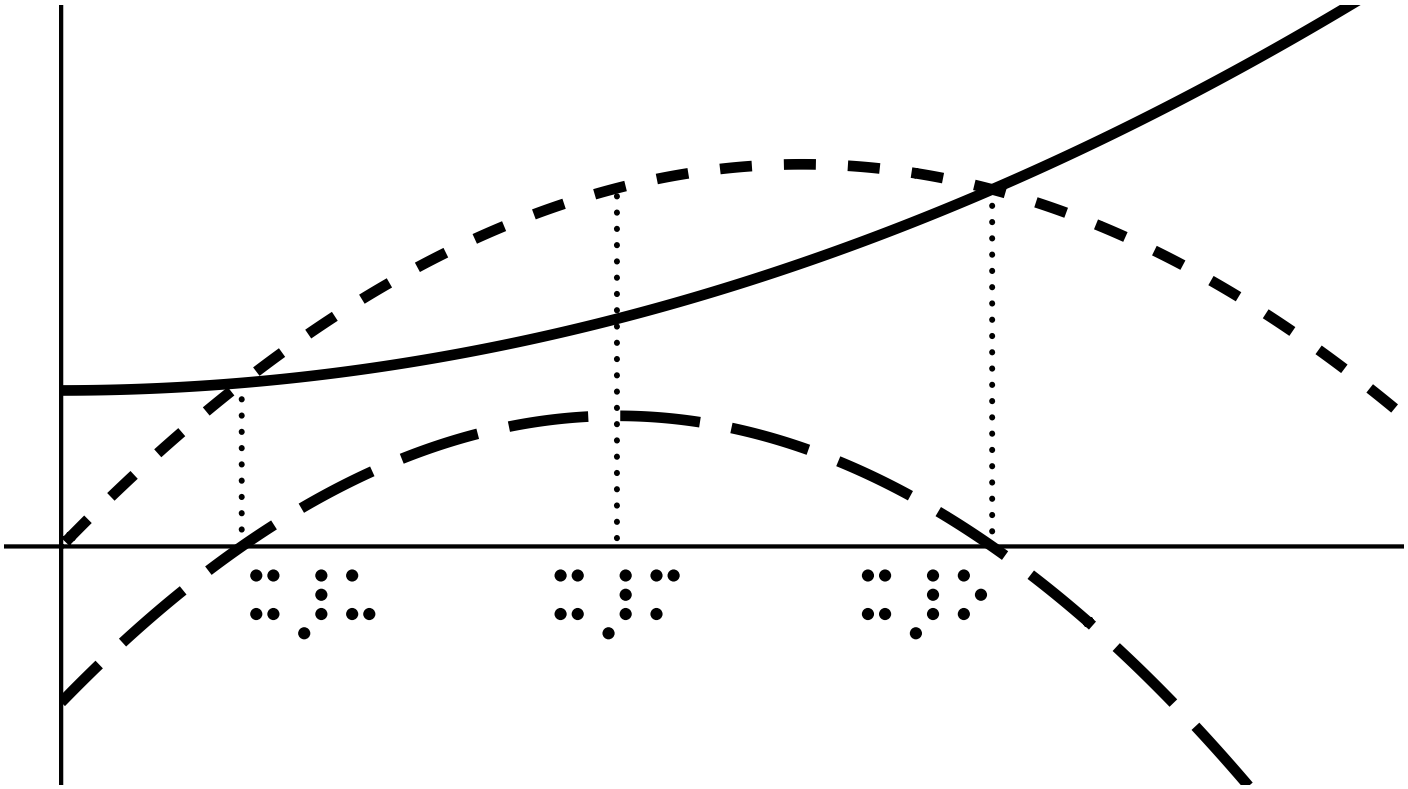
K_s; E; G



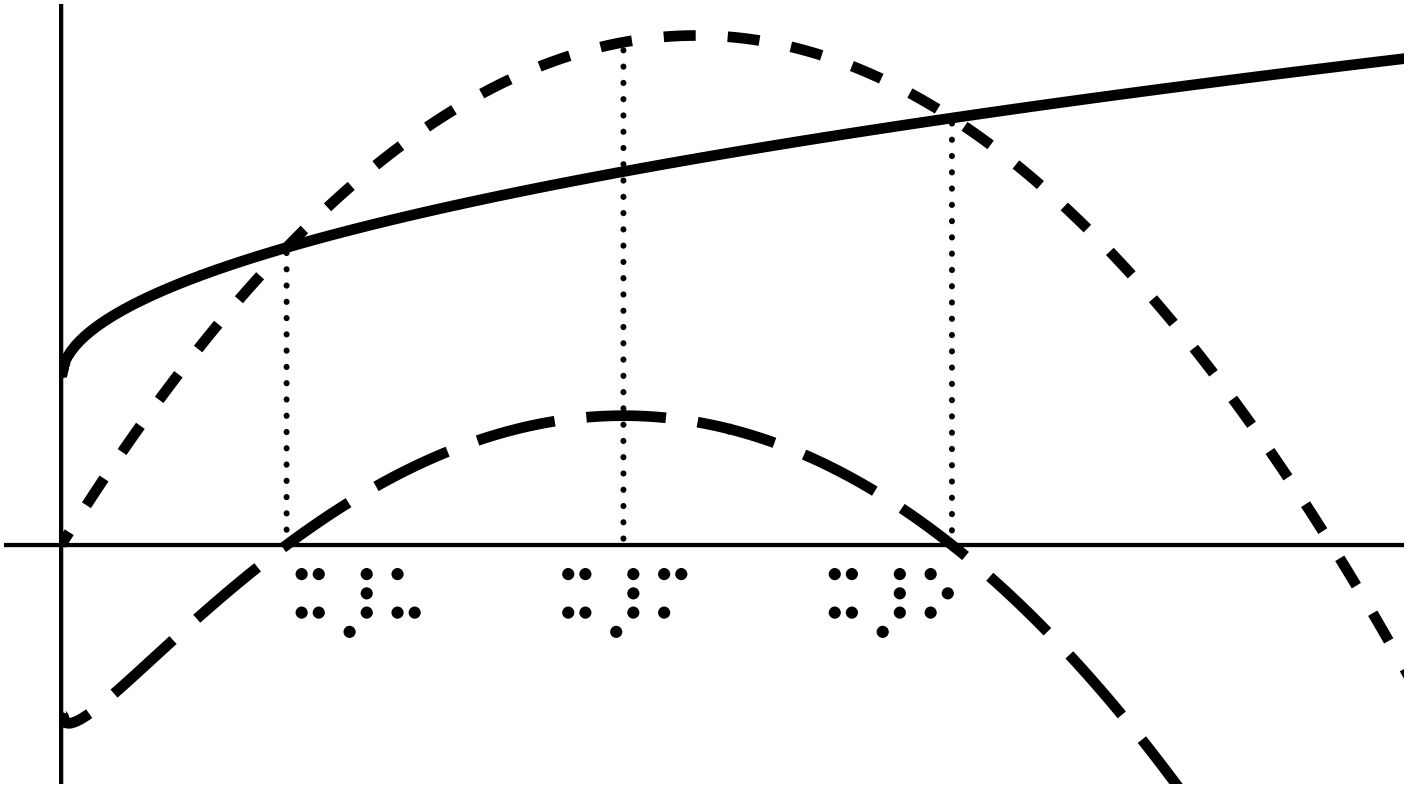
K_l; E_q; G



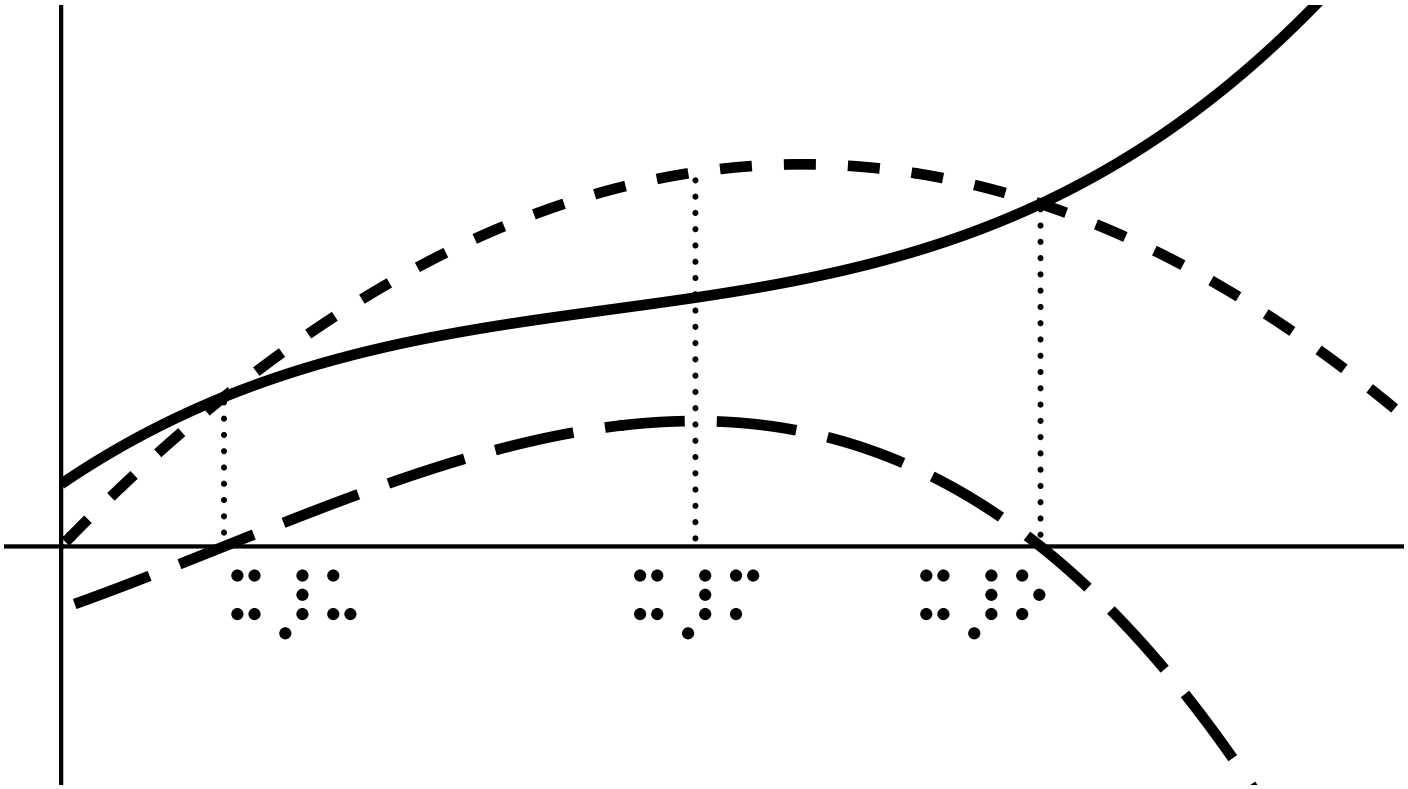
K_p; E_q; G



K_d; E_q; G



K_s; E_q; G



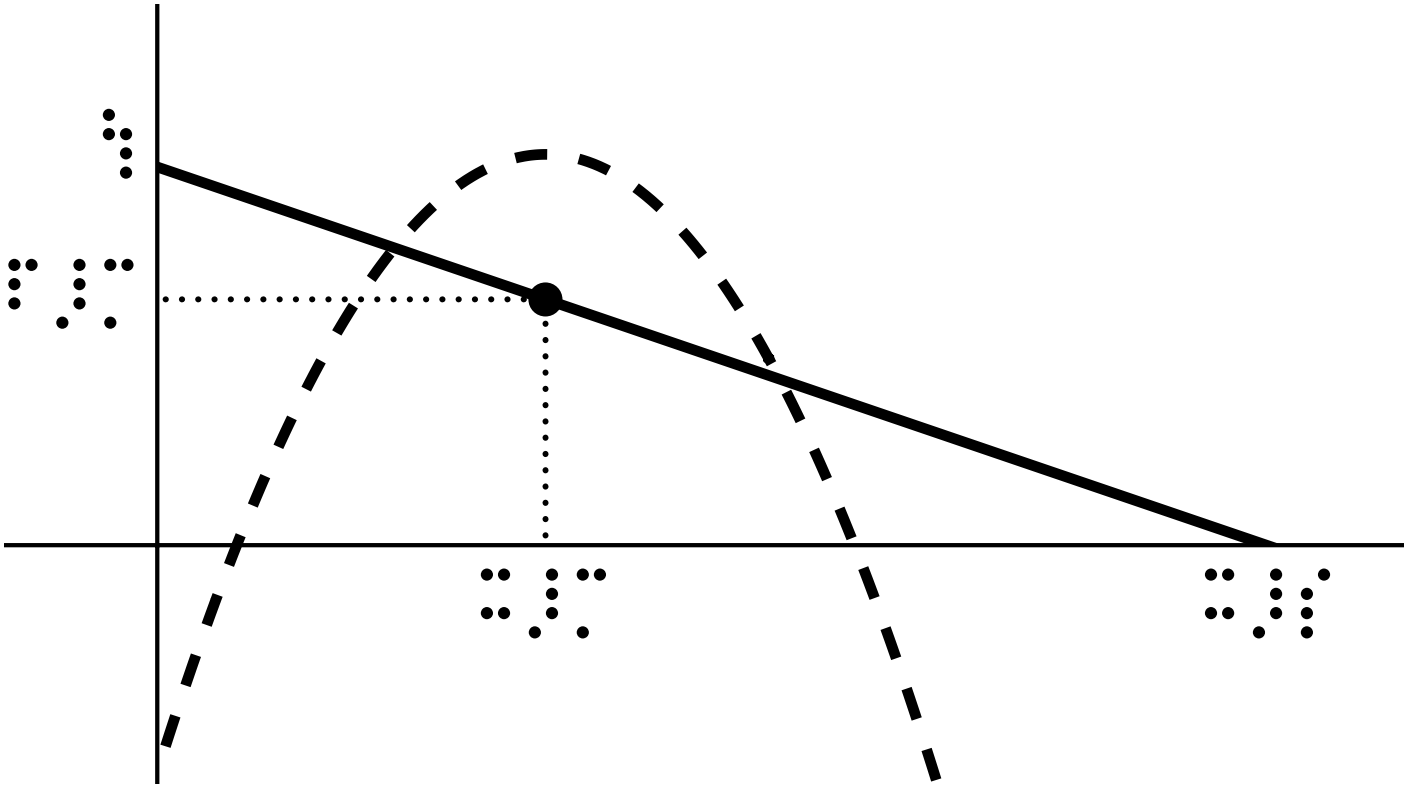
Ergänzungen zu Seite 16

C(x_C|p_C)

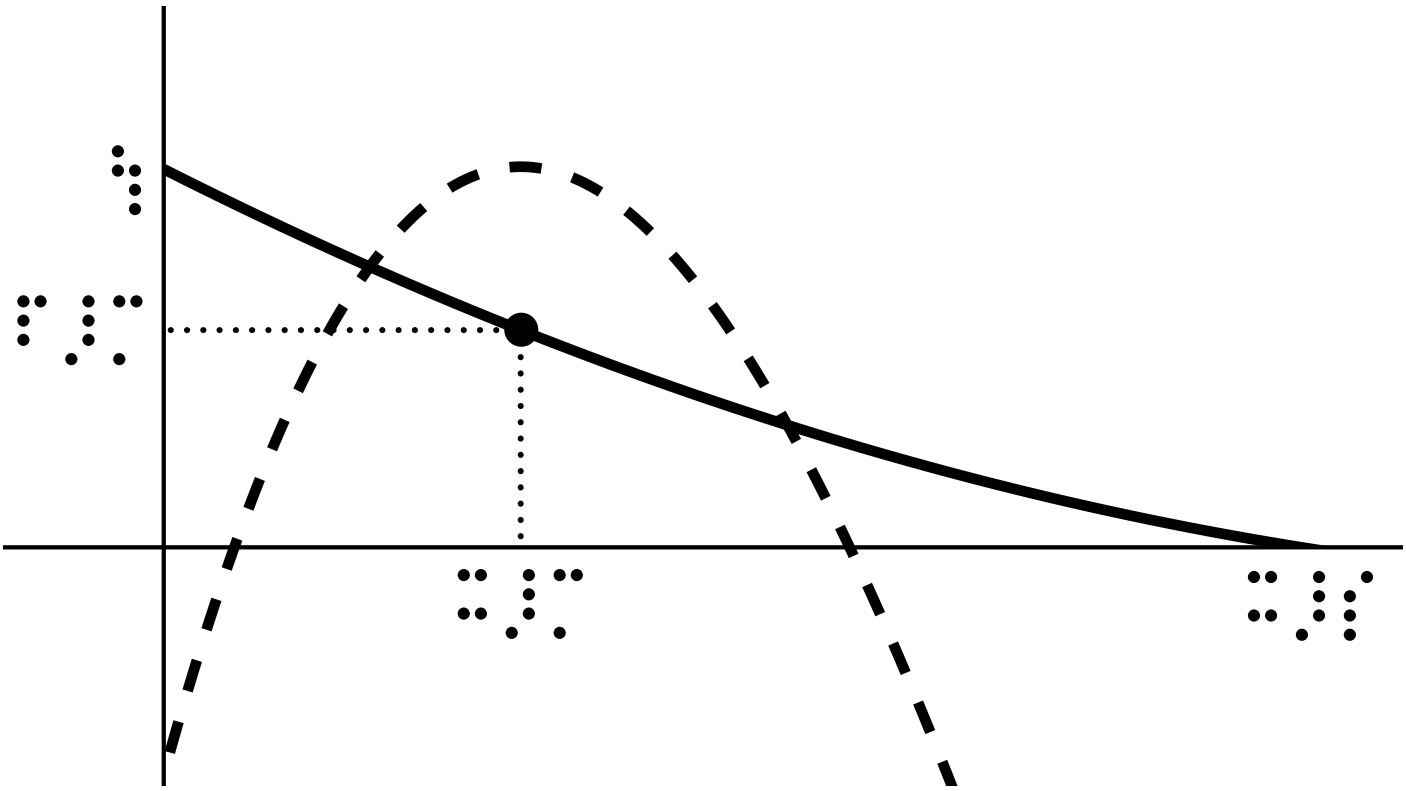
Cournot‘scher Punkt

x_C: gewinnmaximale Produktionsmenge eines Monopolisten

p (x); G



p (x); G



Normalverteilung

Schulstufe 12

Histogramm

Dichtefunktion f

Verteilungsfunktion F

standardisierte Dichtefunktion 'ph

standardisierte Verteilungsfunktion 'Ph

Inhalt

0 Liste der Abkürzungen

1 Histogramm n =6

2 Histogramm n =10

3 Verteilungsfunktion F
Integration der Dichtefunktion

4 Allgem. Dichtefunktion f
Allgem. Verteilungsfunktion F
F(a) =P(X <=a)

5 Allgem. Verteilungsfktn
P(-'ue <=X <='ue) =1

6 Allgem. Dichtefunktion 'ph
P(X <='my) =P(X >='my) =0,5

7 Standard. Dichtefunktion 'ph
'my =^0, 1 *'si =^ 1 *z)

8 Standard. Dichtefunktion 'ph
Skala mit 'my, Skala mit z,

9 Standard. Dichtefunktion 'ph
'ph(z) =f('my +z *'si))

10 Stand. Verteilungsfkt 'Ph

'Ph(z) =P(X <='my +z *'si)

11 Stand. Verteilungsfkt 'Ph

1 -'Ph(z) ='Ph(-z)

12 Stand. Verteilungsfkt 'Ph

2 *'Ph(z) -1

13 Stand. Verteilungsfkt 'Ph

'Ph(z), z <0

14 Stand. Verteilungsfkt 'Ph

1 -'Ph(z), z >0

15 Stand. Verteilungsfkt 'Ph

'Ph(z_2) -'Ph(z_1), z >0

16 Stand. Verteilungsfkt 'Ph

'Ph(z_2) -'Ph(z_1)

17 Stand. Verteilungsfkt 'Ph

'Ph(z_2) -'Ph(z_1), z <0

18 Unterschiedliche Standardabweichungen

Inhalt

0 Liste der Abkürzungen

1 Histogramm n =6

2 Histogramm n =10

3 Verteilungsfunktion F

Integration der Dichte-

funktion

4 Allgem. Dichtefunktion f

Allgem. Verteilungsfunk-

tion F

F(a) =P(X <=a)

5 Allgem. Verteilungsfkt

P(-⡶ue <=X <=⡶ue) =1

6 Allgem. Dichtefunktion

⣤ph

P(X <=⡶my) =P(X >=⡶my)

=0,5

7 Standard. Dichtefunktion

⣤ph

⣤⠍⠽ ⠶⡮⠬⠂ ⠡ ⠔⡶⠎⠊ ⠶⡮ ⠡ ⠔⠵⠴
⠳ ⡎⠞⠁⠝⠙⠁⠗⠙⠄ ⡙⠊⠉⠓⠞⠑⠋⠥⠝⠅⠞⠊⠕⠝
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⡎⠅⠁⠇⠁ ⠍⠊⠞ ⣤⠍⠽⠂ ⡎⠅⠁⠇⠁ ⠍⠊⠞
⠵⠂
⠪ ⡎⠞⠁⠝⠙⠁⠗⠙⠄ ⡙⠊⠉⠓⠞⠑⠋⠥⠝⠅⠞⠊⠕⠝
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⣤⠏⠓⠦⠵⠴ ⠤⠋⠦⣤⠍⠽ ⠖⠵ ⠔⡶⠎⠊⠴⠴
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⣤⡏⠓

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⣤⡏⠓

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⣤⡏⠓

⠣ ⠔⡶⡏⠓⠦⠵⠴ ⠤⠡
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⣤⡏⠓

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⣤⡏⠓

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⠡⠳⠀⡥⠝⠞⠑⠗⠎⠉⠓⠊⠑⠙⠇⠄

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Normalverteilung - Abkürzungen

Index

f:	(Wahrscheinlichkeits-) Dichtefunktion von X
F:	Verteilungsfunktion von X F(a) =P(X <=a)
P:	Wahrscheinlichkeit
P(a_i):	Wahrscheinlichkeit für a_i
X:	diskrete Zufallsvariable
z:	(x -'my)/'si
'my:	Erwartungswert
'ph:	(Wahrscheinlichkeits) Dichtefunktion der Standardnormalverteilung von X
'Ph:	Verteilungsfunktion der Dichtefunktion der Standardnormalverteilung von X
'si:	Standardabweichung

⡝⠕⠗⠍⠁⠇⠧⠑⠗⠞⠑⠊⠇⠥⠝⠛ ⠤ ⡁⠃⠅⢳⠗⠤
⠵⠥⠝⠛⠑⠝

⡊⠝⠙⠑⠭

⠋⠒ ⠦⡺⠁⠓⠗⠎⠉⠓⠑⠊⠝⠇⠊⠉⠓⠅⠑⠊⠞⠎⠤⠴
⡙⠊⠉⠓⠞⠑⠋⠥⠝⠅⠞⠊⠕⠝ ⠧⠕⠝ ⡭

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⡋⠦⠁⠴ ⠶ ⡏⠦⡭ ⠰⠶⠁⠴

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⡏⠦⠁⡸⠊⠴⠒ ⡺⠁⠓⠗⠎⠉⠓⠑⠊⠝⠇⠊⠉⠓⠅⠑⠊⠞
⠋⢳⠗ ⠁⡸⠊

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⠵⠒ ⠦⠭ ⠤⡶⠍⠽⠴⠲⡶⠎⠊

⣤⠍⠽⠒ ⡑⠗⠺⠁⠗⠞⠥⠝⠛⠎⠺⠑⠗⠞

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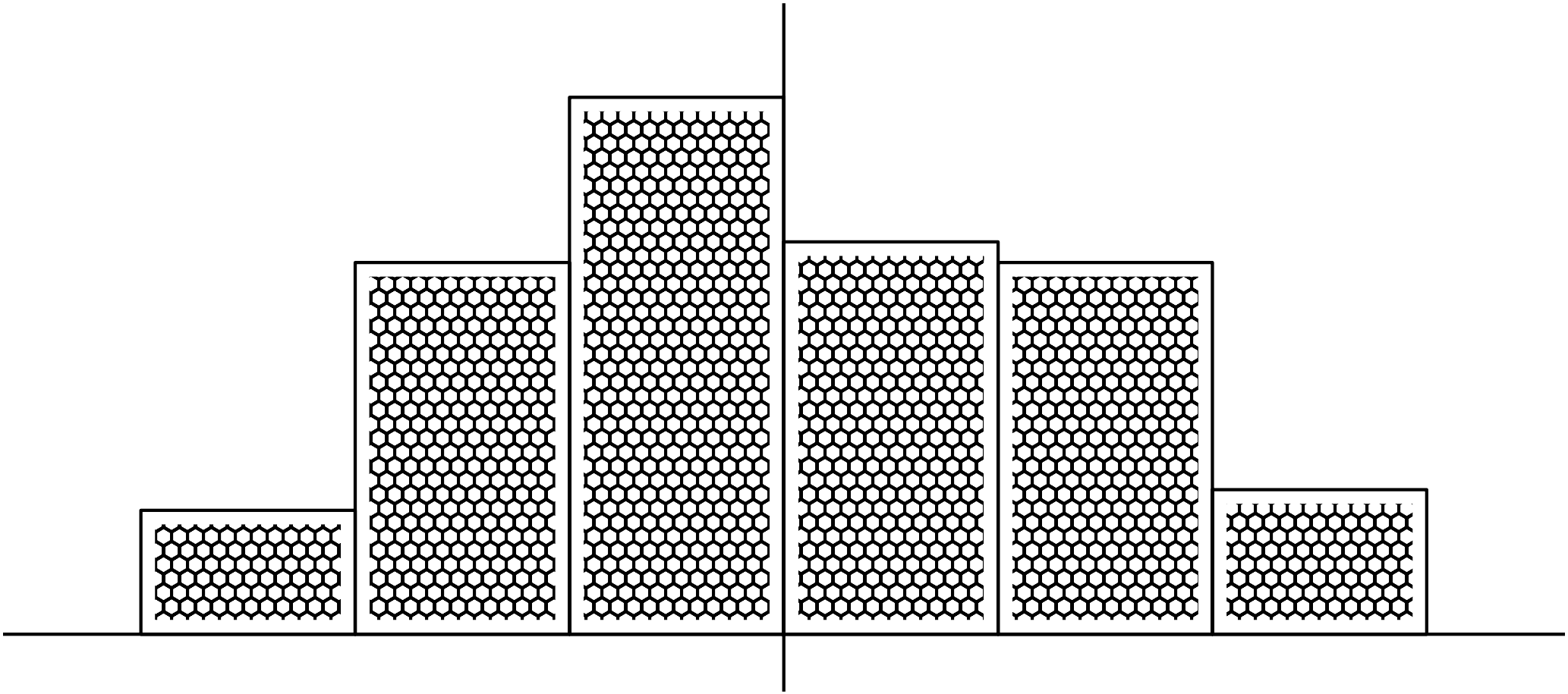
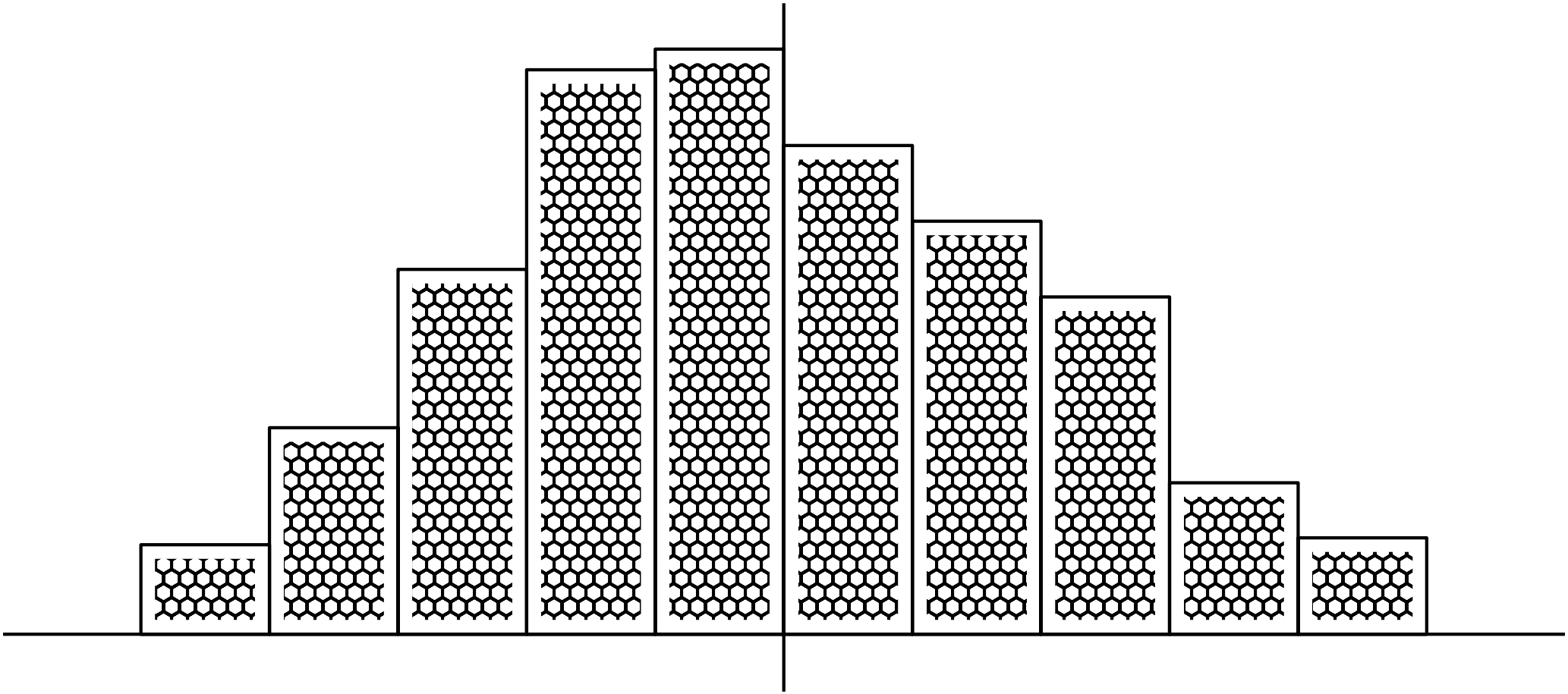
NV Histogramm, 1/18

Histogramm, n = 6

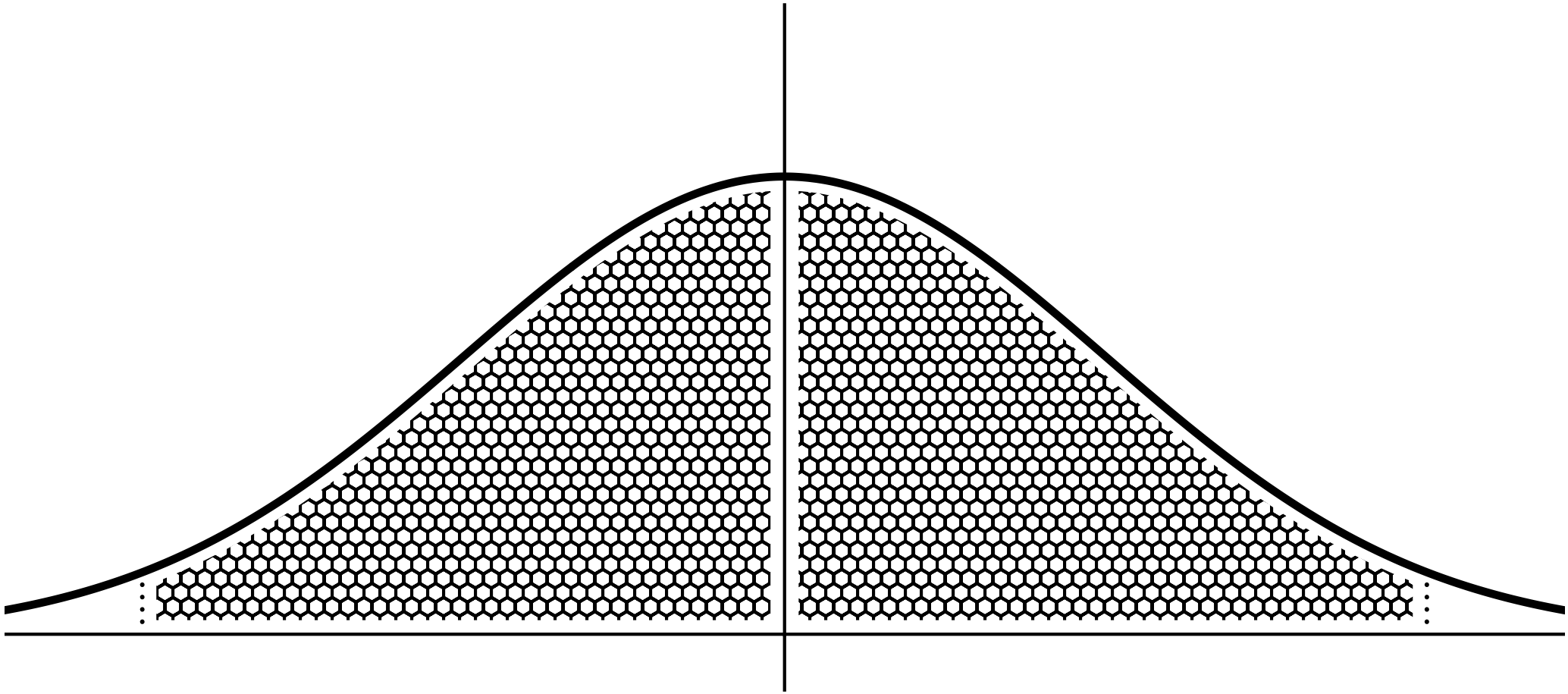


Histogramm, n = 10

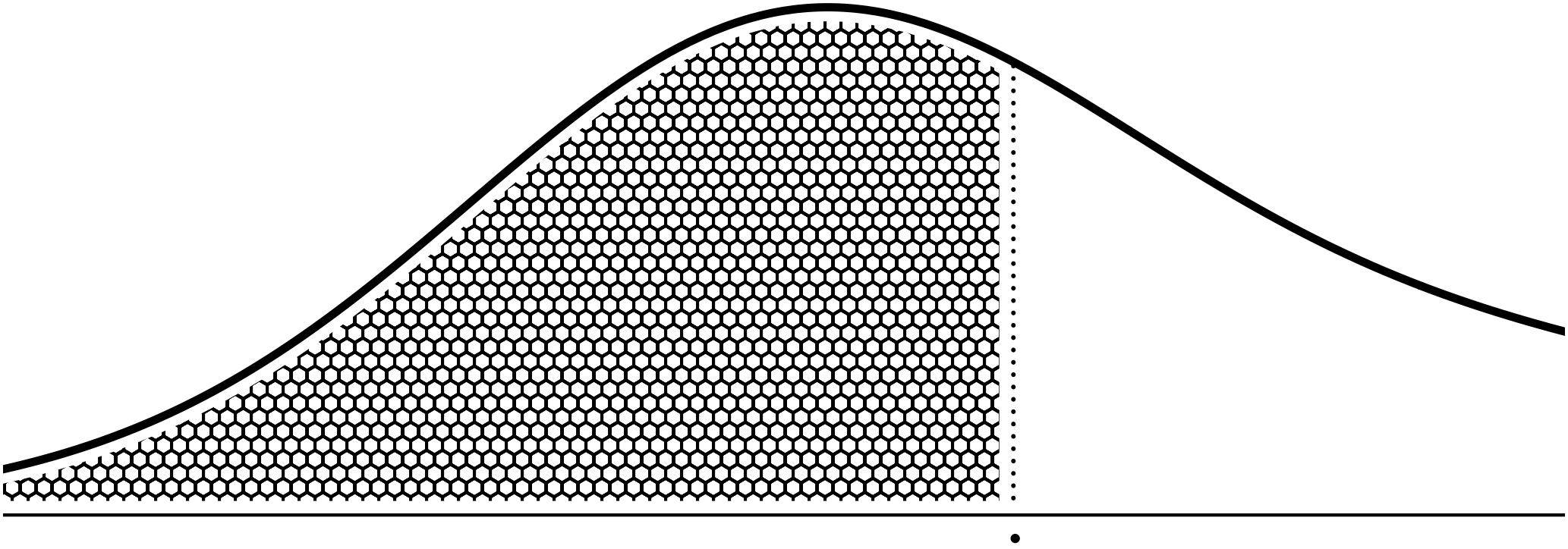


NV Histogramm, 3/18

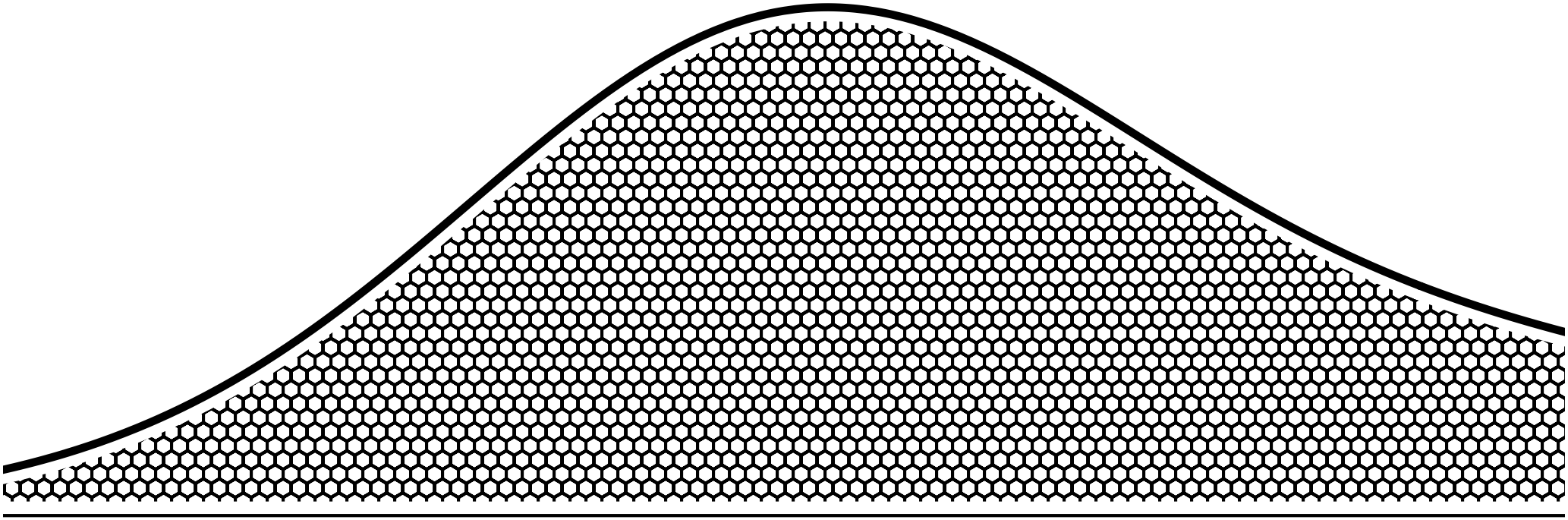
Verteilungsfunktion



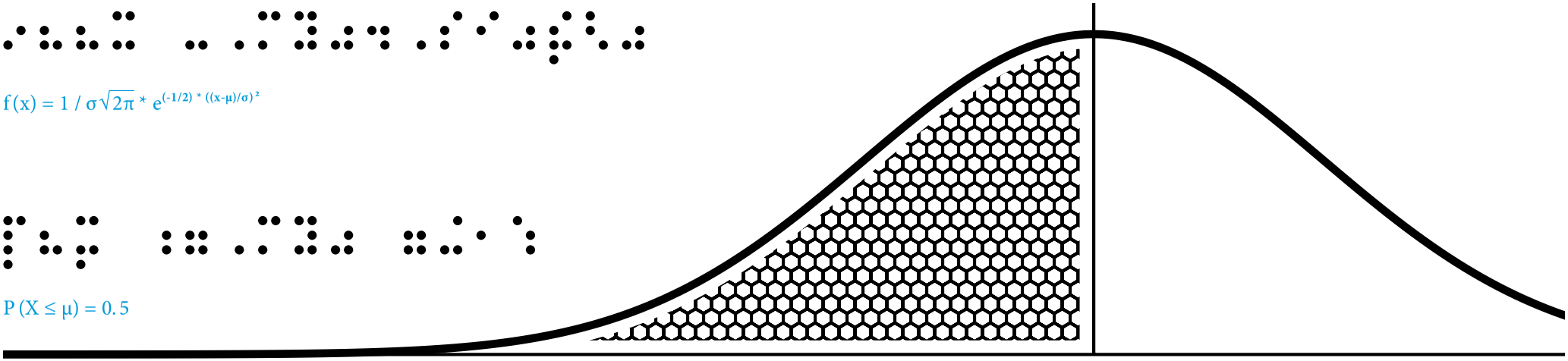
$P(X \leq a) = F(a)$ allgemeine Dichtefkunktion f, allgemeine Verteilungsfkunktion F



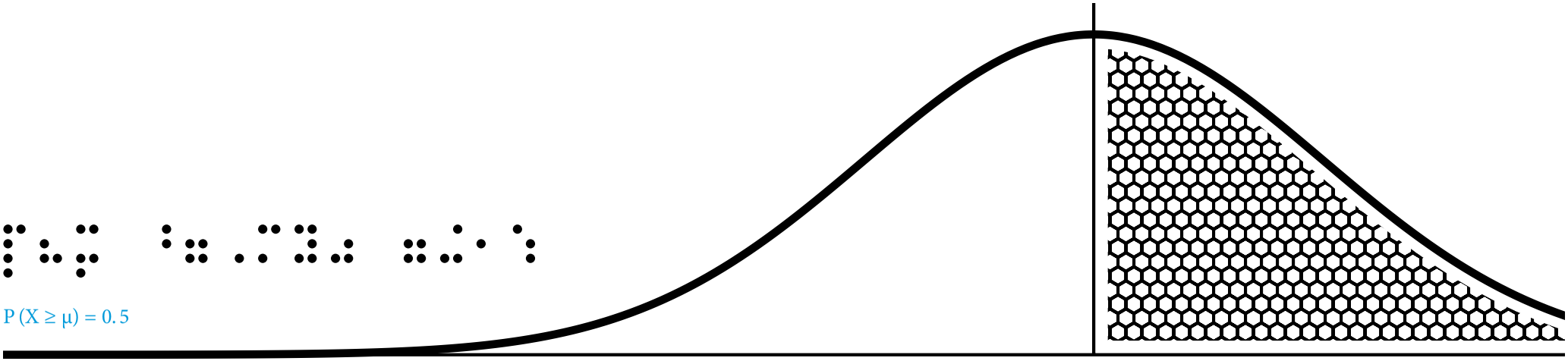
$$P(-\infty < x < +\infty) = 1$$



$$f(x) = 1 / \sigma\sqrt{2\pi} * e^{(-1/2) * ((x-\mu)/\sigma)^2}$$

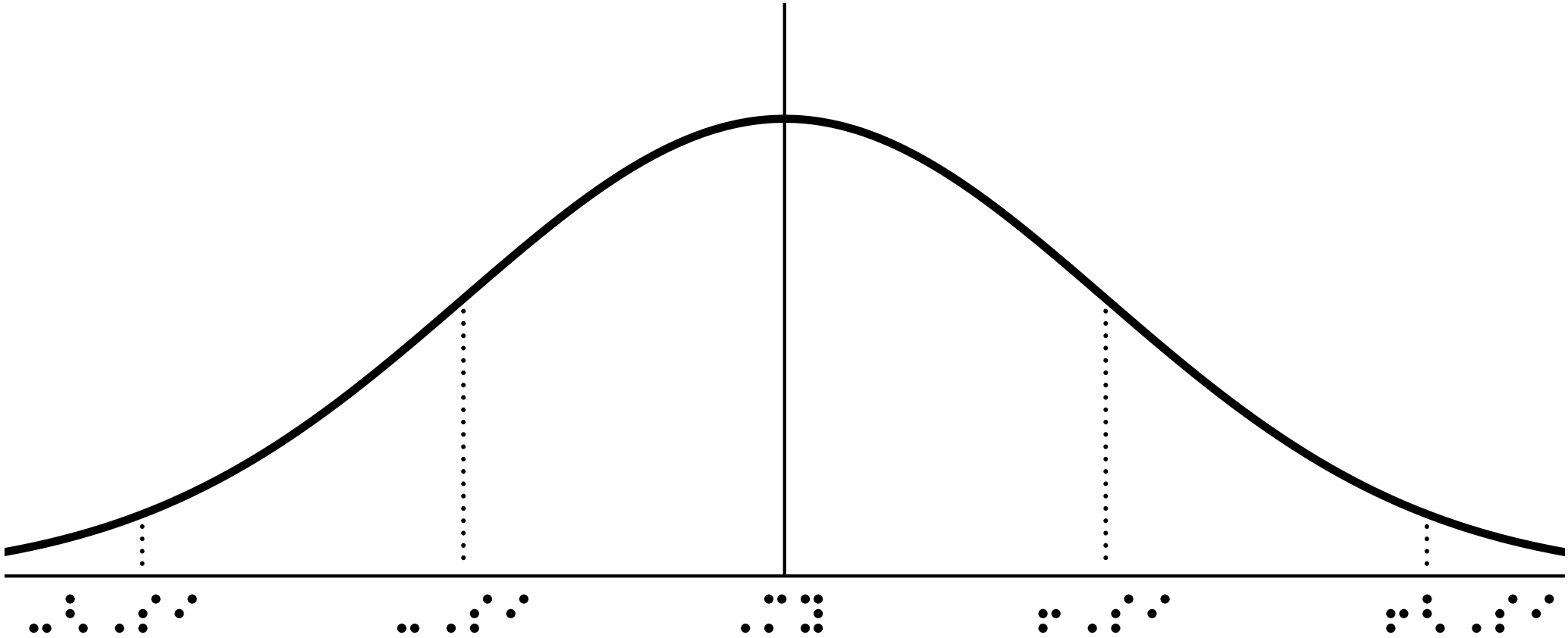


$P(X \leq \mu) = 0.5$

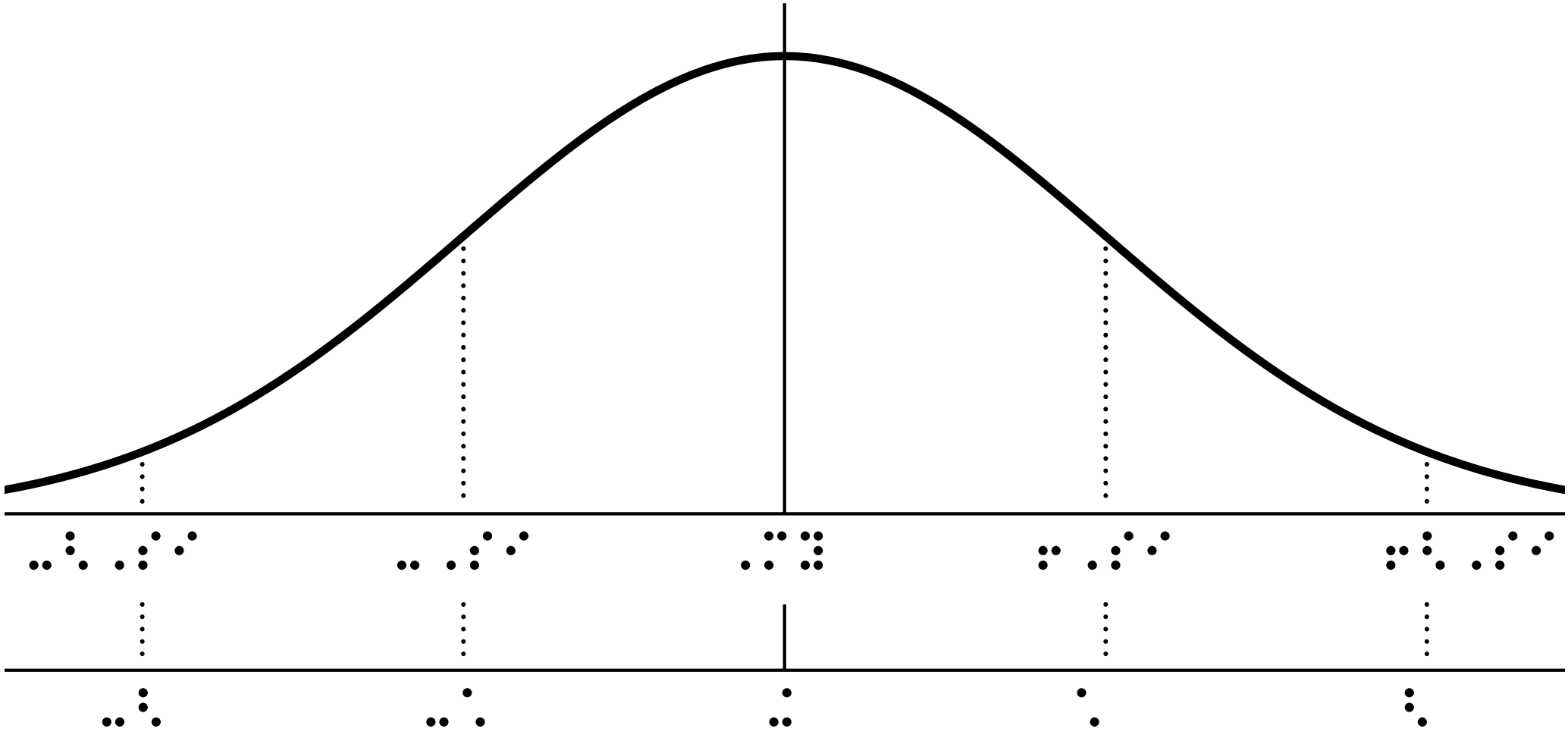


$P(X \geq \mu) = 0.5$

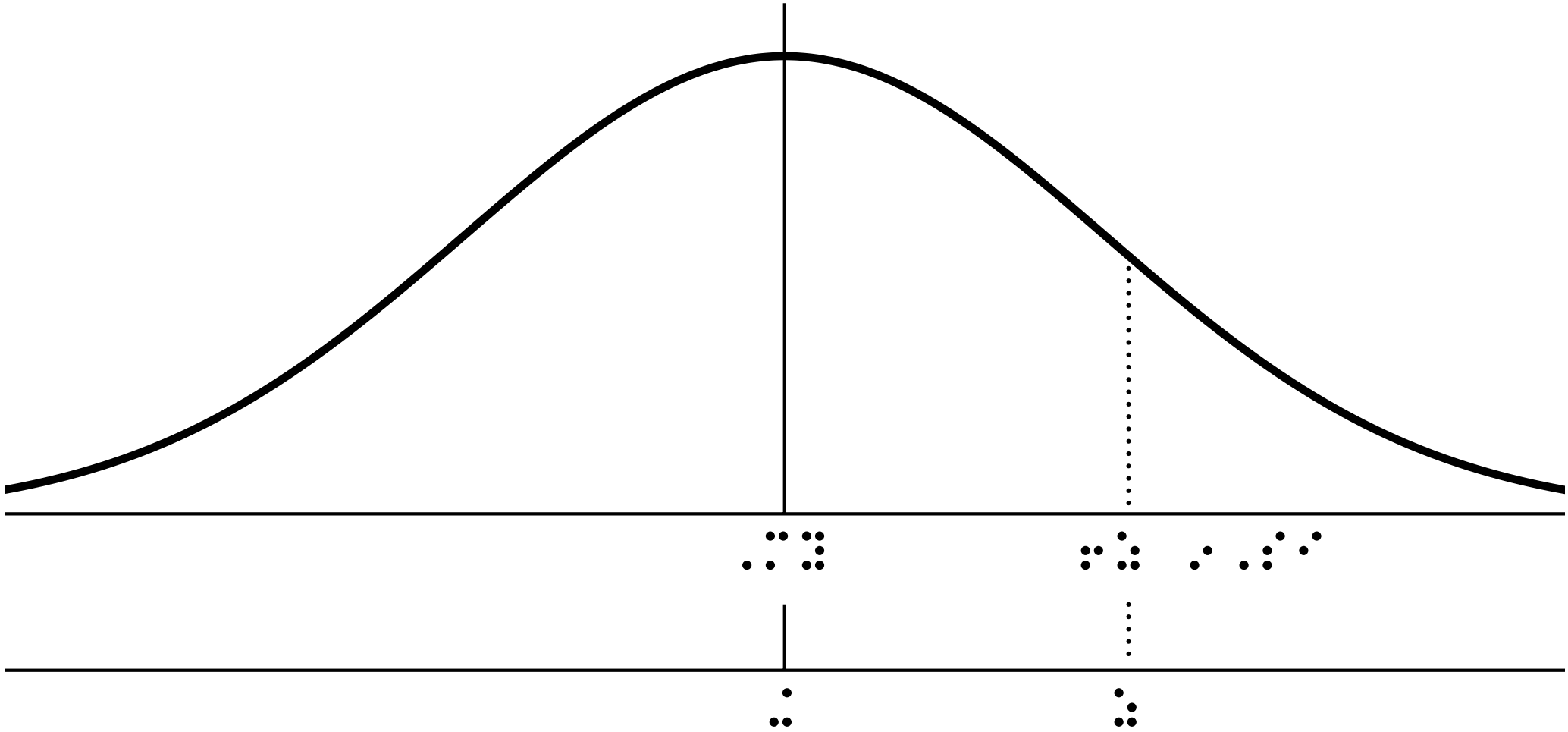
$$f(x) = 1 / \sigma\sqrt{2\pi} * e^{(-1/2) * ((x-\mu)/\sigma)^2}$$



$\varphi(z) = 1/\sqrt{2\pi} * e^{-z^2/2}$

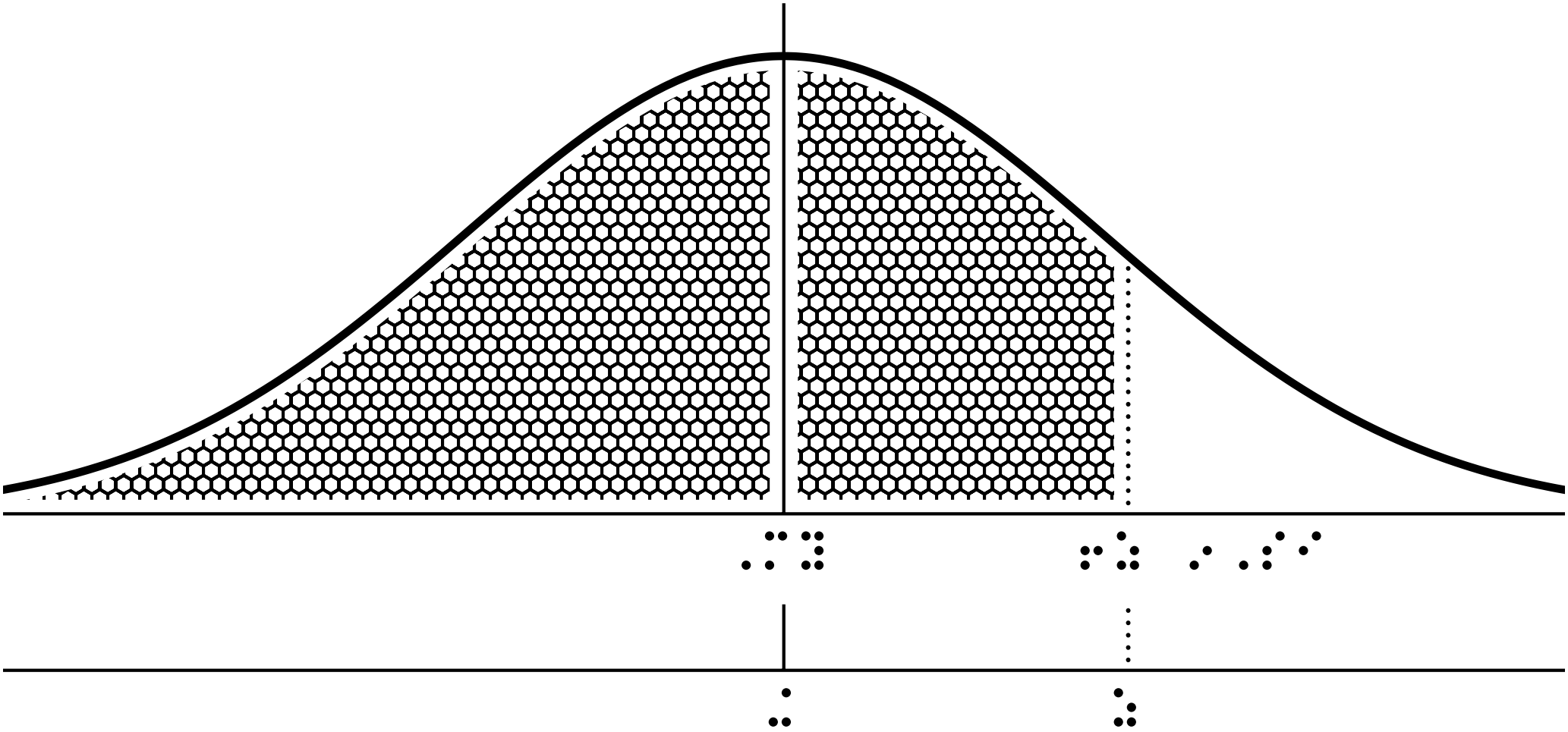


$\varphi(z) = f(\mu + z\sigma)$

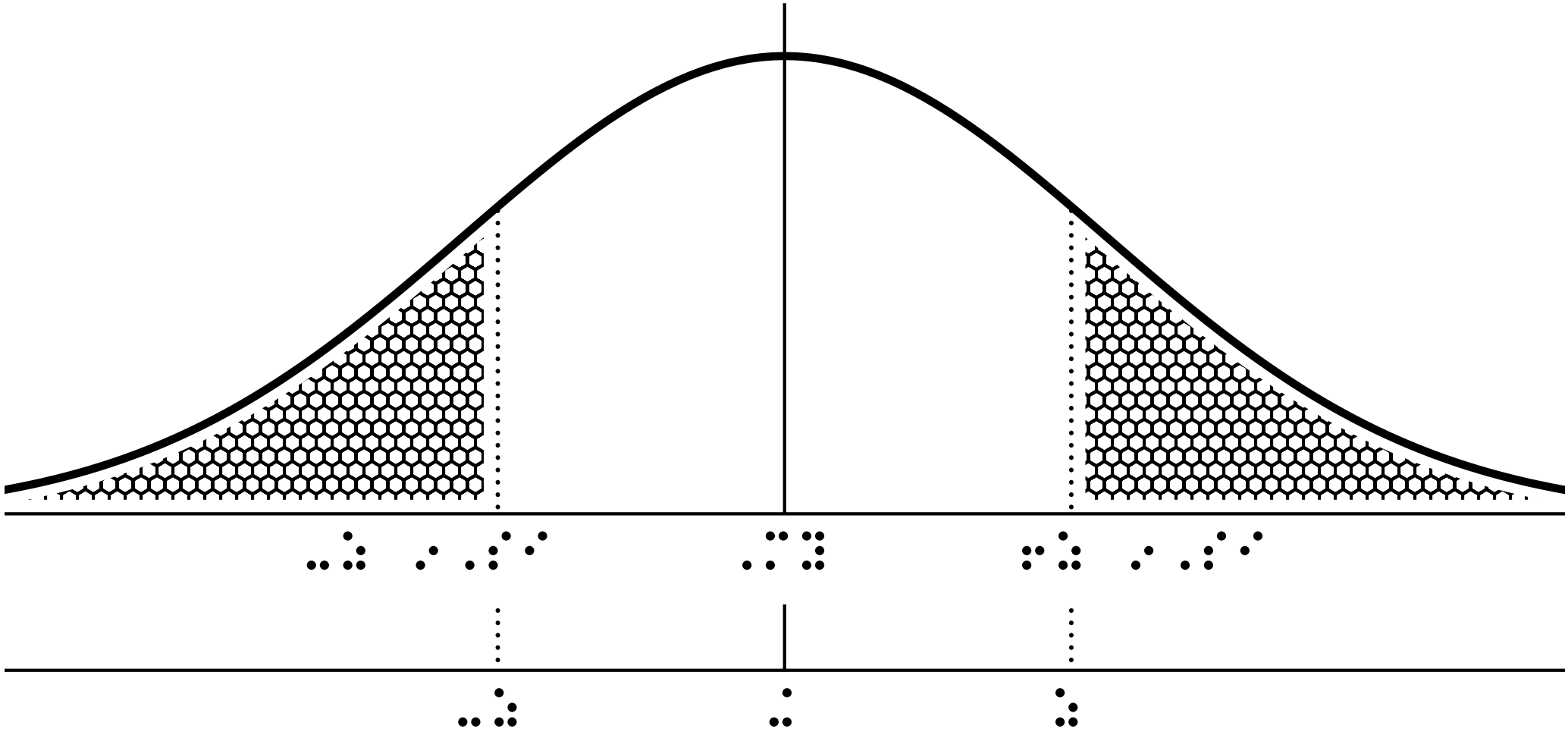


NV Verteilungsfunktion Standard, 10/18

$\Phi(z) = P(X \leq \mu + z\sigma)$

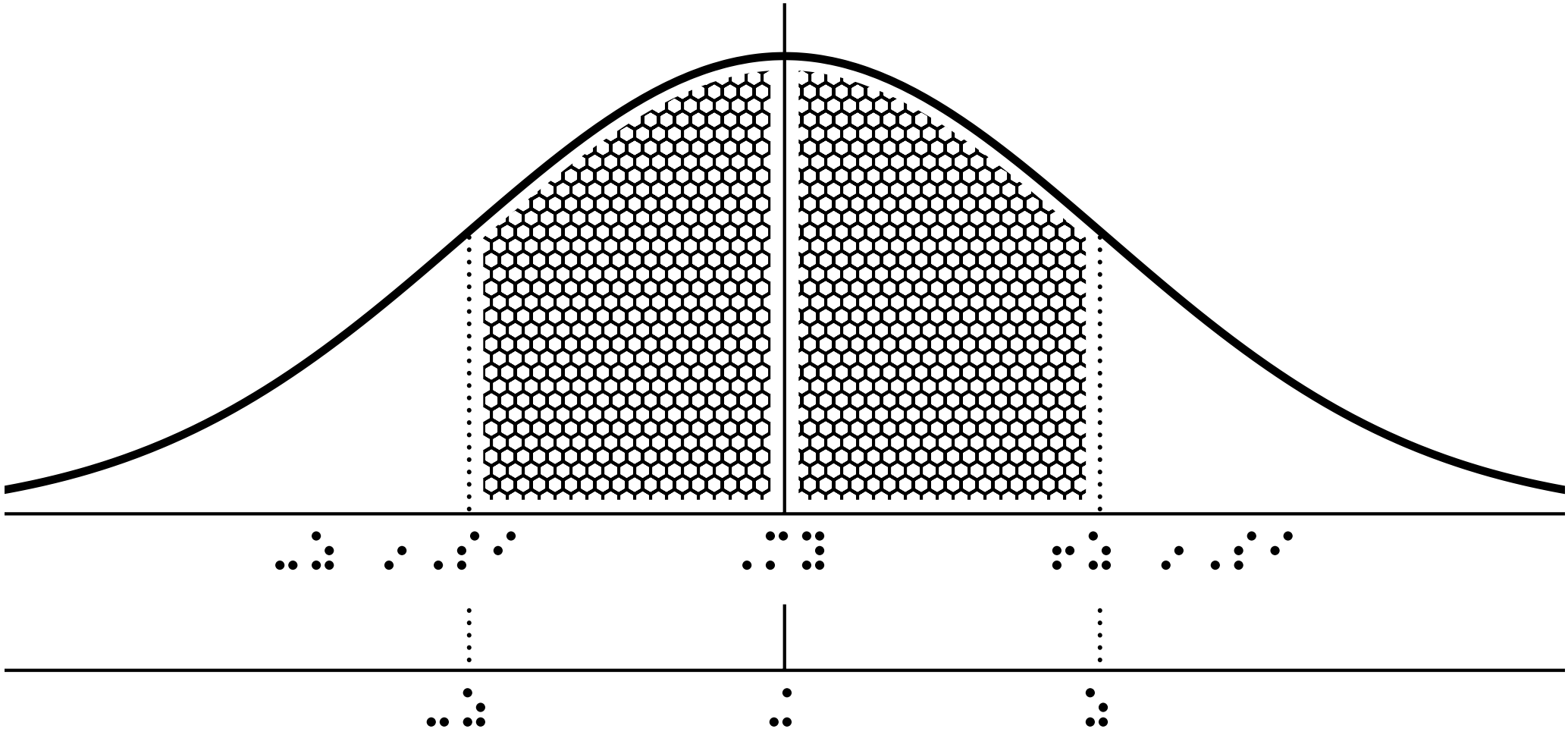


$1 - \Phi(z) = \Phi(-z)$

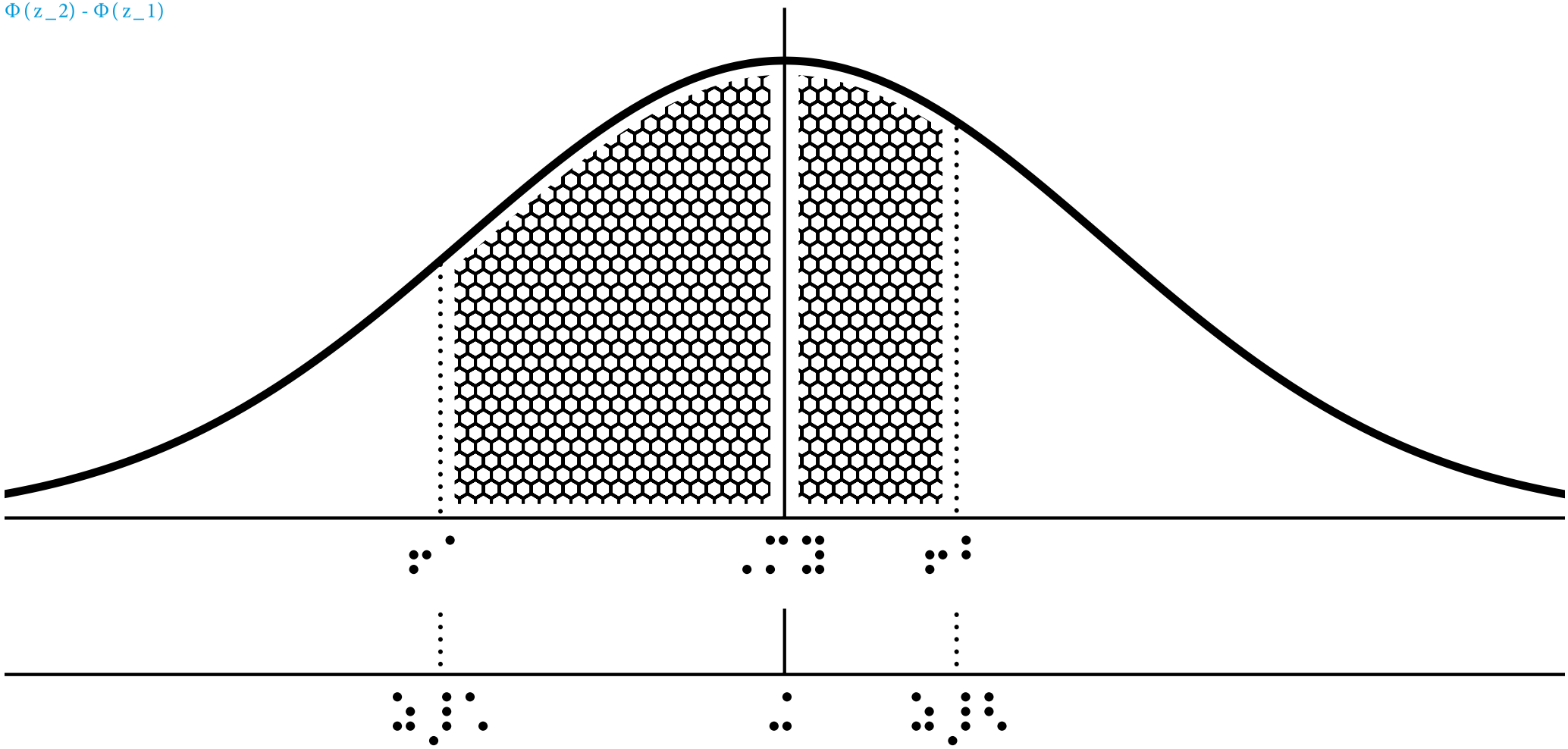


NV Verteilungsfunktion Standard, 12/18

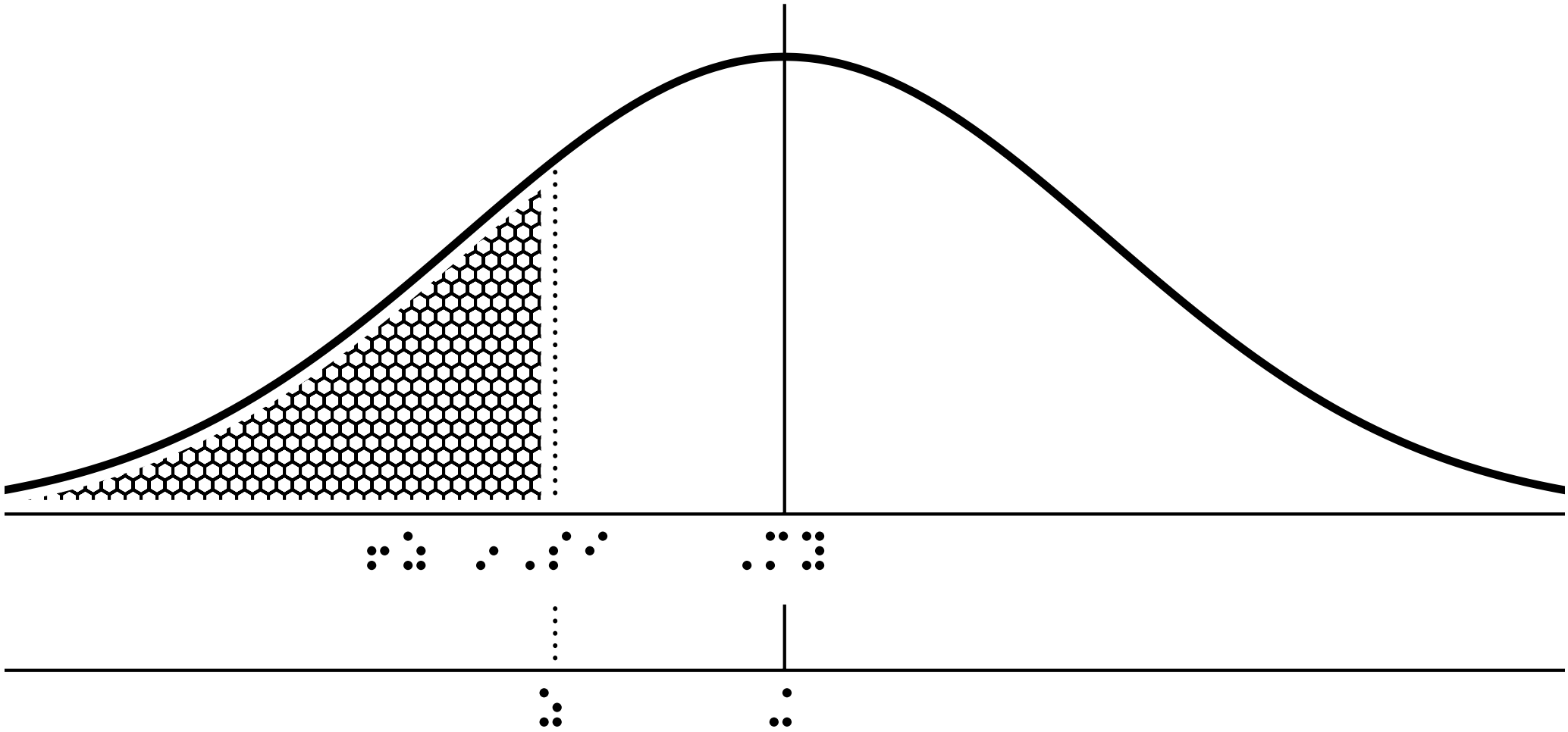
2 Φ(z) - 1



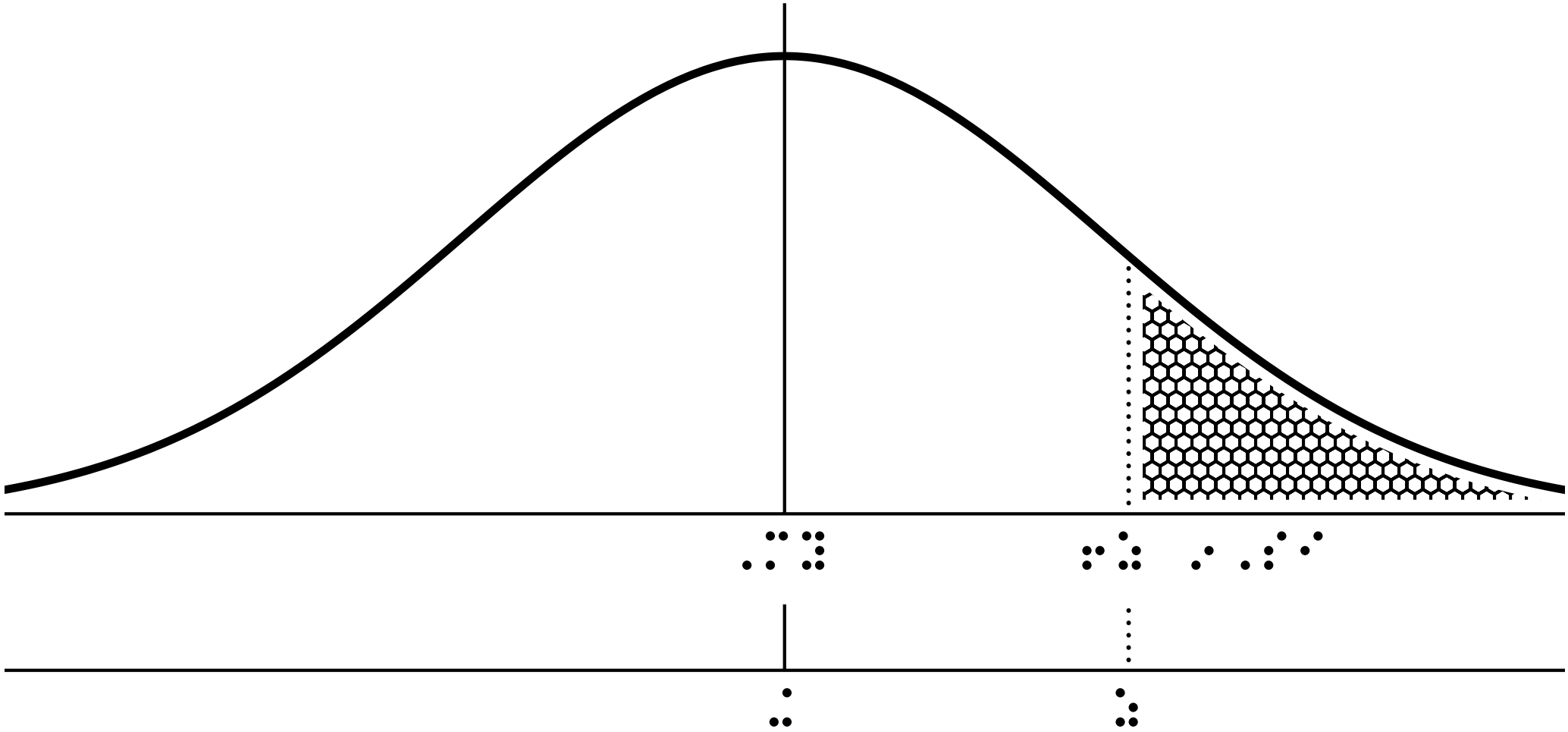
$\Phi(z\_2) - \Phi(z\_1)$



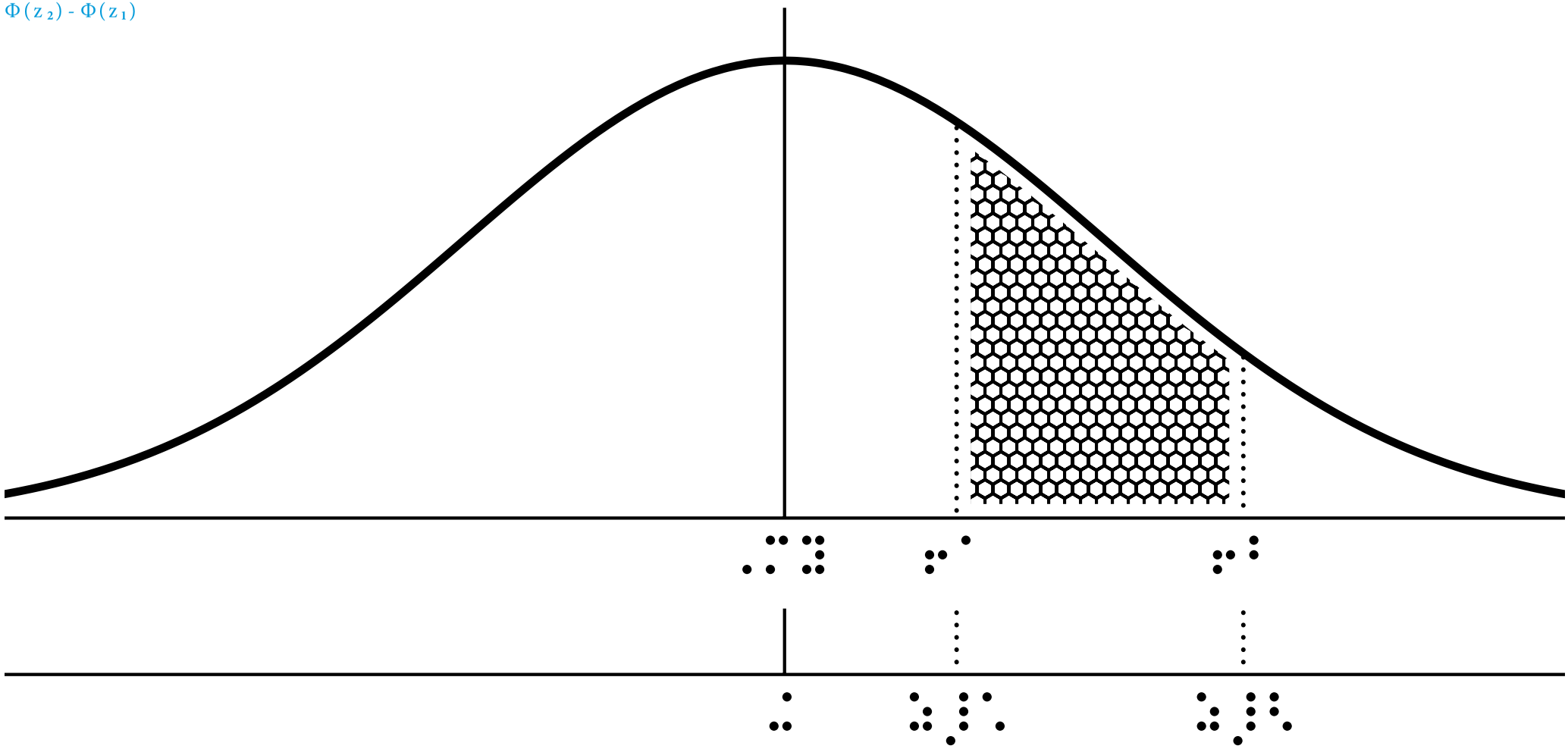
$\Phi(z)$



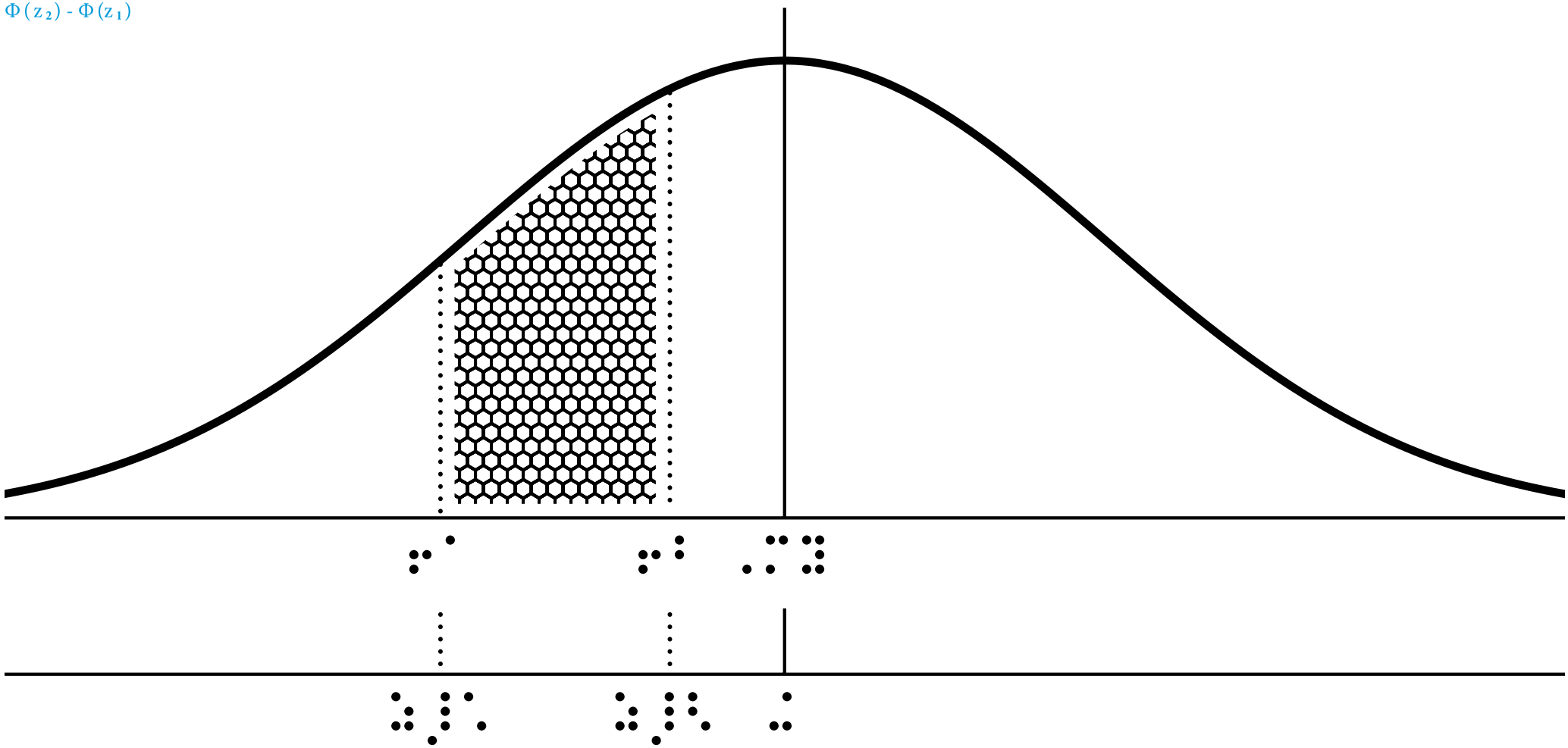
1 - Φ(z)



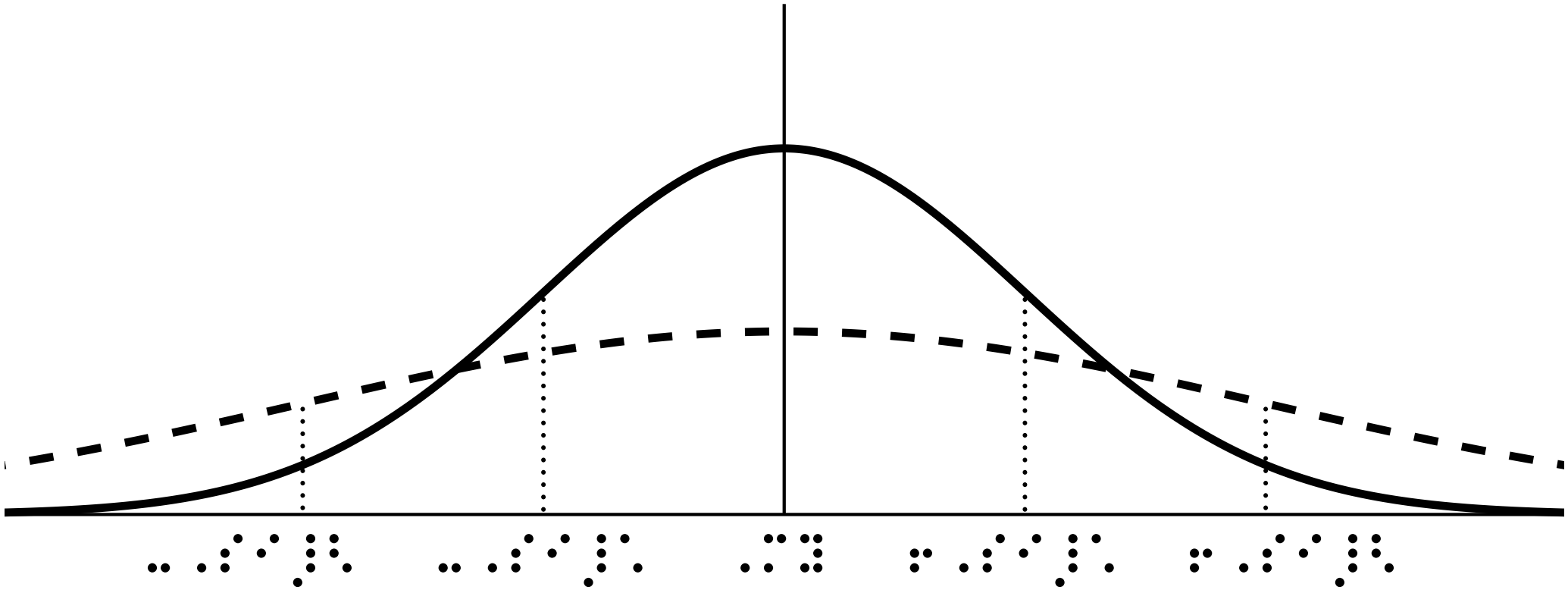
$\Phi(z_2) - \Phi(z_1)$



$\Phi(z_2) - \Phi(z_1)$



$\mu_1 = \mu_2 \quad \sigma_1 < \sigma_2$



Reelle Funktionen

Schulstufe 12

Typische Funktionen (TF)

- Potenzfunktionen
- Polynomfunktionen
- Exponentialfunktionen

Vergleichen von Funktionen (VF)

Änderungsmaße (ÄM)

⠠⠠⠠

Inhalt

Typische reelle Funktionen

1 Potenzfunktionen
a *x^r mit a =1, r 'el 'Z

2 Polynomfunktionen
Grad 1 Geraden

3 Polynomfunktionen
Grad 2 Parabeln

4 Polynomfunktionen
Grad 3 mit „Entartungen“

5 Polynomfunktionen Grad 4

6 Polynomfunktionen
Grad 4 mit „Entartungen“

7 Exponentialfunktionen a^x
a >1; 0 <a <1

Vergleichen von reellen Funktionen

8 Mult: a *f(x)
a >1; 0 <a <1;

9 Mult: a *f(x)
a =-1; a <-1; -1 <a <0;

10 Mult: f(b *x)

b >1; 0 <b <1;

11 Add: f(x) +d

d >0; d <0;

12 Add: f(x +c)

c >0; c <0

Änderungsmaße

13 f(x) =k *x +d

f(x +1) =f(x) +k

f(x +h) =f(x) +k *h

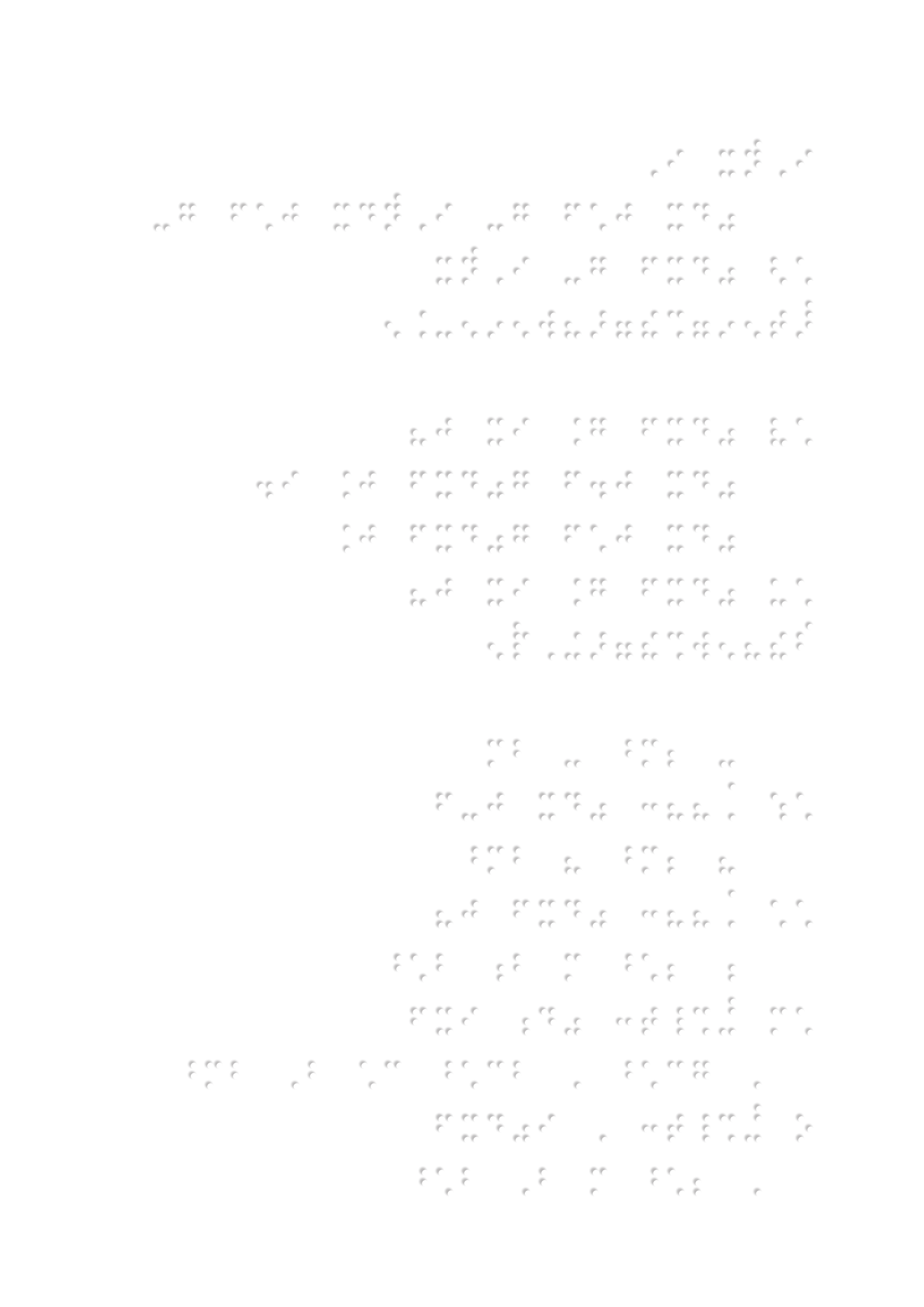
14 f(x) =k *x +d

Steigungsdreiecke

15 f(x) =c *a^x

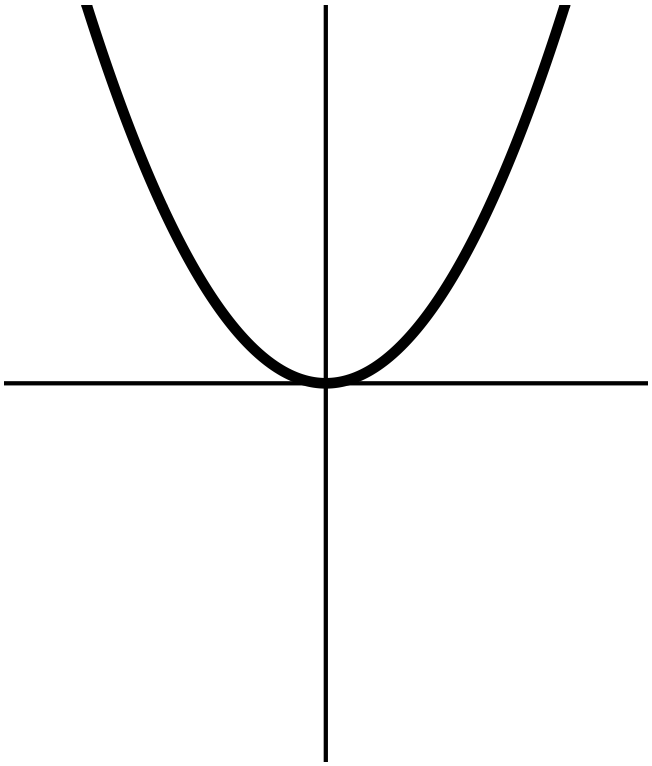
f(x +1) =c *a^(x +1) =c *a^x *a

f(x +h) =c *a^(x +h) =c *a^x *a^h

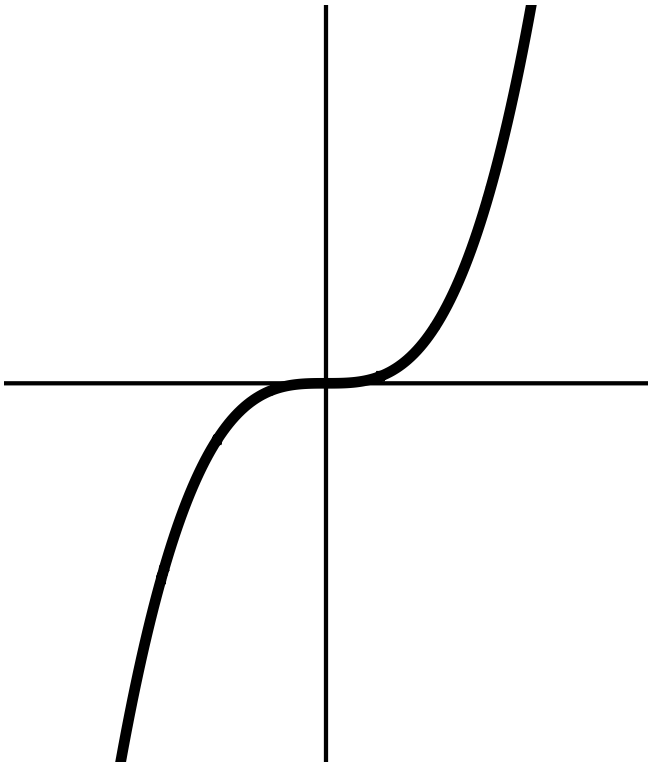


⠀⠀⠀⠋⠦⠭ ⠖⠓⠴ ⠶⠉ ⠔⠁⡮⠦⠭ ⠖⠓⠴ ⠶⠉
⠔⠁⡮⠭ ⠔⠁⡮⠓

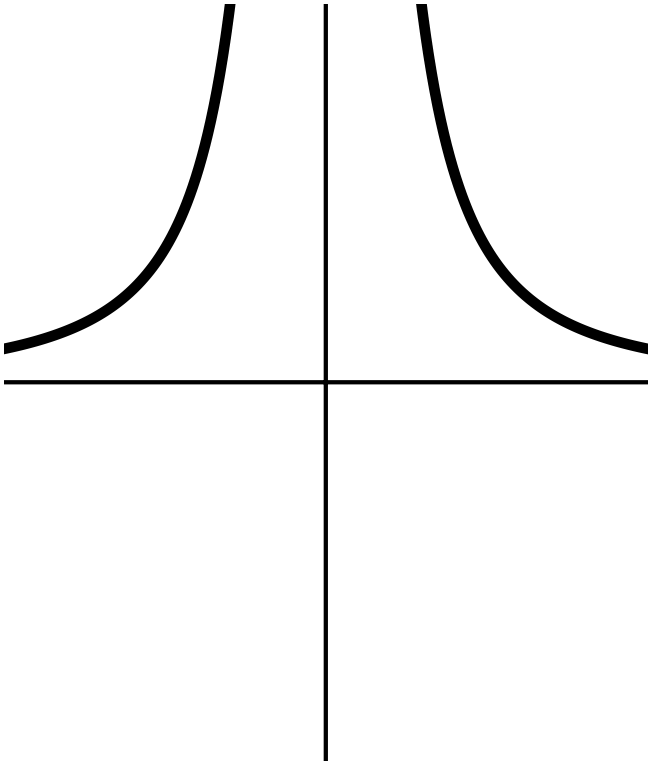
$f(x) = x^2$



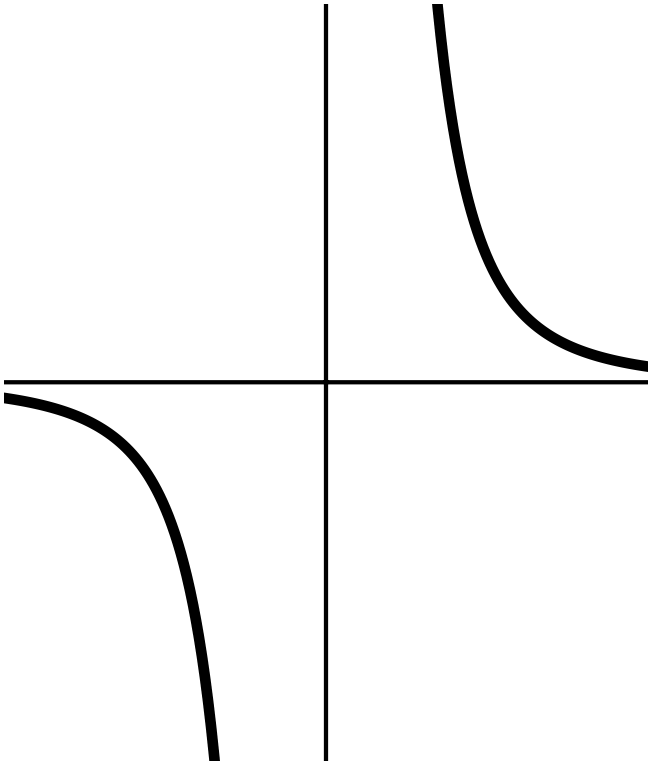
$f(x) = x^3$



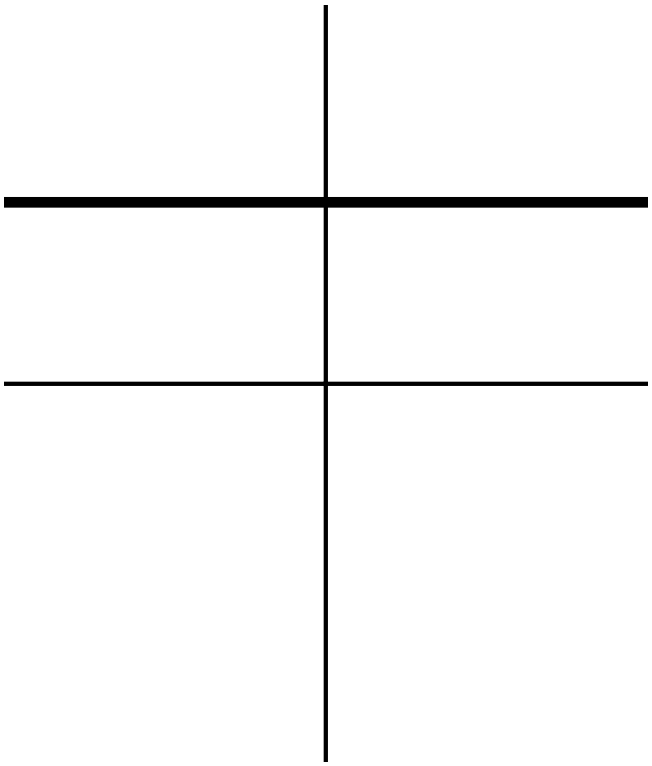
$f(x) = 1/x^2$

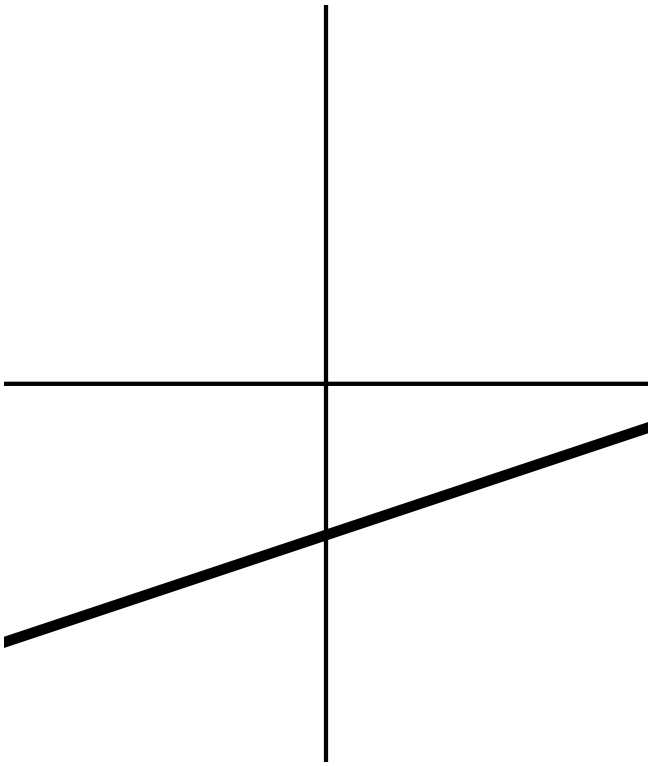


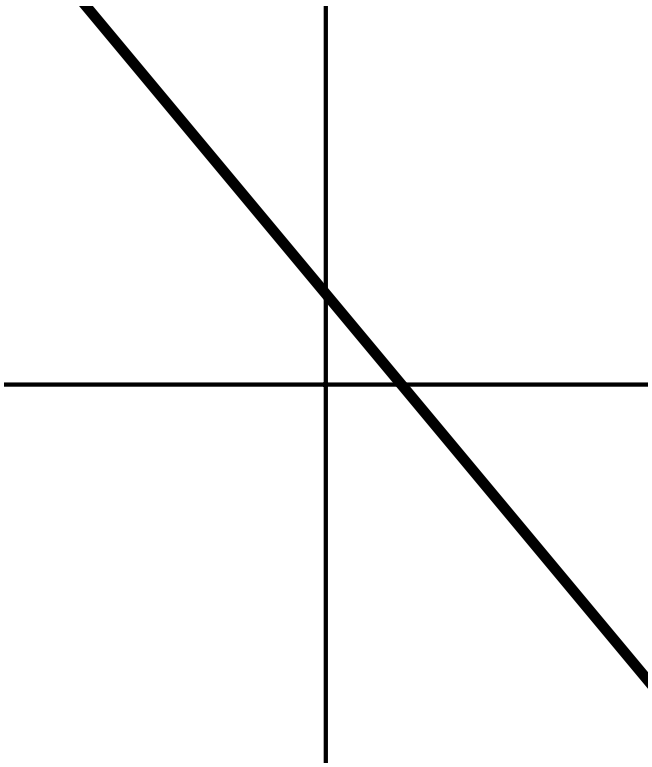
$f(x) = 1/x^3$

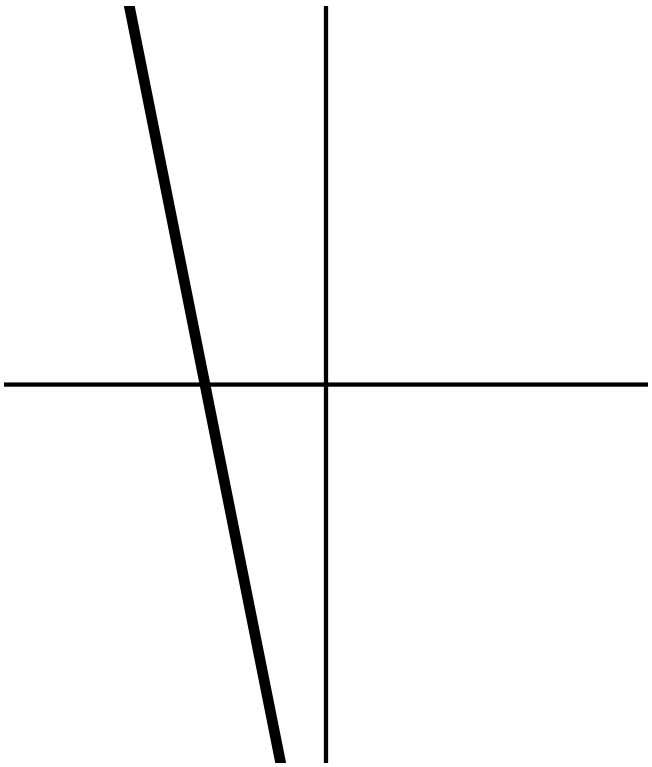


Grad 1: Gerade

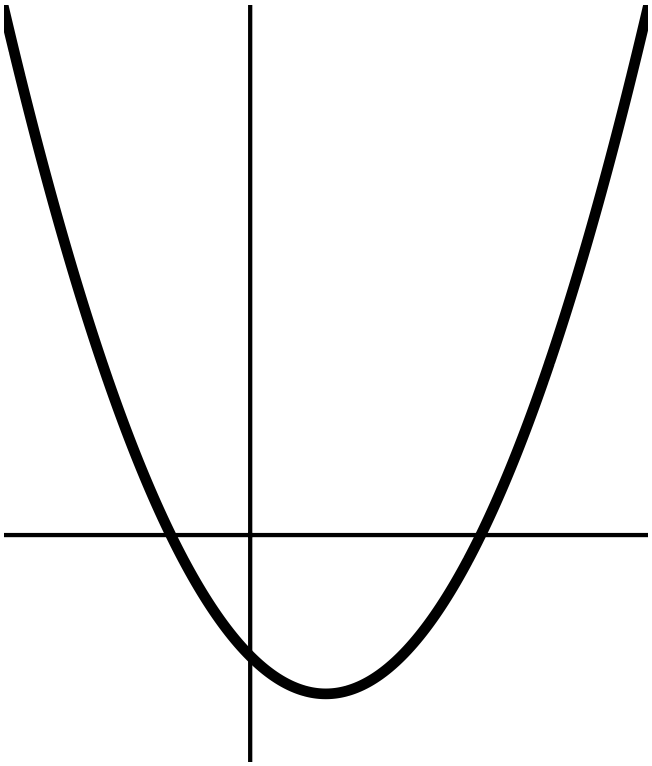


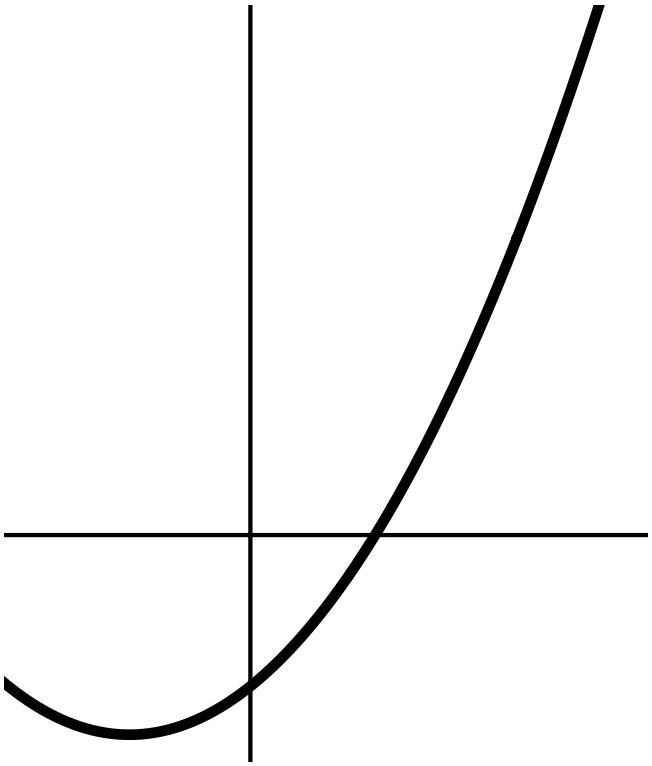


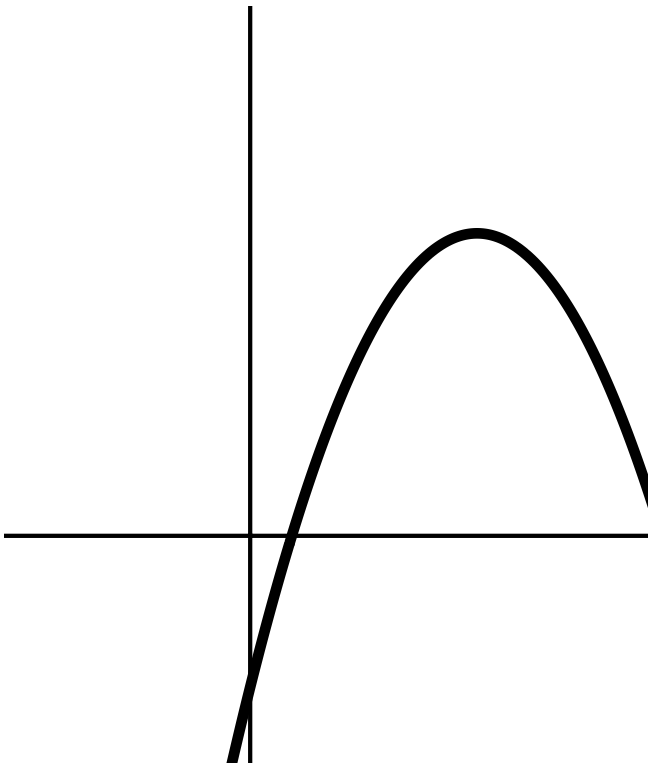


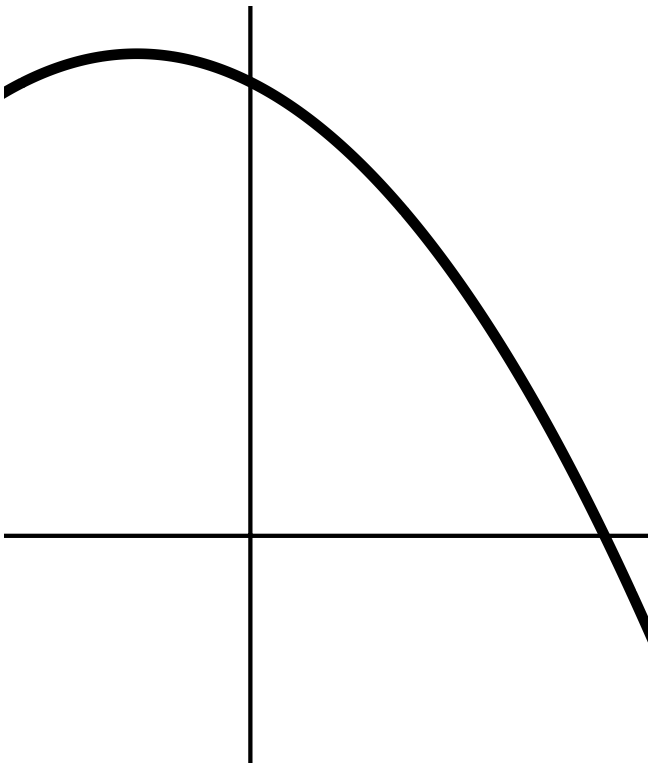


Grad 2: Parabel

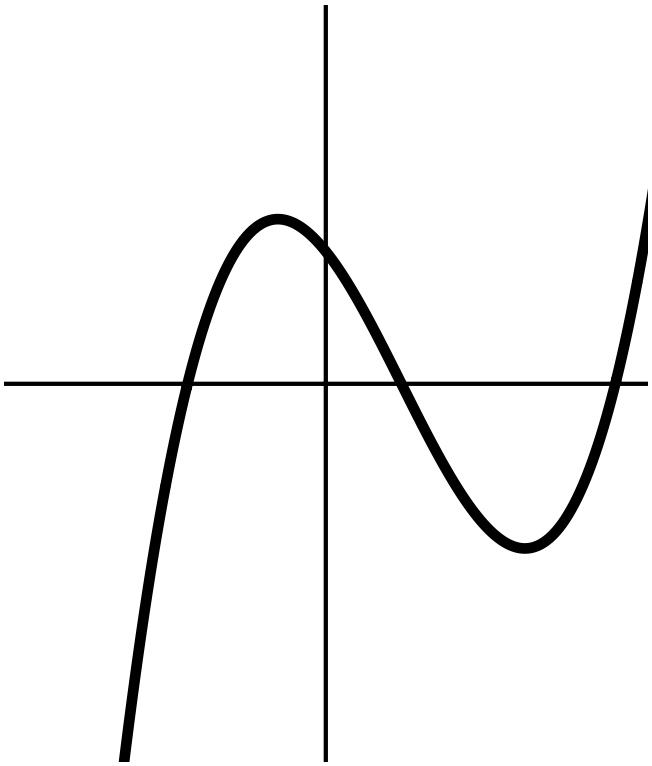


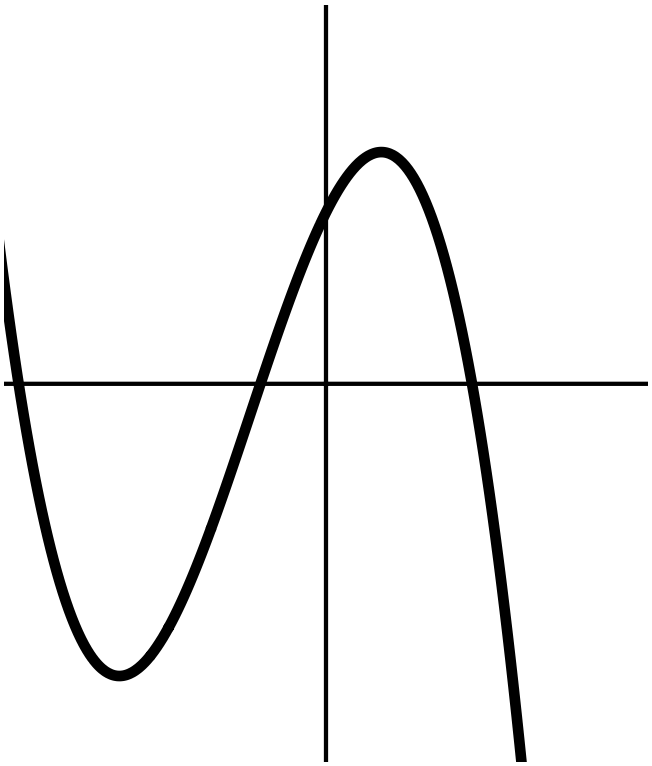


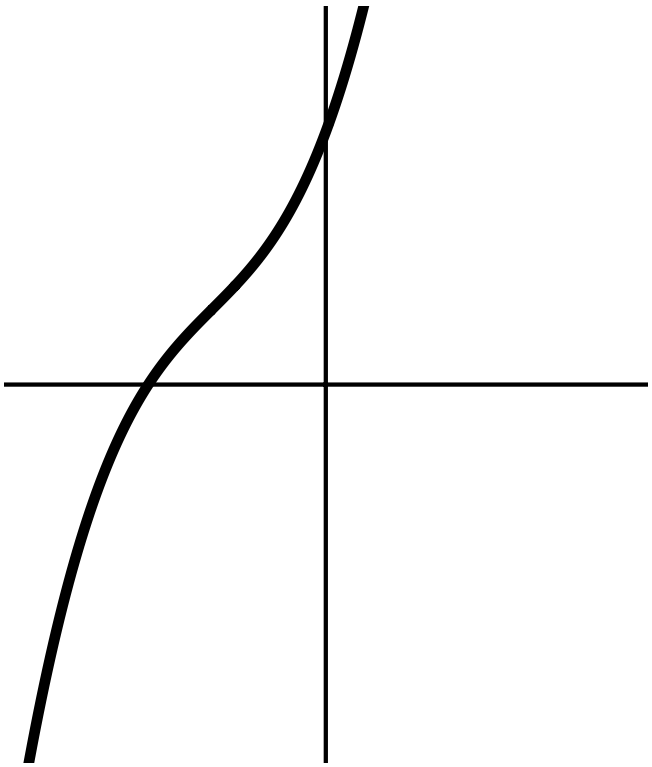


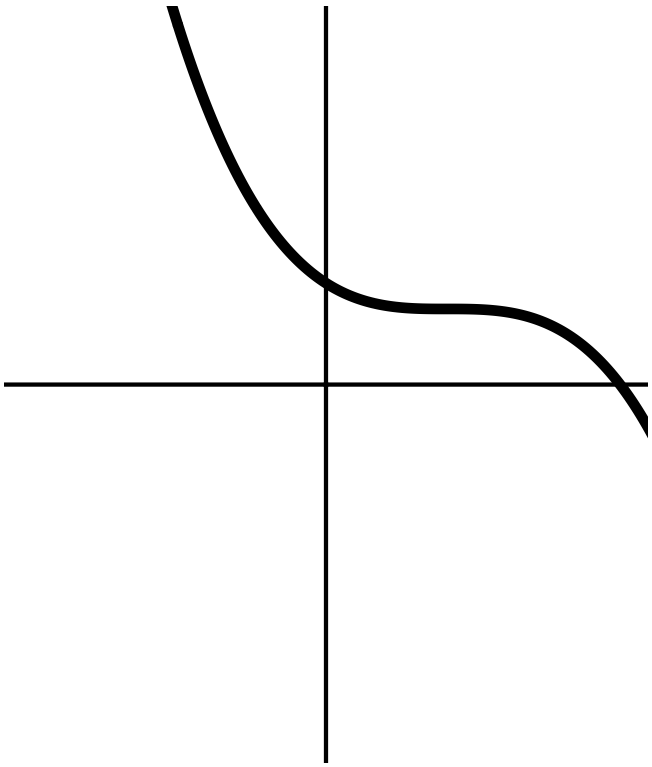


Grad 3: „S-Kurve“ mit „Entartungen“

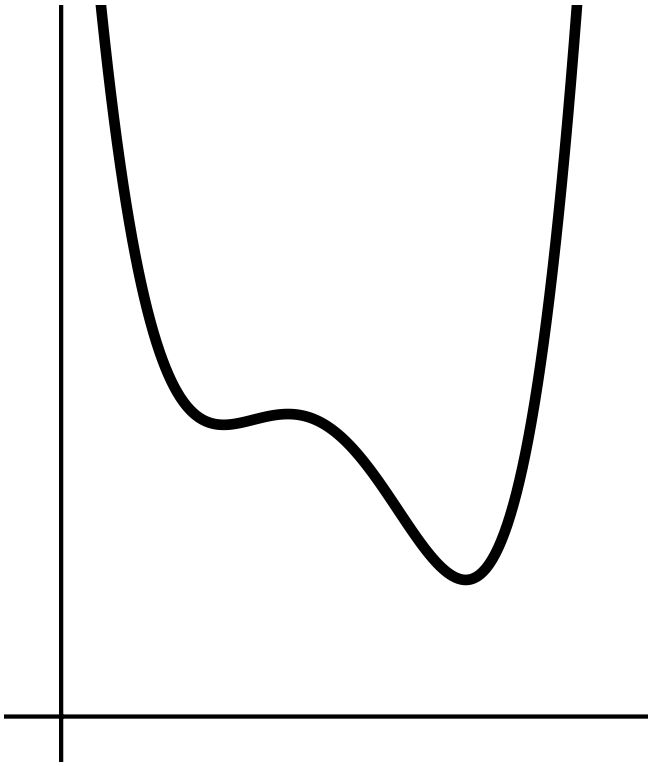


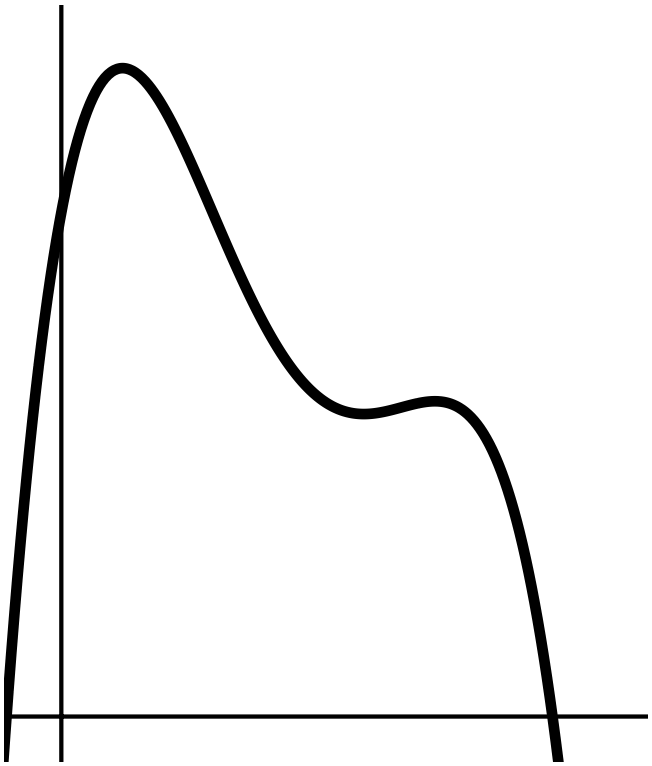


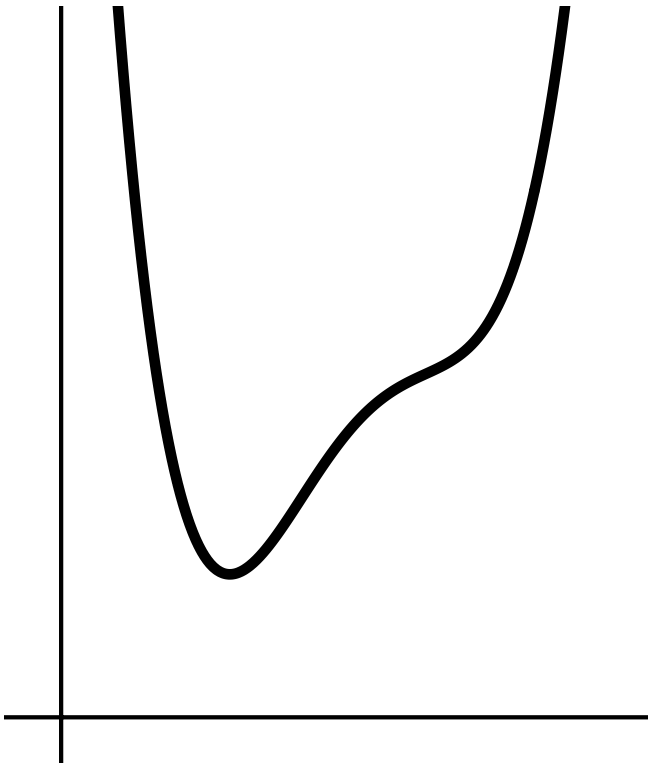


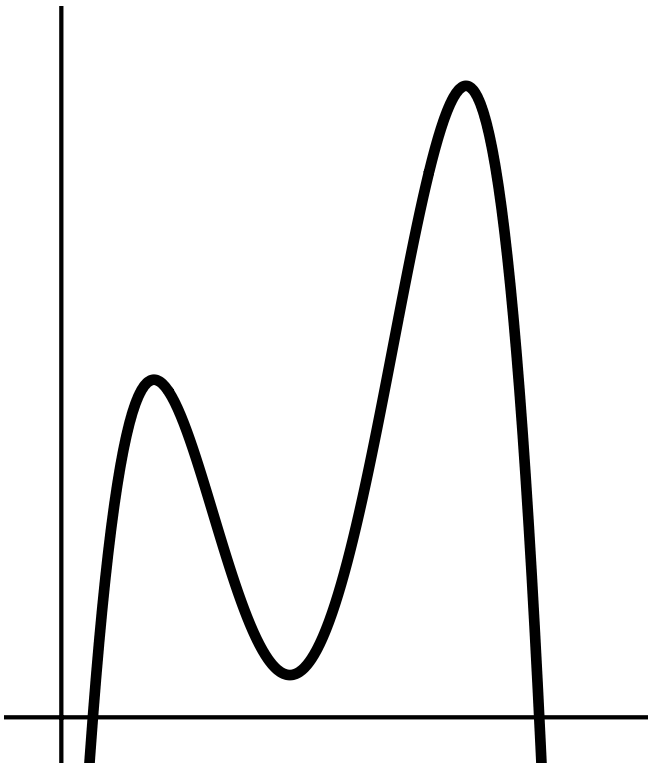


Grad 4: „Doppel-S-Kurve“

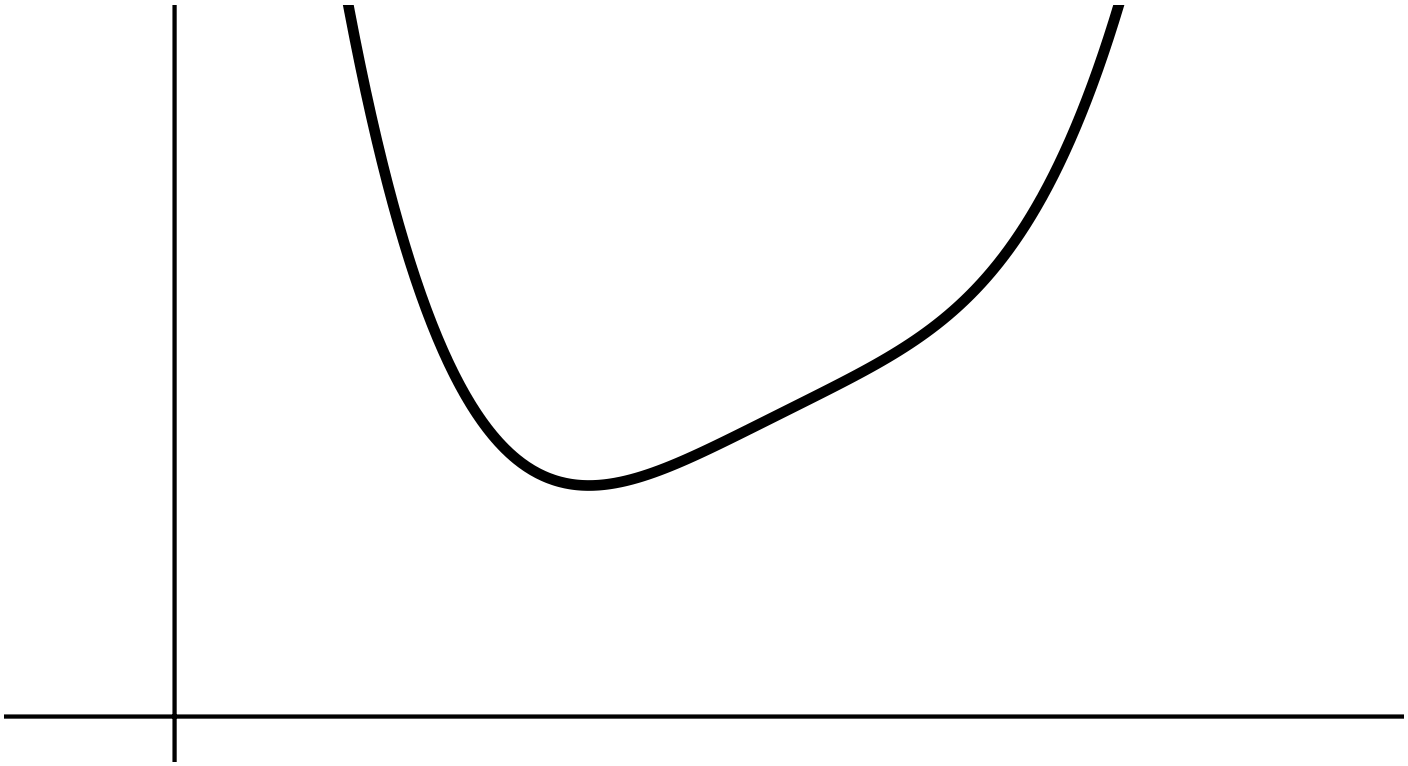


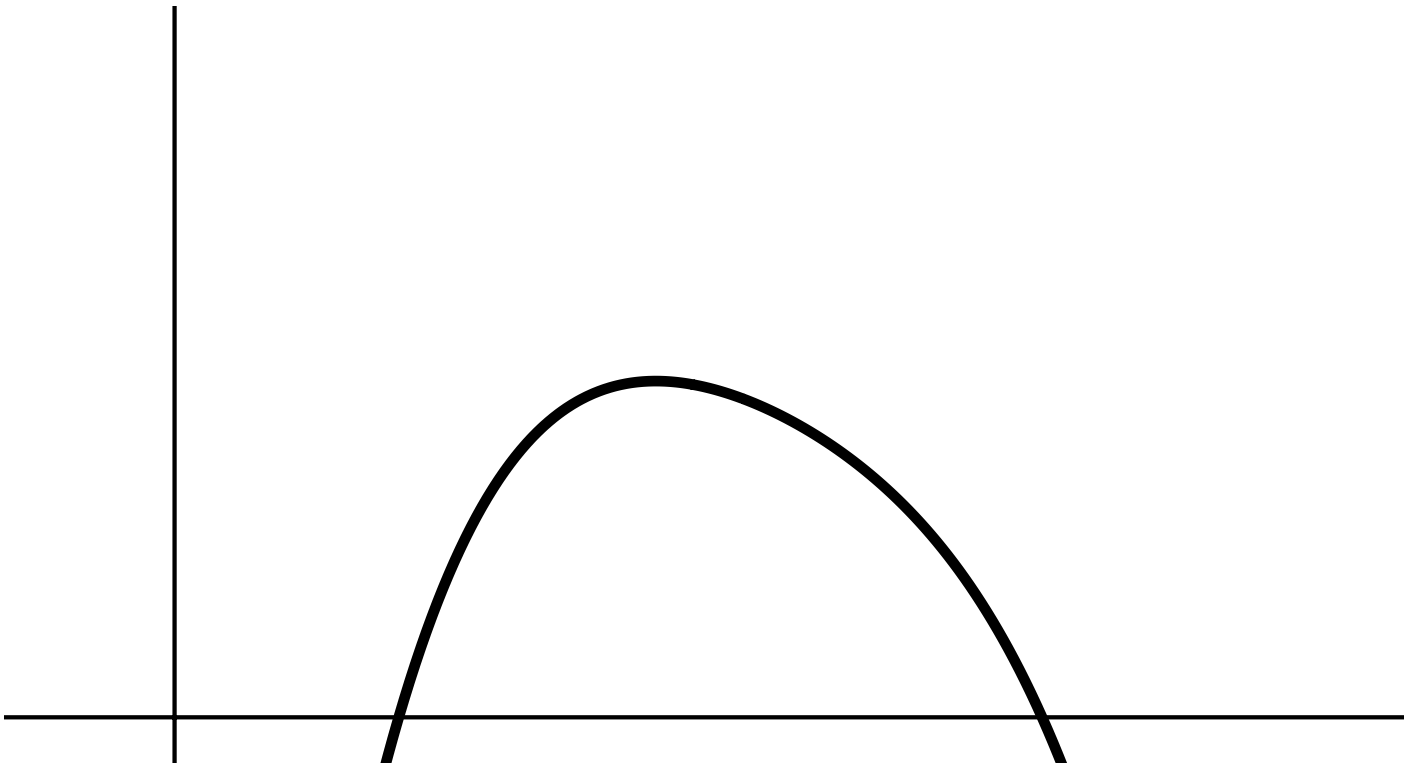




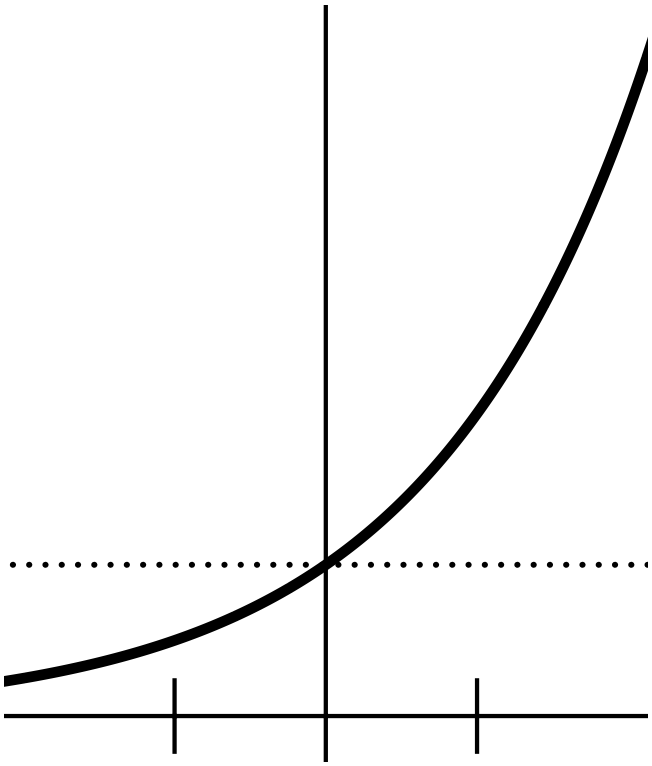


Grad 4: „Doppel-S-Kurve“ mit „Entartungen“

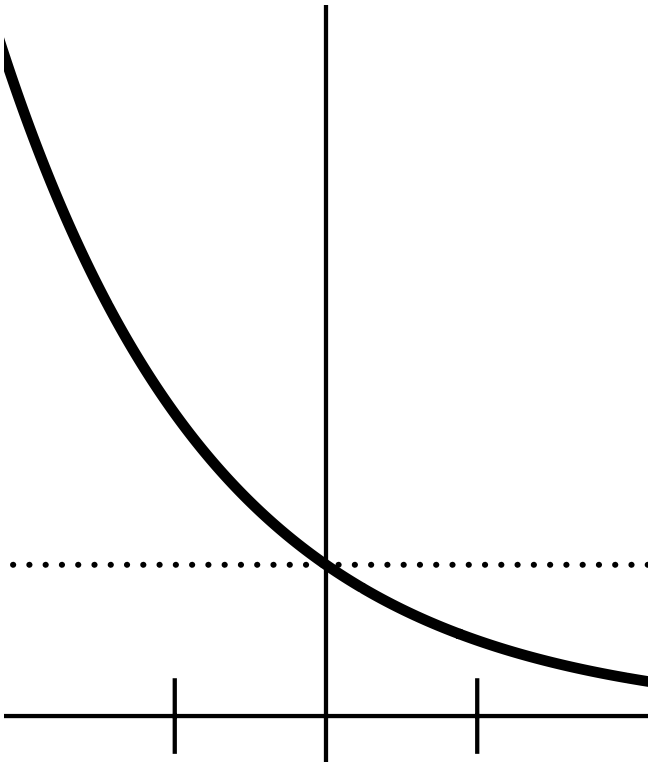




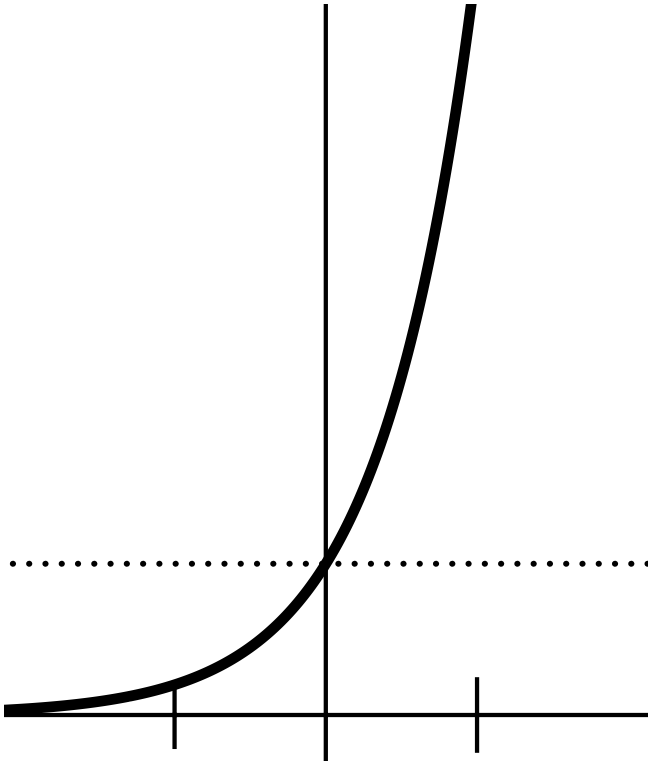
$f(x) = 2^x$



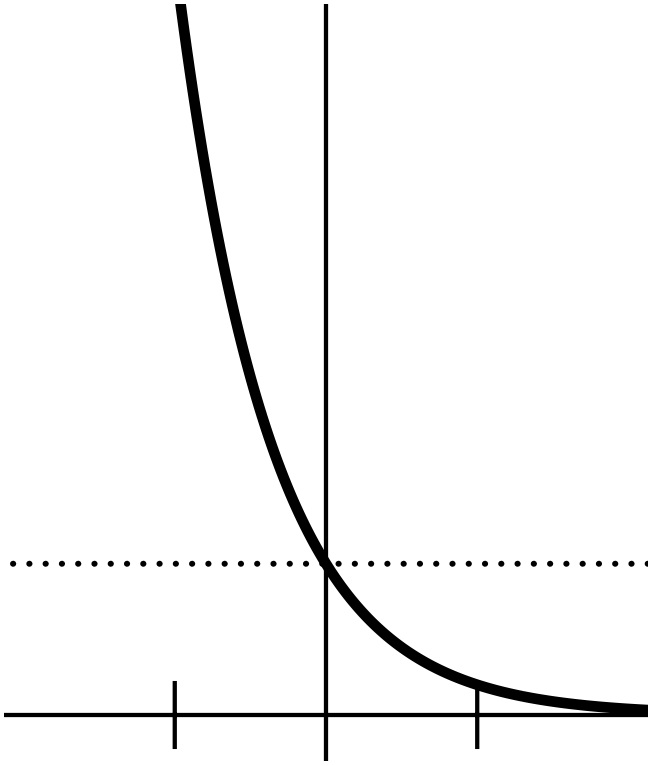
$f(x) = (1/2)^x$



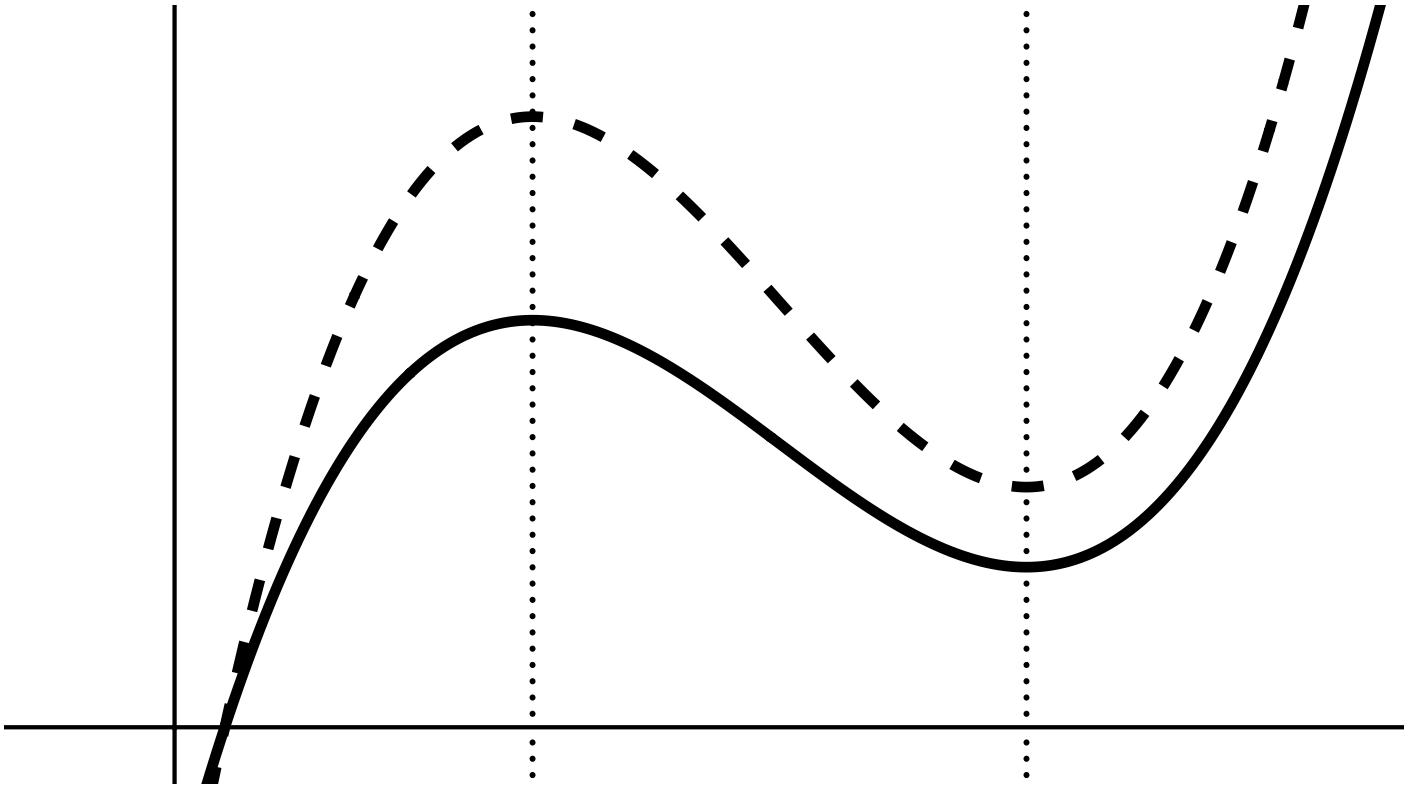
$f(x) = 5^x$



$f(x) = (1/5)^x$



f; g: a >1



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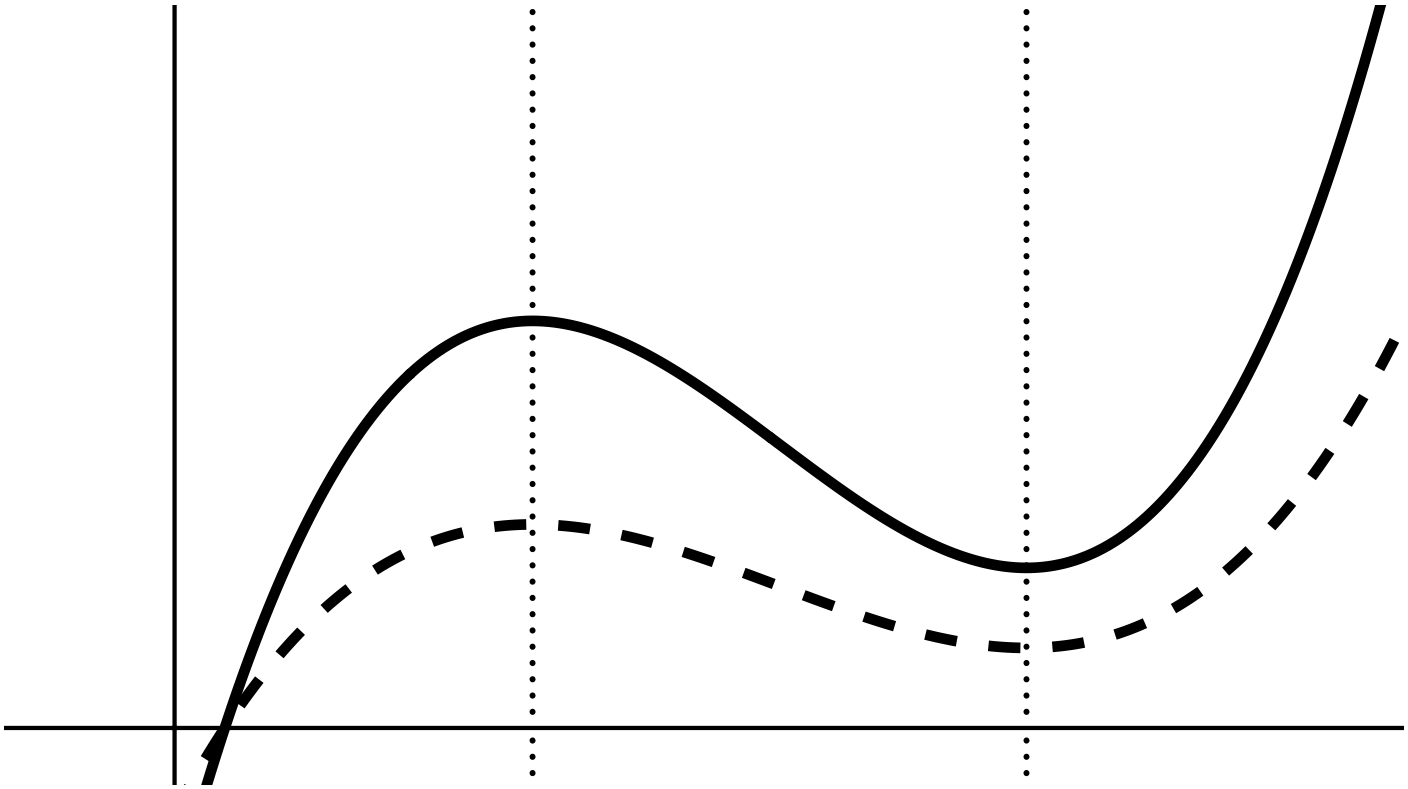

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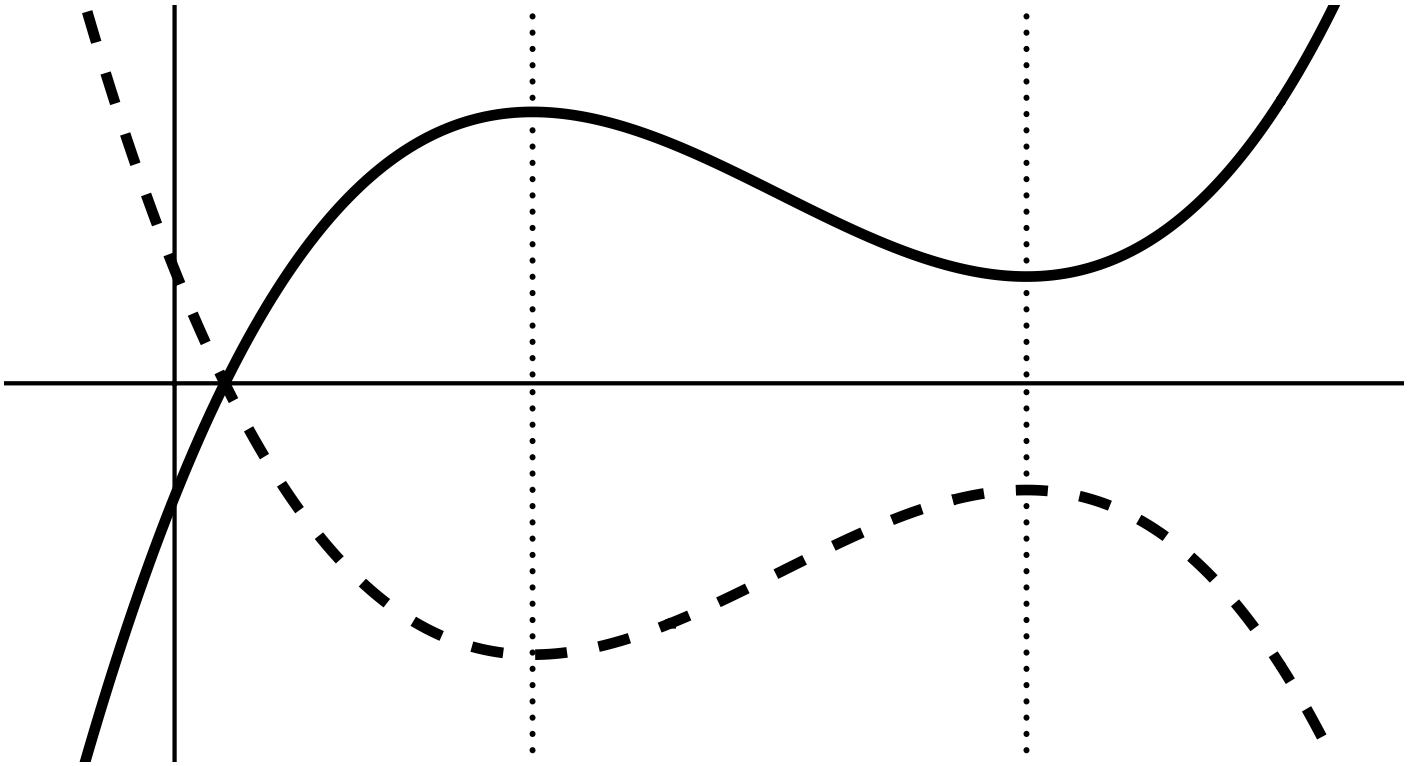

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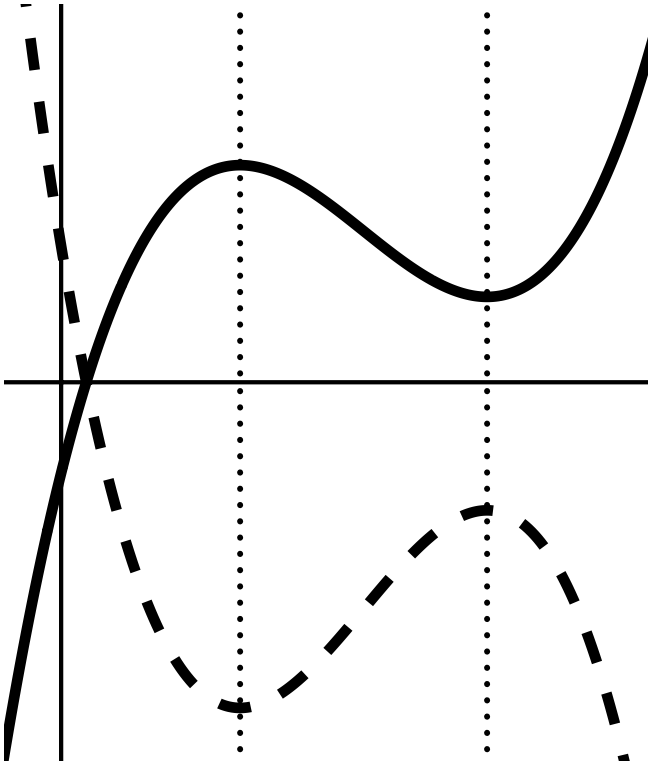

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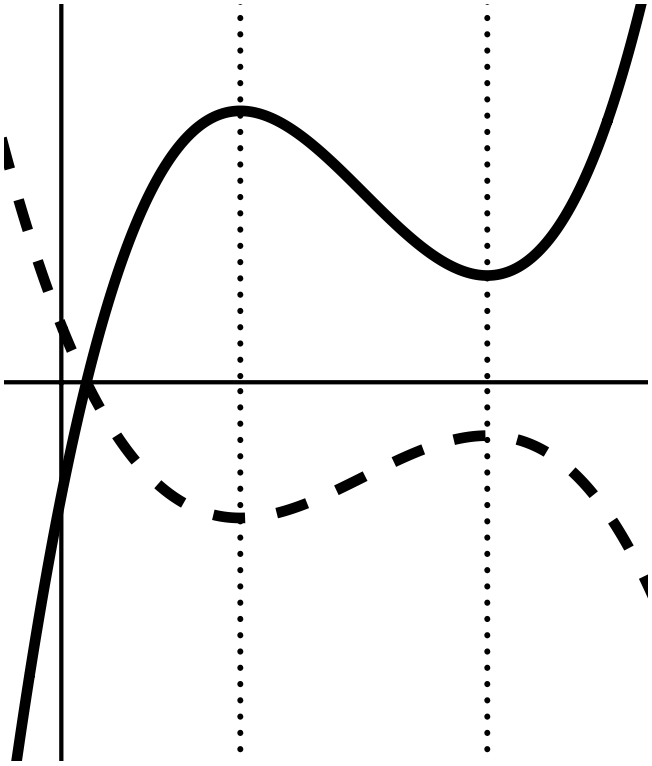

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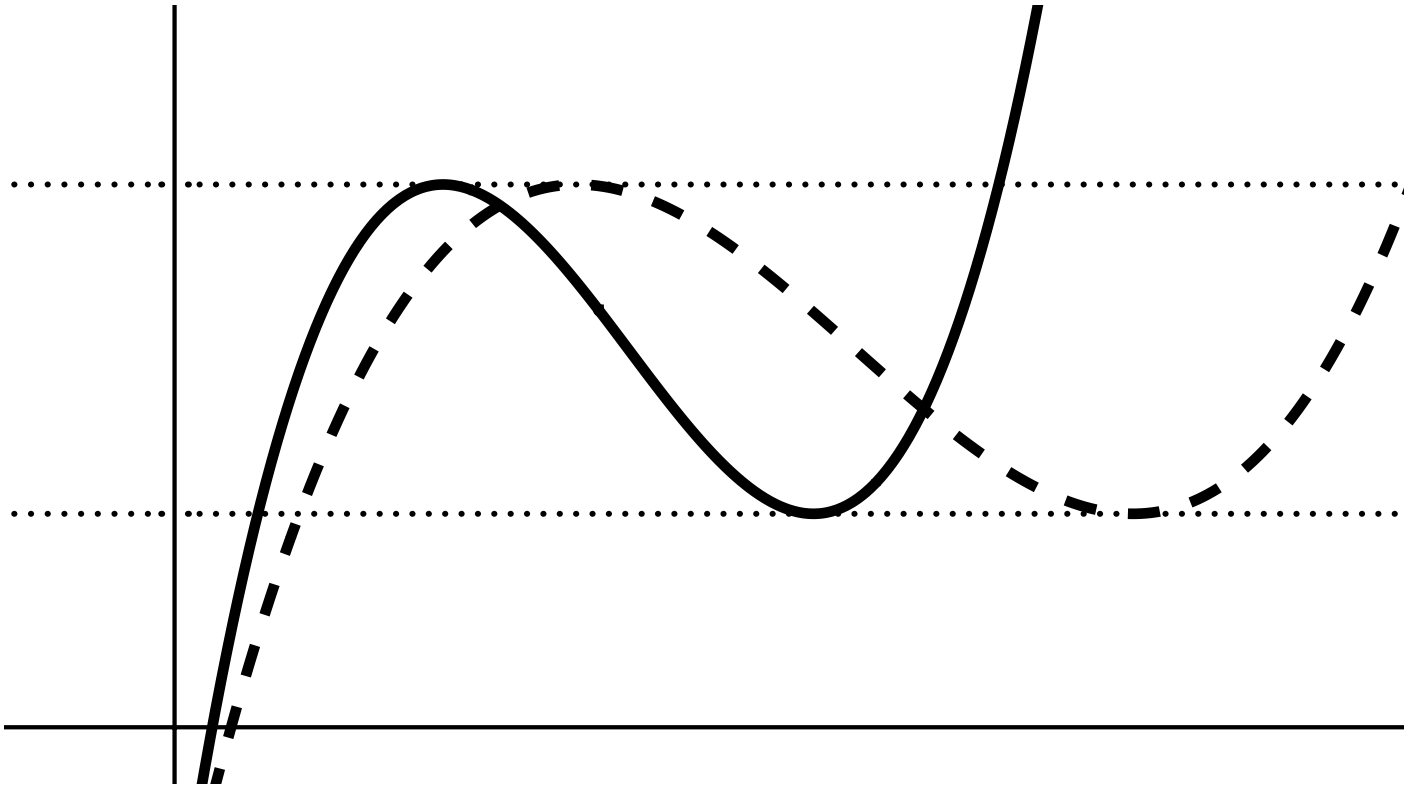

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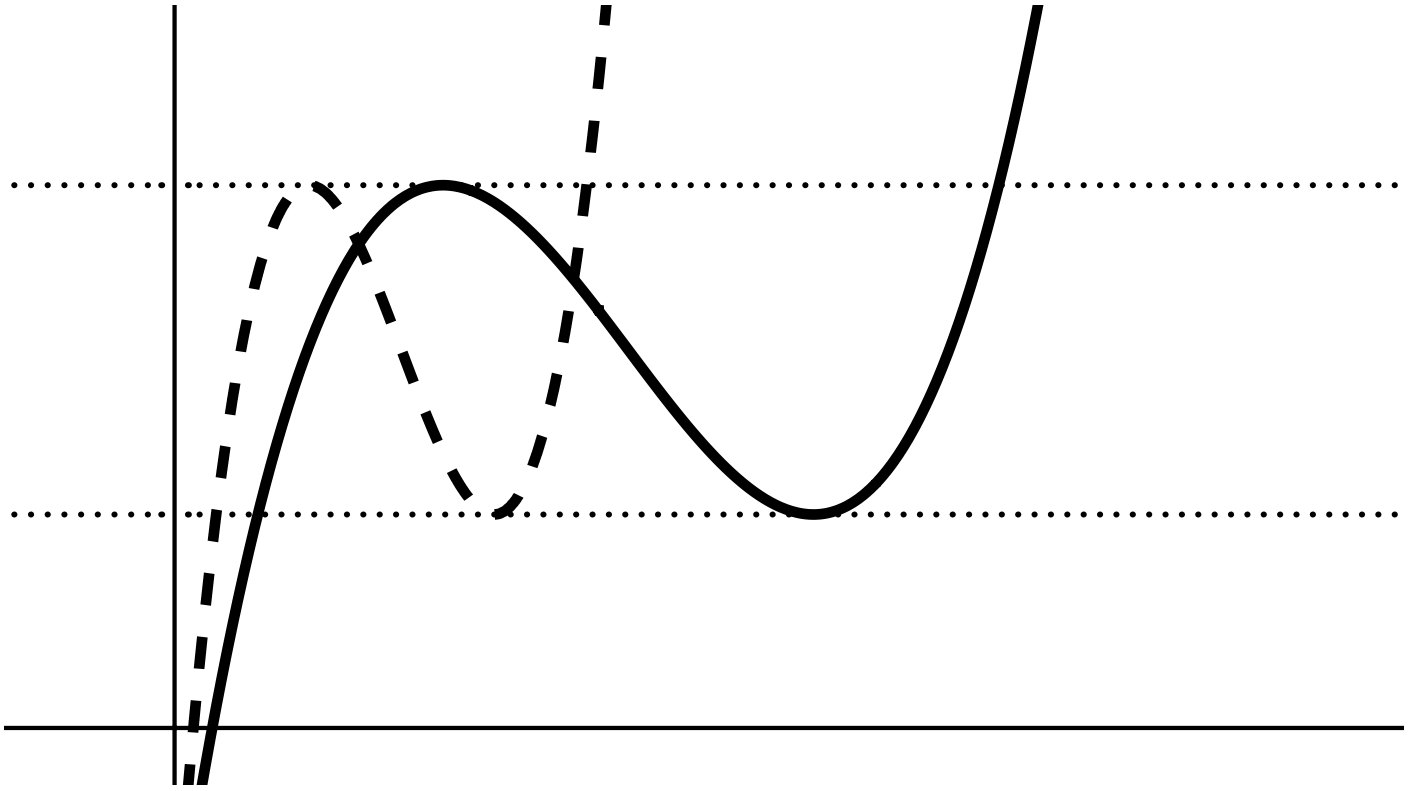

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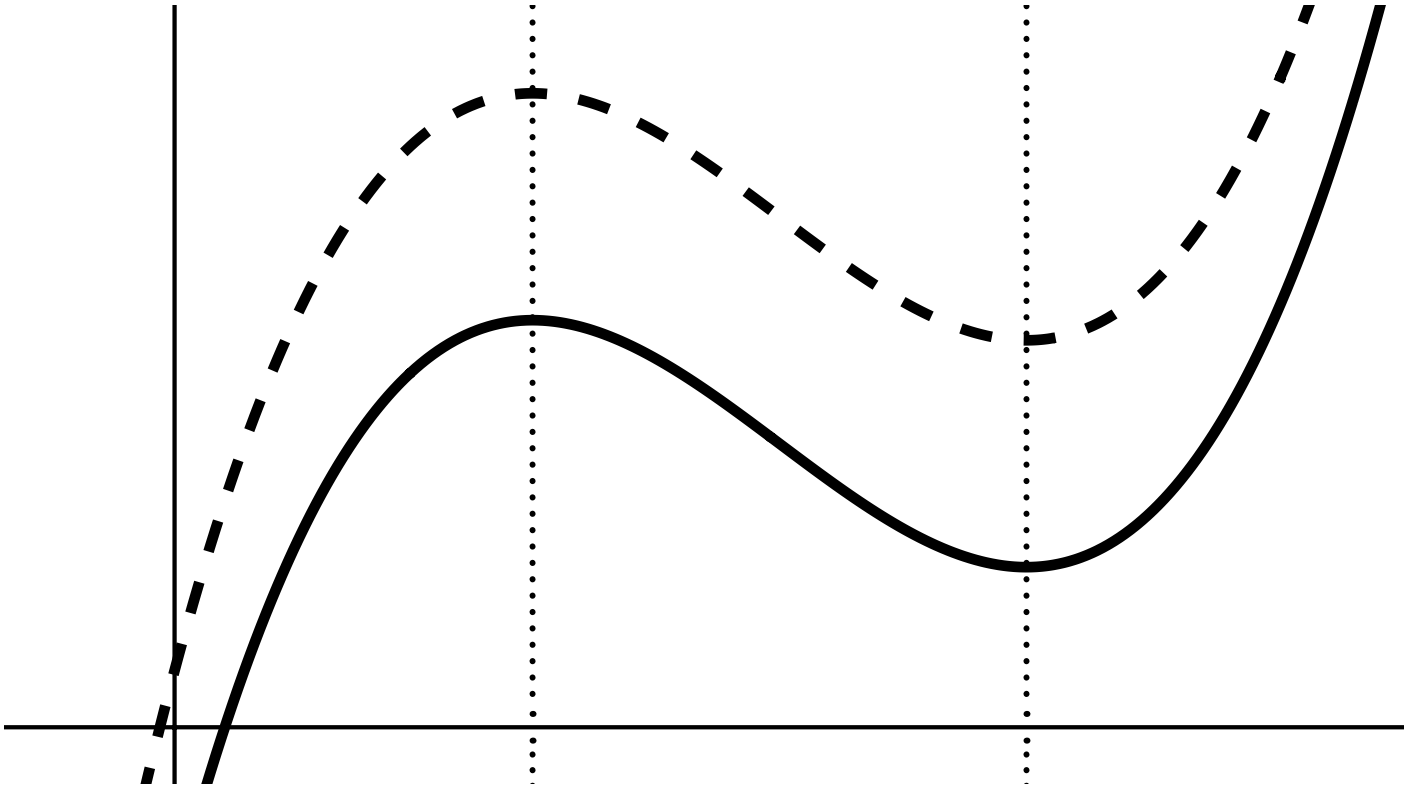

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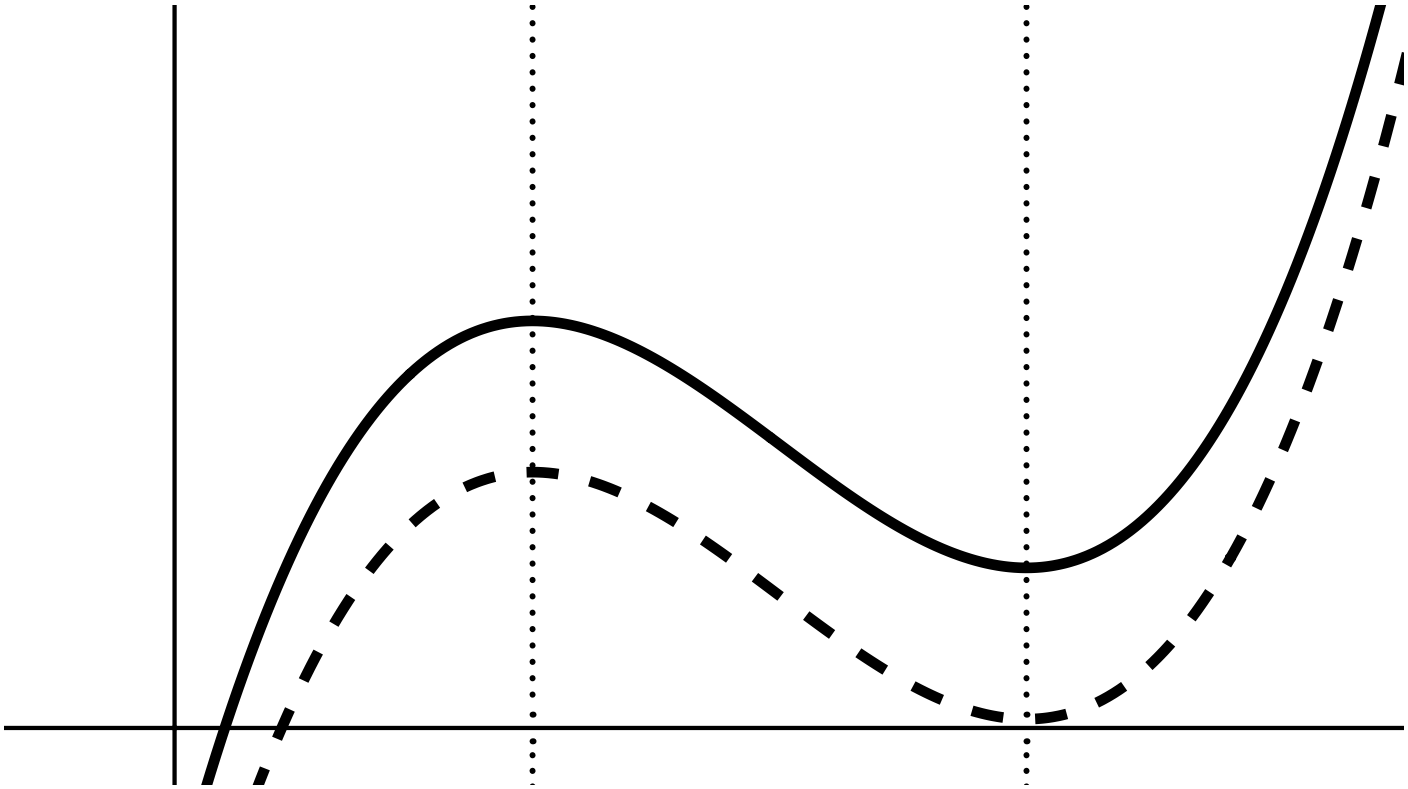

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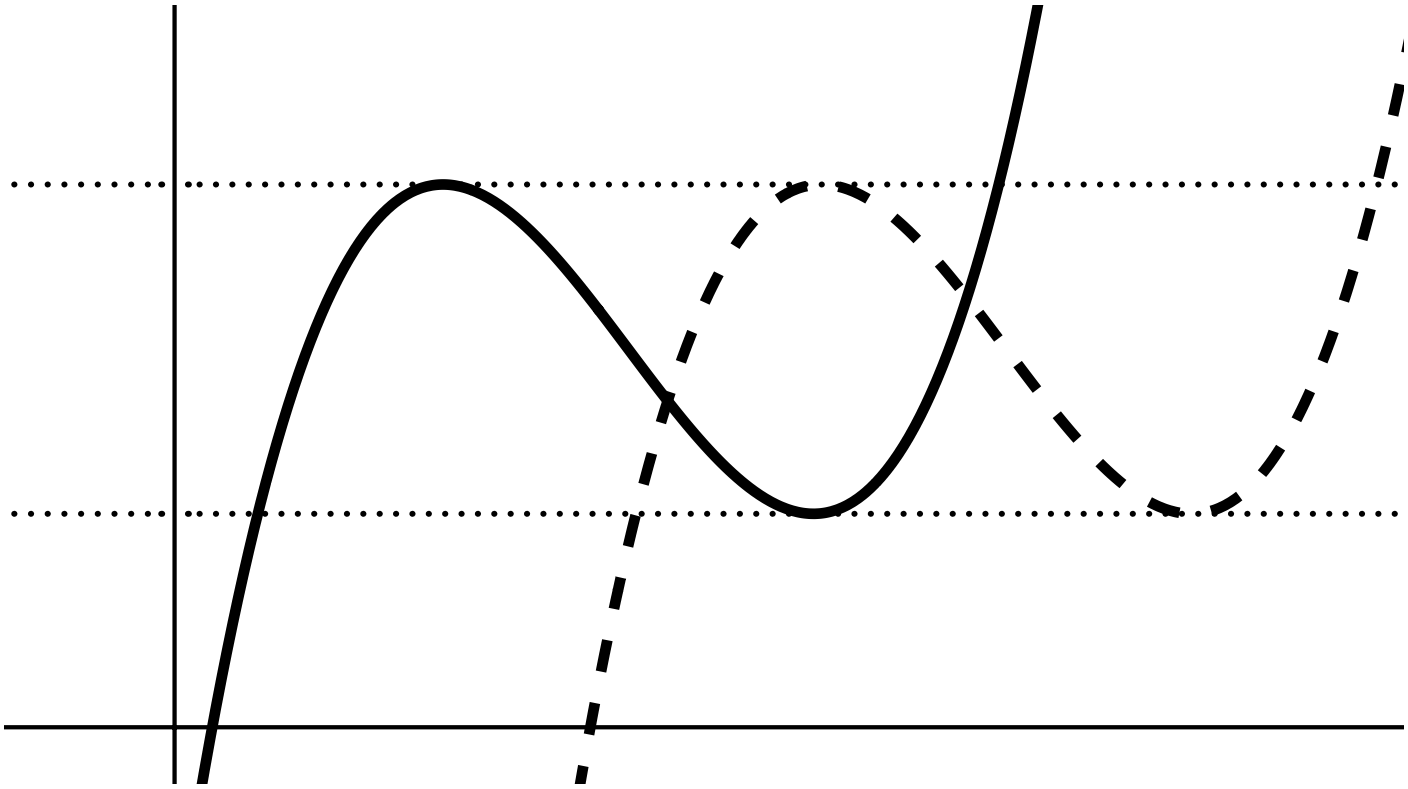

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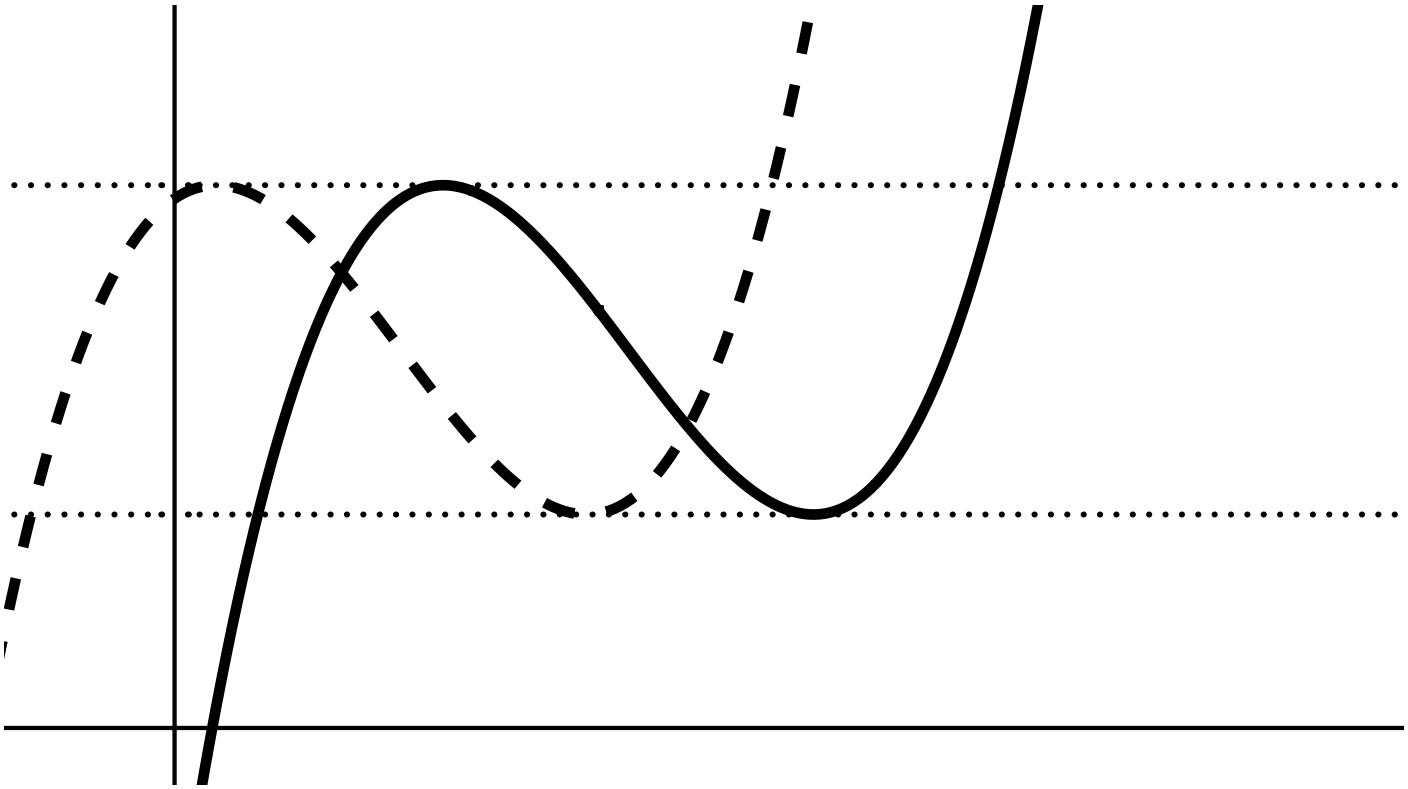

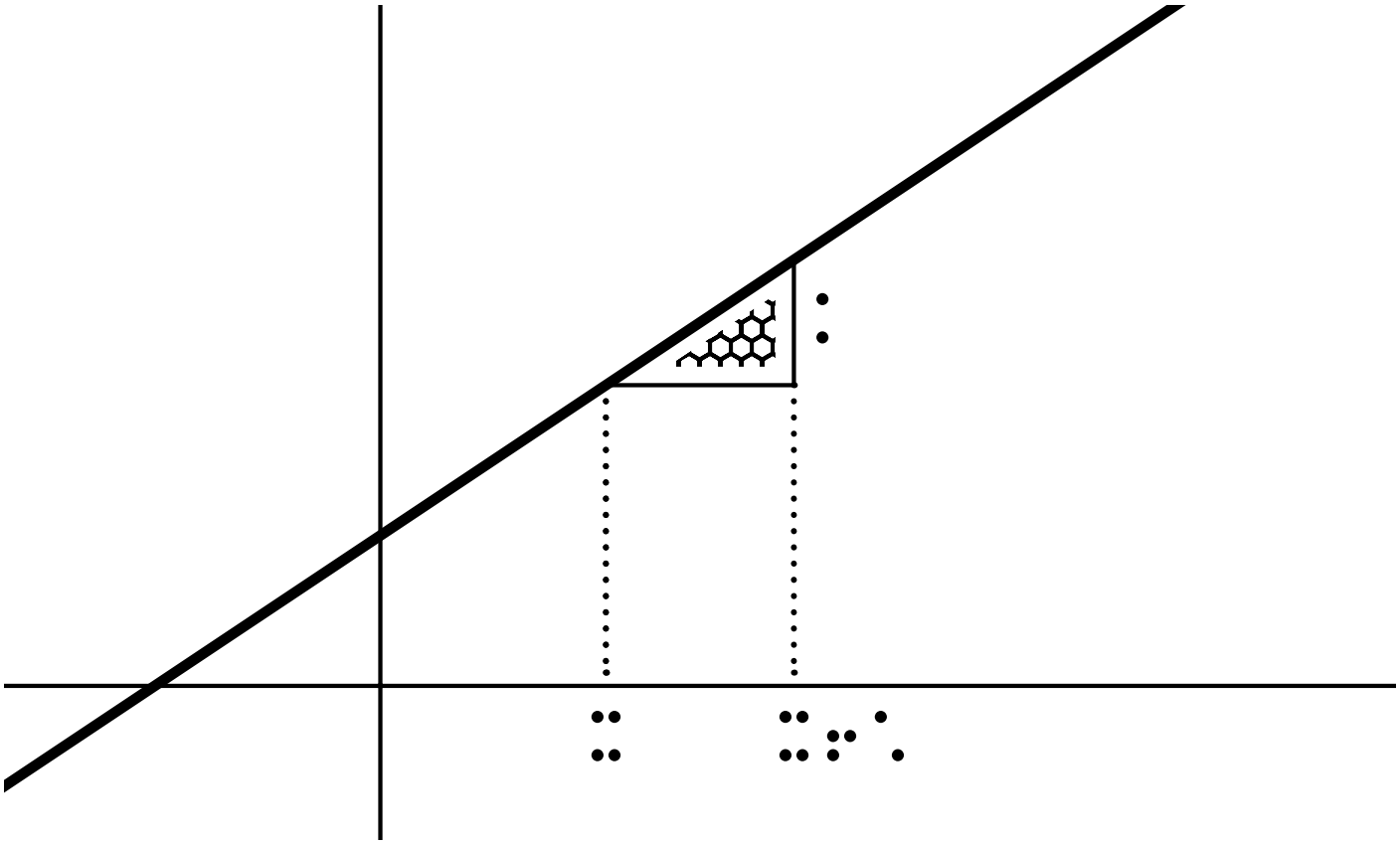

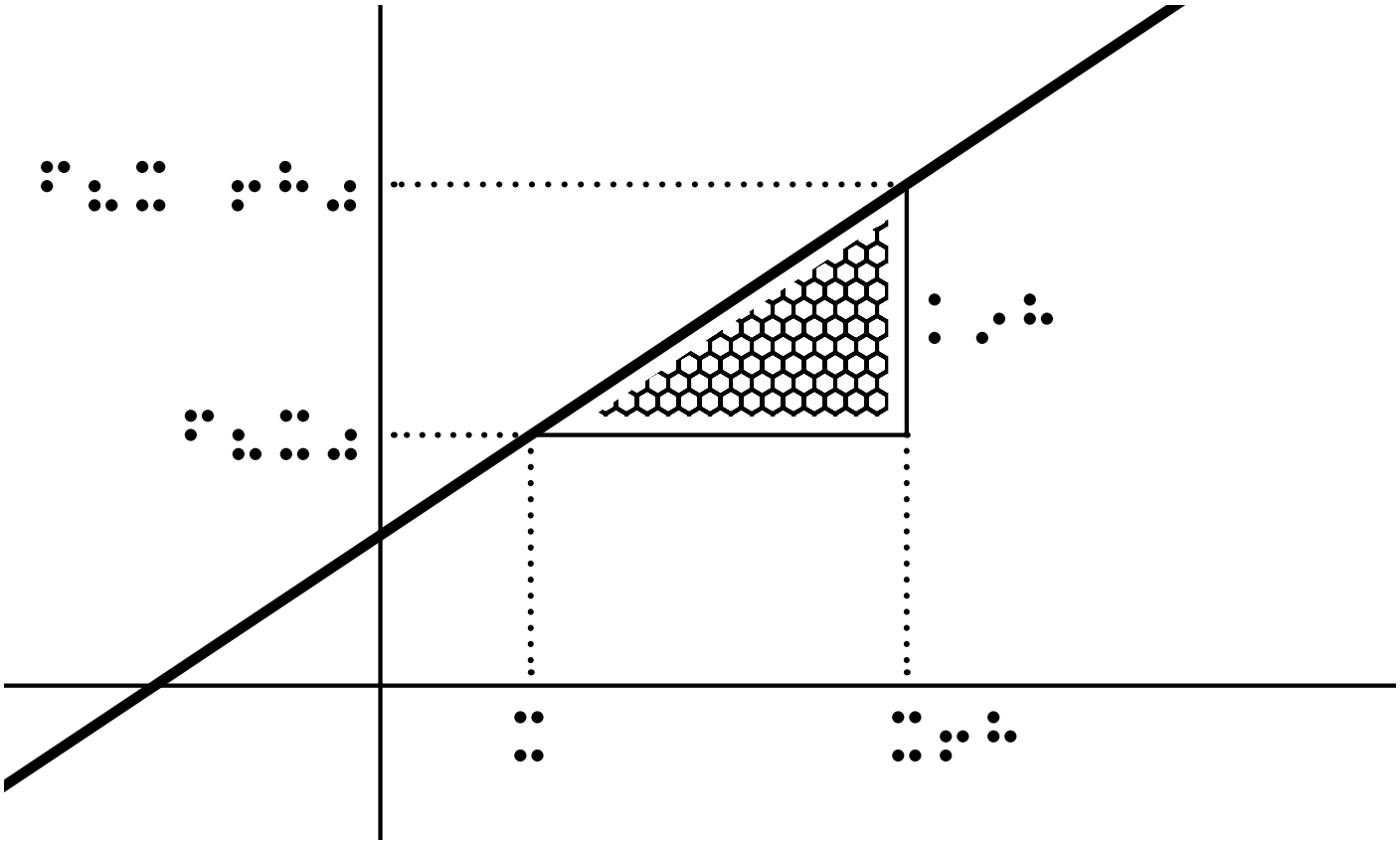

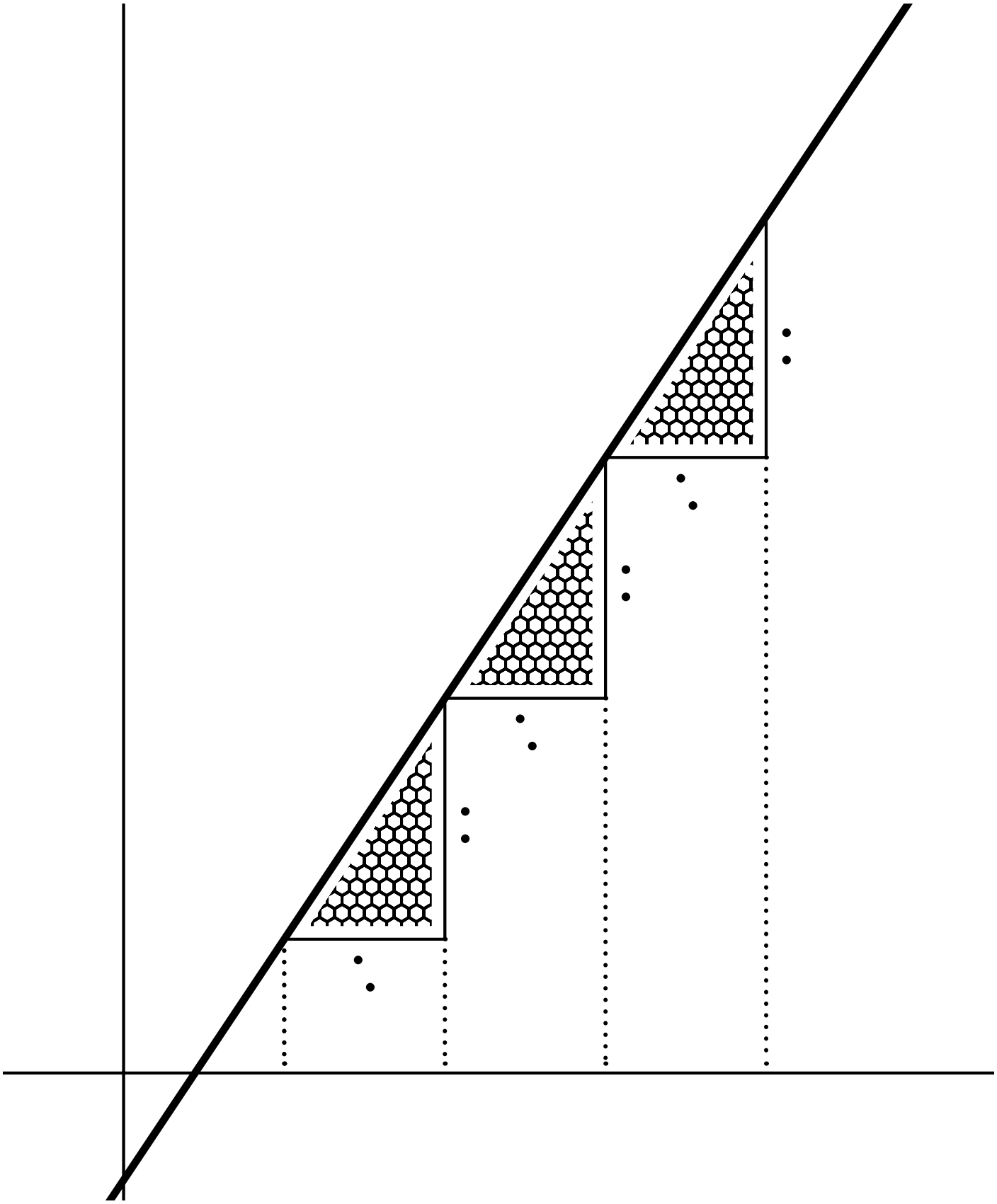

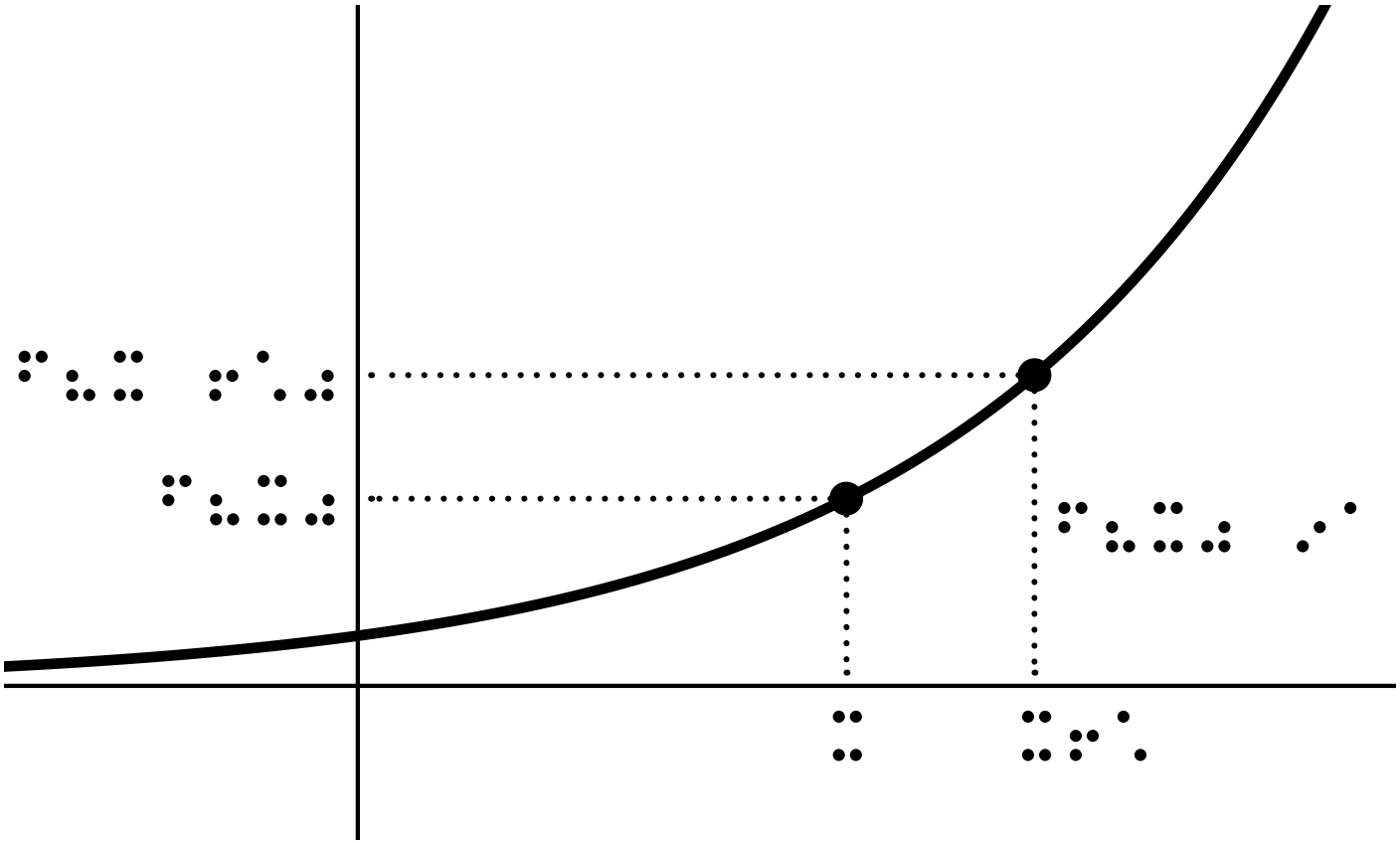

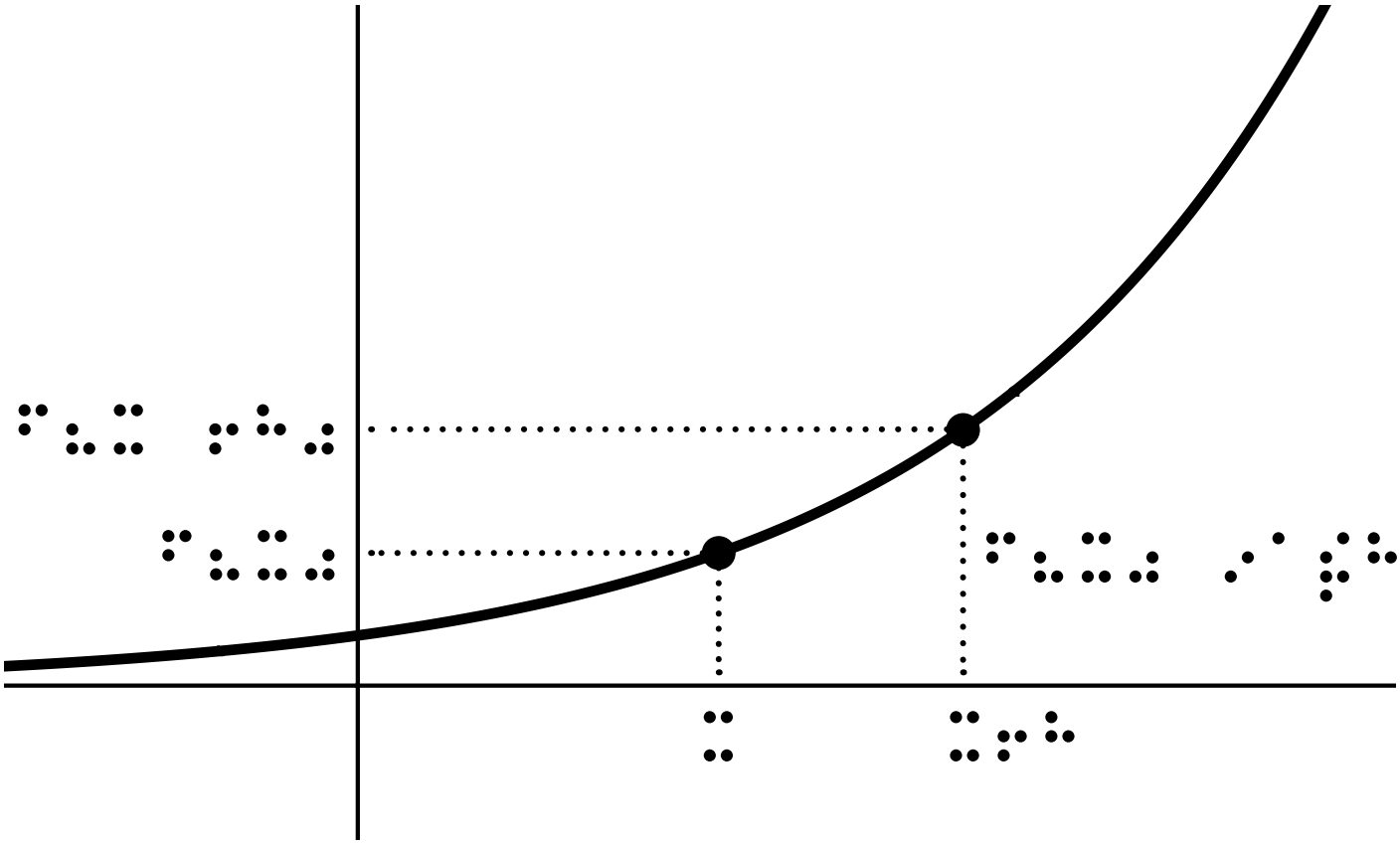

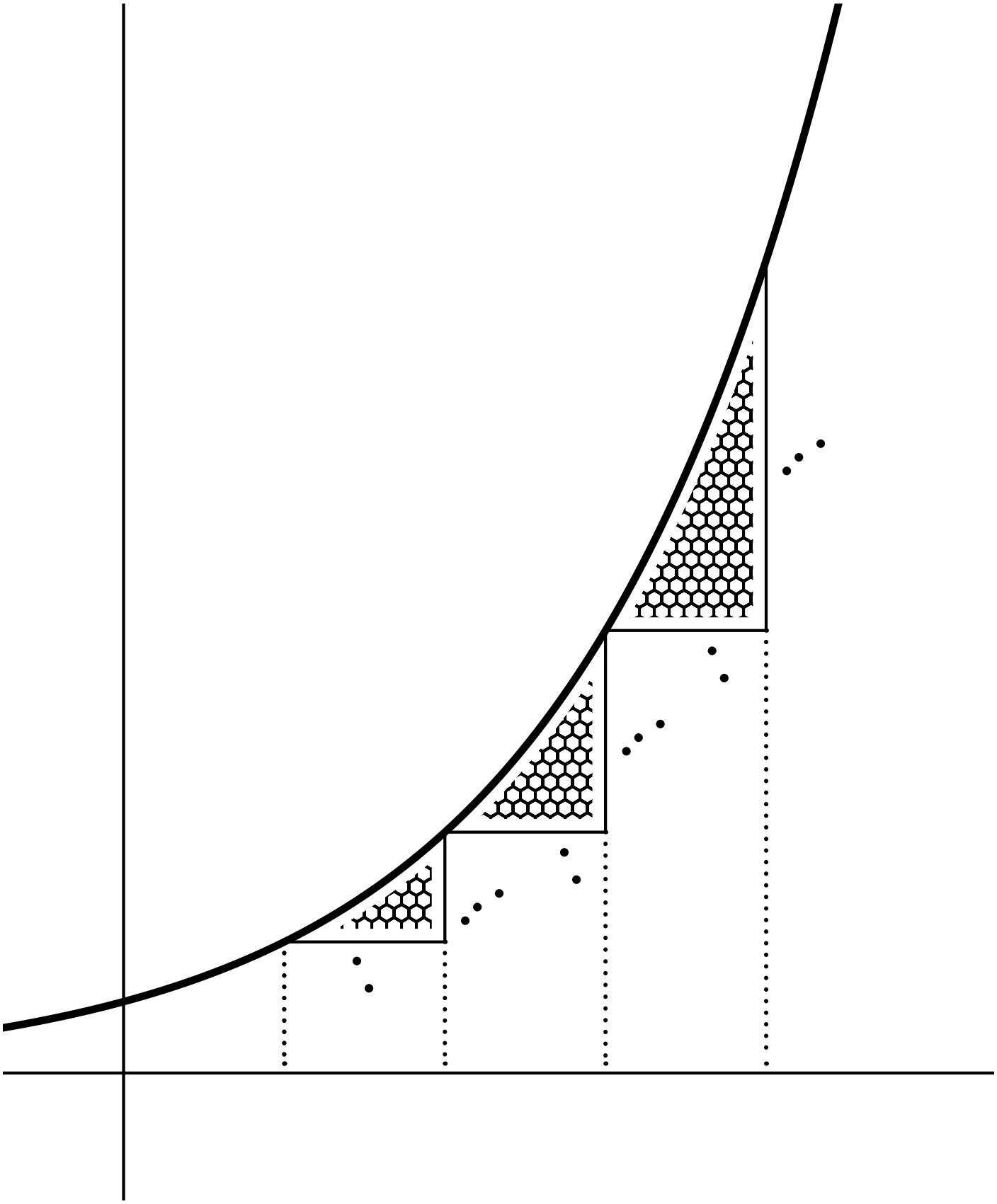

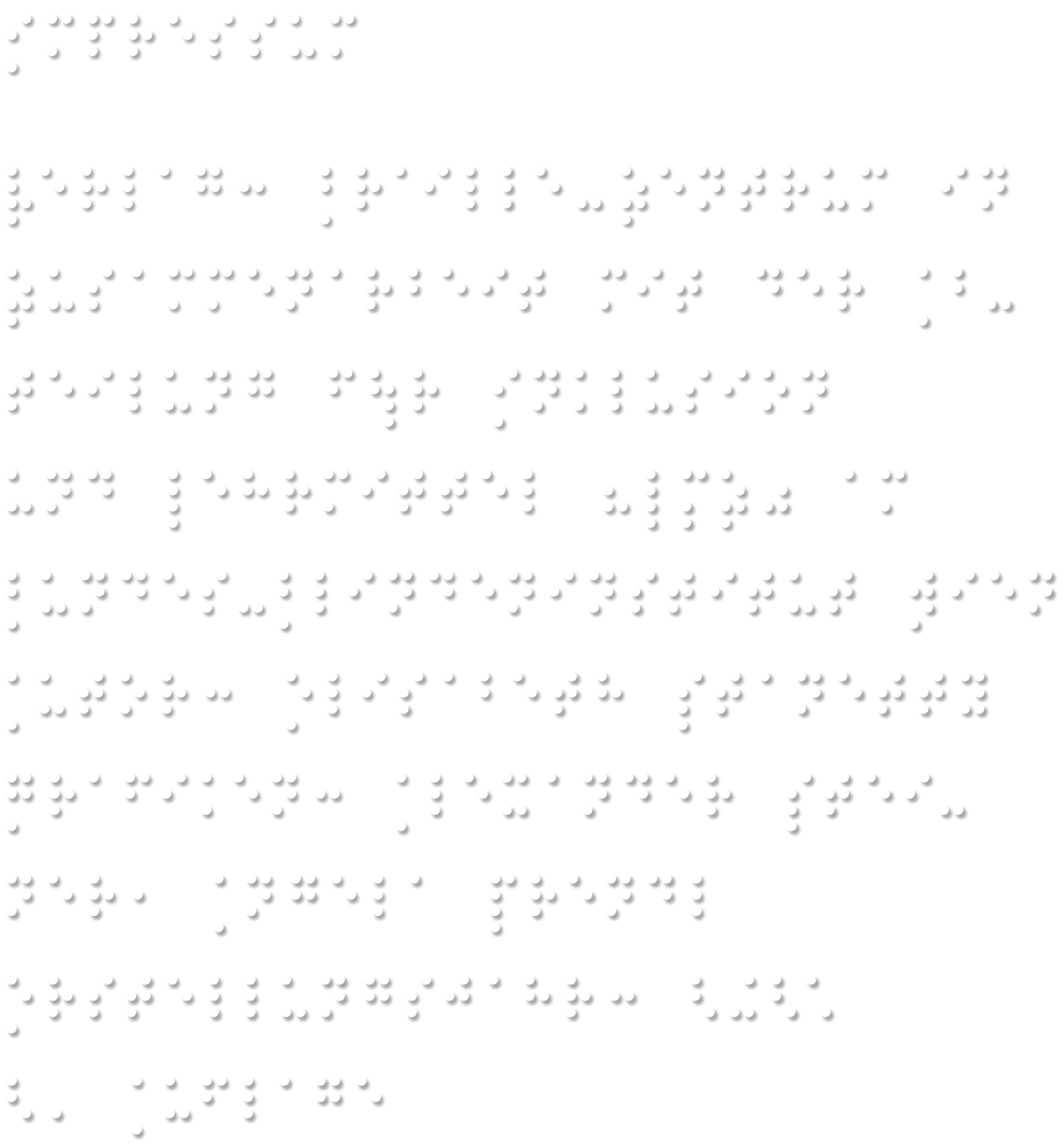

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