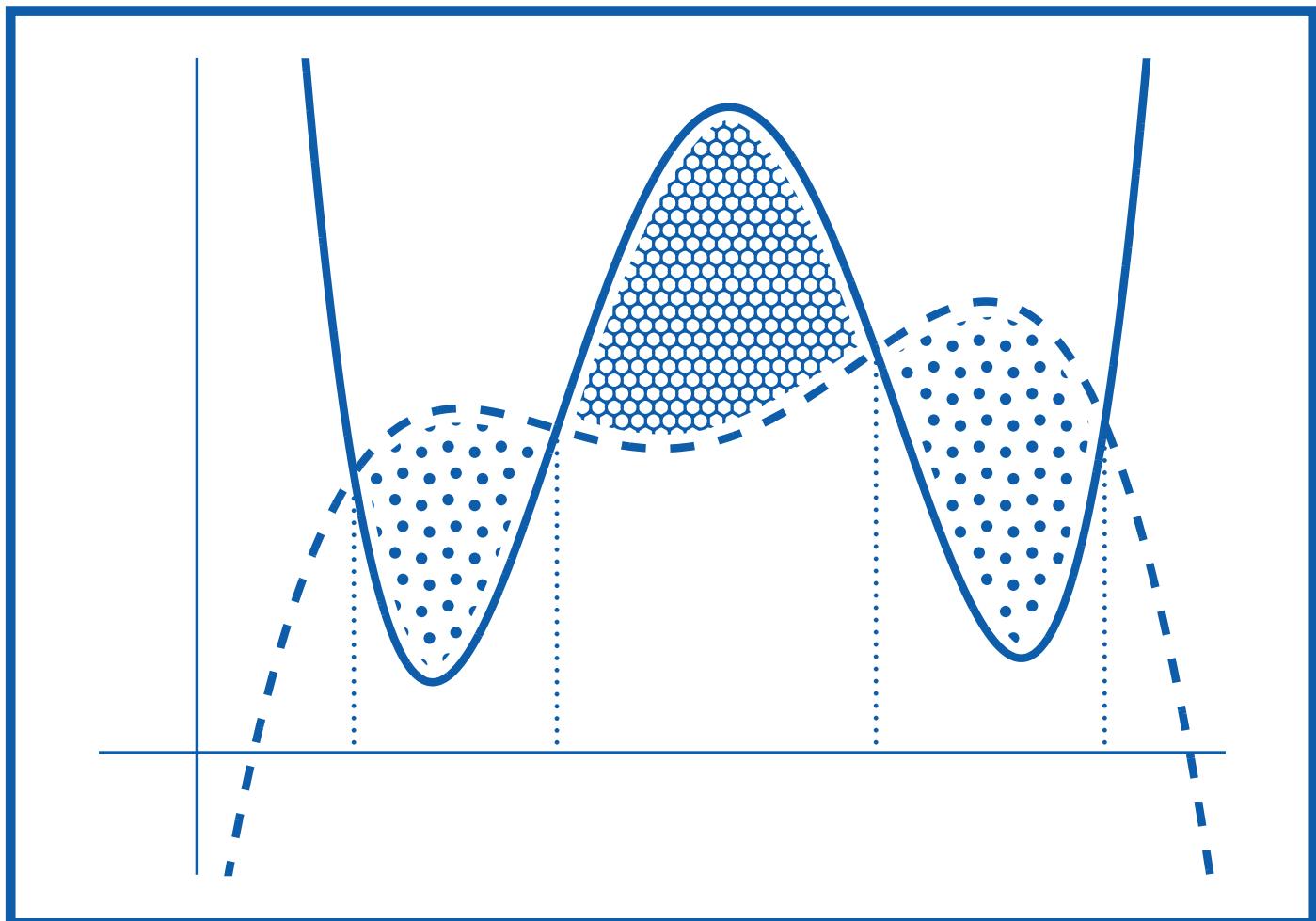


# Grafikkatalog

## 12. Schulstufe (8. AHS)

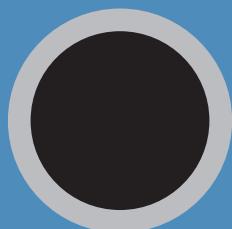
**Autor: Elisabeth Stanetty • Grafiken: Alexander Steiner, Angela Prendl**





Grafiikkatalog  
12. Schulstufe (8. AHS)

**12**





## Inhalt

- 1** Stammfunktionen
- 2** Integrale
- 3** Wirtschaft
- 4** Normalverteilung
- 5** Reelle Funktionen

# **Stammfunktionen**

**Schulstufe 12**

Mögliche Stammfunktionen (F)

diverse Funktionen (f)



# Inhalt

**1**  $f(x) = d; d > 0;$

**2**  $f(x) = d; d < 0;$

$f(x) = x;$

**3**  $f(x) = x + d; d > 0; d < 0;$

**4**  $f(x) = -x;$

$f(x) = -x + d; d > 0;$

**5**  $f(x) = -x + d; d < 0;$

$f(x) = x^2;$

**6**  $f(x) = x^2 + d; d > 0; d < 0;$

**7**  $f(x) = -x^2 + d; d > 0; d < 0;$

**8**  $f(x) = x^3;$

$f(x) = \sin(x)$

**9**  $f(x) = \cos(x)$

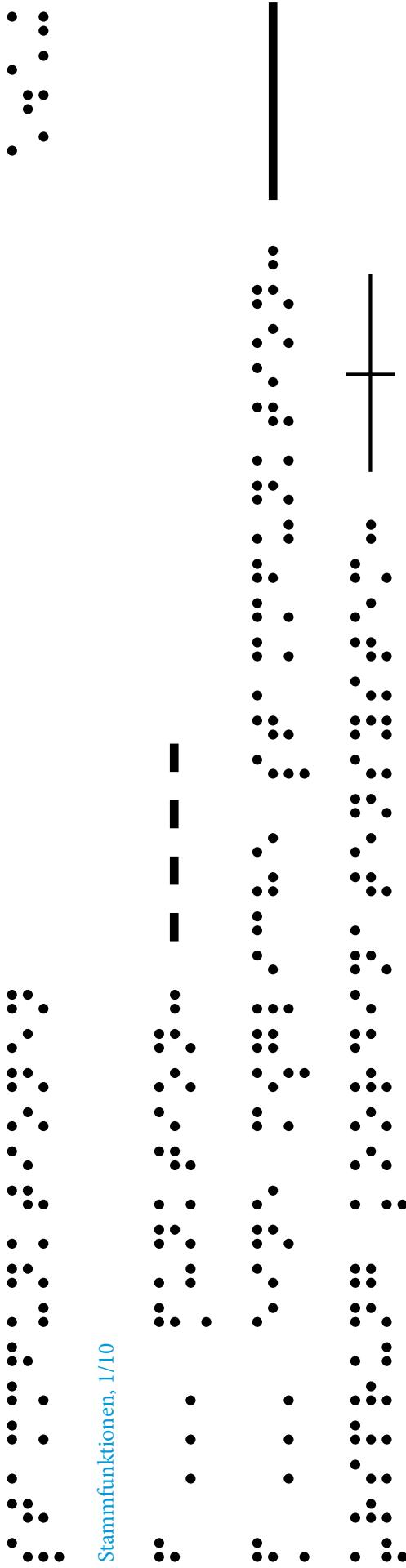
$f(x) = 2^x$

**10**  $f(x) = 1/x$

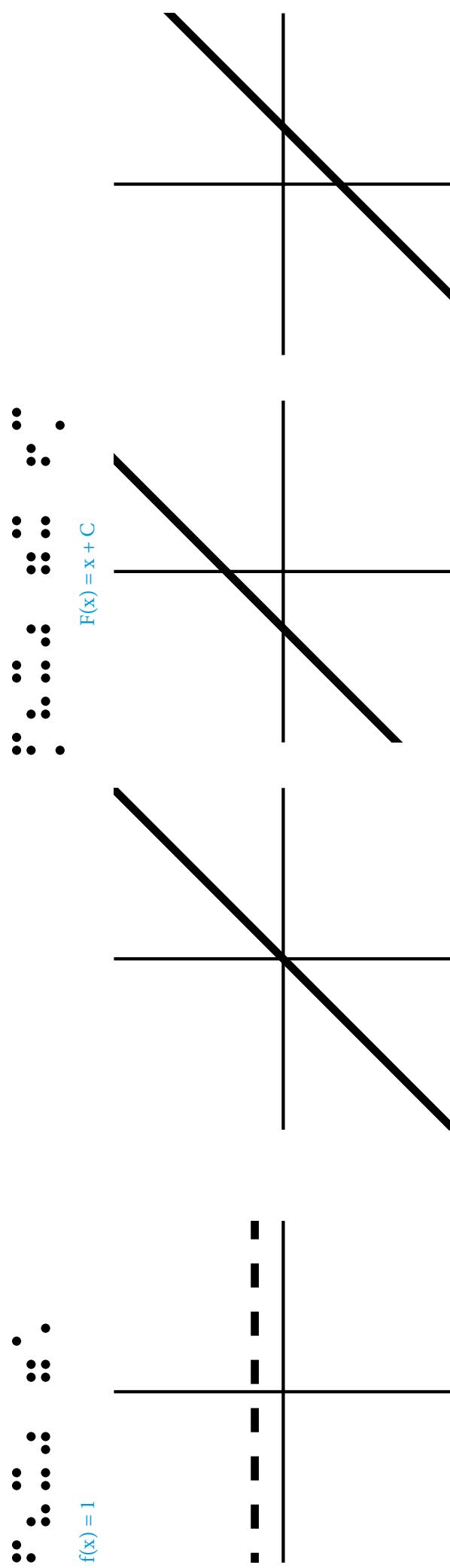
$f(x) = \ln(x)$

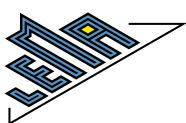


## Stammfunktionen, 1/10

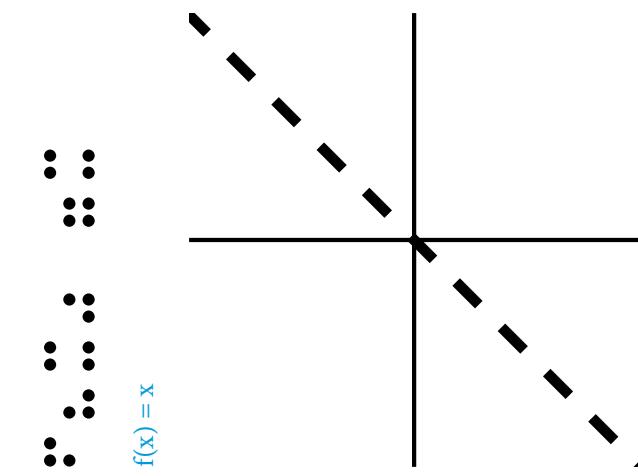
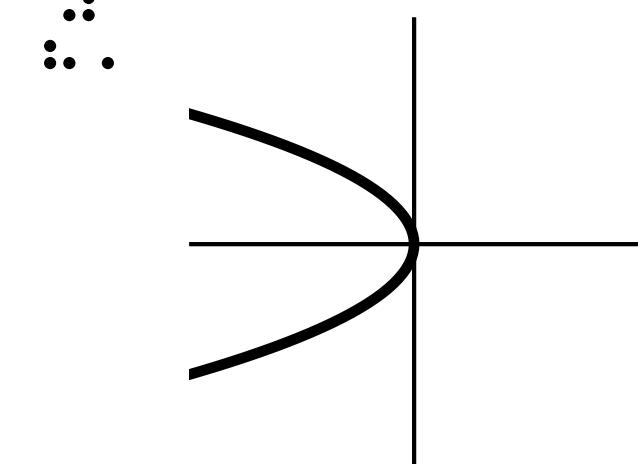
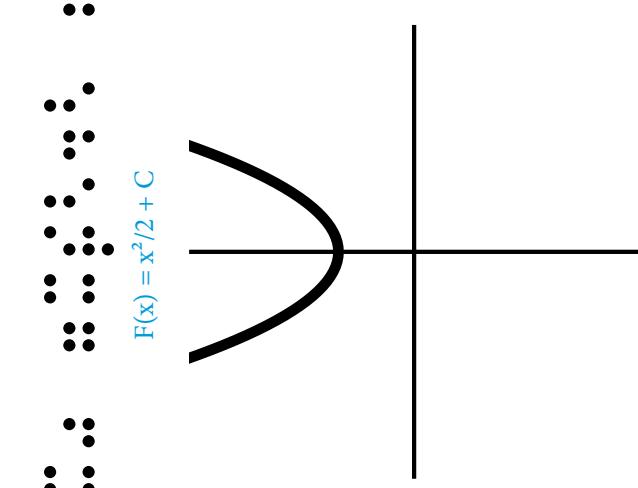
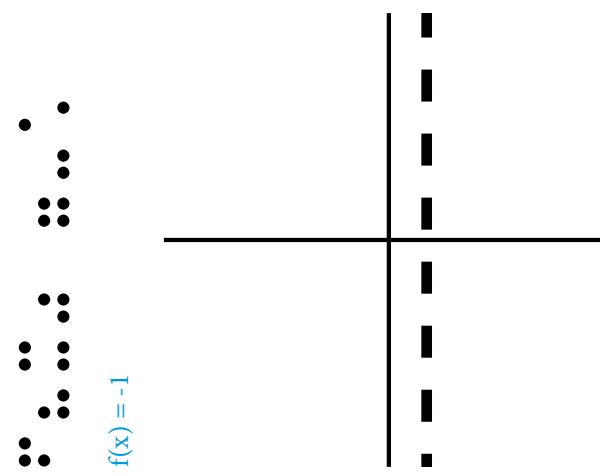
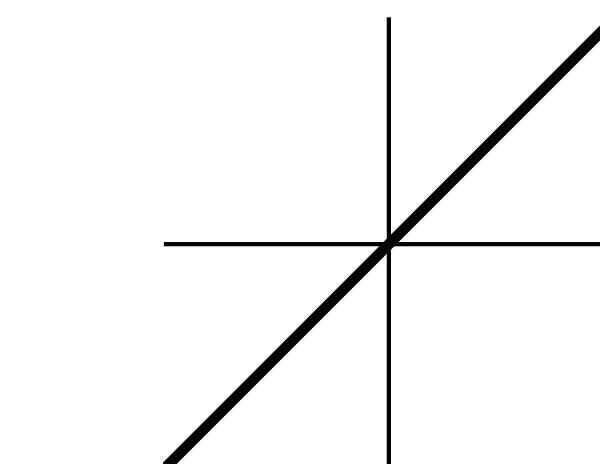
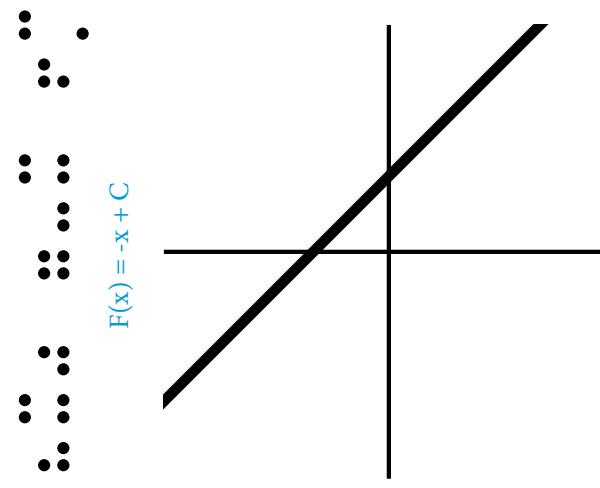
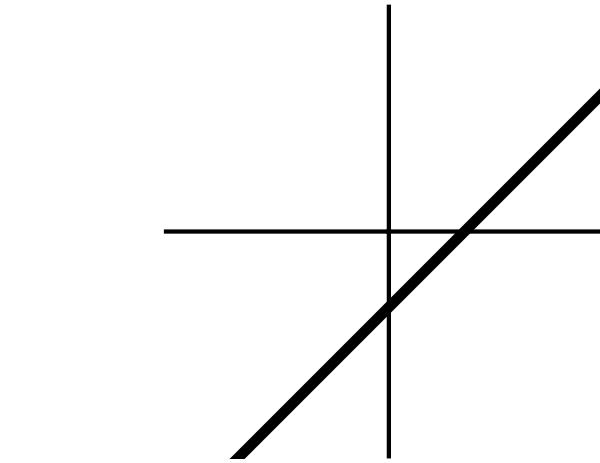


$F \dots$  eine mögliche Stammfunktion



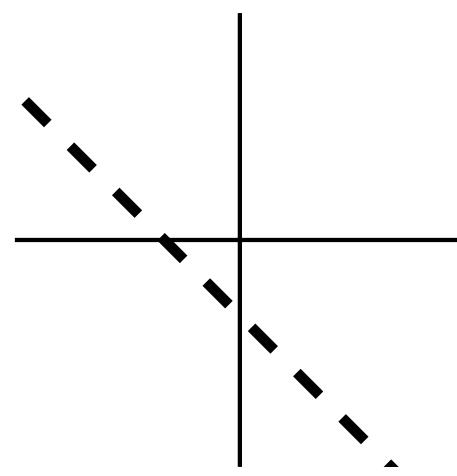


## Stammfunktionen, 2/10

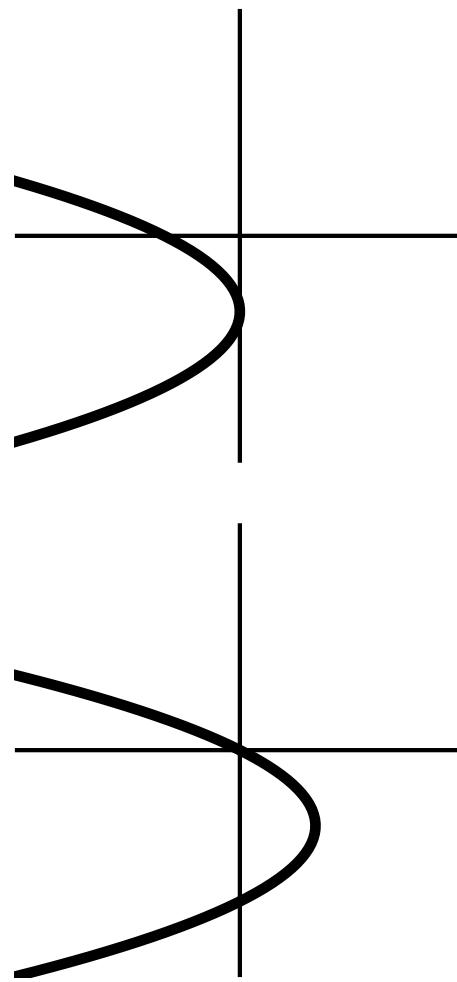


## Stammfunktionen, 3/10

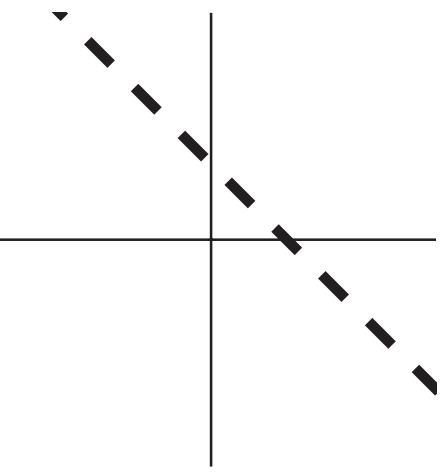
$$f(x) = x + 2$$



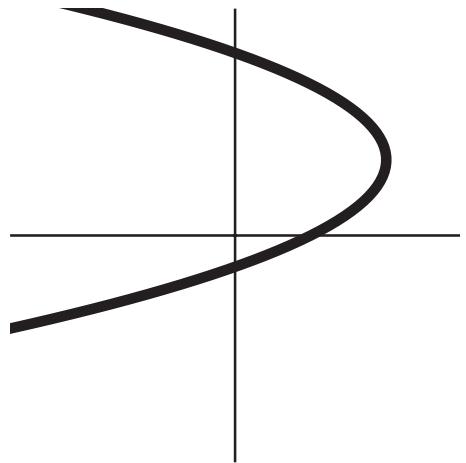
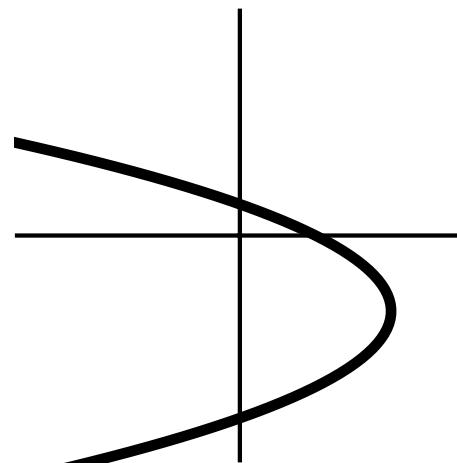
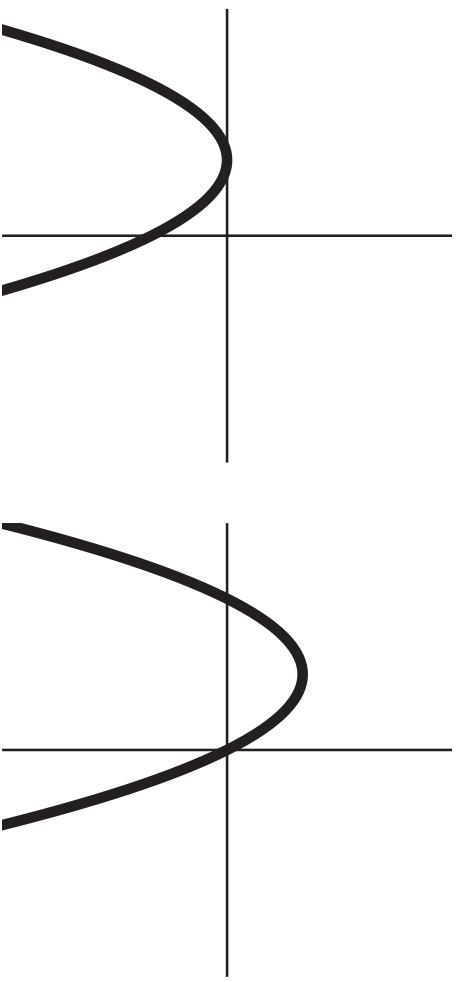
$$F(x) = x^2/2 + 2x + C$$



$$f(x) = x - 2$$

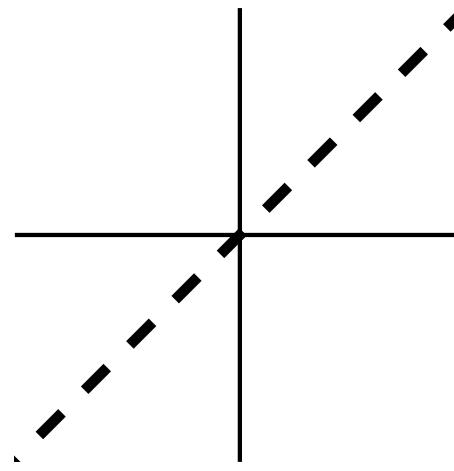


$$F(x) = x^2/2 - 2x + C$$

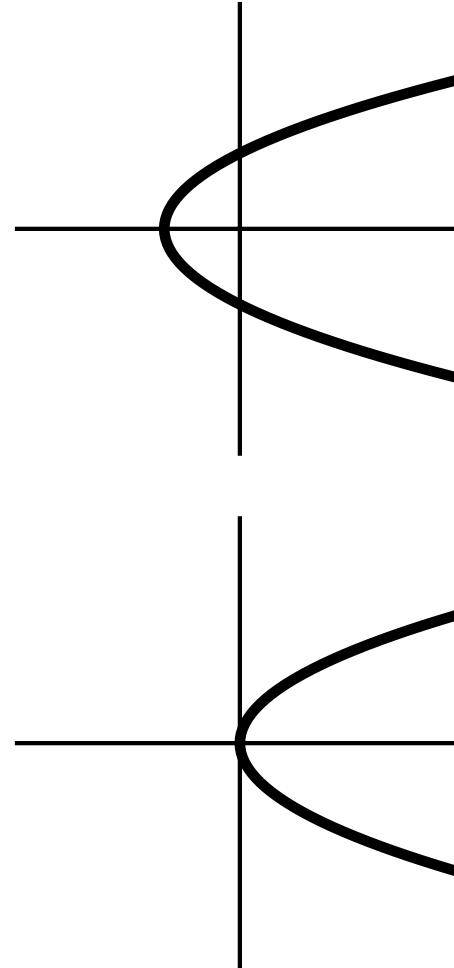


## Stammfunktionen, 4/10

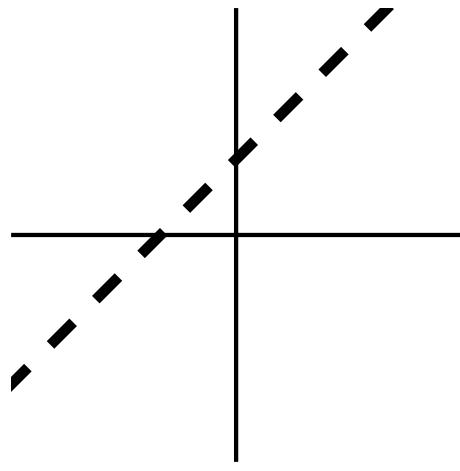
$$f(x) = -x$$



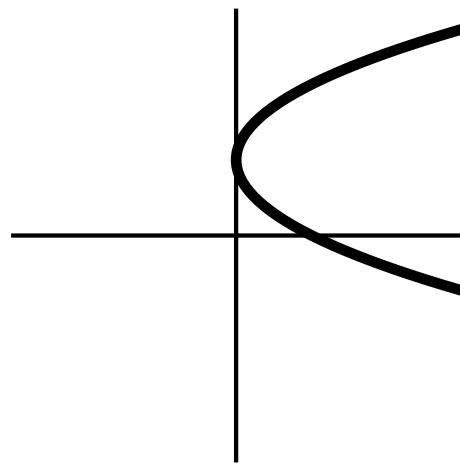
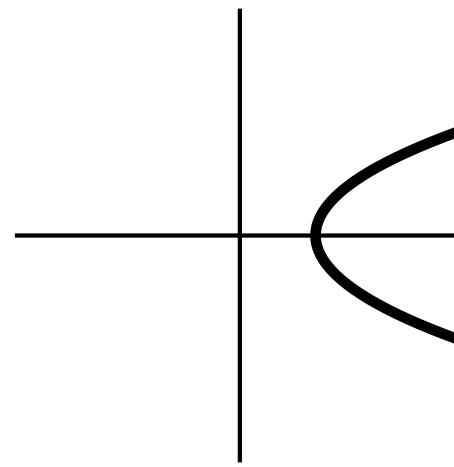
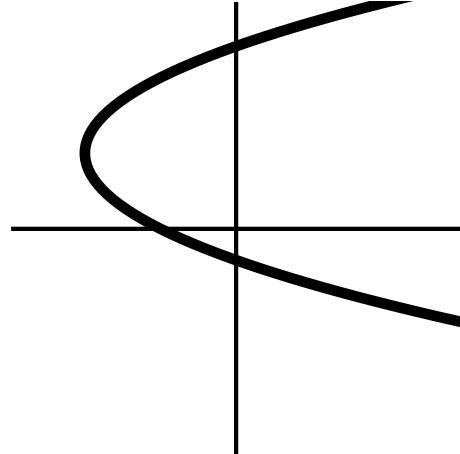
$$F(x) = -x^2/2 + C$$



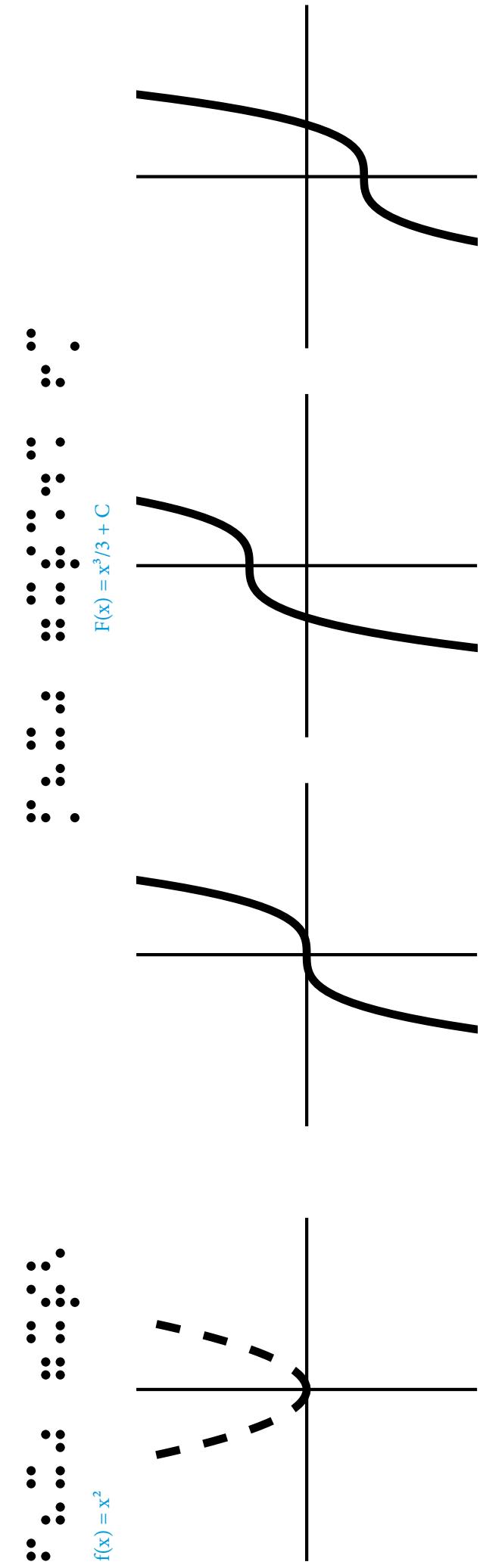
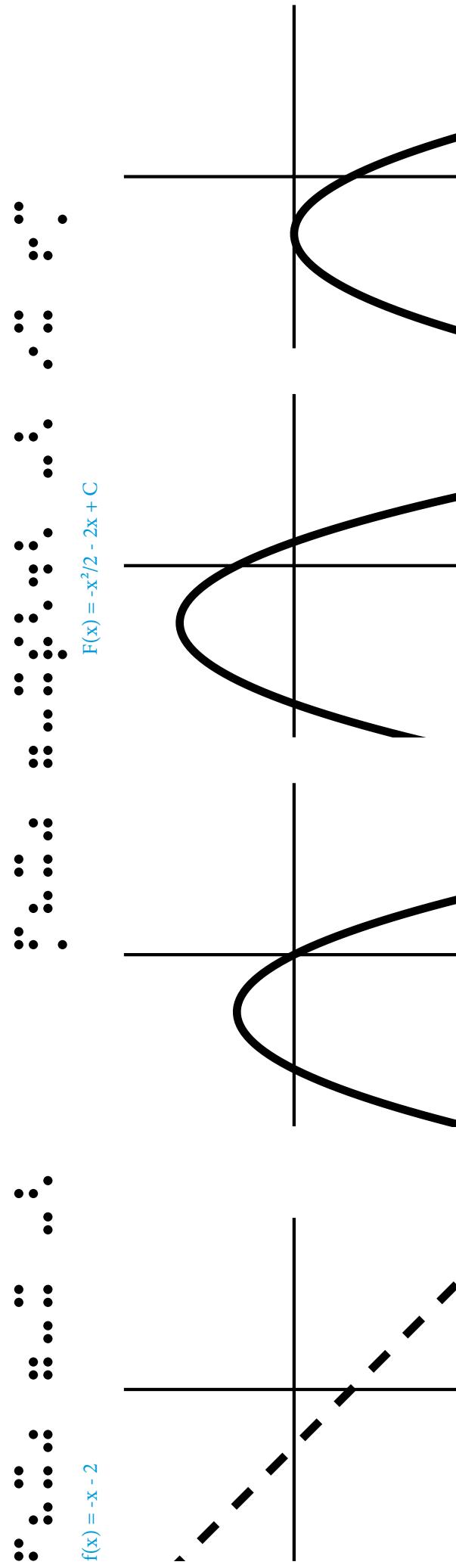
$$f(x) = -x + 2$$



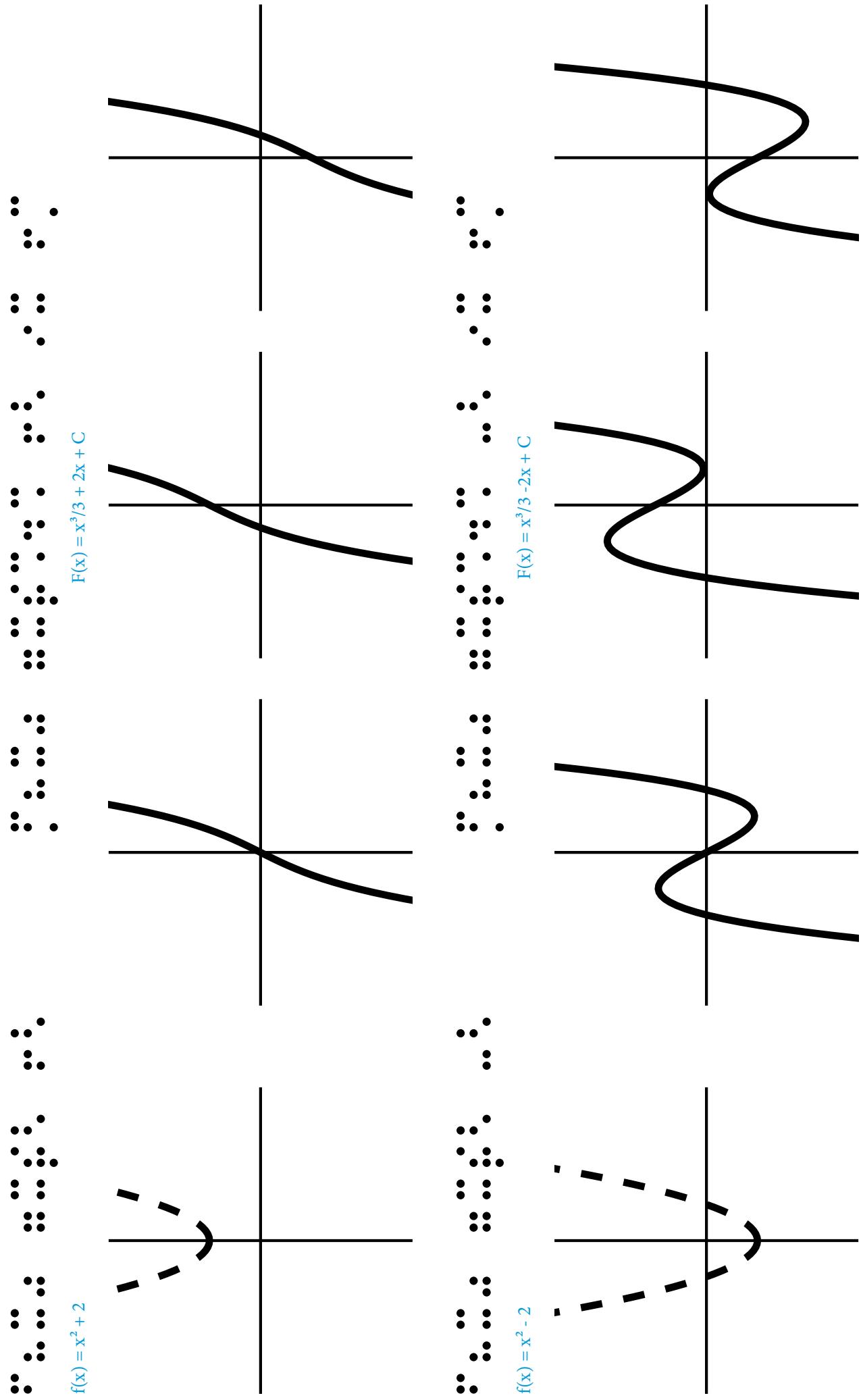
$$F(x) = -x^2/2 + 2x + C$$



## Stammfunktionen, 5/10

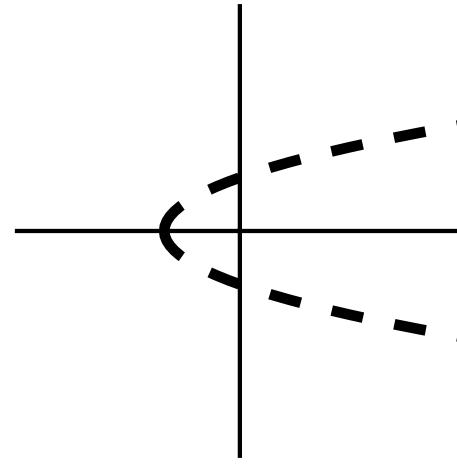


## Stammfunktionen, 6/10

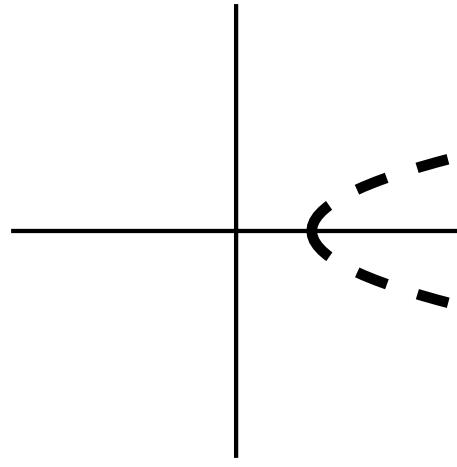


## Stammfunktionen, 7/10

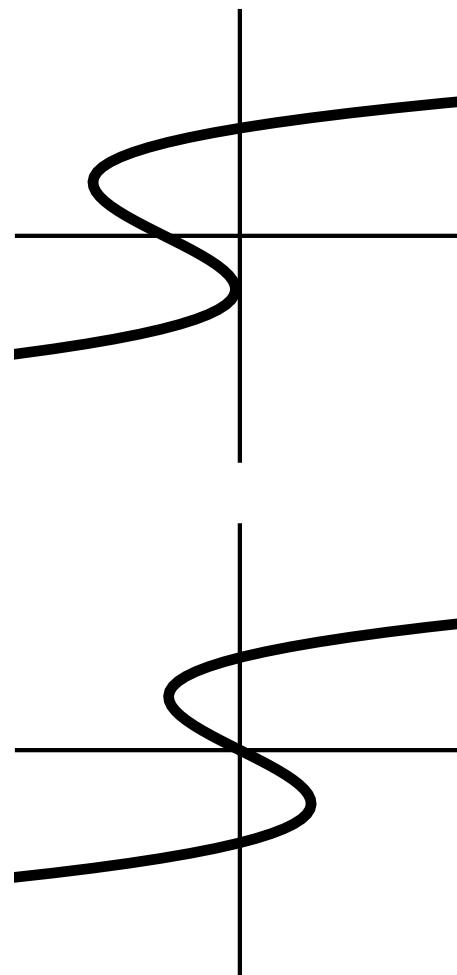
$$f(x) = -x^2 + 2$$



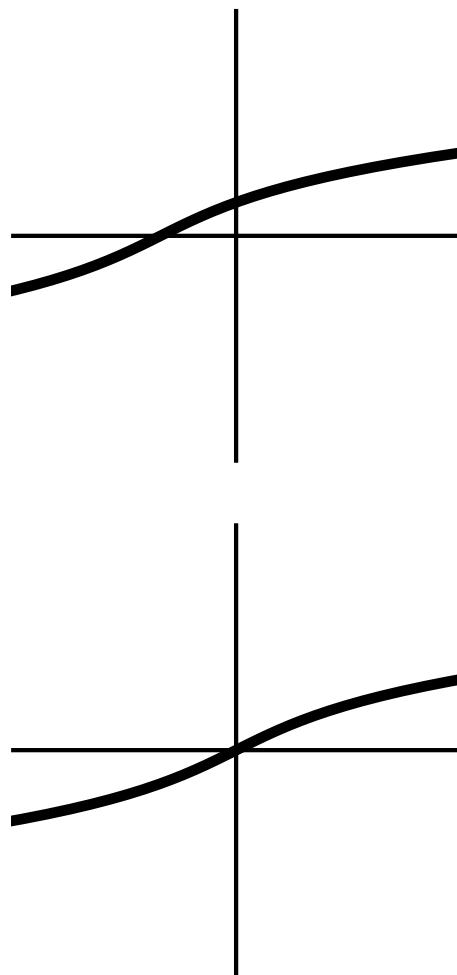
$$f(x) = -x^2 - 2$$



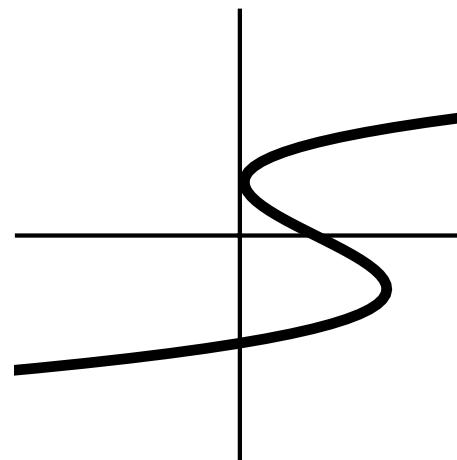
$$f(x) = -x^2 + 2$$



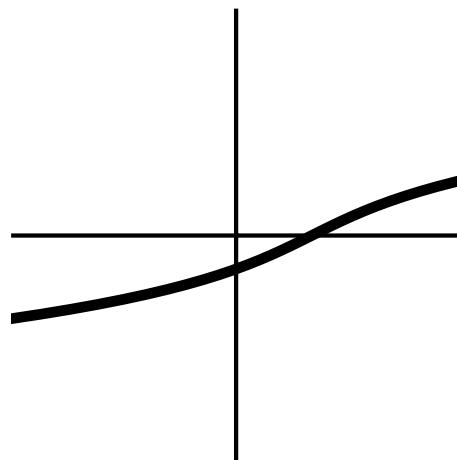
$$f(x) = -x^2 - 2$$

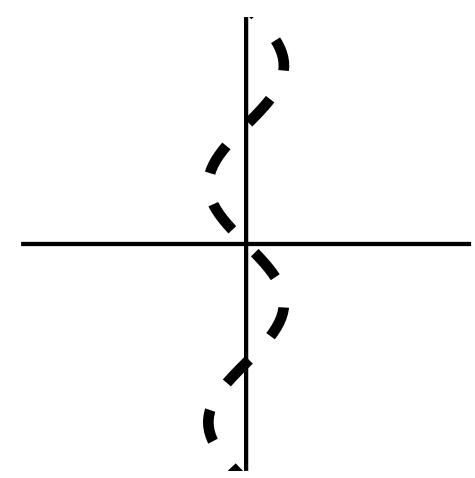
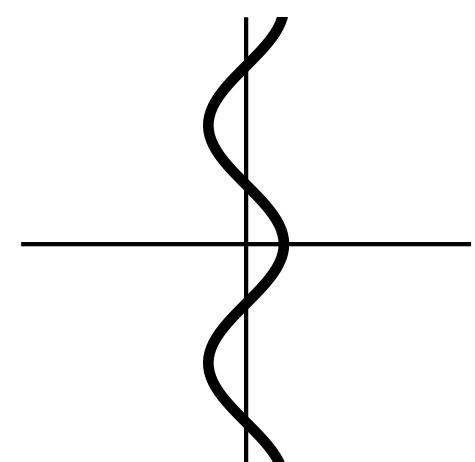
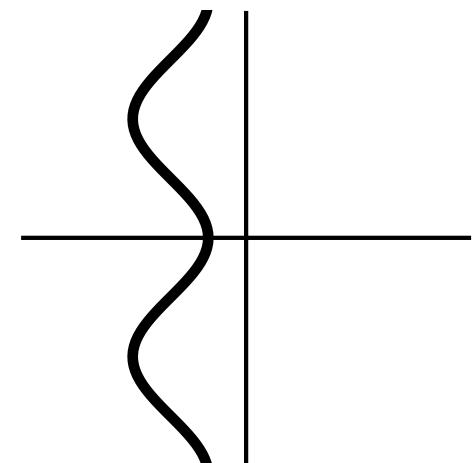
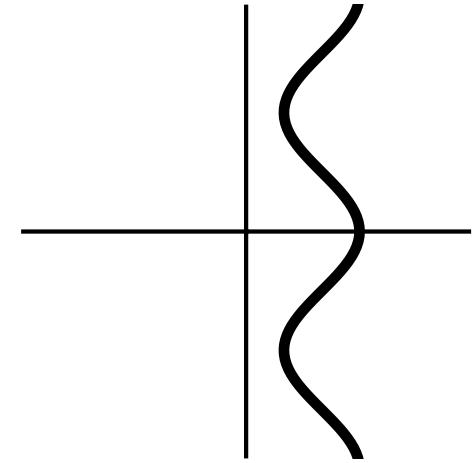
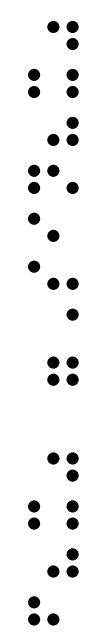
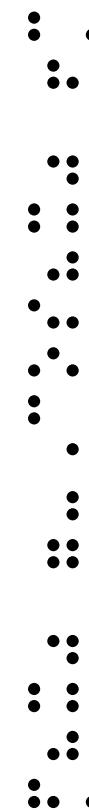
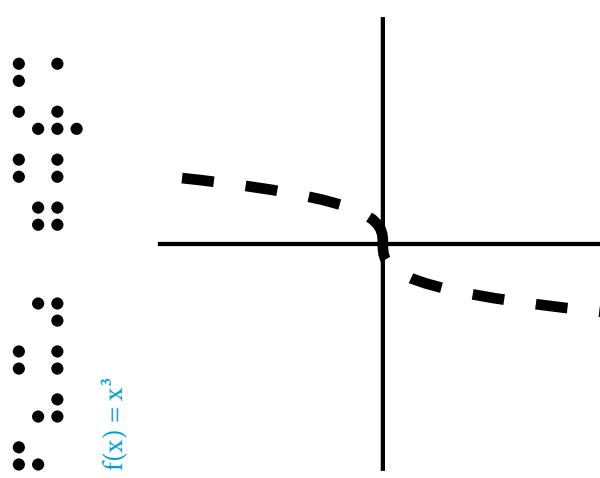
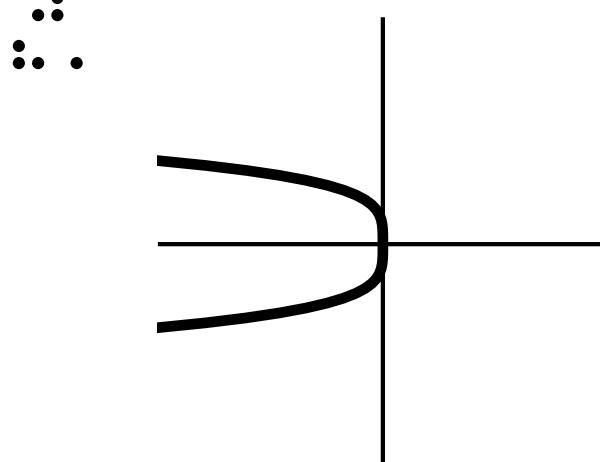
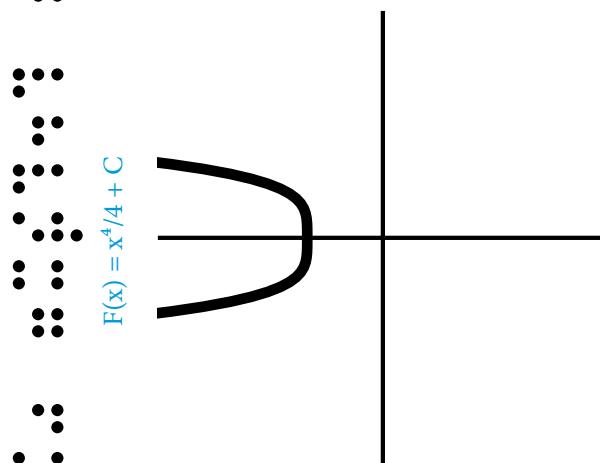
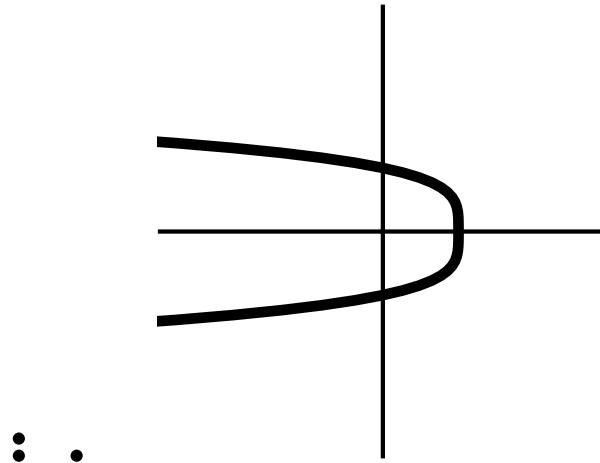


$$f(x) = -x^2 + 2$$



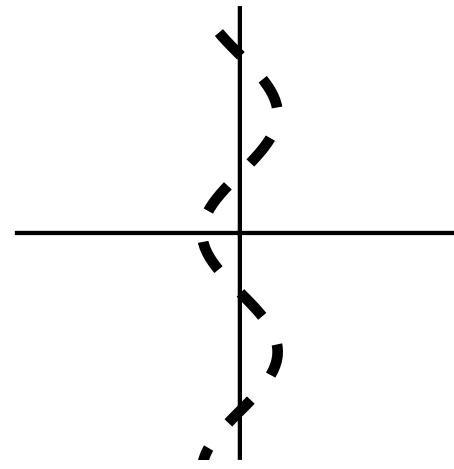
$$f(x) = -x^2 - 2$$



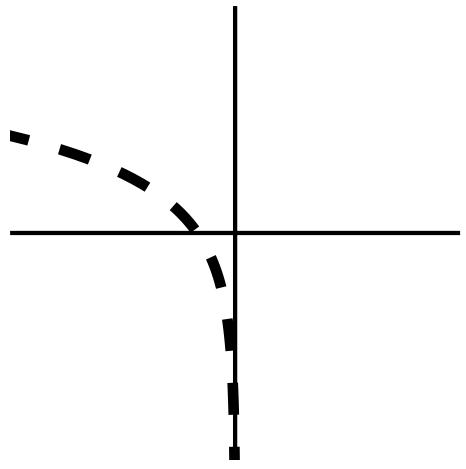


## Stammfunktionen, 9/10

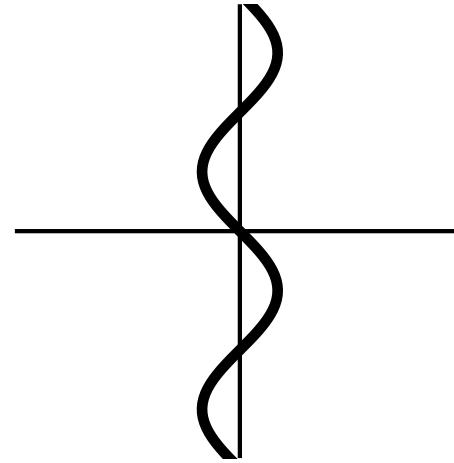
$$f(x) = 2^x$$



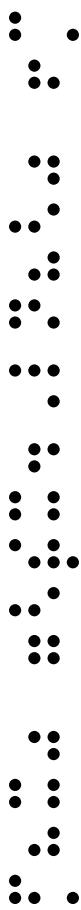
$$F(x) = 2^x$$



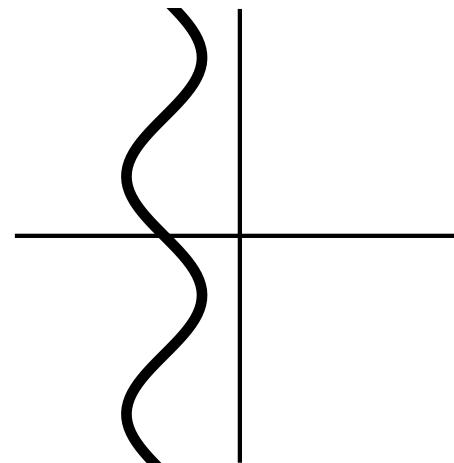
$$f(x) = \cos(x)$$



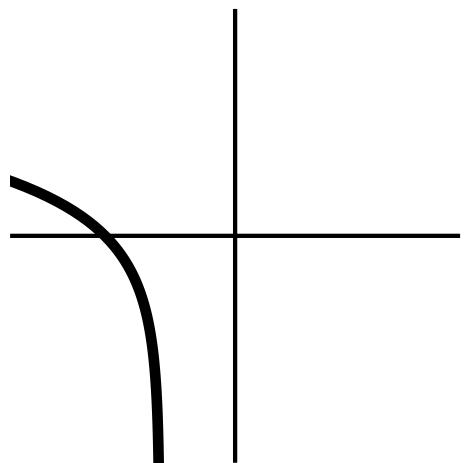
$$F(x) = \sin(x) + C$$



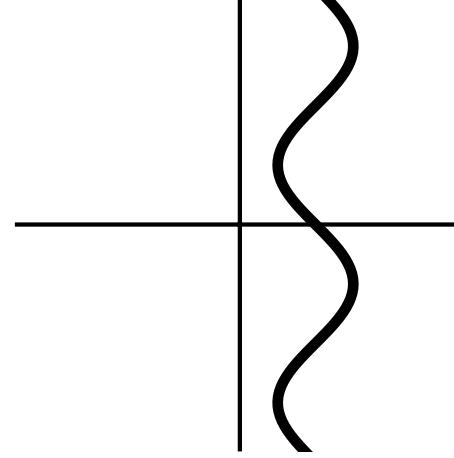
$$f(x) = \sin(x) + C$$



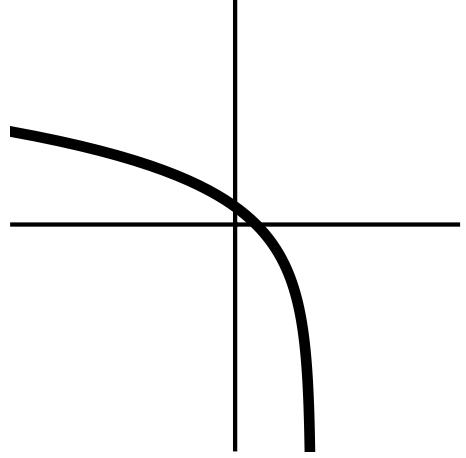
$$F(x) = 2^x/\ln(2) + C$$



$$f(x) = \sin(x)$$

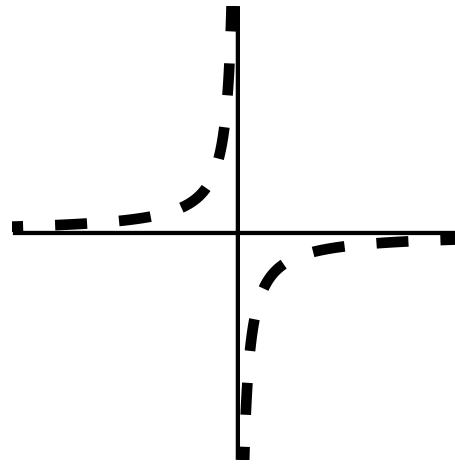


$$F(x) = -\cos(x) + C$$

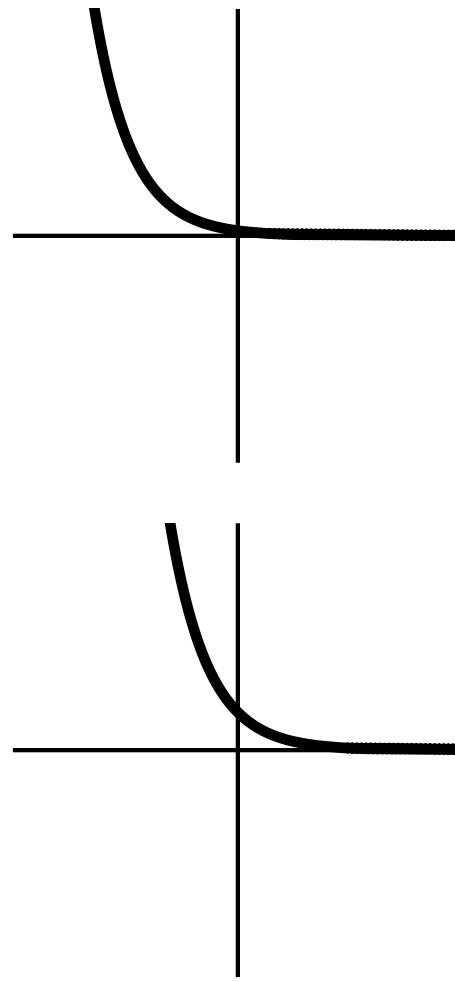


## Stammfunktionen, 10/10

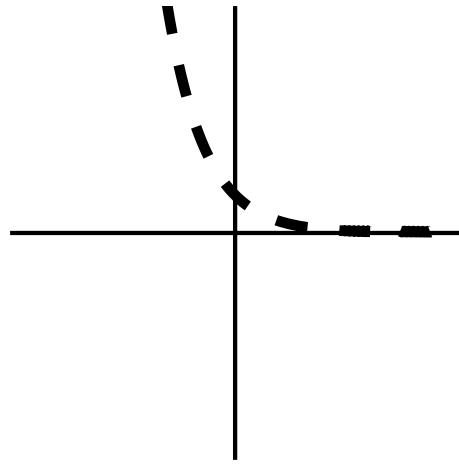
$$f(x) = \frac{1}{x}$$



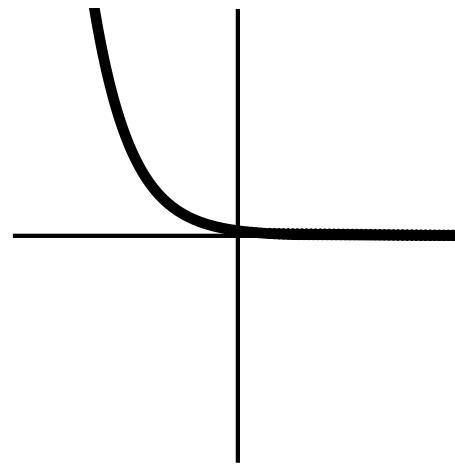
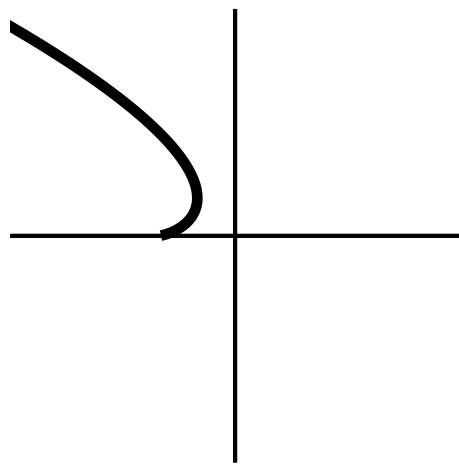
$$F(x) = \ln(x) + C$$



$$f(x) = \ln(x)$$



$$F(x) = x \cdot \ln(x) - x + C$$



# **Integrale**

## **Schulstufe 12**

Darstellung und Berechnung von Flächen

Weglängen

Volumina



# Inhalt

## 1 - 9 Flächenberechnungen

- 1 Fläche zwischen Kurve und x- Achse
- 2 Unter- und Obersumme n =3
- 3 Unter- und Obersumme n =5
- 4  $A = \int[a; b], f(x) > 0$
- 5  $A = \int[a; b], f(x) < 0$
- 6  $A = |\int[a; b]| + |\int[b; c]|$
- 7  $A = \int[a; b](f(x) - g(x))'dx$
- 8  $A = \int[a; b](g(x) - f(x))'dx$
- 9 Abschnittweises Integrieren

## 10-12 Streckenberechnungen

- 10  $s = \int[t_1; t_2](v(t) 'dt)$  mit v konstant
- 11  $s = \int[t_1; t_2](v_1(t)'dt) + \int[t_2; t_3](v_2(t)'dt)$   
mit  $v_1, v_2$  konstant
- 12  $s = \int[t_1; t_2](v(t) 'dt)$   
mit v nicht konstant

## **13 - 24 Volumsberechnungen**

**13** Kreis  $x^2 + y^2 = r^2$

**14** Kreisrotation um x-Achse

**15** Kreisrotation um y-Achse

**16** Ellipse  $b^2x^2 + a^2y^2 = a^2b^2$

**17** Ellipsenrotation um x-Achse

**18** Ellipsenrotation um y-Achse

**19** Hyperbel  $b^2x^2 - a^2y^2 = a^2b^2$

**20** Hyperbelrotation um x-Achse

**21** Hyperbelrotation um y-Achse

**22** Parabel  $y^2 = 2px$

**23** Parabelrotation um x-Achse

**24** Parabelrotation um y-Achse







Integral Flächen

Integral Flächen



Integral Flächen, 1/24

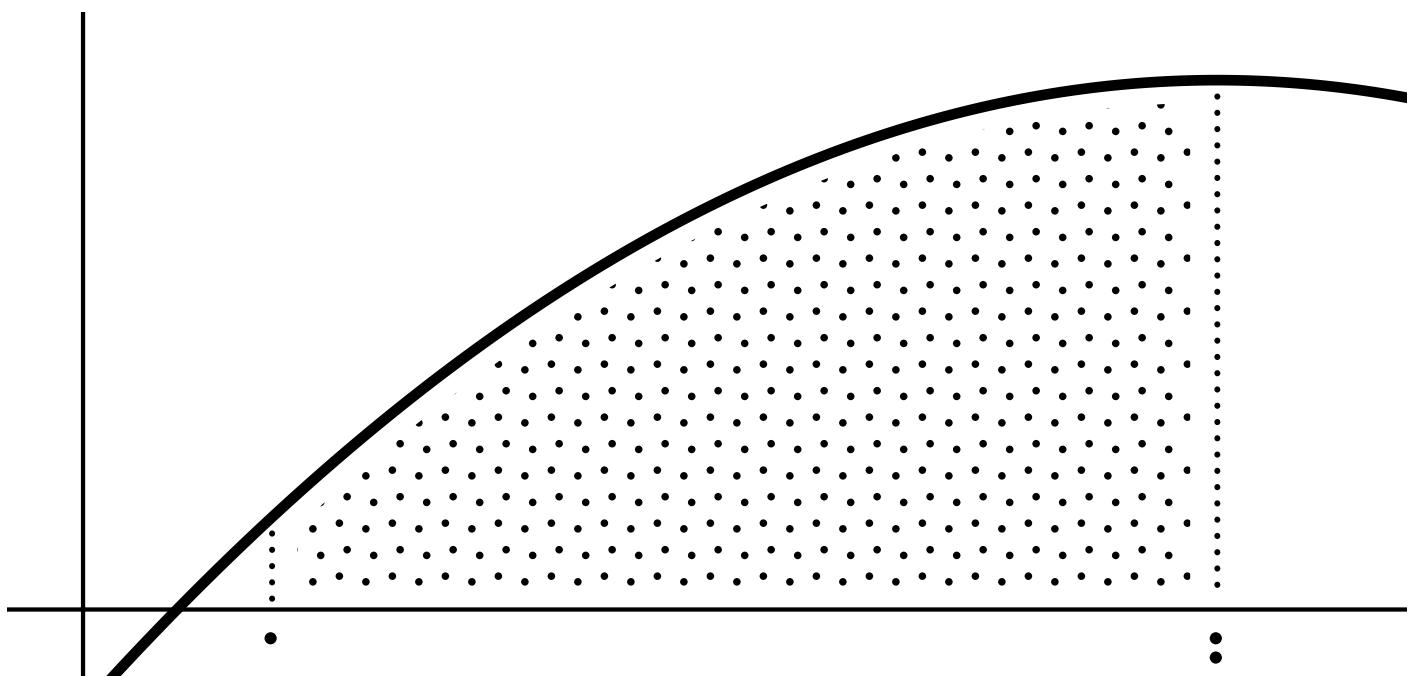
f(x) ... Funktion    a, b ... Grenzen

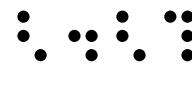
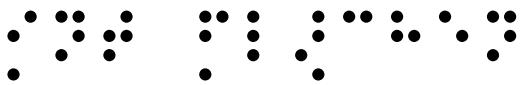
Integral

f ... Funktion    a, b ... Grenzen

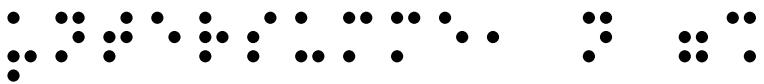
Integral

Integral

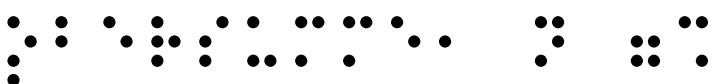
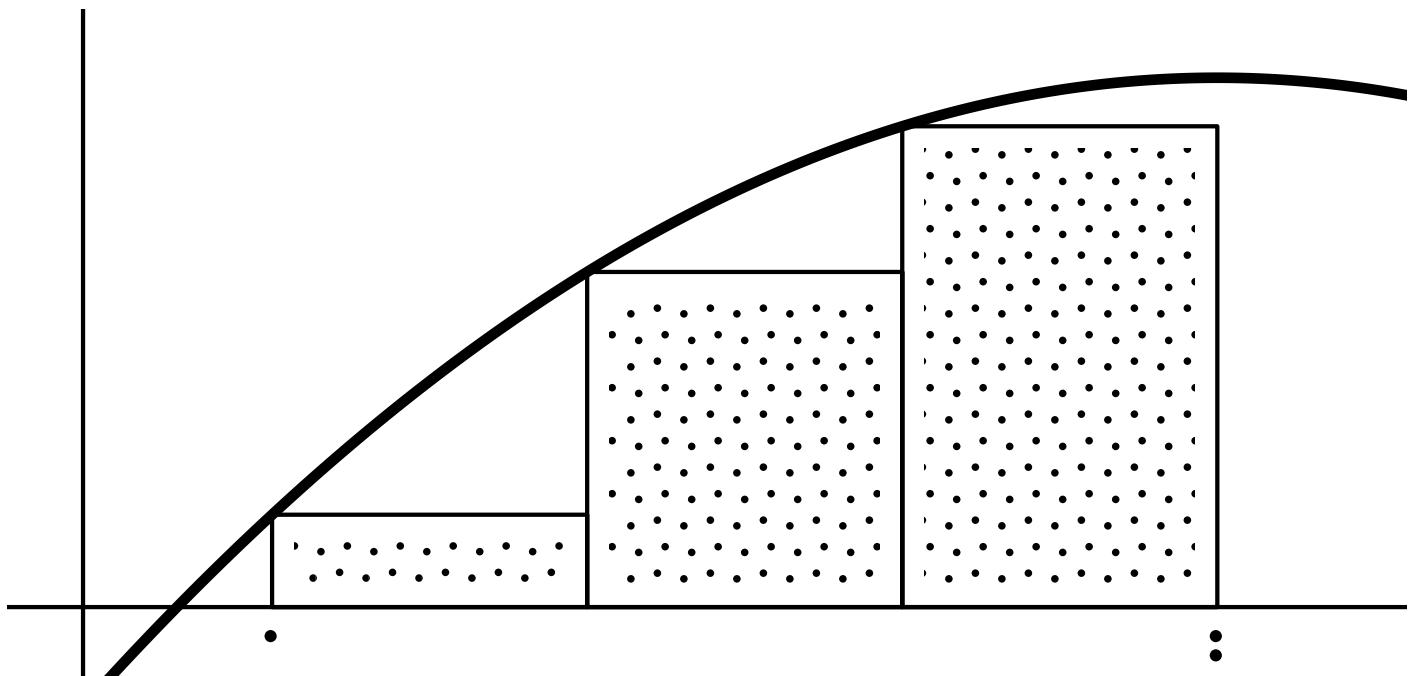




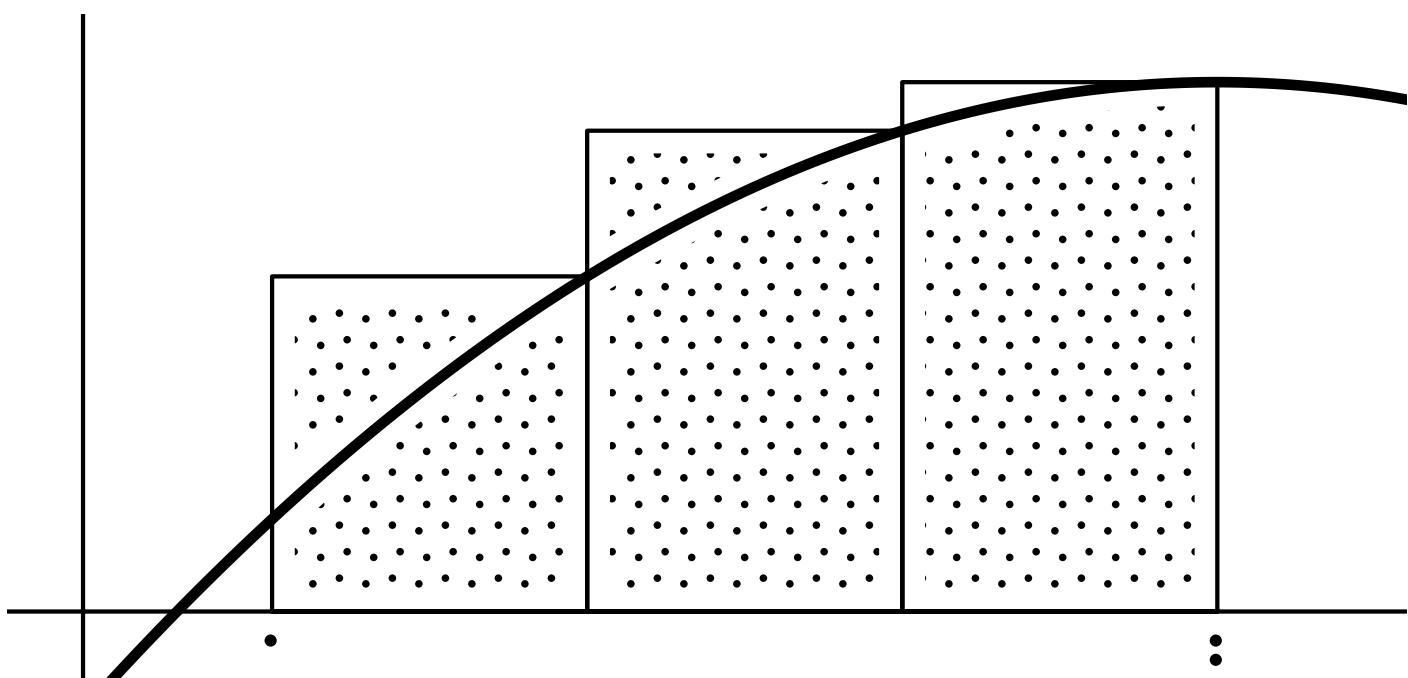
Integral Flächen, 2/24

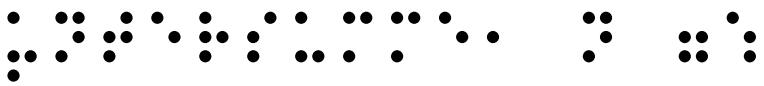
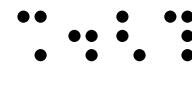
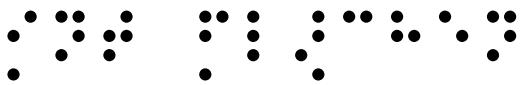


Untersumme,  $n = 3$

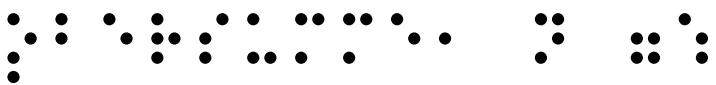
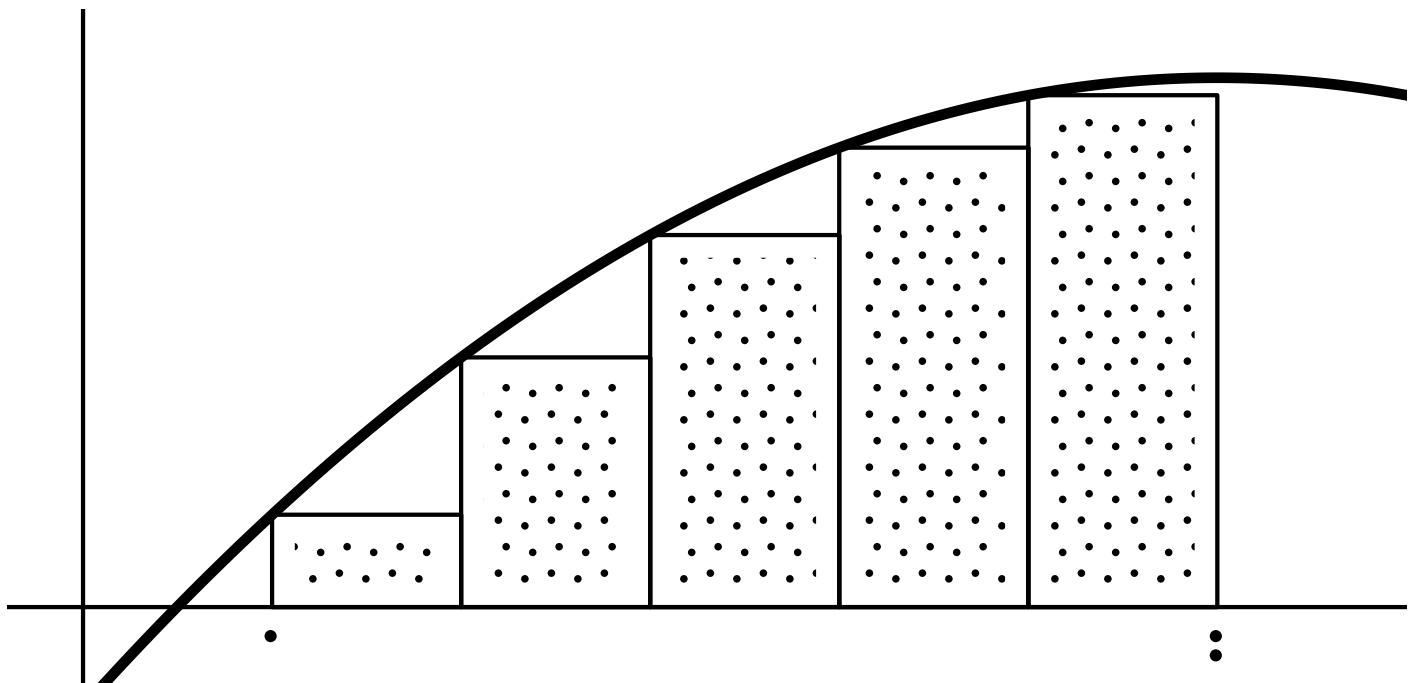


Obersumme,  $n = 3$

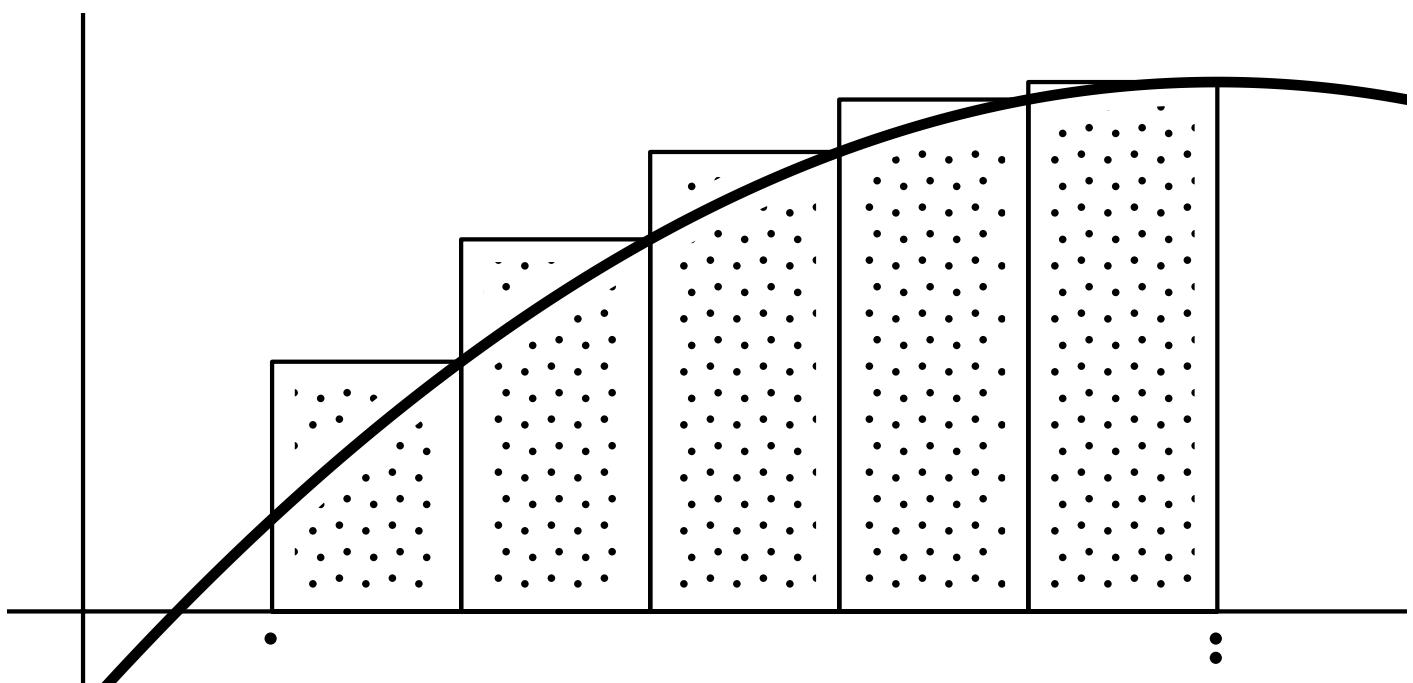


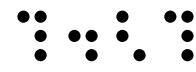
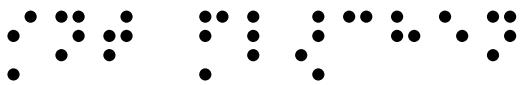


Untersumme,  $n = 5$

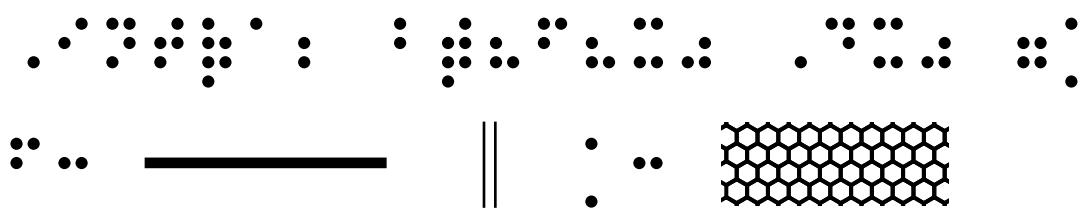


Obersumme,  $n = 5$

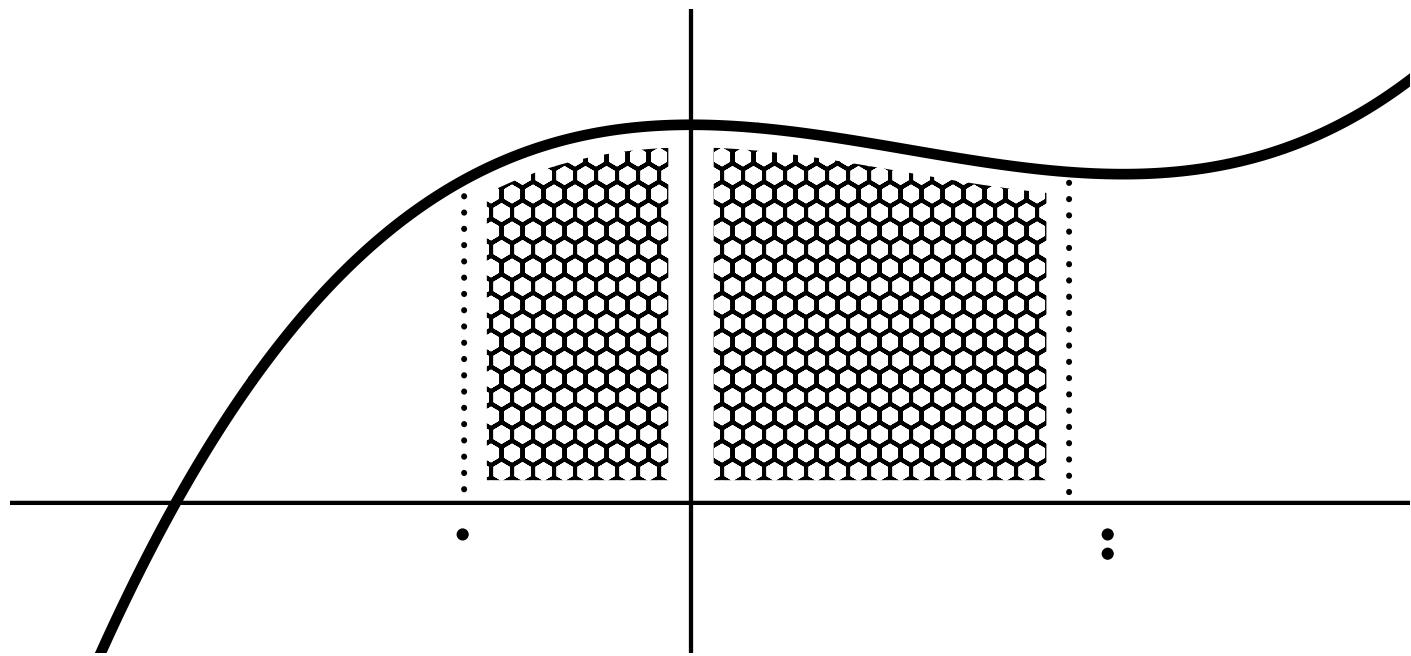
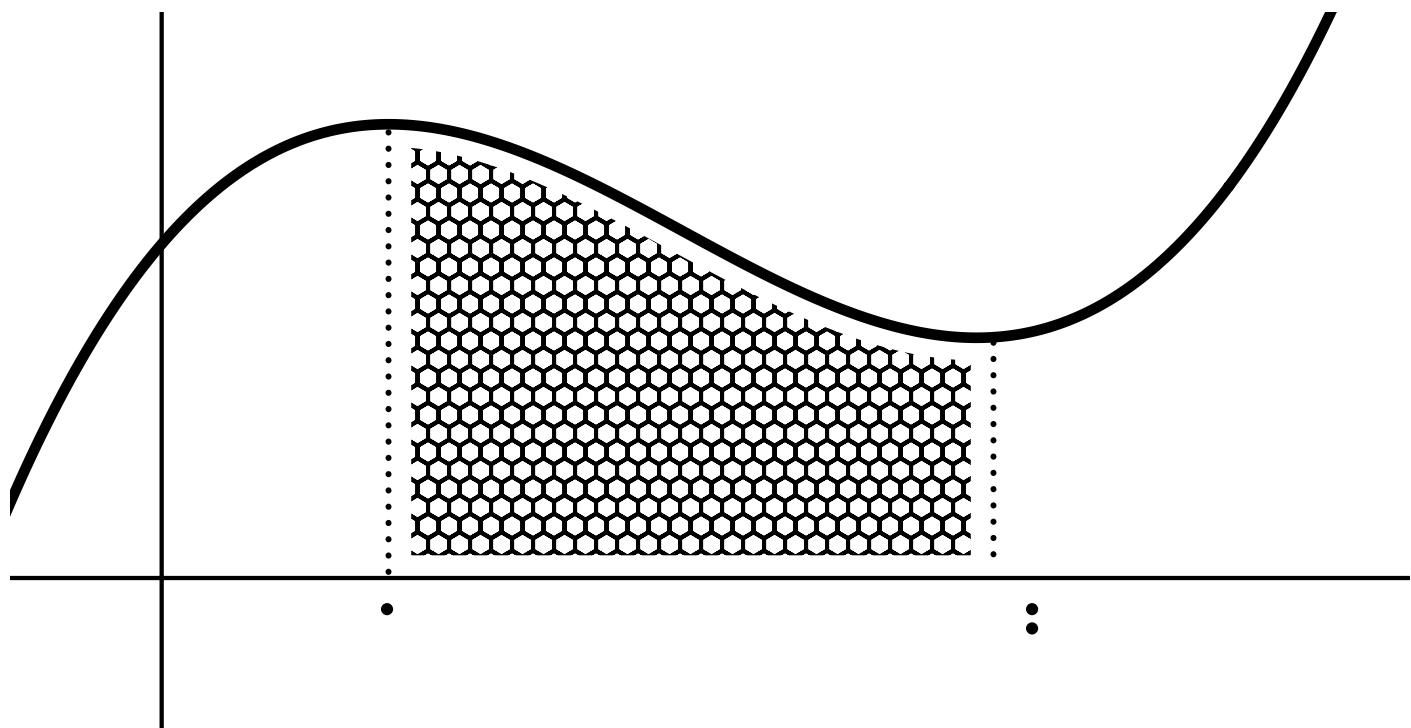


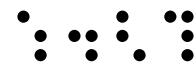
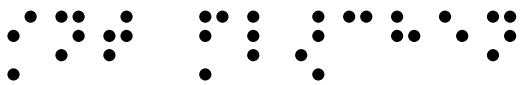


## Integral Flächen, 4/24

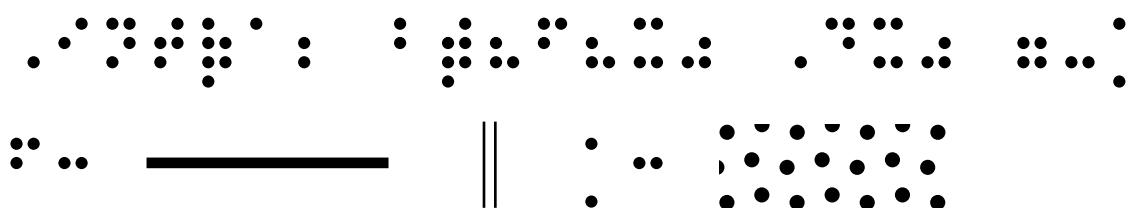


$$\int [a; b] (f(x) dx) = A$$

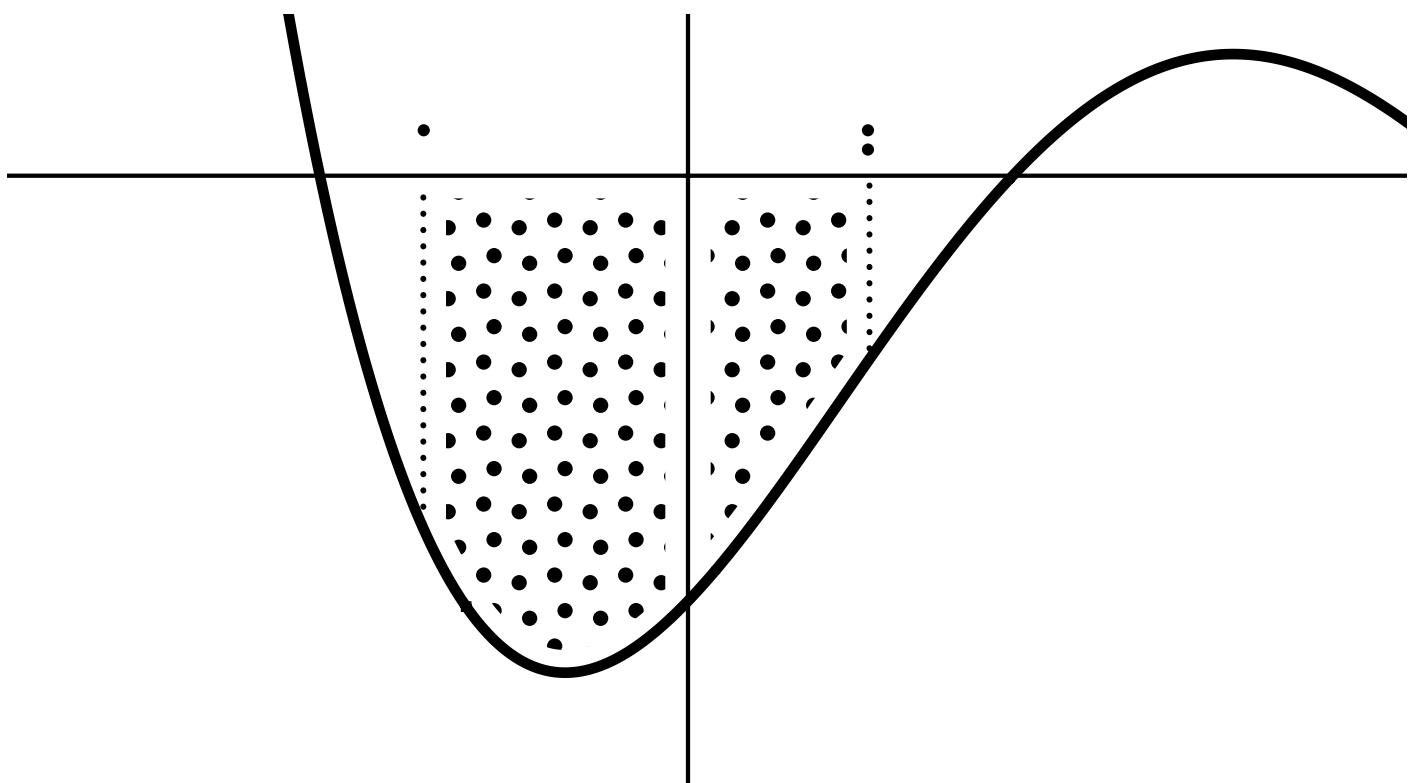
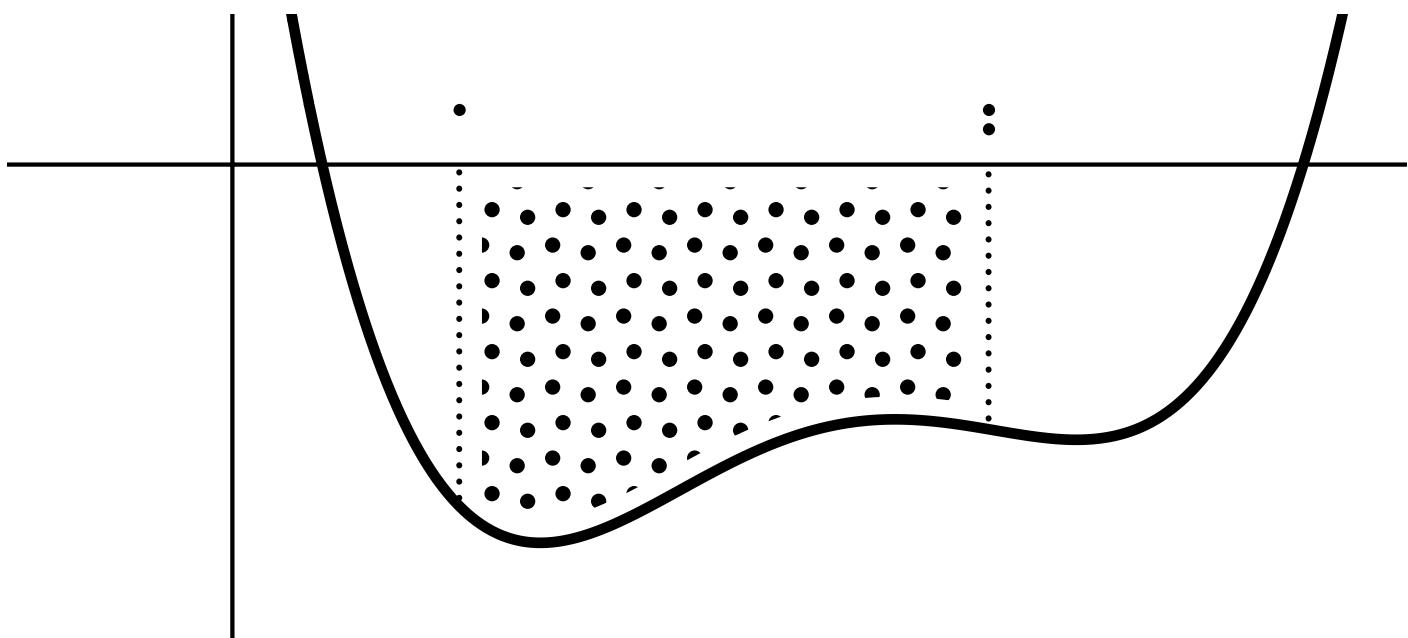


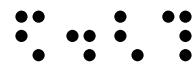
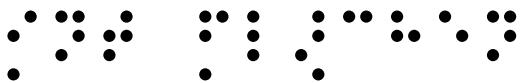


## Integral Flächen, 5/24



$$\int_{a}^{b} f(x) dx = -A$$



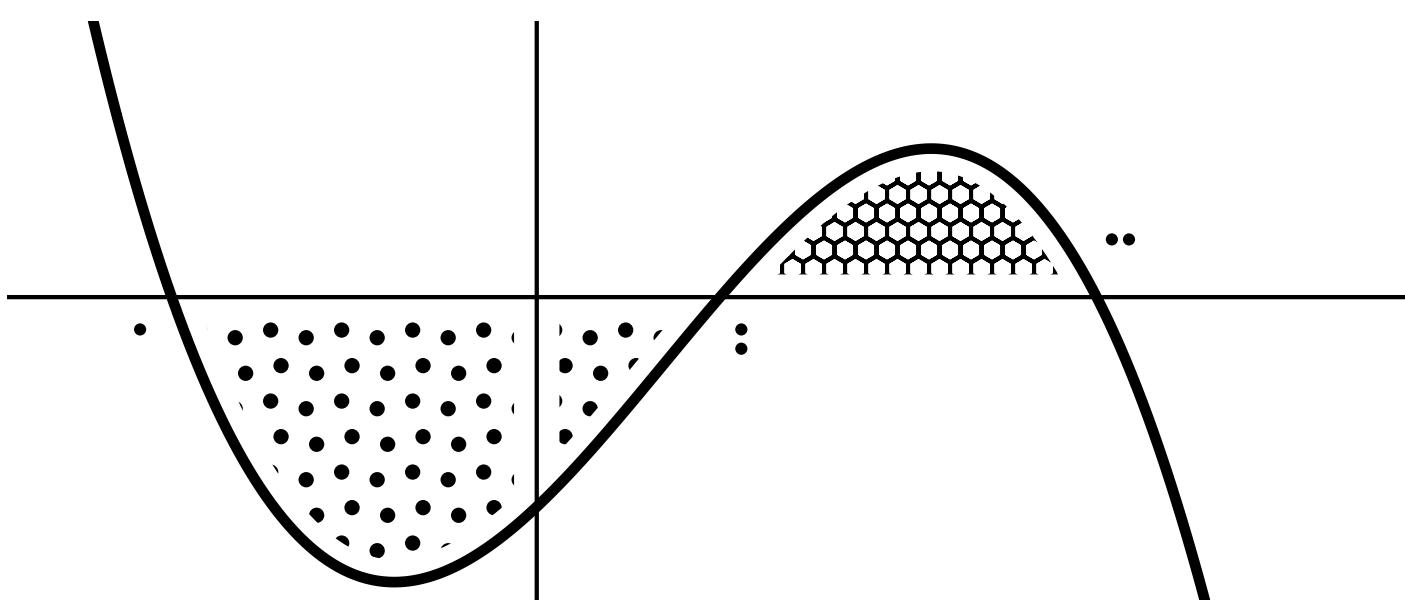
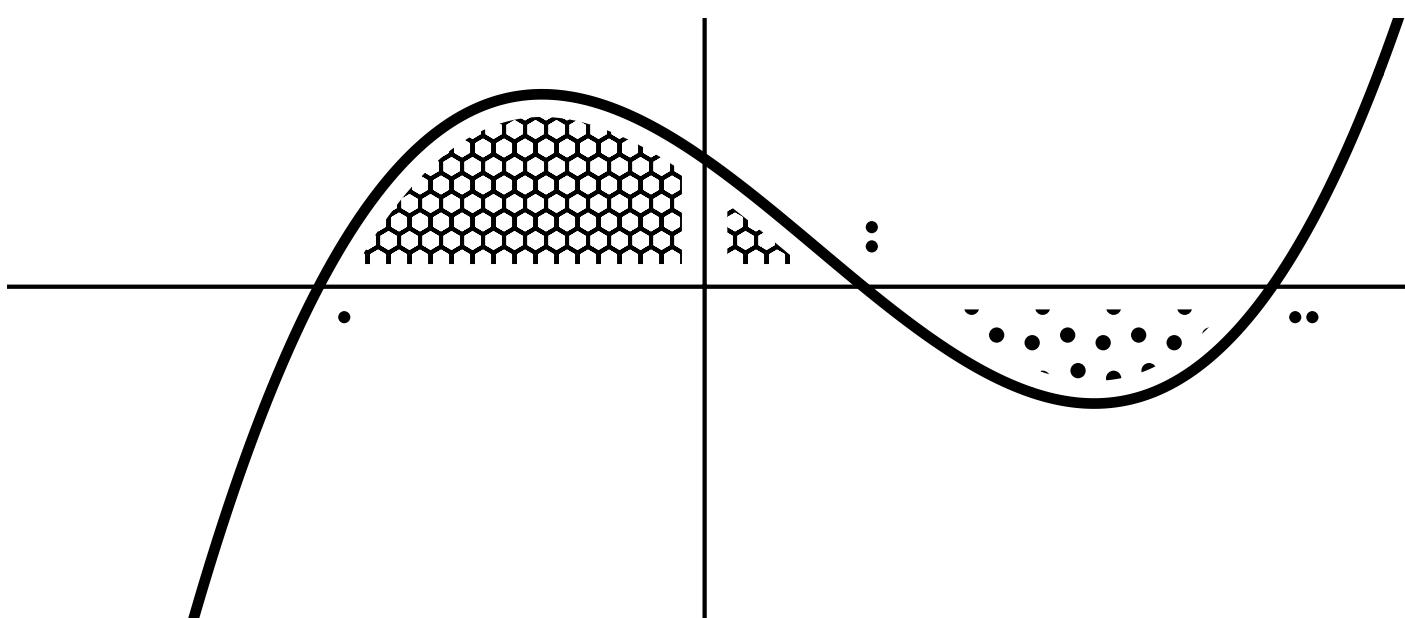


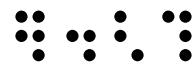
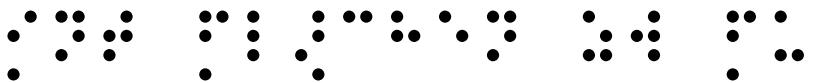
## Integral Flächen, 6/24

Die Fläche unter einer Funktion ist die Fläche, die der Graph der Funktion über dem Intervall  $[a; b]$  mit der  $x$ -Achse bildet.

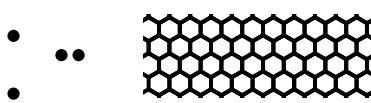
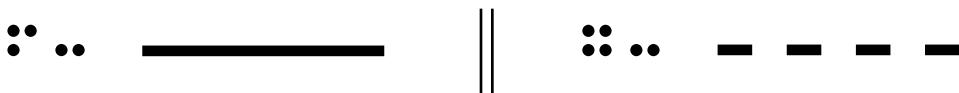
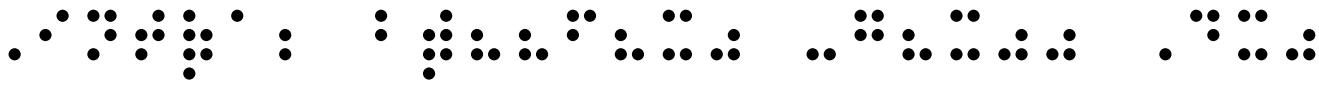
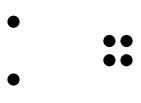


$$A_1 = \left| \int [a; b] (f(x) dx) \right| \quad A_2 = \left| \int [b; c] (f(x) dx) \right| \quad A = A_1 + A_2$$

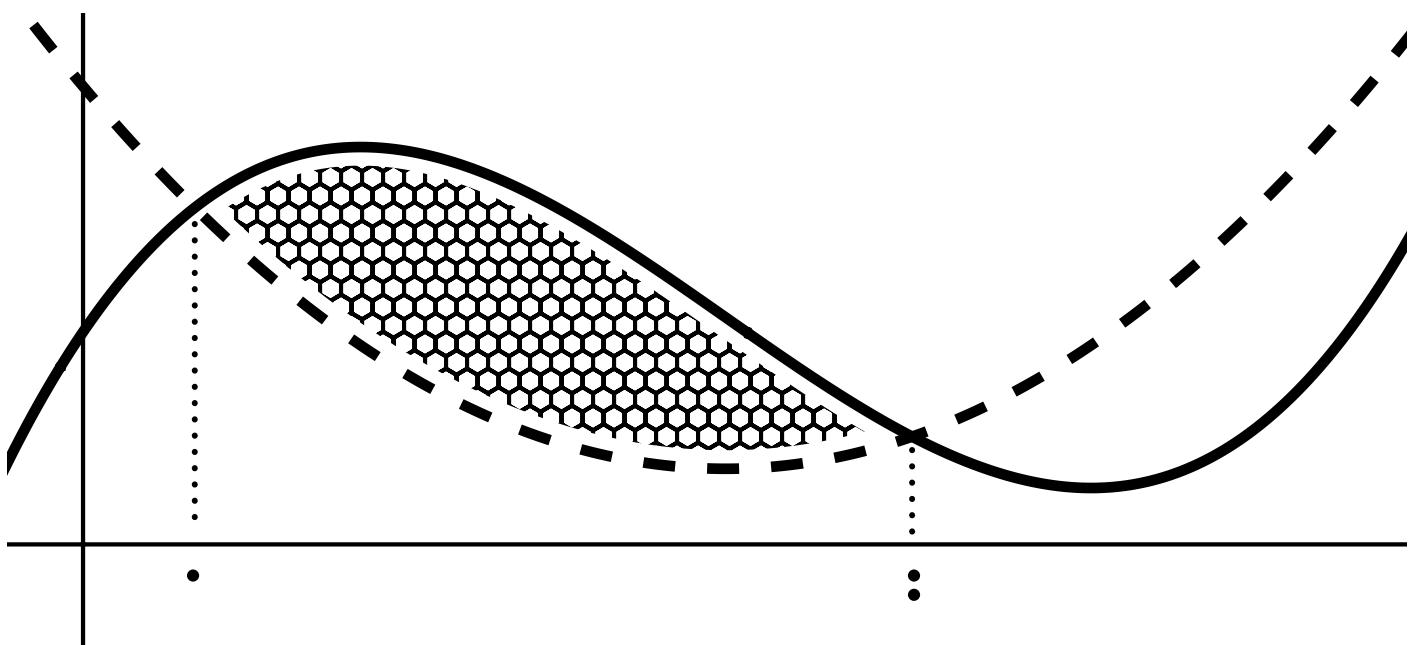
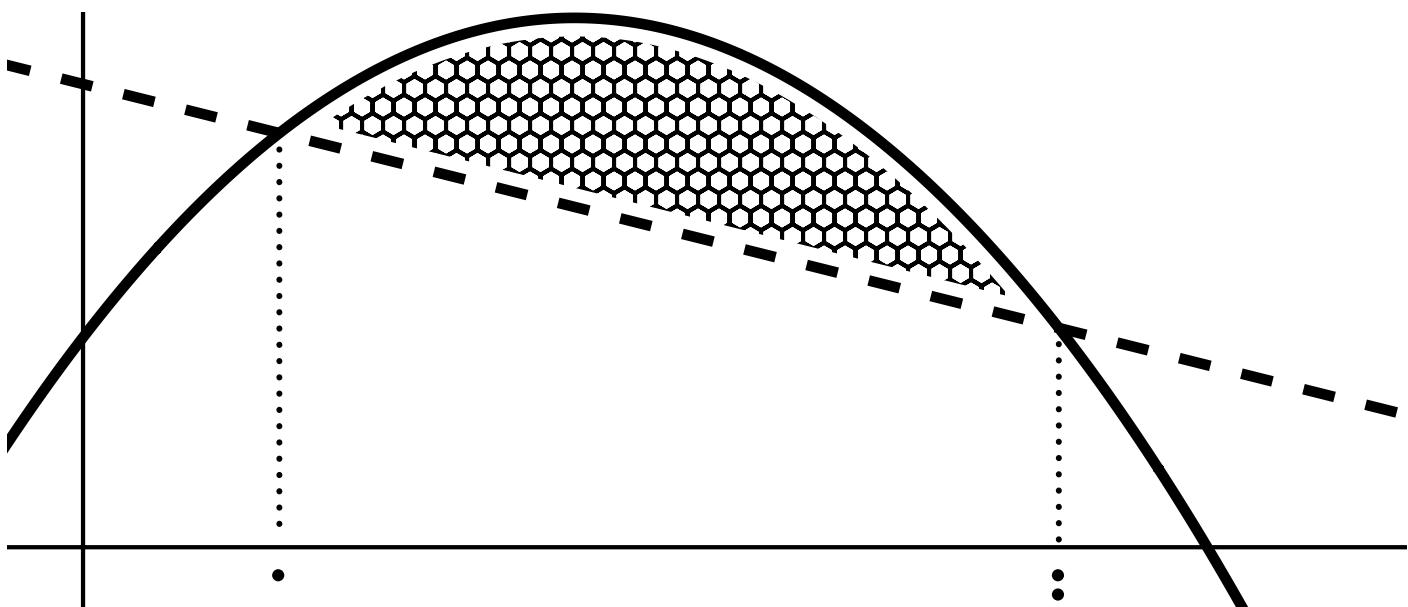




Int Flächen zwischen Funktionen, 7/24



$$A = \int [a; b] ((f(x) - g(x)) dx)$$



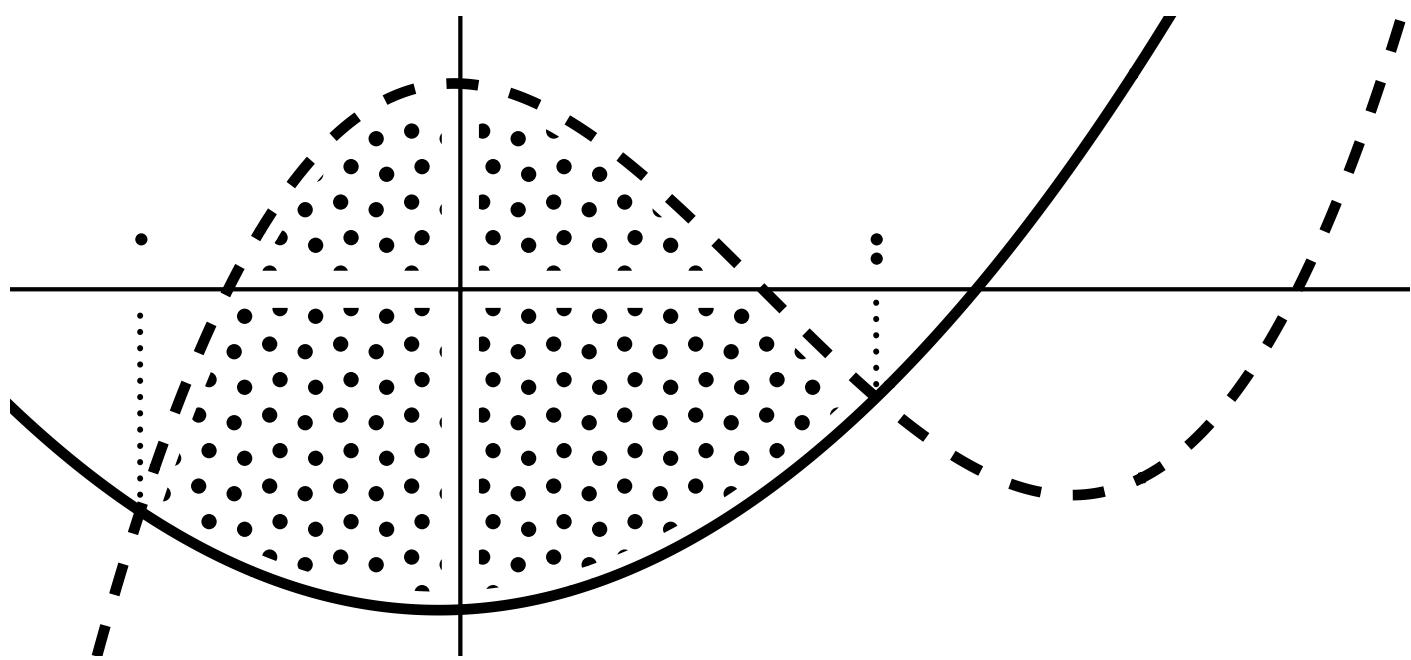
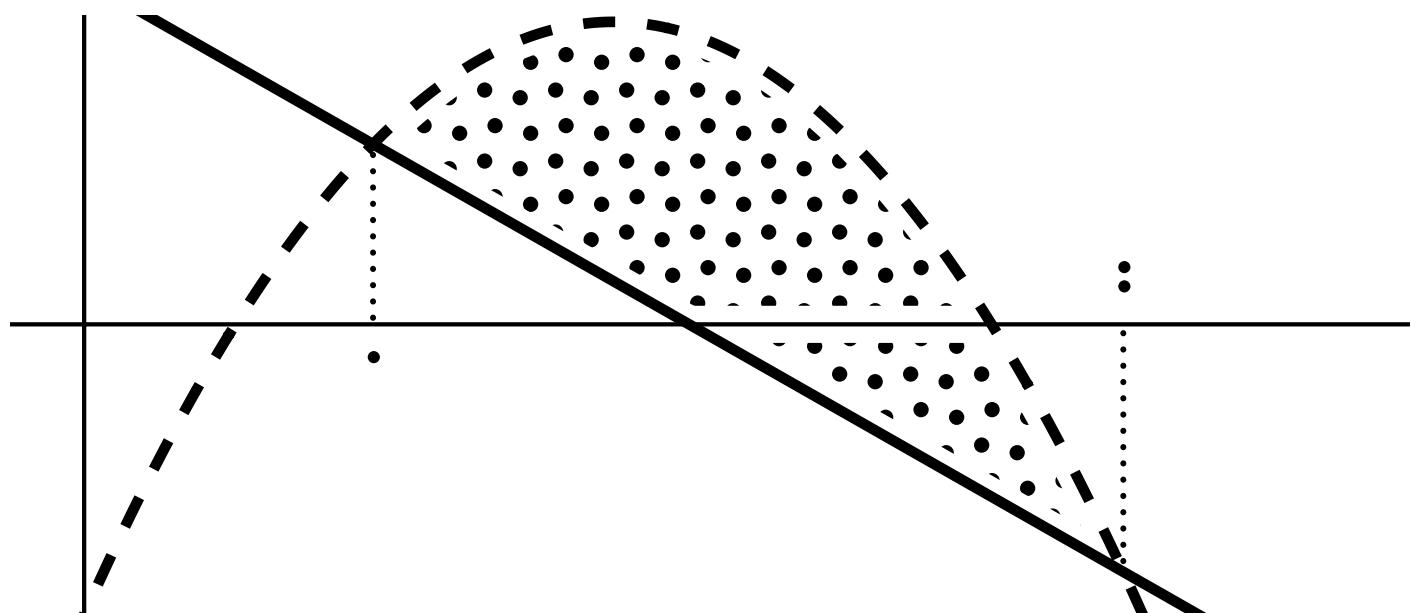


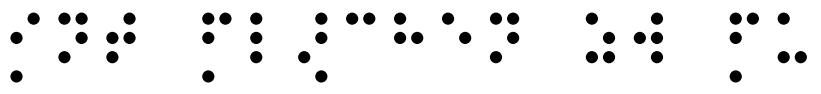
1

A musical staff with two measures. The first measure contains a dotted half note followed by a long horizontal bar line. The second measure contains a vertical double bar line, a dotted half note, and a dashed horizontal bar line.

The figure consists of two parts. The left part shows a sparse collection of black dots scattered across a white background. The right part shows a regular grid of black dots arranged in a rectangular pattern.

$$A = \int [a; b] ((g(x) - f(x)) dx)$$



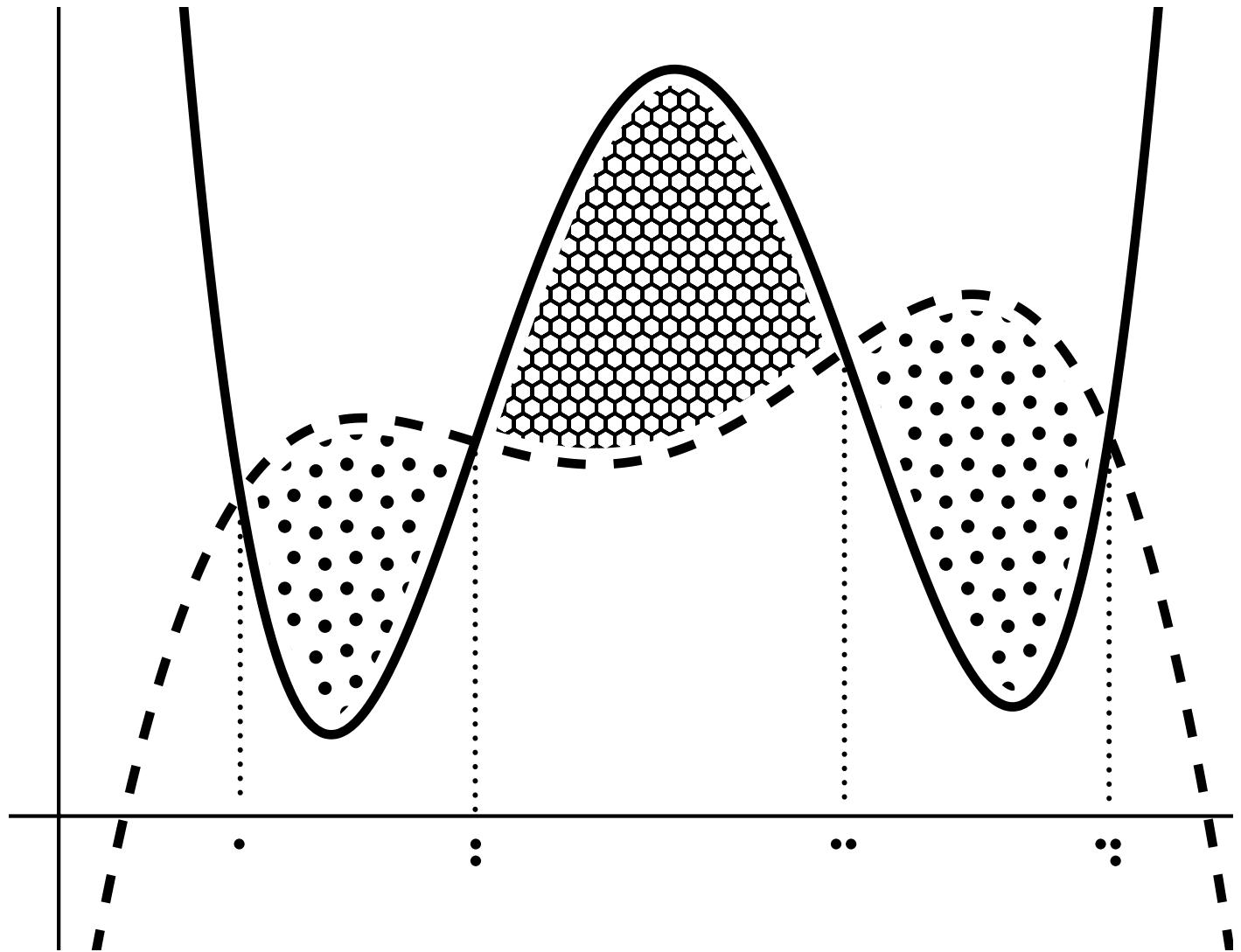


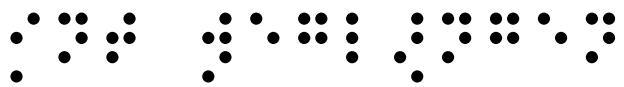
10

The diagram consists of two horizontal lines separated by a vertical double bar. The left line features a pattern of dots above and below the line, followed by a short horizontal dash. The right line features a similar pattern of dots above and below the line, followed by a series of five short horizontal dashes.

The image displays three distinct patterns of points arranged in a grid-like structure. The first pattern on the left is a hexagonal lattice, consisting of a regular arrangement of hexagonal cells. The second pattern in the middle is a triangular lattice, where each point is connected to its six nearest neighbors forming equilateral triangles. The third pattern on the right is a square lattice, where points are arranged in a regular grid and connected by straight lines forming squares.

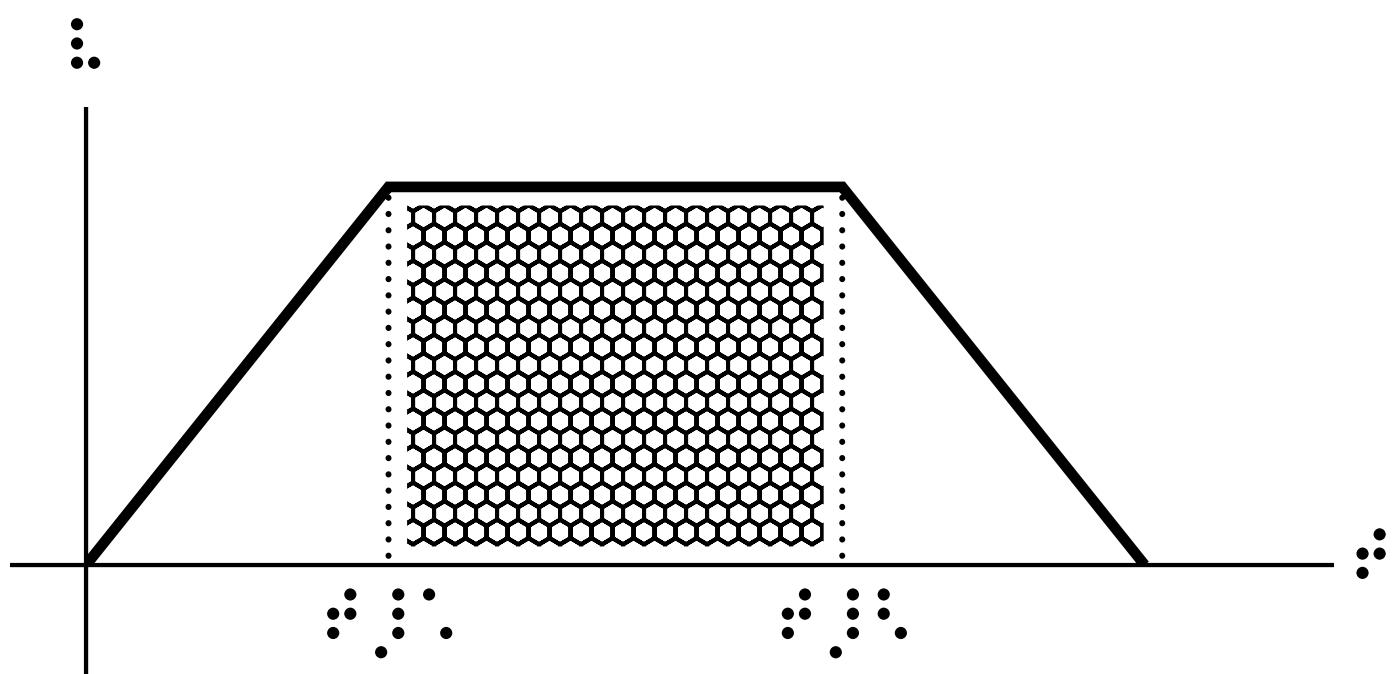
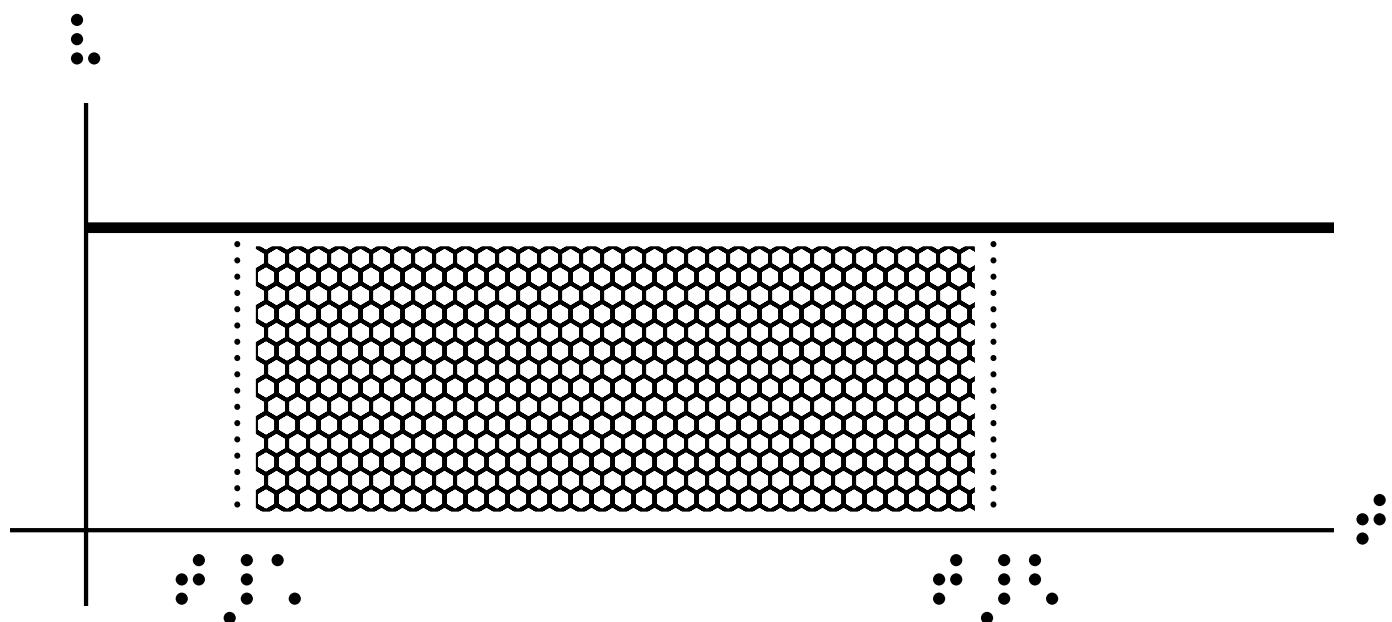
$$A = \int [a; b] ((g(x) - f(x)) dx) + \int [b; c] ((f(x) - g(x)) dx) + \int [c; d] ((g(x) - f(x)) dx)$$

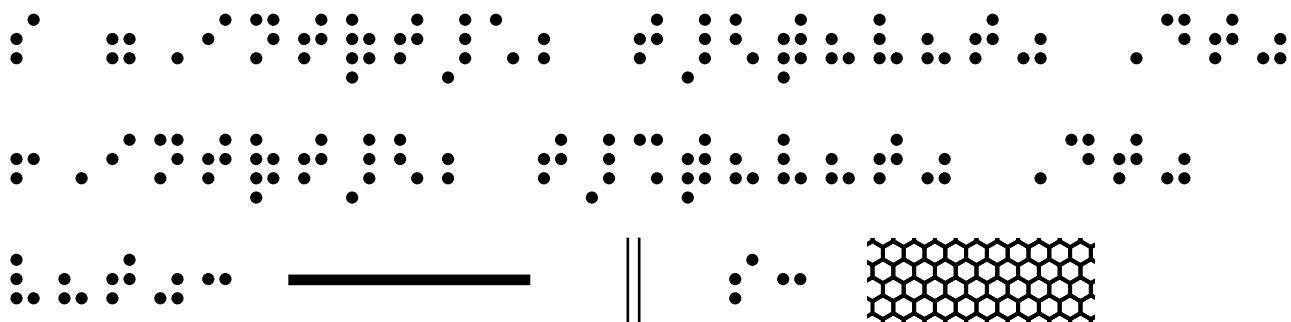
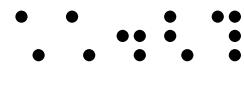
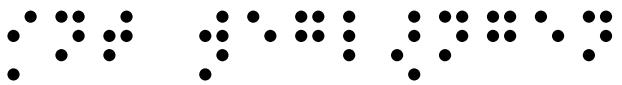




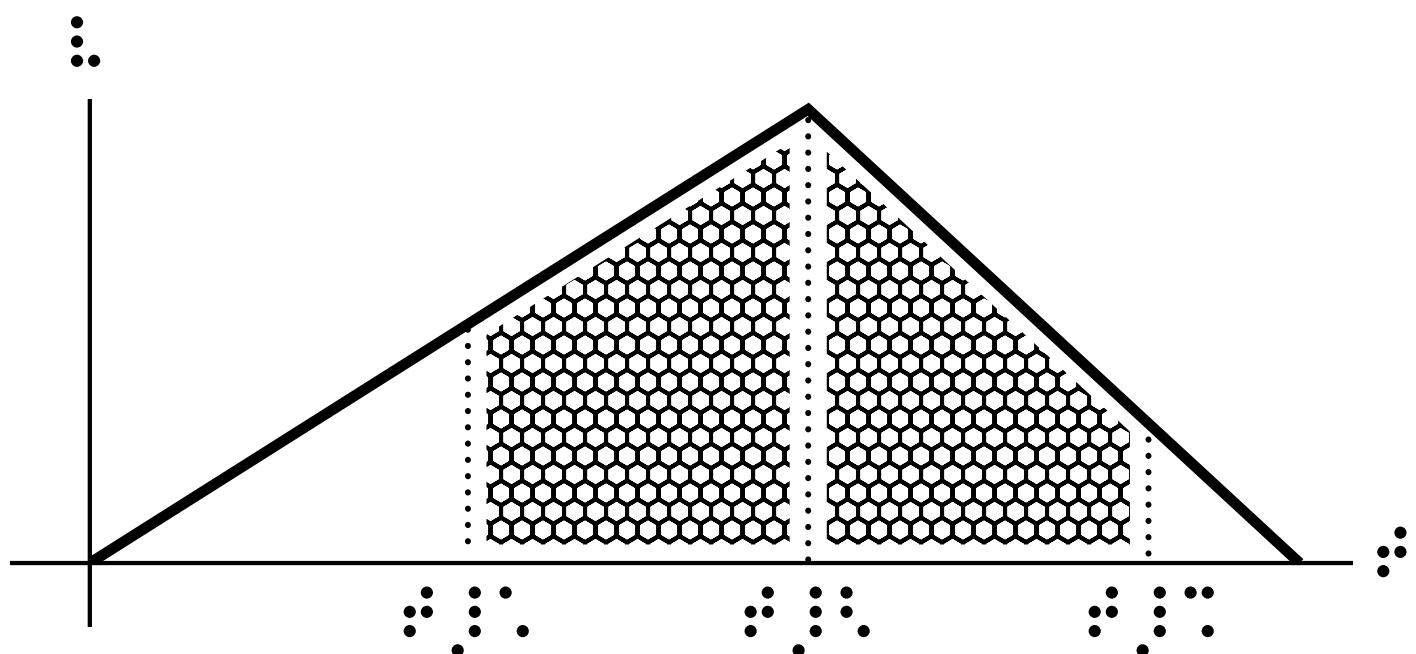
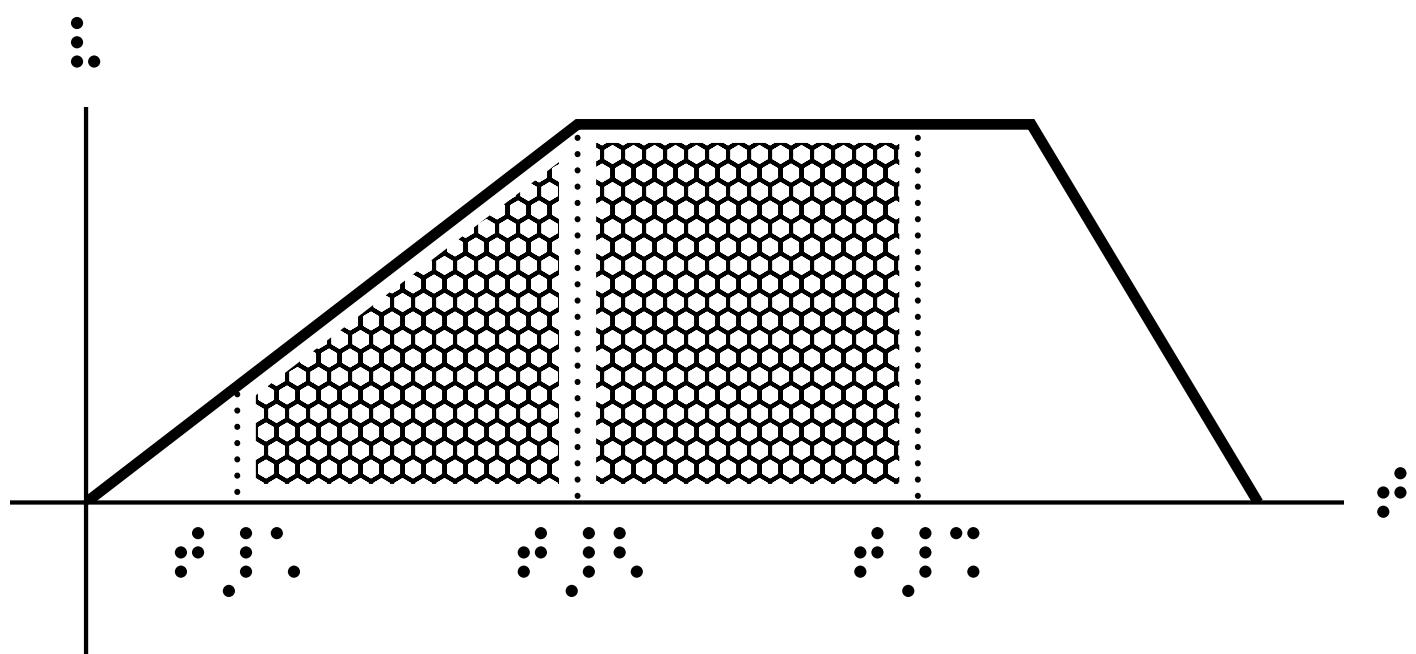
The image shows a sequence of symbols. It starts with a row of five black dots. This is followed by a thick horizontal black bar. After a short gap, there are two vertical black bars. Following another gap, there is another row of five black dots. The sequence concludes with a large area filled with a pattern of black hexagons.

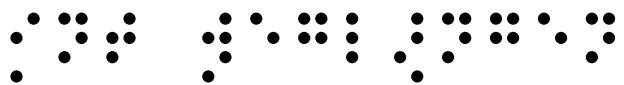
$$s = \int [t_1; t_2] (v(t) dt)$$



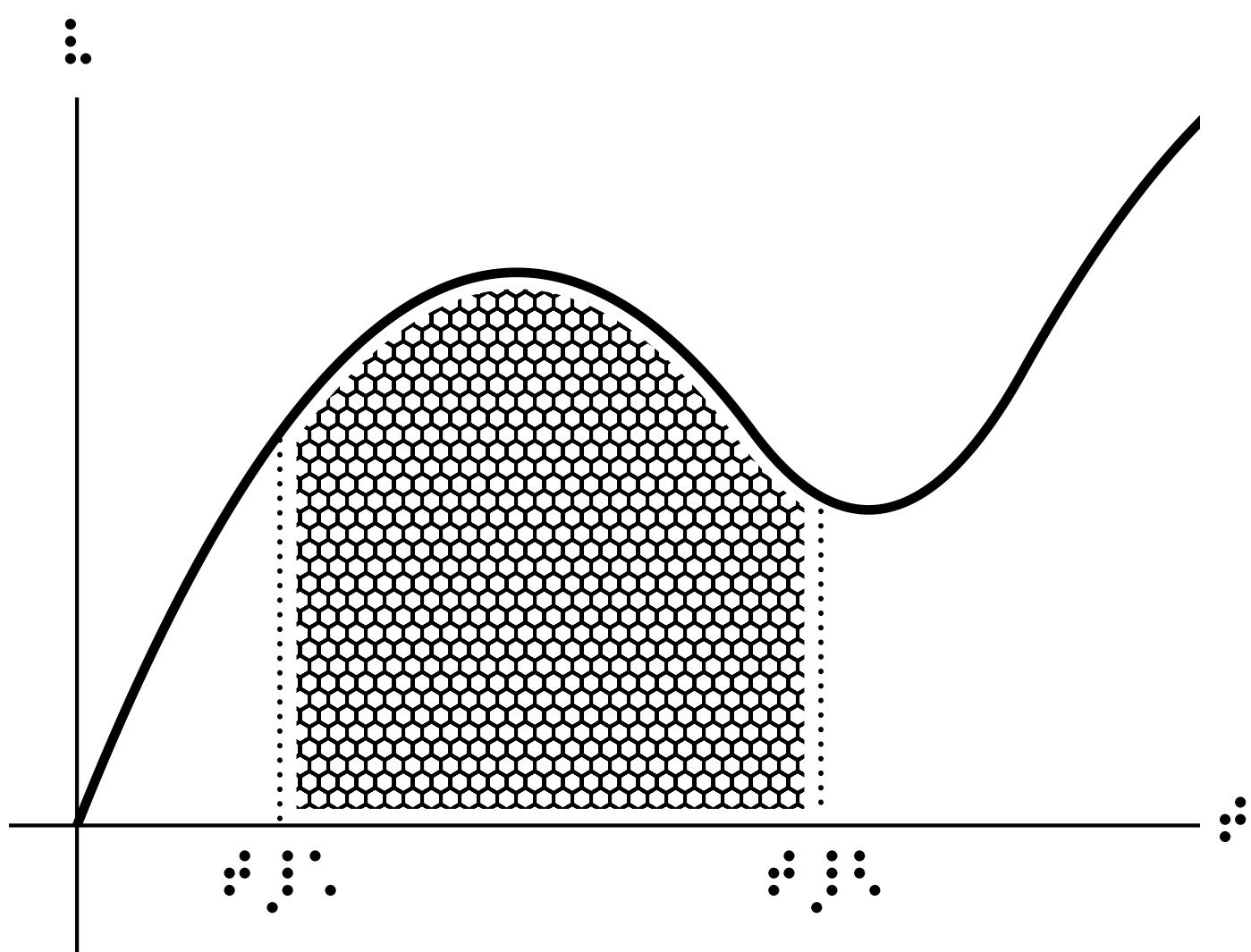


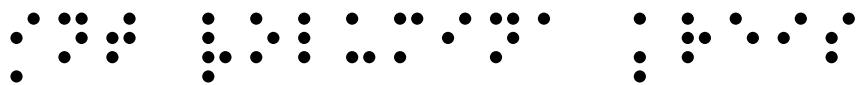
$$s = \int_{t_1}^{t_2} v(t) dt + \int_{t_2}^{t_3} v(t) dt$$



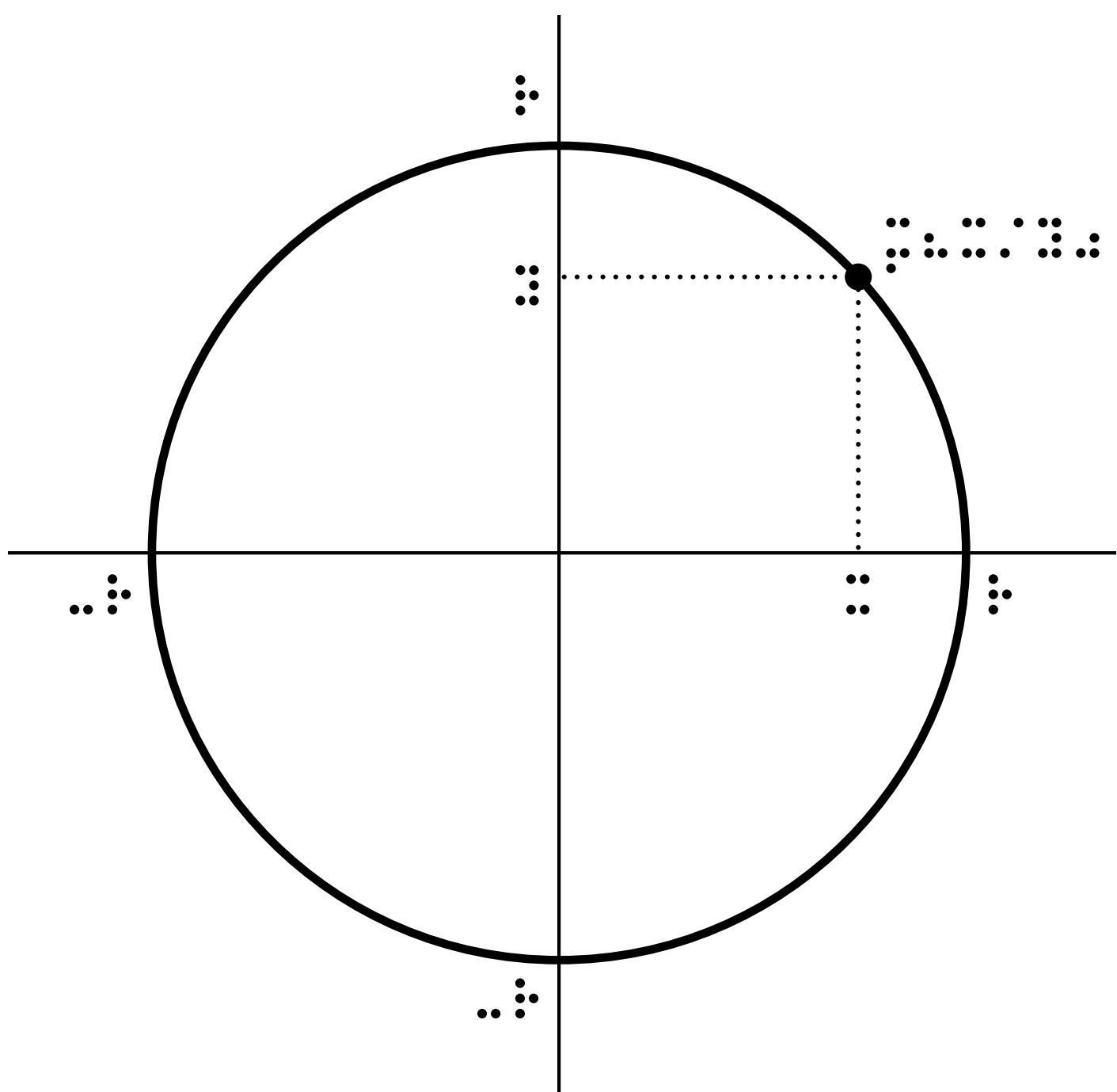


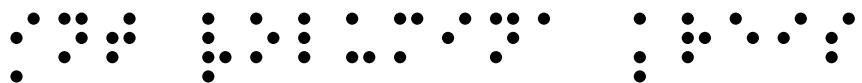
$$s = \int [t_1; t_2] (v(t) dt)$$





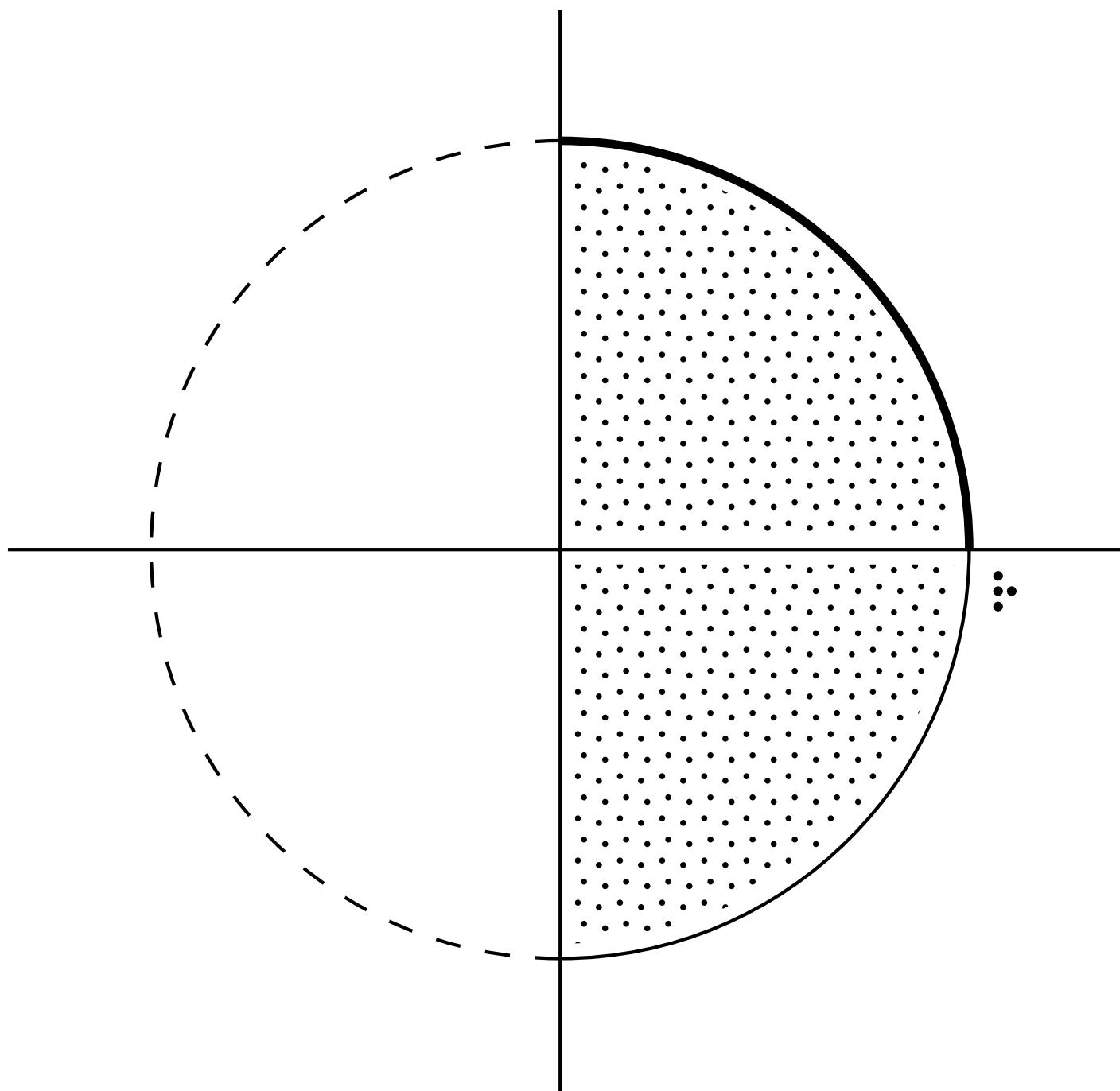
$$\text{Kreis: } x^2 + y^2 = r^2$$

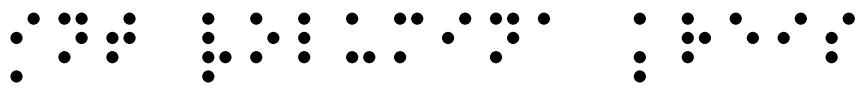




The image shows three groups of Braille characters. Each group consists of a vertical column of four dots at the top and a vertical column of four dots at the bottom, with a central dot connecting them. The first group has one dot in the middle column. The second group has two dots in the middle column. The third group has three dots in the middle column. These represent the Braille characters for the digits 1, 2, and 3 respectively.

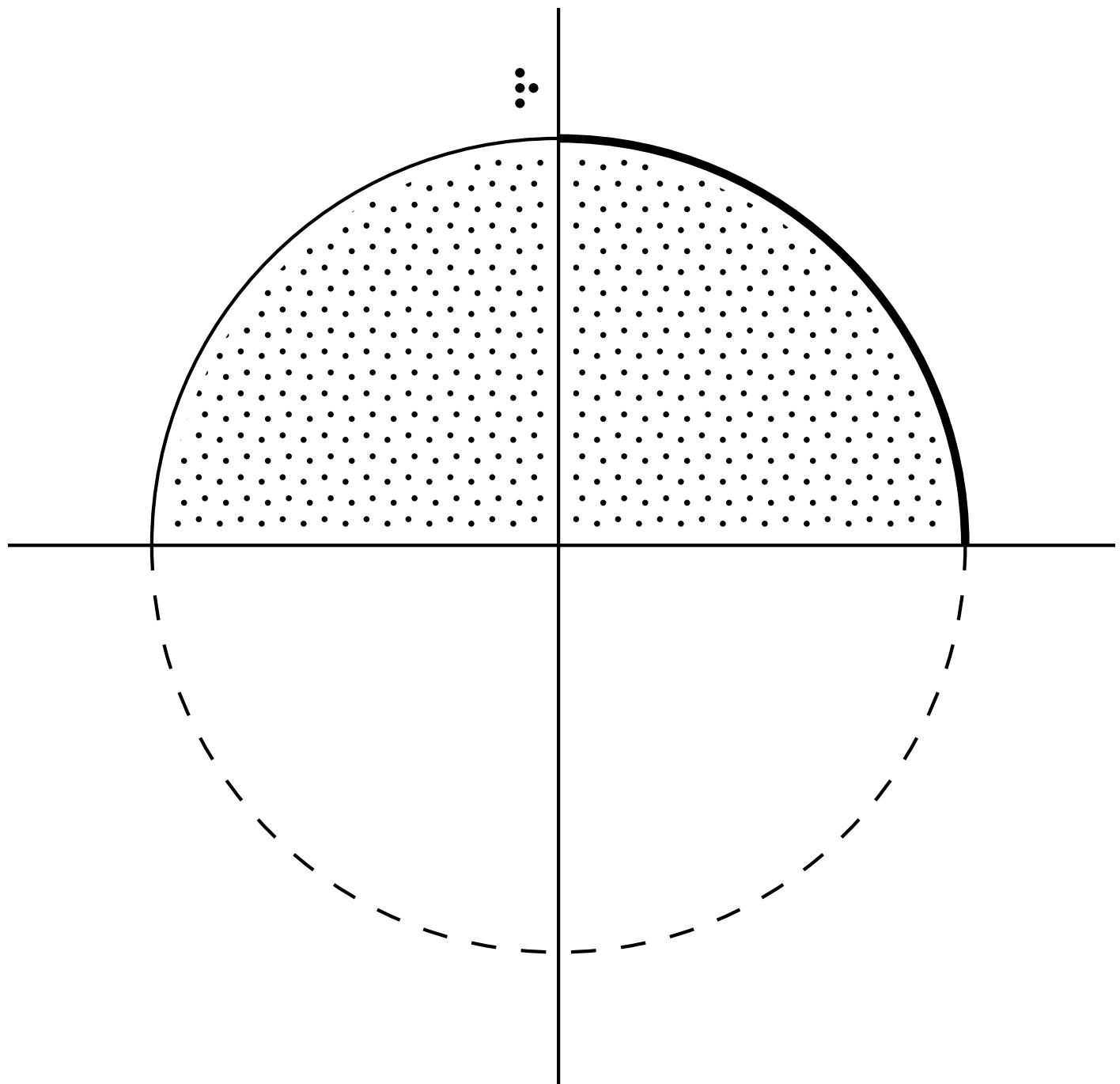
$$y^2 = r^2 - x^2 \quad V/2 = \pi * \text{int}[0; r] (y^2 dx)$$

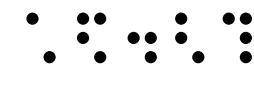
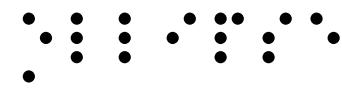
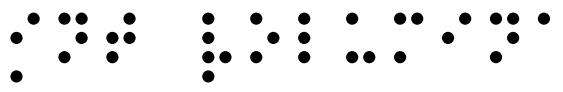




The image shows three identical Braille characters side-by-side. Each character consists of a 2x5 grid of dots. The top row has dots at positions (1,1), (1,3), (1,5), (2,1), and (2,3). The bottom row has dots at positions (1,1), (1,2), (1,3), (1,4), and (1,5). This pattern repeats for each of the three characters.

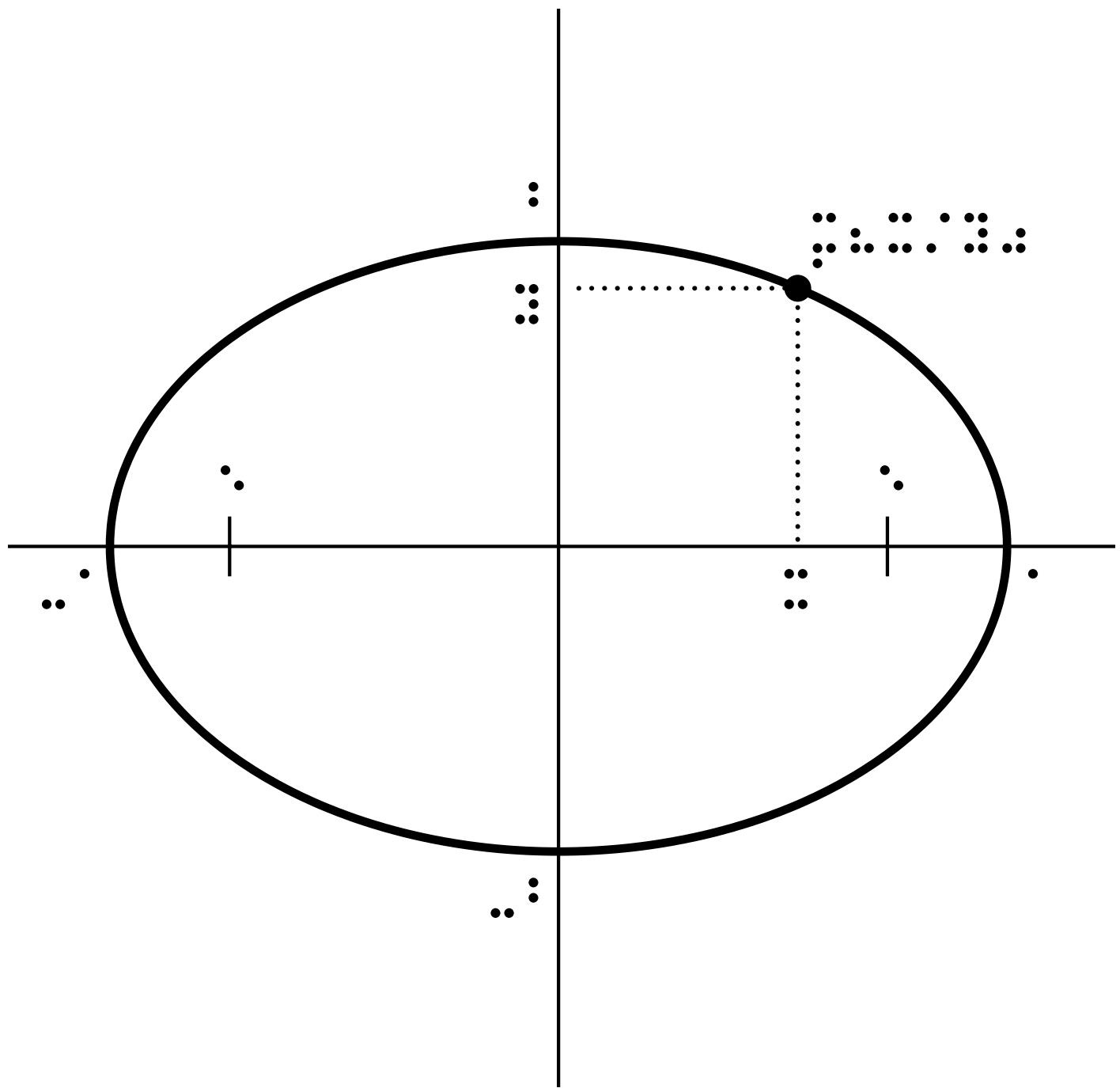
$$x^2 = r^2 - y^2 \quad V/2 = \pi * \text{int}[0; r] (x^2 dy)$$

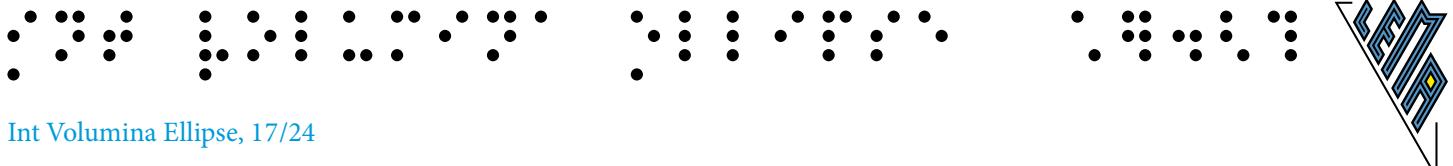




The image shows a sequence of seven Braille characters. The first character has two dots in the top row and three dots in the middle row. The second character has four dots in the top row and one dot in the middle row. The third character has two dots in the top row and five dots in the middle row. The fourth character has one dot in the top row and two dots in the middle row. The fifth character has one dot in the top row and three dots in the middle row. The sixth character has one dot in the top row and four dots in the middle row. The seventh character has one dot in the top row and five dots in the middle row.

$$\text{Ellipse: } b^2 \cdot x^2 + a^2 \cdot y^2 = b^2 \cdot a^2 \quad e^2 = a^2 - b^2$$





Int Volumina Ellipse, 17/24

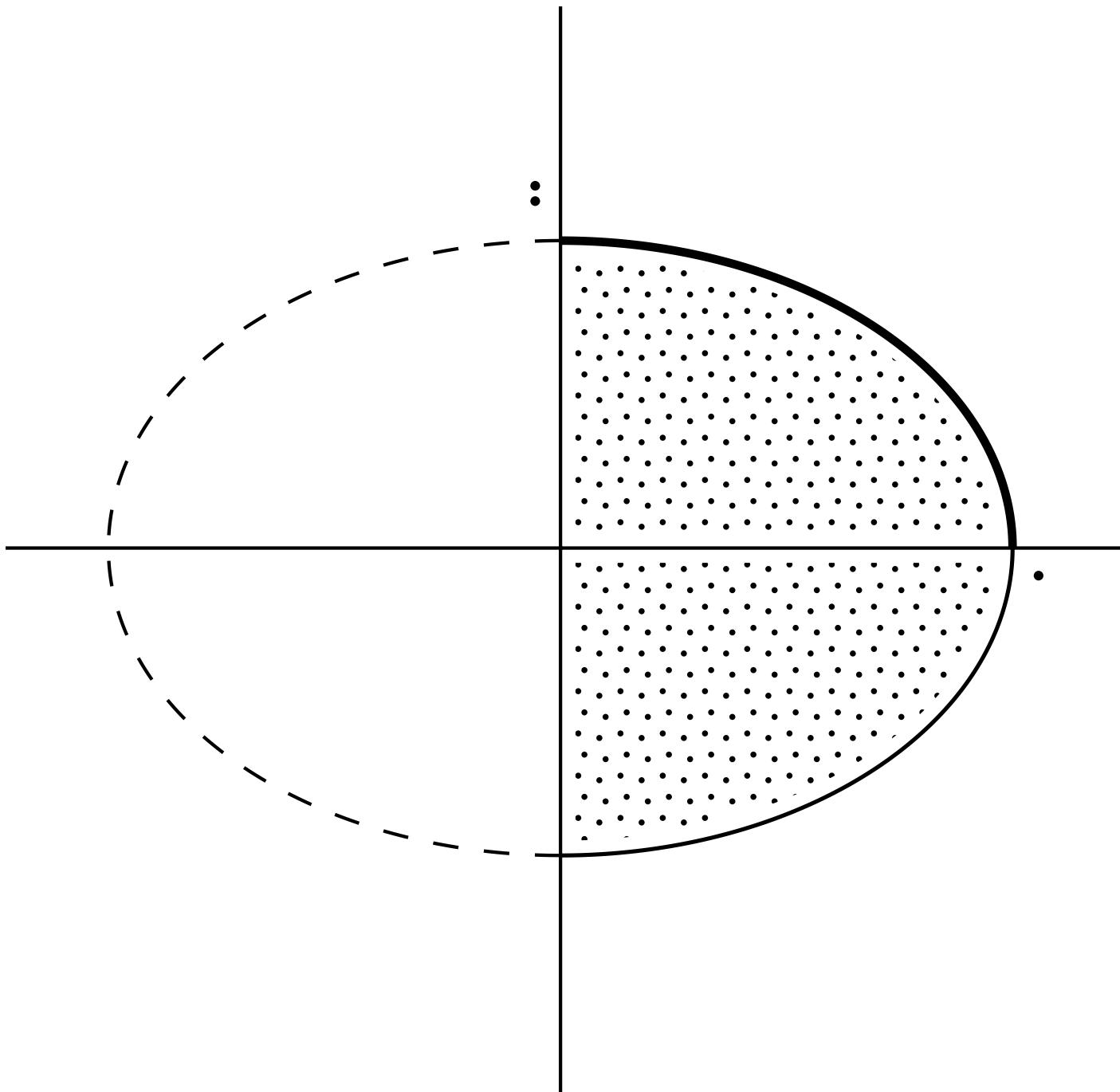
Elliptische Formeln für Ellipse und Kreis

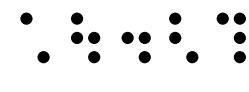
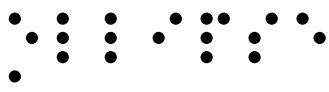
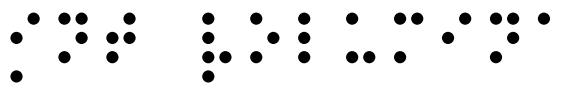
Fläche:

$\pi \cdot a \cdot b$  oder  $\pi \cdot \sqrt{a^2 - b^2} \cdot b$

$$y^2 = b^2 - b^2/a^2 * x^2$$

$$V/2 = \pi * \int [0; a] (y^2 dx)$$

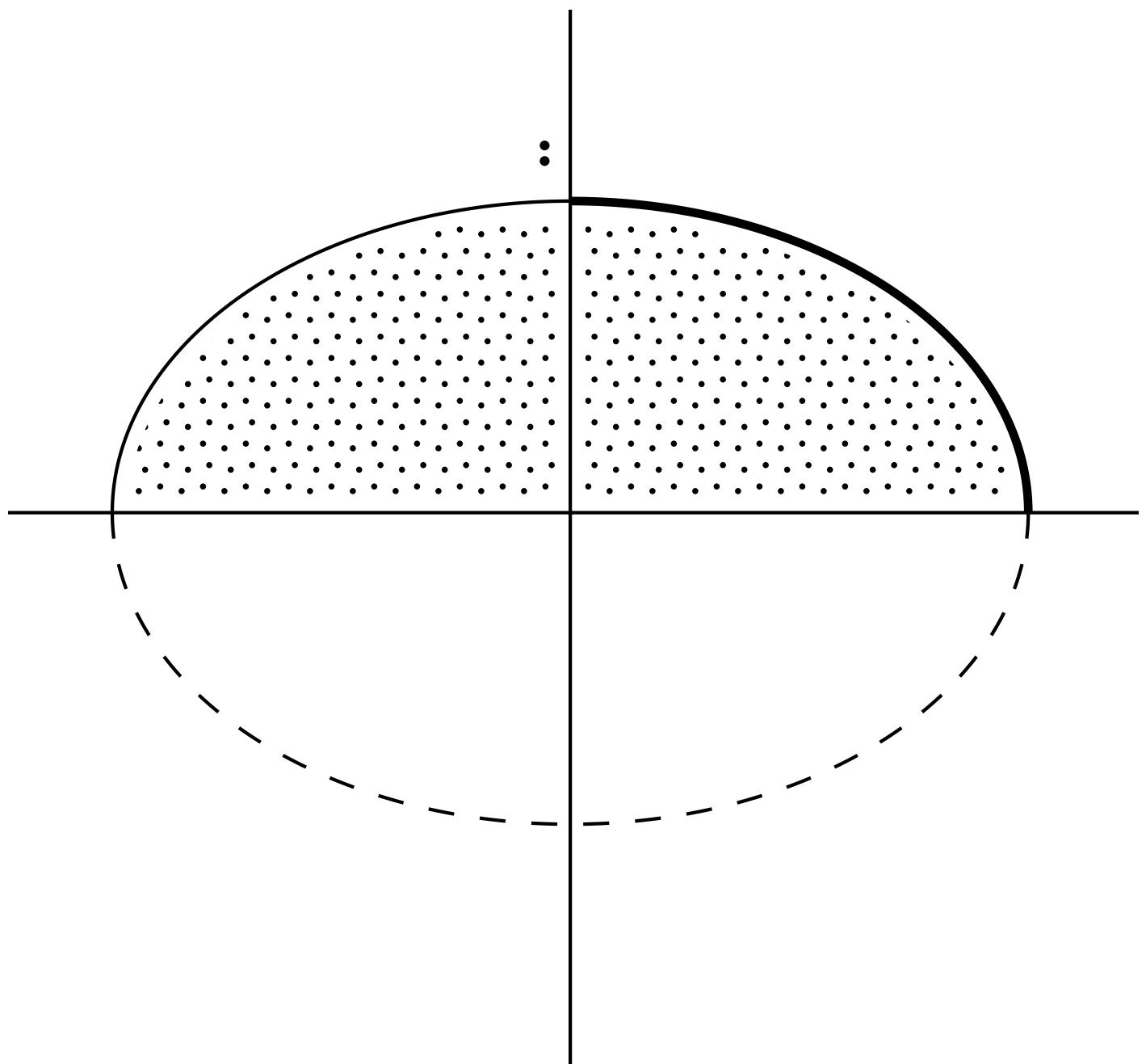


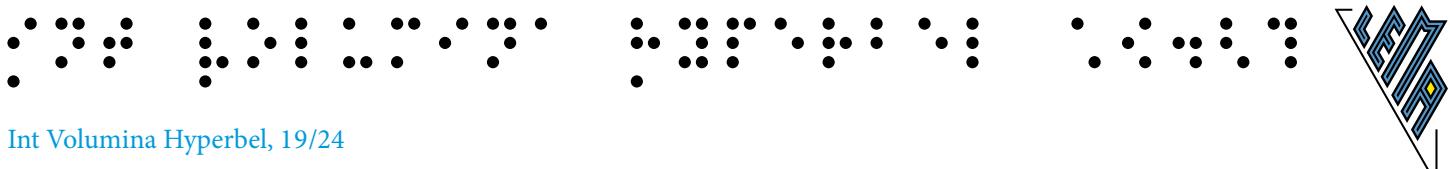


Int Volumina Ellipse, 18/24

A 4x4 grid of 16 black dots arranged in four rows and four columns.

$$x^2 = a^2 - a^2 / b^2 * y^2 \quad V/2 = \pi * \text{int}[0; b] (x^2 dy)$$



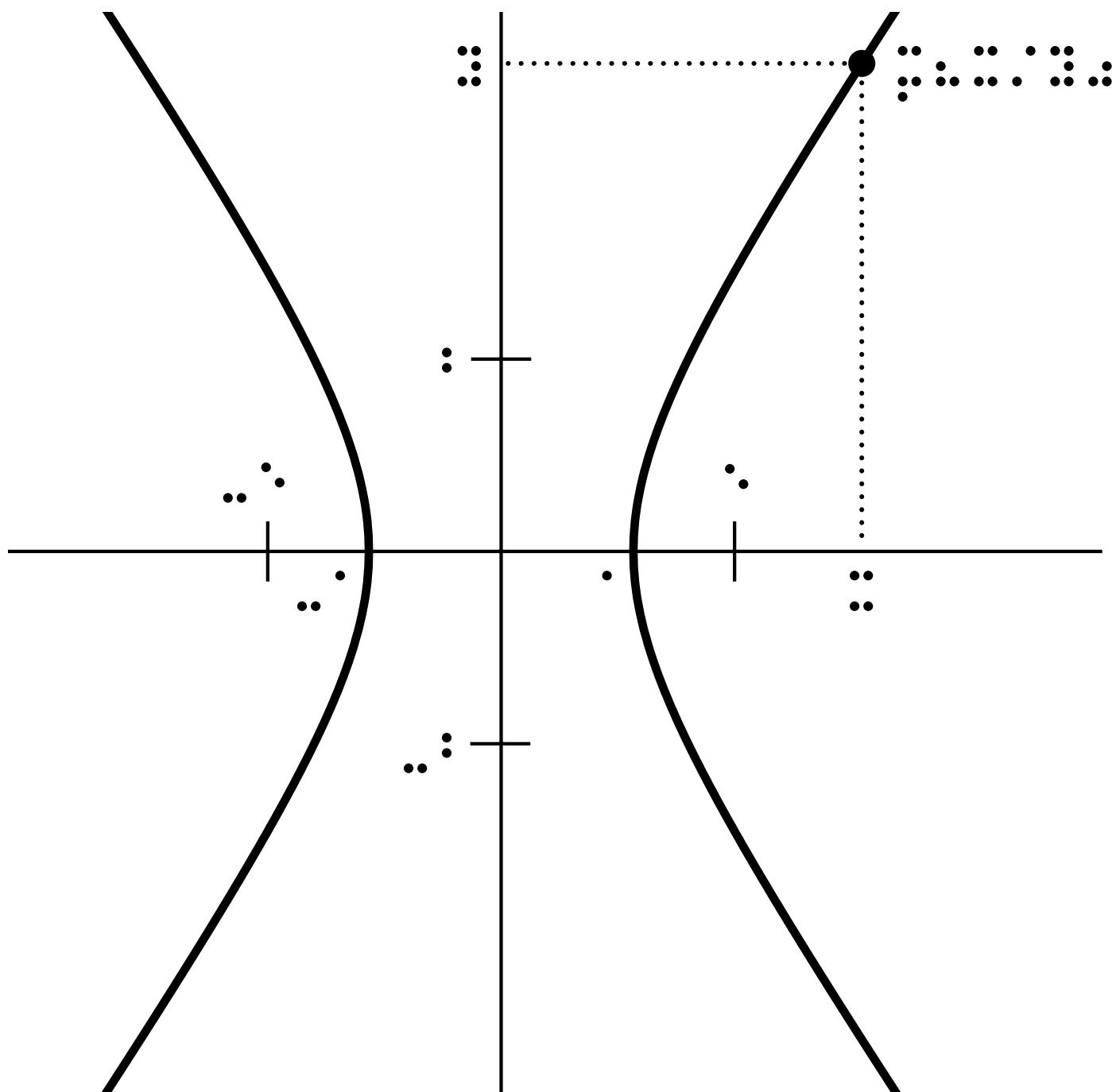


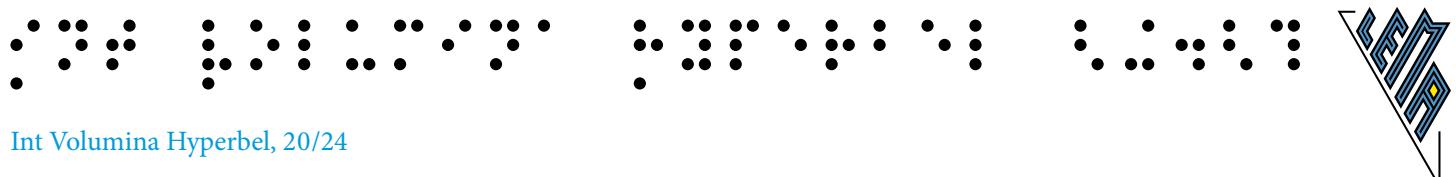
•••••

••••• ••••• ••••• ••••• ••••• •••••

••••• ••••• •••••

$$\text{Hyperbel: } b^2 \cdot x^2 - a^2 \cdot y^2 = b^2 \cdot a^2 \quad e^2 = a^2 + b^2$$

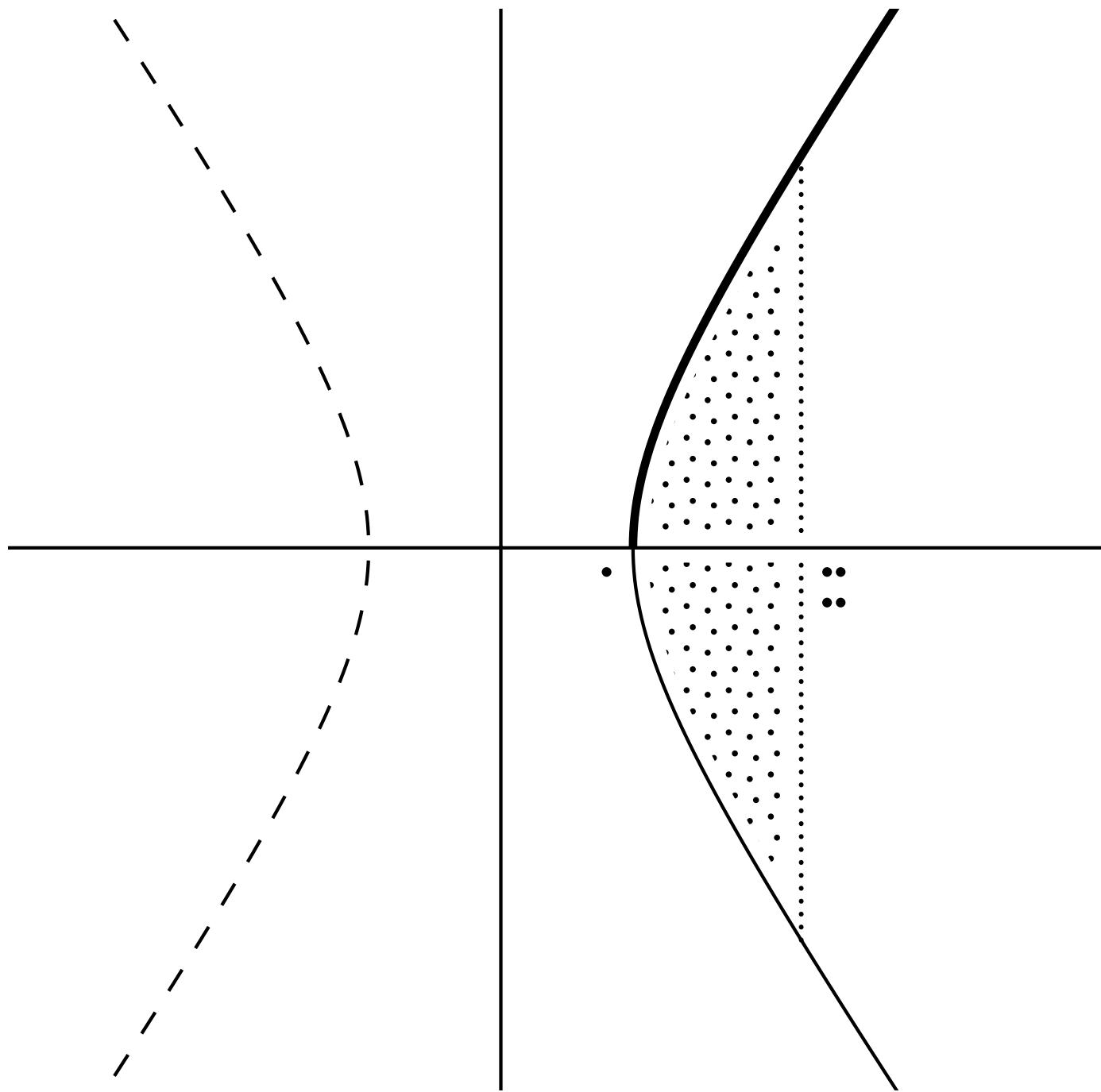


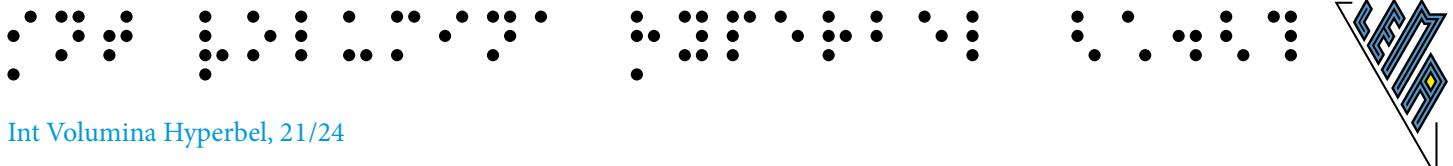


Int Volumina Hyperbel, 20/24

••••• , •••••••••• : •••••••••• . •••••

$$y^2 = b^2 + b^2/a^2 * x^2 \quad V/2 = \pi * \text{int}[a; x] (y^2 dx)$$





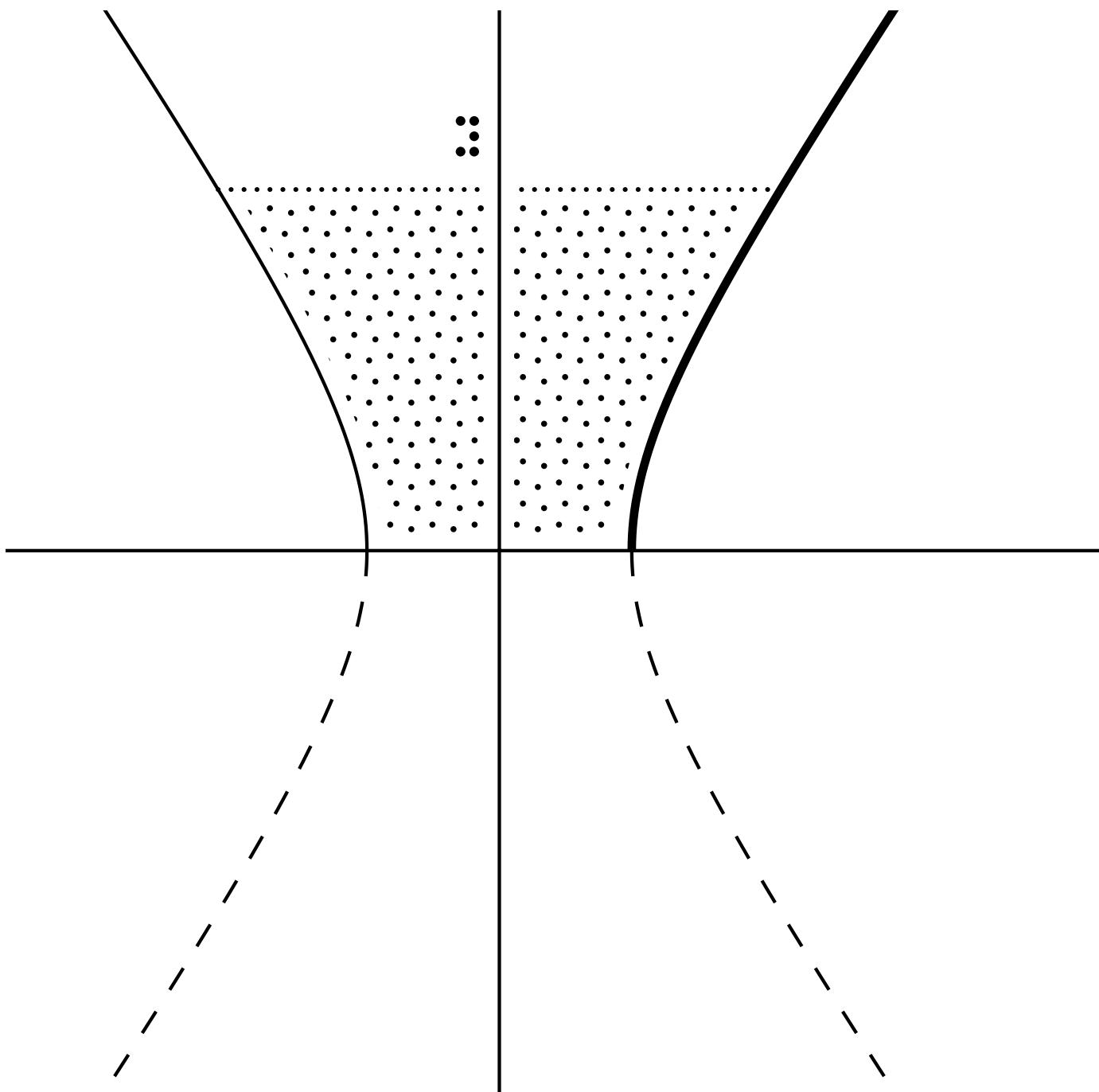
Int Volumina Hyperbel, 21/24

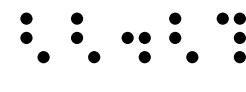
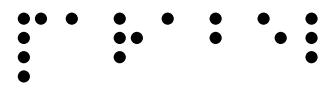
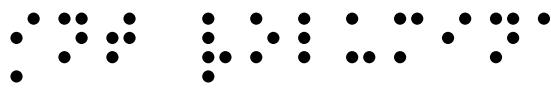
Hyperbel mit einem Schwerpunkt

Hyperbel

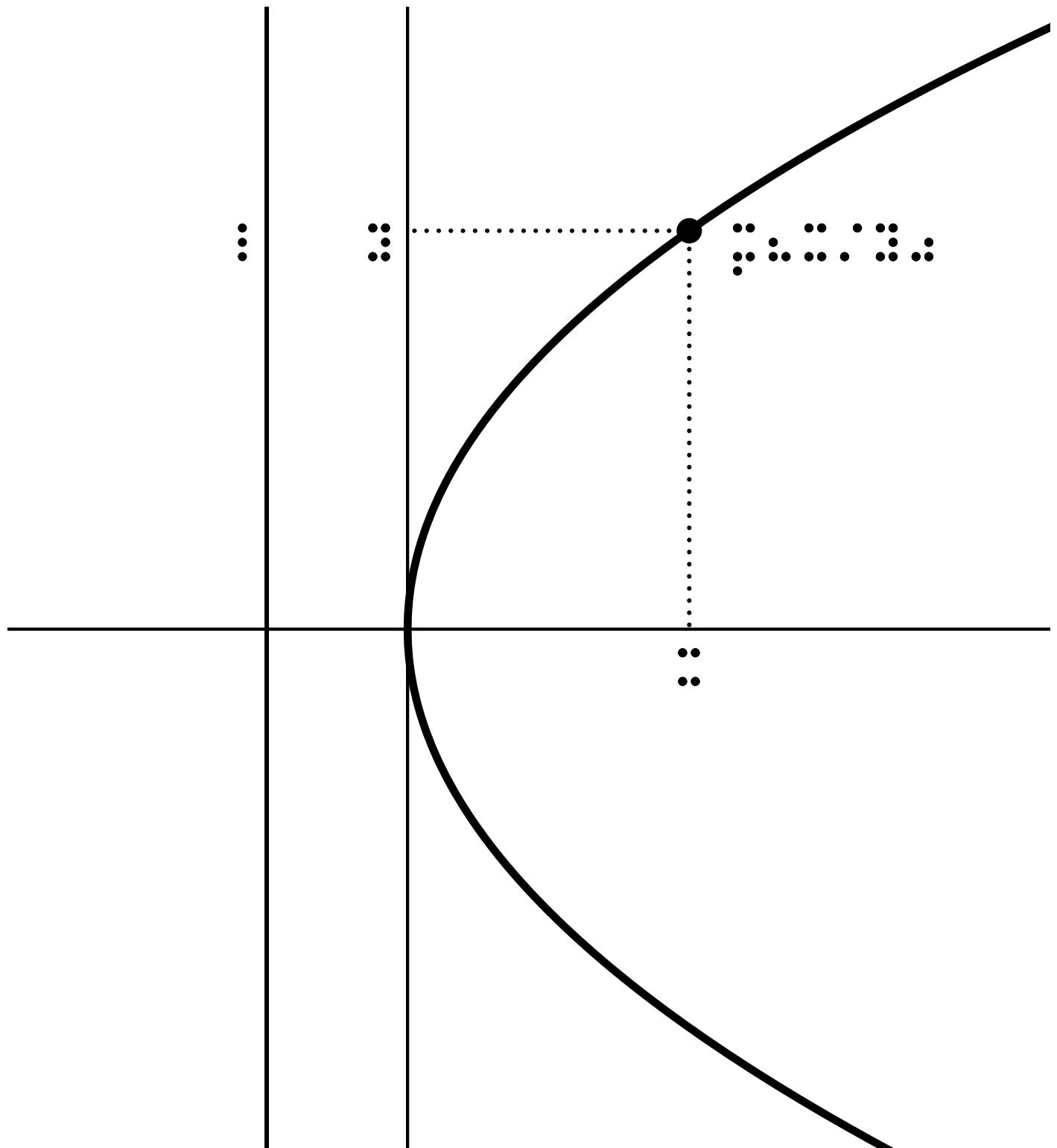
Fläche unter einer Hyperbel

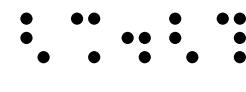
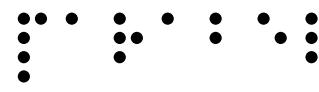
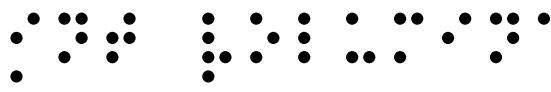
$$x^2 = a^2 + a^2 / b^2 * y^2 \quad V/2 = \pi * \text{int}[0; y] (x^2 dy)$$





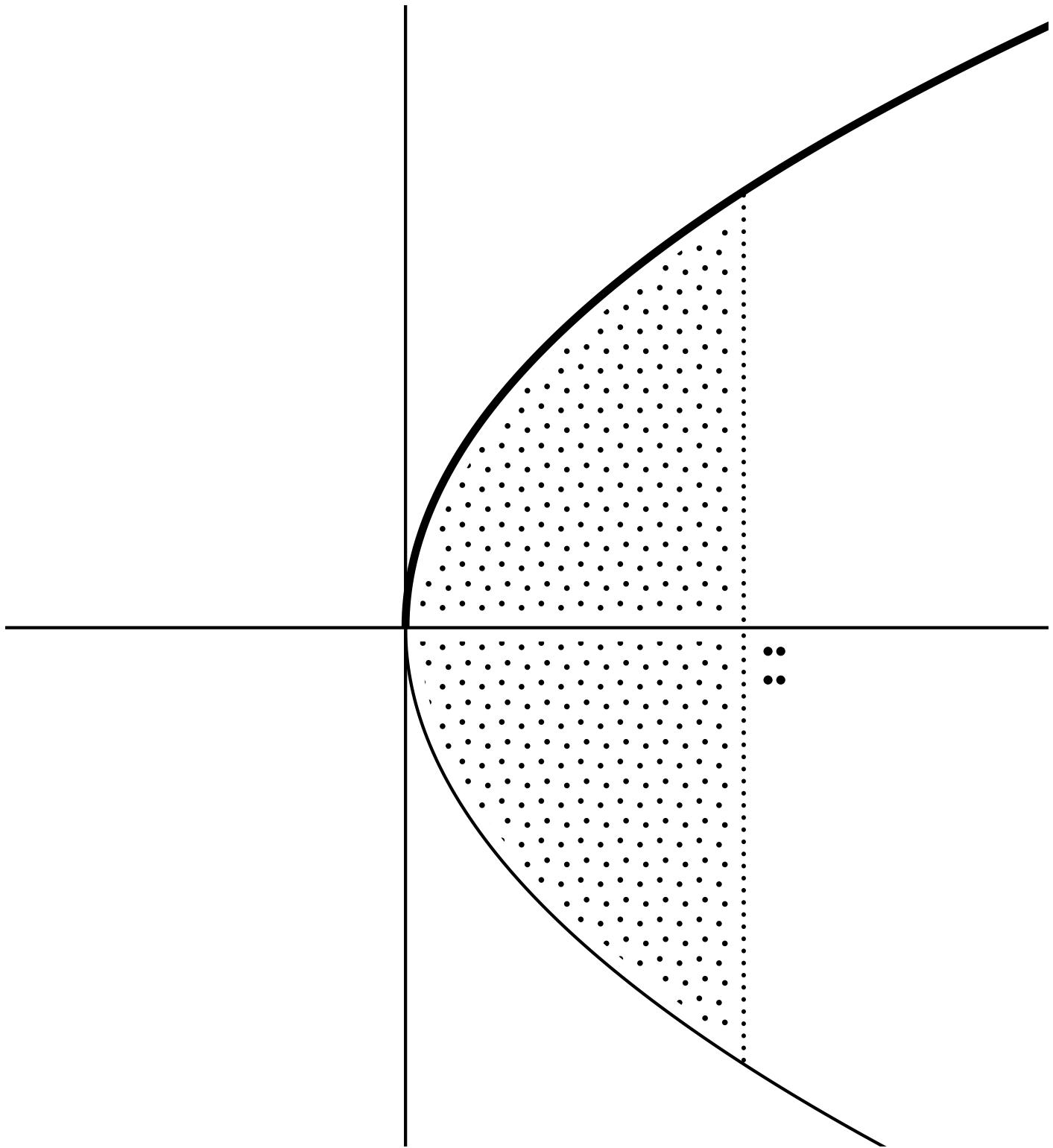
$$\text{Parabel: } y^2 = 2 \cdot p \cdot x \quad e = p/2; l = -p/2$$

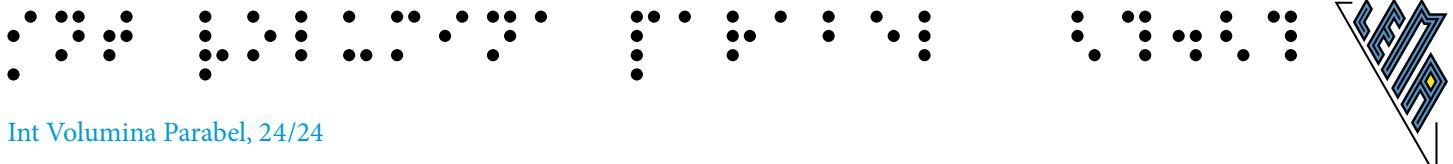




••••• •••• ••••• •••

$$y^2 = 2 * p * x \quad V = \pi * \text{int}[0; x] (y^2 dx)$$

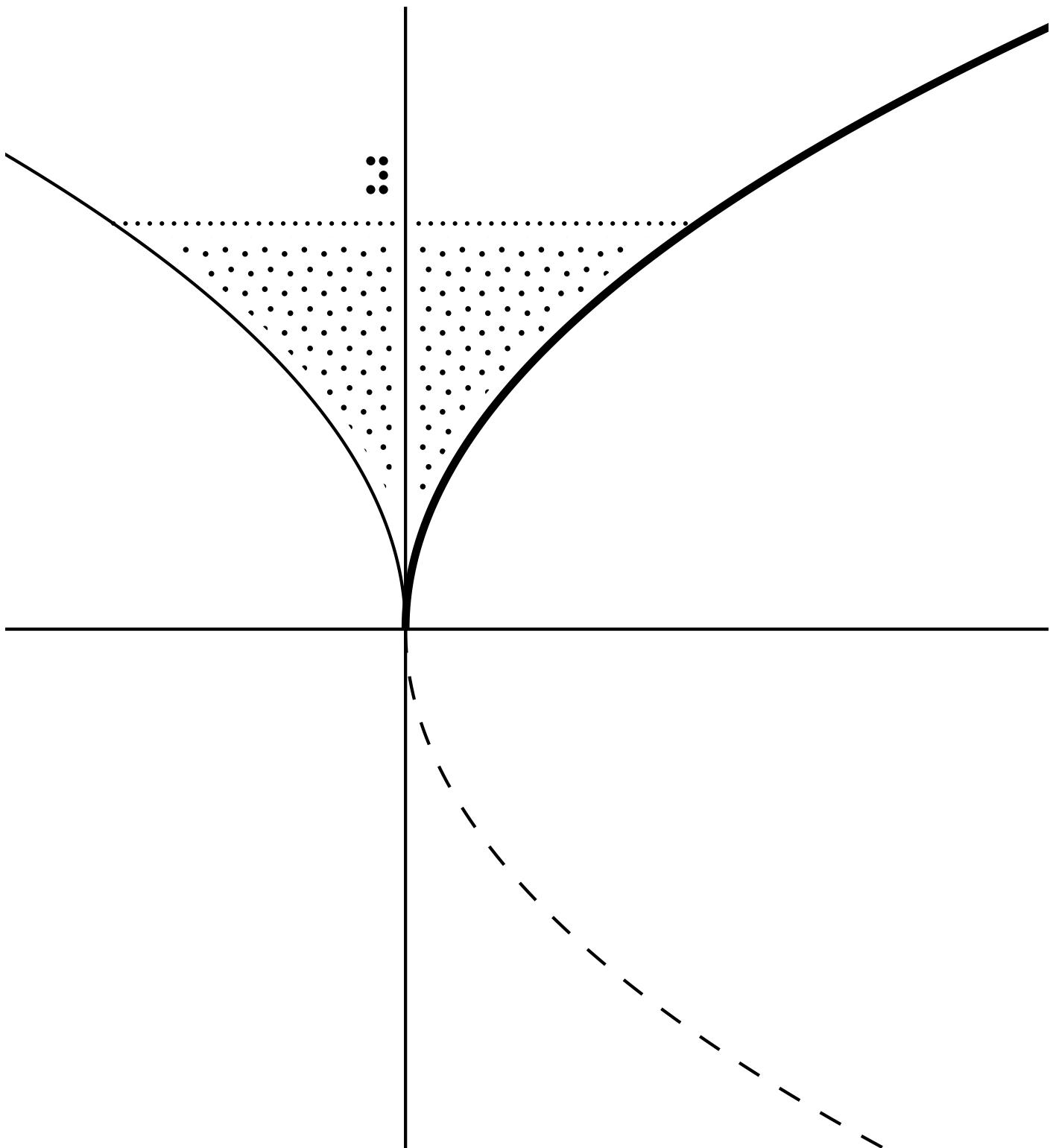




Parabel unter der x-Achse

Parabel unter der x-Achse

$$x^2 = (y^2/2p)^2 \quad V = \pi * \int [0; y] (x^2 dy)$$



# **Wirtschaft**

## **Schulstufe 12**

Kostenfunktionen

Grenzkostenfunktionen

Stückkostenfunktionen

Betriebsoptimum

Preis- und Erlösfunktionen

Gewinngrenzen

Gewinnmaximierung

Gewinnfunktionen

Cournot'sche Punkt



# Inhalt

**0** Liste der Abkürzungen

**1** Kostenfunktionen K

Variable Kosten KV

K, KV (4 Arten: lin,  
prog, degr, s-förm)

**2** Grenzkostenfunktionen K'

K und K' (4 Arten)

**3** Stückkostenfunktionen K/x

K und K/x (4 Arten)

**4** Betriebsoptimum  $x_{\text{opt}}$

$K'(x) = K(x)/x$

(K<sub>s</sub>, K<sub>s'</sub>, K<sub>s/x</sub>)

**5** Preisfunktionen (p, p(x))

(p\_konst, p\_lin.fall)

Erlösfunktionen (E, Eq)

(E\_lin, E\_quadr)

**6 - 9 Gewinngrenzen K = E**

**6** Gewinngrenzen

(K\_lin, K\_prog, E\_lin)

**7** Gewinngrenzen  
(K\_deg, K\_s-förm, E\_lin)

**8** Gewinngrenzen  
(K\_lin, K\_prog, E\_quad)

**9** Gewinngrenzen  
(K\_deg, K\_s-förm, E\_quad)

## **10 - 11 Gewinnmaximierung**

**10** Gewinnmaximierung  $x_m$   
(K\_prog, E\_lin)

**11** Gewinnmaximierung  $x_m$   
(K\_s-förm, E\_lin)

## **12 - 15 Gewinnfunktionen**

**12** Gewinnfunktionen  $G$   
 $G = K - E$   
(K\_lin, K\_prog, E\_lin)

**13** Gewinnfunktionen  $G$   
(K\_deg, K\_s-förm, E\_lin)

**14** Gewinnfunktionen  
(K\_lin, K\_prog, E\_quad)

**15** Gewinnfunktionen  
(K\_deg, K\_s-förm, E\_quad)

**16** Cournot'sche Punkt C  
C ( $x_m/p(x_m)$ )  
( $p(x)$ , G,  $x_C = x_m$ )







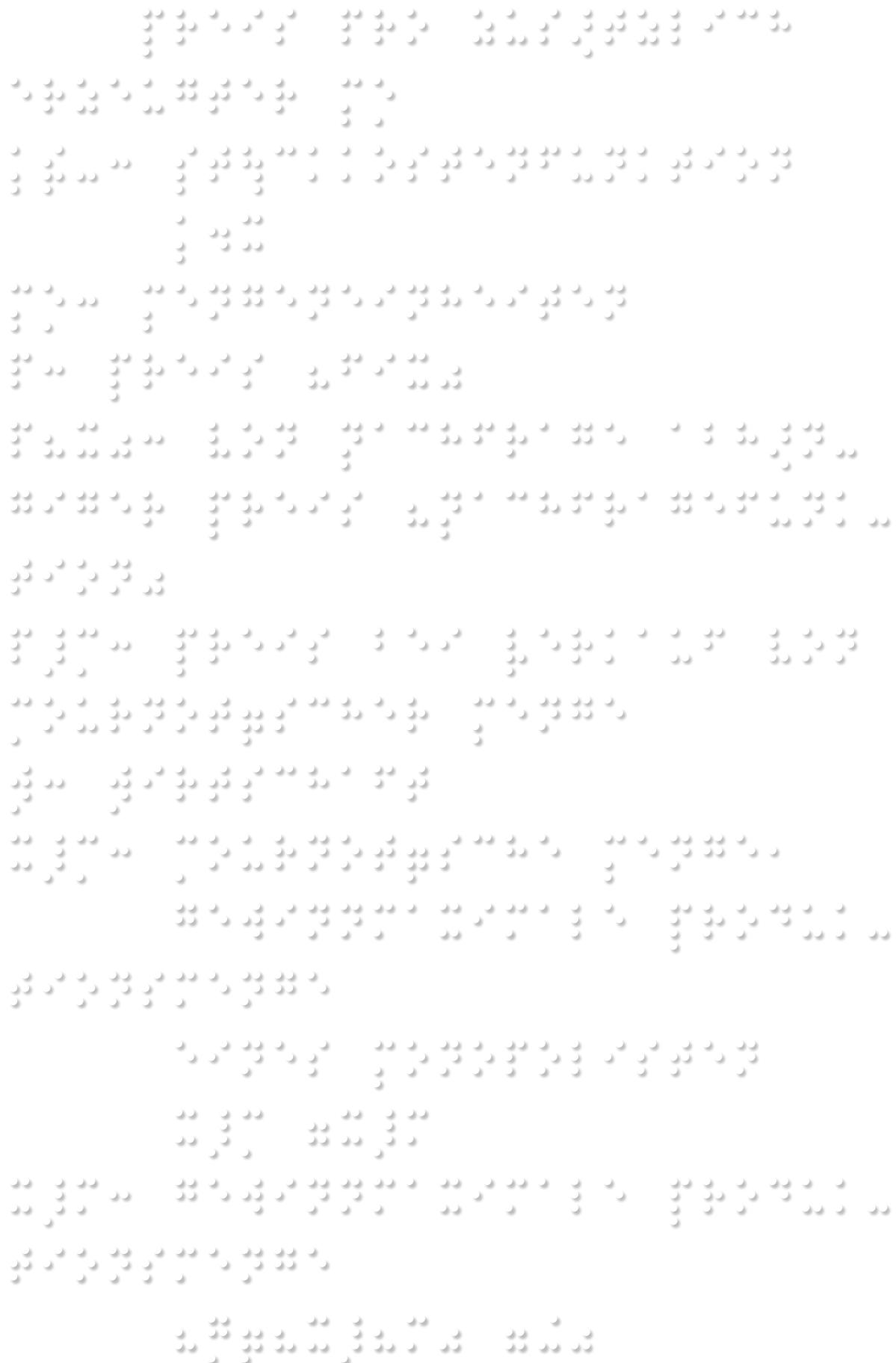
# Wirtschaft - Abkürzungen

## Index

- C:** Cournot'sche Punkt (liegt auf der Preisfunktion mit dem x-Wert:  $x_m$ )
- E:** Erlösfunktion ( $p * x$ )
- Eq:** E\_quad, quadrat. E
- F:** Fixkosten
- G:** Gewinnfunktion (Erlös minus Kosten)
- GE:** Geldeinheiten
- H:** Höchstpreis
- K:** Kostenfunktion:
- Kl:** K\_lin, lineare K,
- Kp:** K\_prog, progressive K
- Kd:** K\_deg, degressive K
- Ks:** K\_s-förm, S-förmige K
- K':** Grenzkostenfunktion  
Preis pro zusätzlich erzeugter ME
- K^-:** Stückkostenfunktion  
 $K/x$
- ME:** Mengeneinheiten
- p:** Preis (fix)
- p(x):** von Nachfrage abhängiger Preis (Nachfragefunktion)

- p\_C:** Preis bei Verkauf von Cournot'scher Menge
- w:** Wirtschaft
- x\_C:** Cournot'sche Menge,  
gewinnmaximale Produktionsmenge  
eines Monopolisten  
 $x_C = x_m$
- x\_m:** gewinnmaximale Produktionsmenge  
( $G'(x_m) = 0$ )
- x\_o:** obere Gewinngrenze  
(2. Break-Even-Point)
- x\_s:** Sättigungsmenge
- x\_u:** untere Gewinngrenze  
(1. Break-Even-Point)
- x\_W** Kostenkehre  
Wendestelle von K
- x\_(opt):** Produktionsmenge  
mit minimalen Stückkosten  
 $K'(x) = K^-$







## Ergänzungen zu Seite 1

**Kl:** linearer Kostenverlauf

**F:** Fixkosten

**KVL:** linearer Kostenverlauf ohne Fixkosten

**Kp:** progressiver Kostenverlauf

linksgekrümmt (pos. gekrümmmt), strg mon st,  
überproportional

$$K' > 0; K'' > 0$$

**Kd:** degressiver Kostenverlauf

rechtsgekrümmt (neg. gekrümmmt), strg mon st,  
unterproportional

$$K' > 0; K'' < 0$$

**Ks:** S-förmiger Kostenverlauf

erst rechtsgekrümmt (neg. gekrümmmt),  
dann linksgekrümmt (pos. gekrümmmt)  
 $x_W$  = Kostenkehre (Wendestelle)

$$K''(x_W) = 0$$

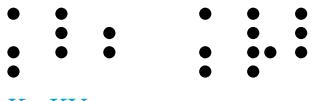
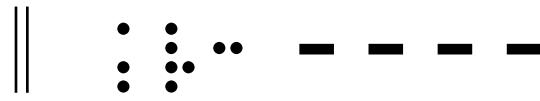
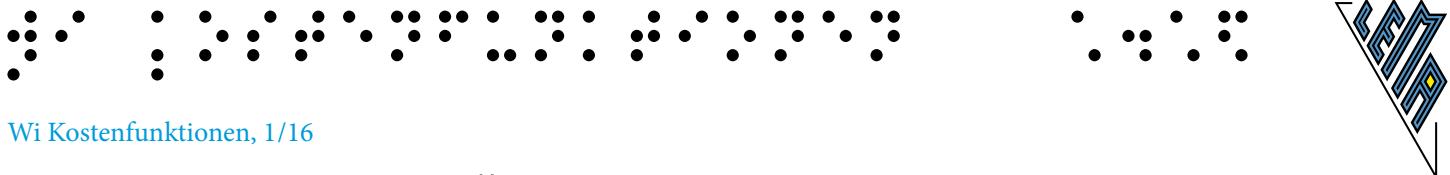
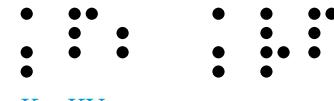
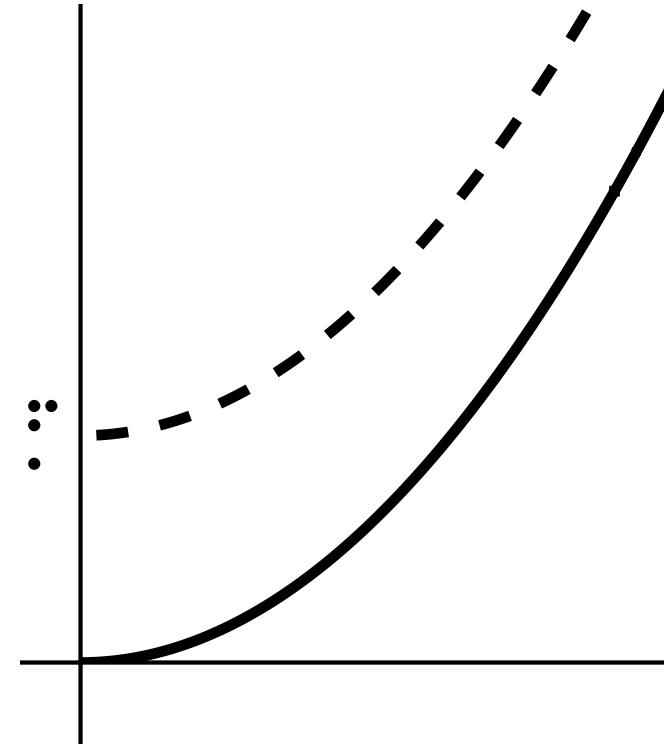
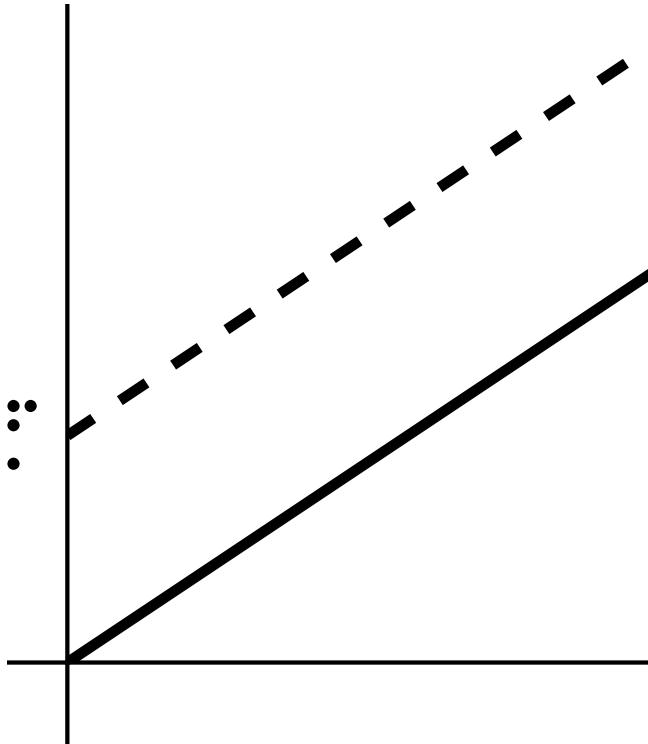
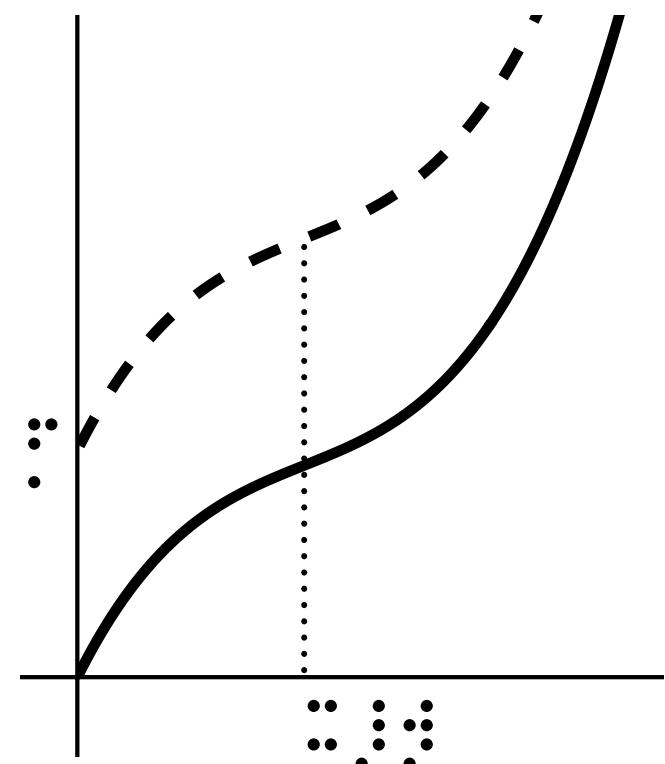
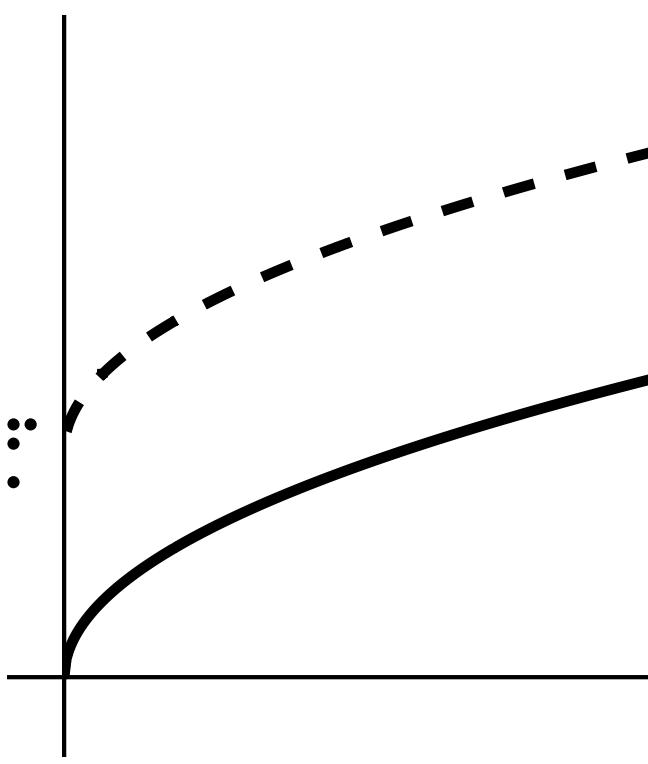


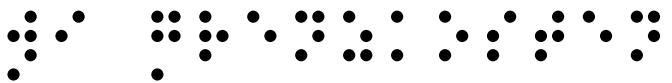
A decorative border consisting of a repeating pattern of small white dots arranged in a diamond grid. The border is set against a light blue background and runs horizontally across the page.

A decorative border consisting of a repeating pattern of small circles arranged in a grid-like fashion. The circles are white with black outlines, set against a light gray background. The pattern is continuous and covers the entire border area.

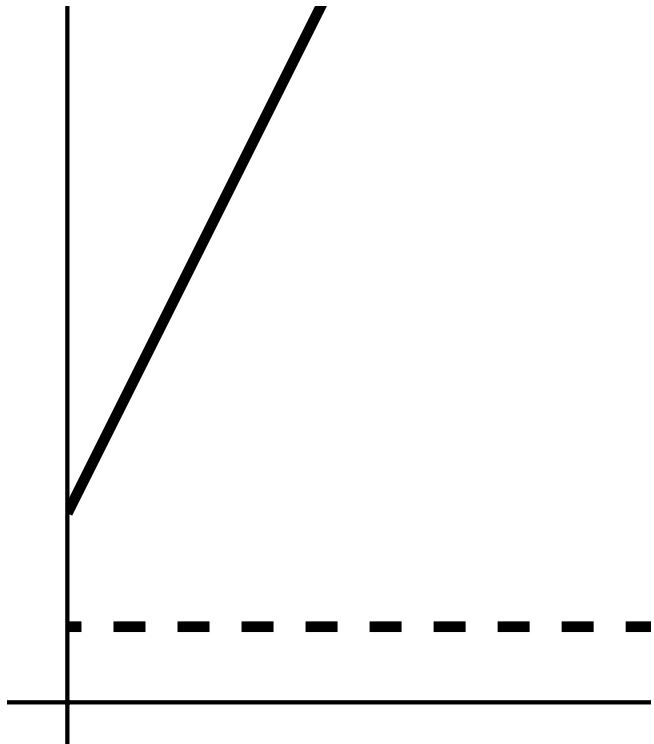
This image shows a decorative border consisting of a repeating pattern of small circles and dots arranged in a grid-like fashion. The pattern is composed of two types of symbols: a solid circle and a smaller dot. They are arranged in rows and columns to create a textured, woven appearance. The border is symmetrical and covers the entire frame.



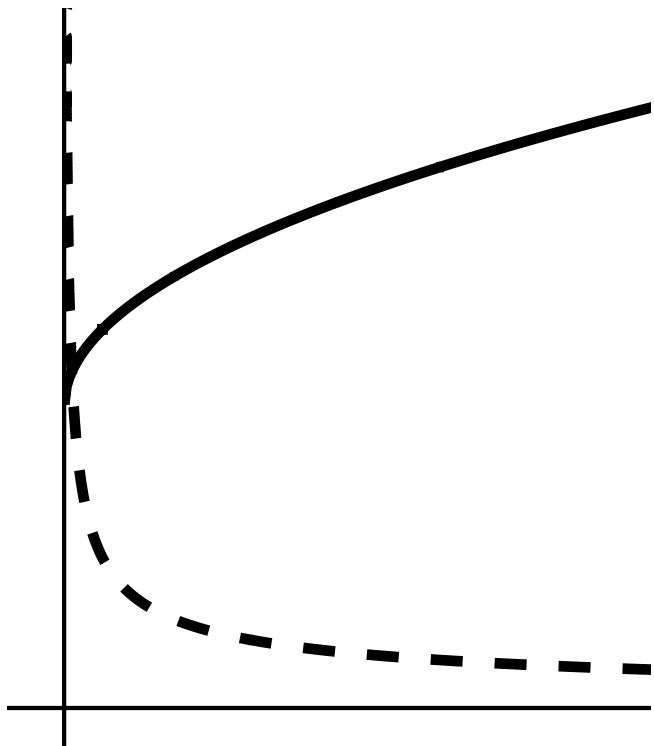
 $K_I; KV_I$  $K_p; KV_p$  $K_d; KV_d$  $K_s; KV_s$ 



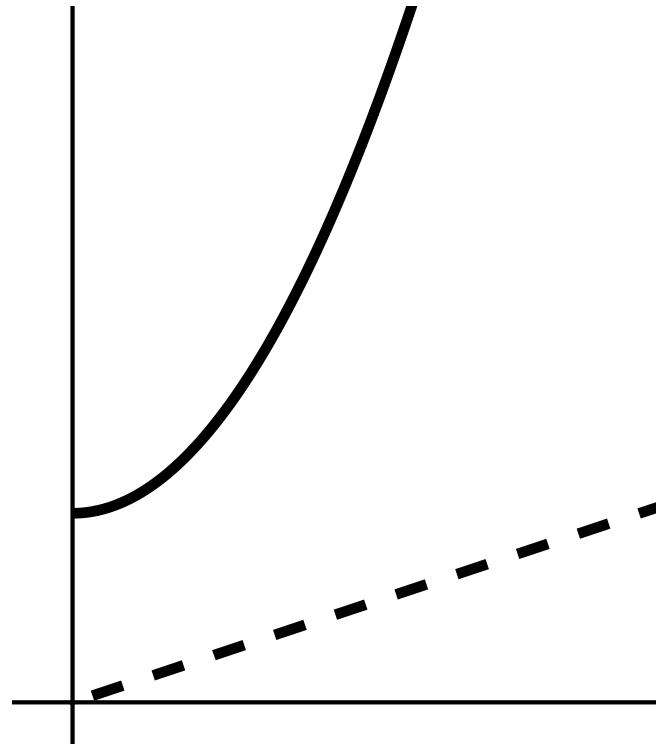
$K_I; K_I'$



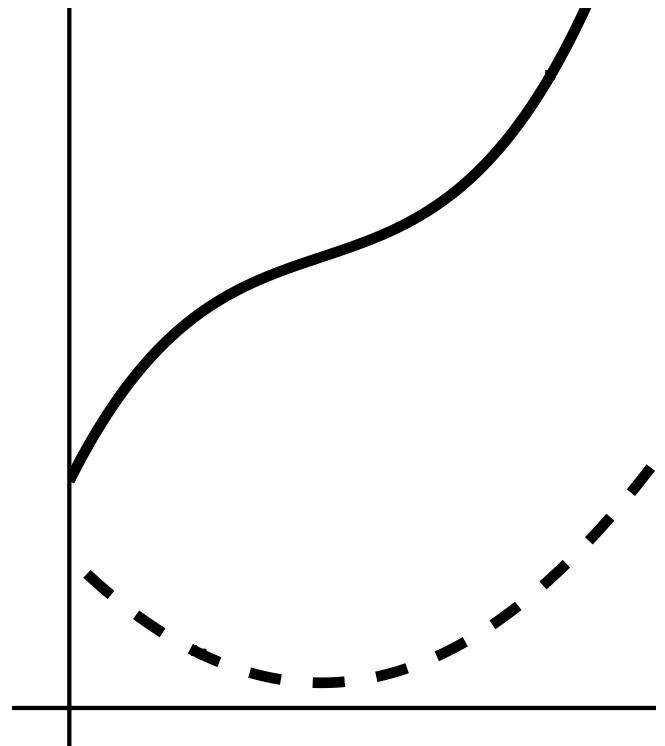
$K_d; K_d'$



$K_p; K_p'$



$K_s; K_s'$



Wi Stückkosten, 3/16

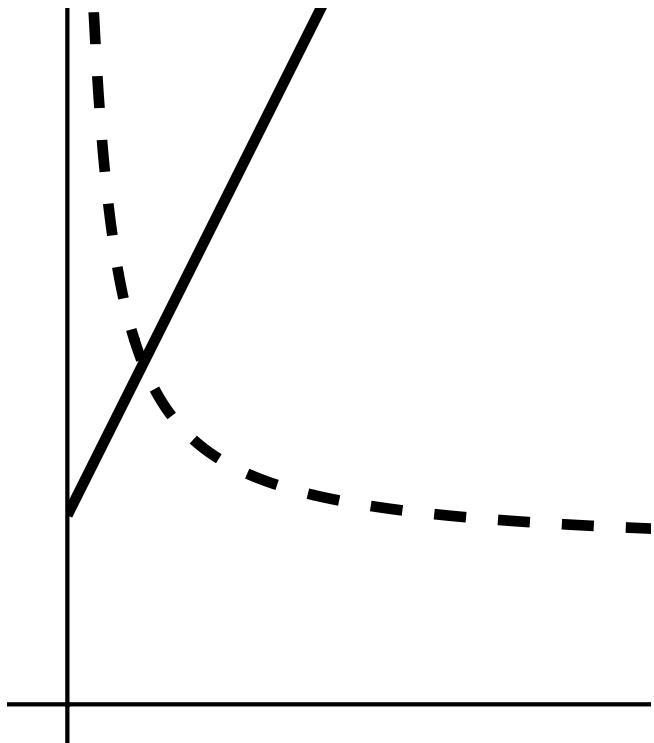


Wi Stückkosten, 3/16

||

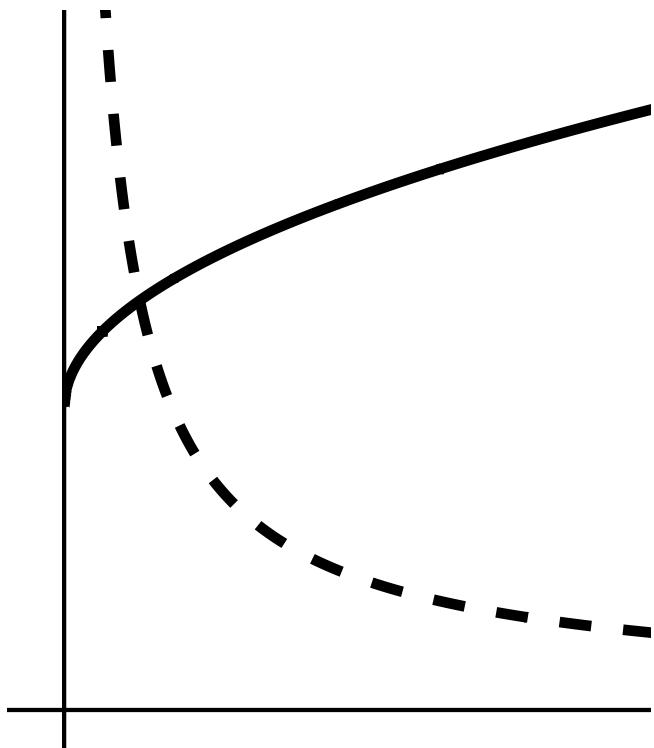
||

$K_I; K_I/x$



||

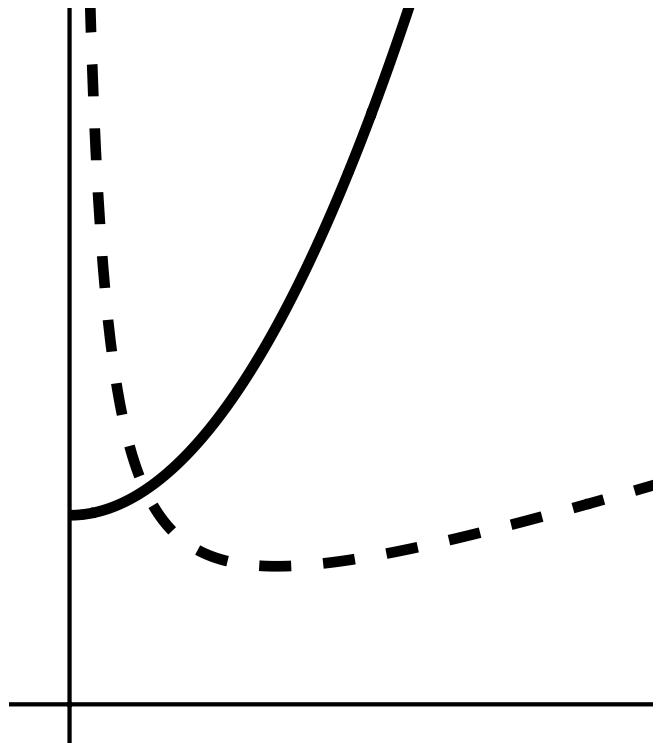
$K_p; K_p/x$



||

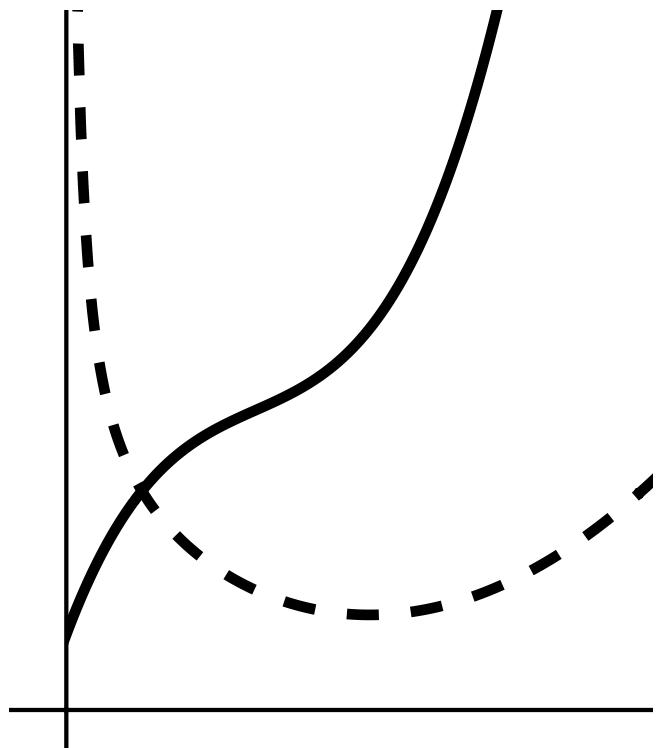
||

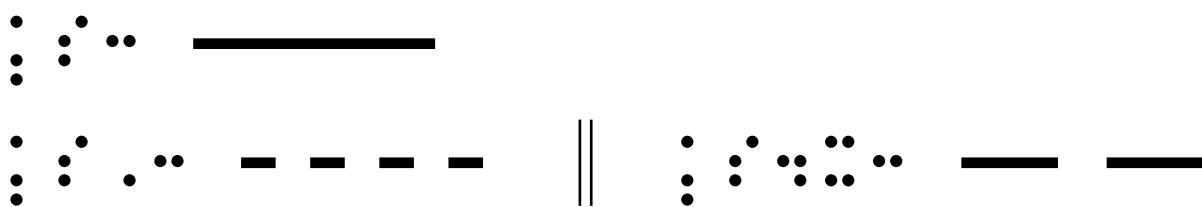
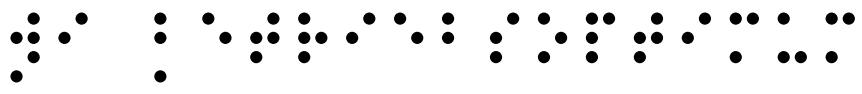
$K_p; K_p/x$



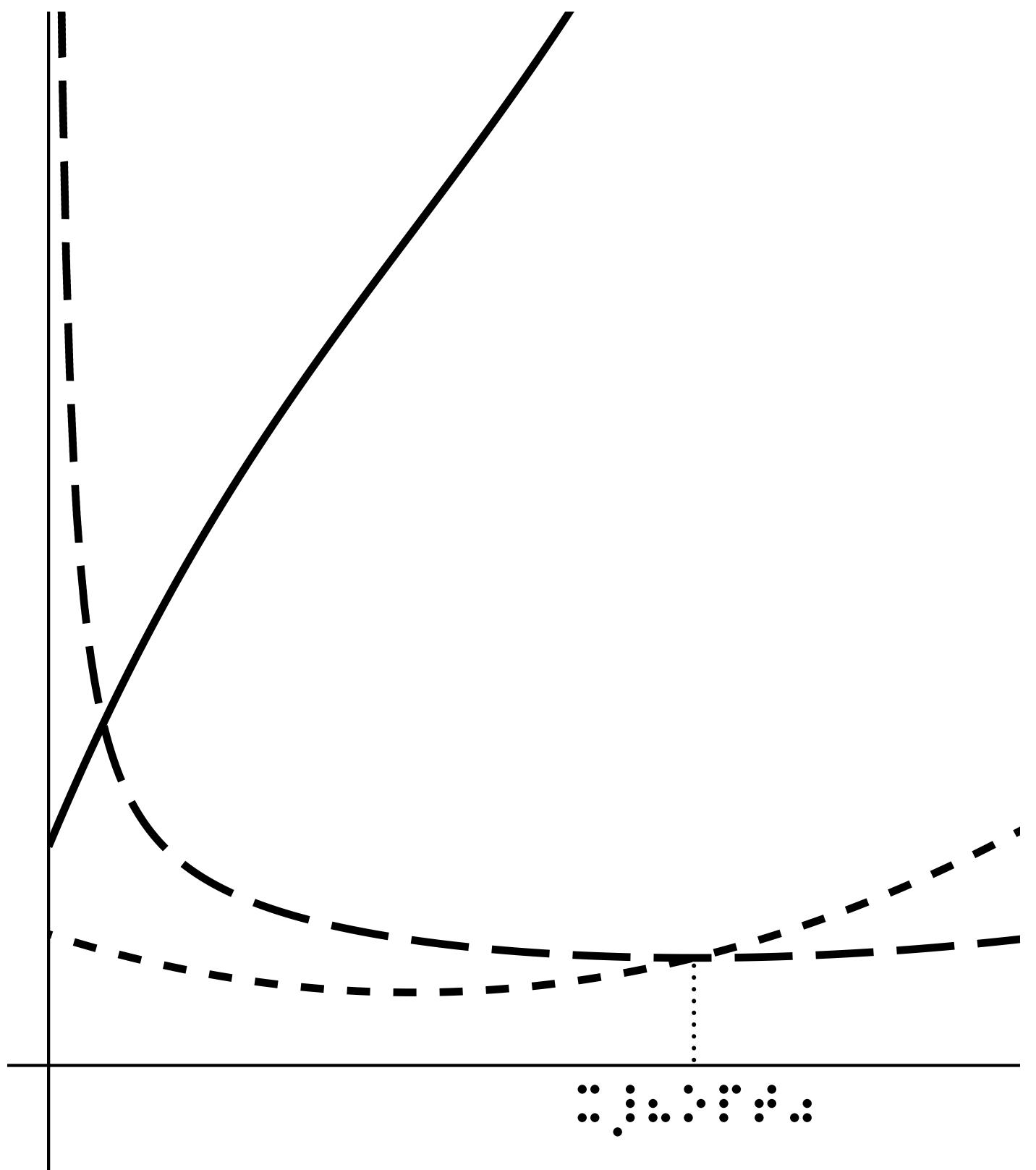
||

$K_s; K_s/x$





$K_s; K_s'; K_s/x$



## Ergänzungen zu Seite 5

Fixpreis:  $p$

Erlös linear steigend

Nachfragefunktion:  $p(x)$

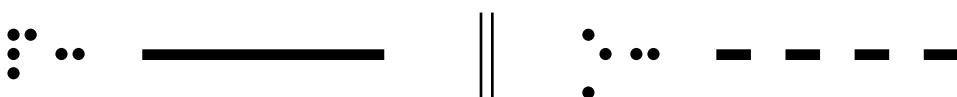
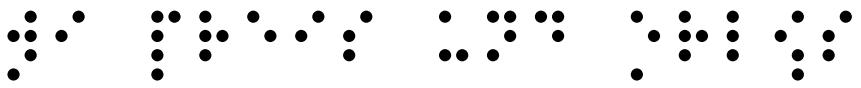
$H(0|p(0))$

Höchstpreis  $H$  = Schnittpunkt mit der y-Achse

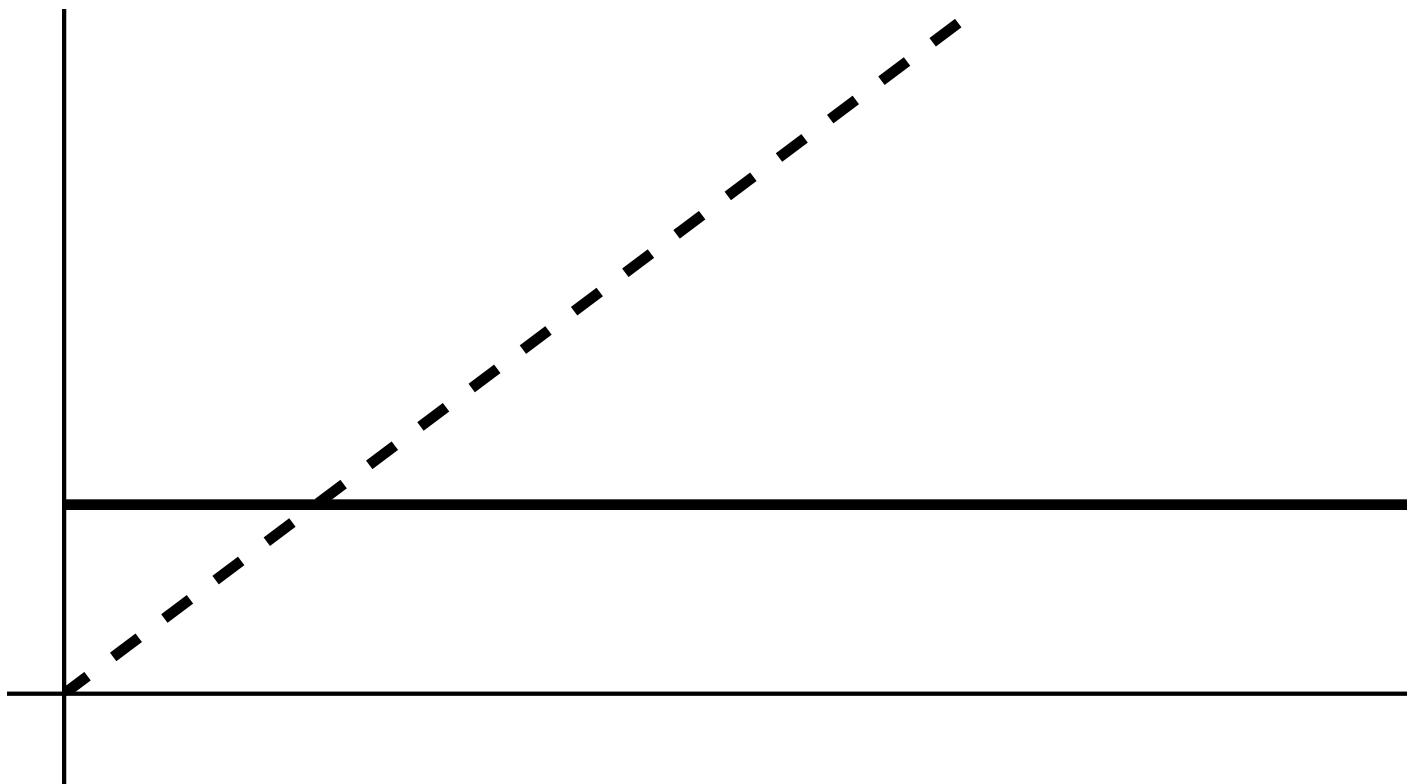
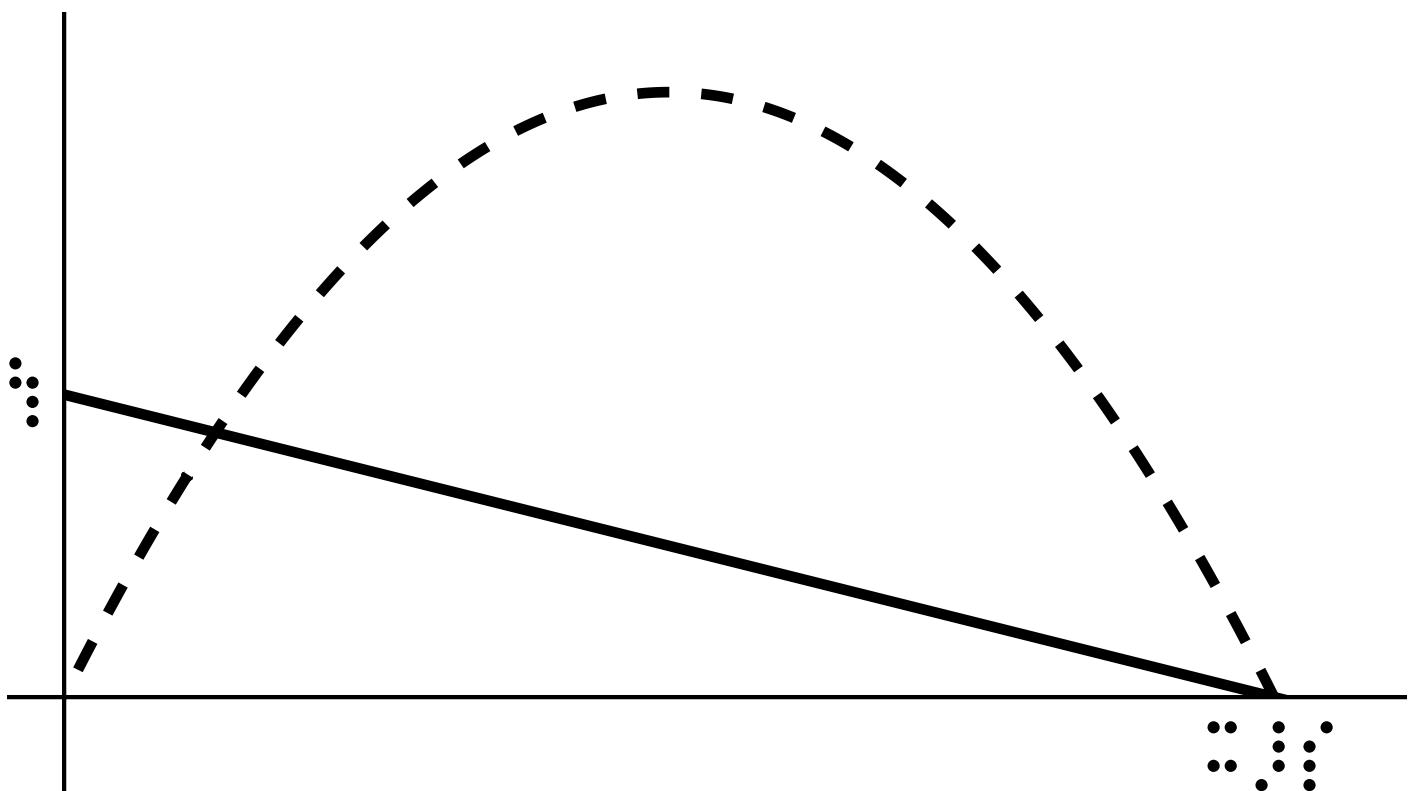
$S(x_S|0)$

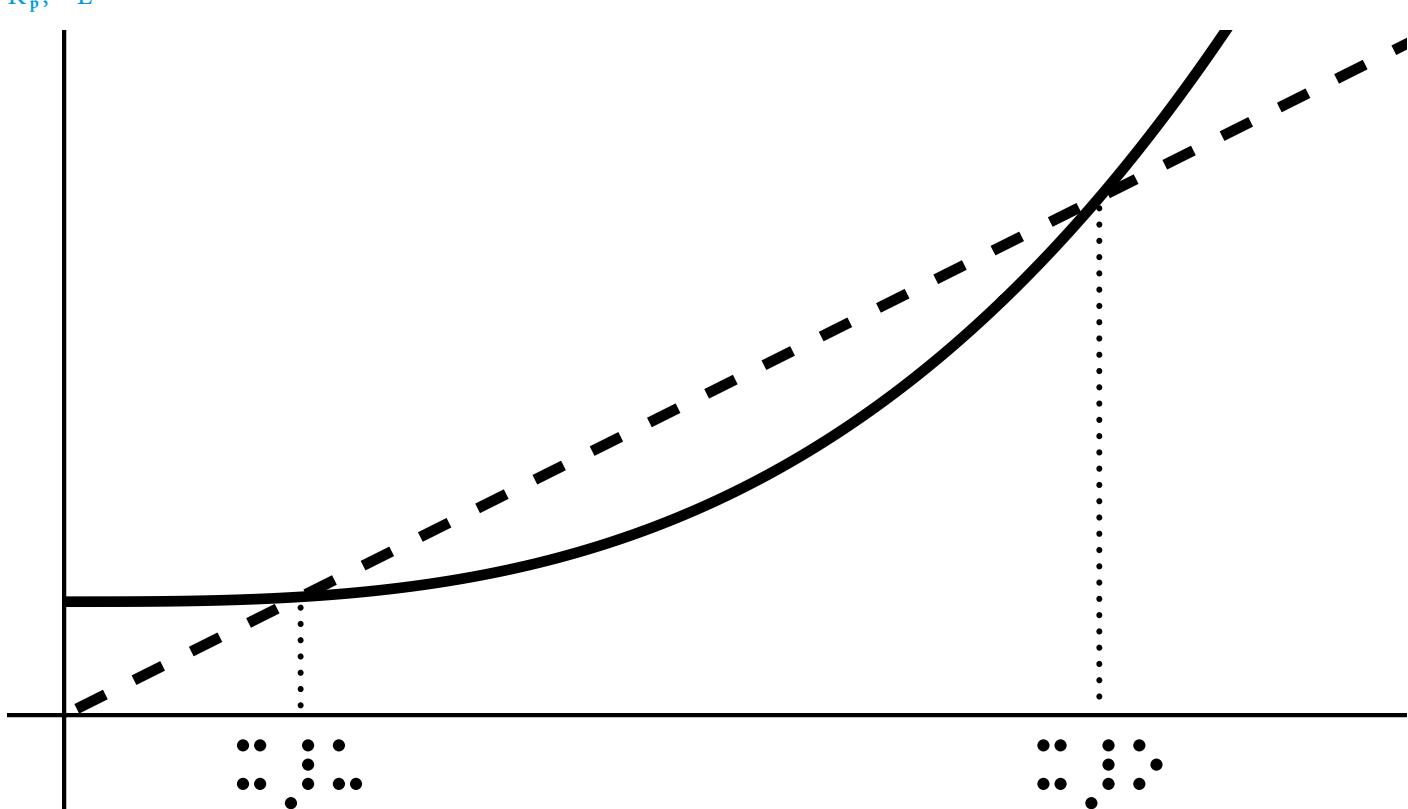
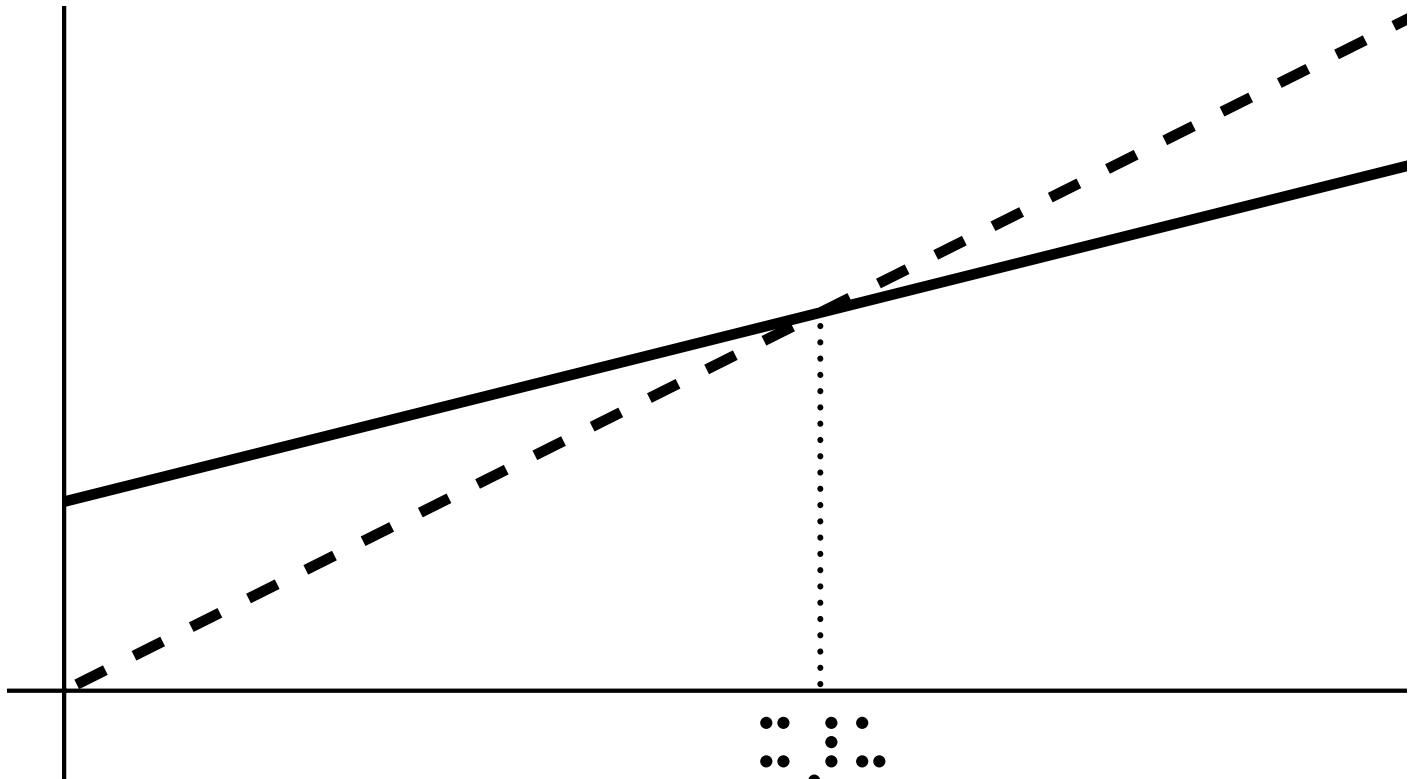
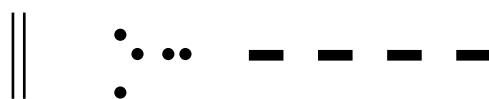
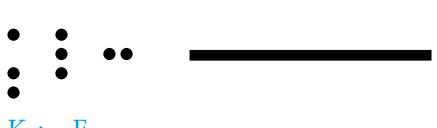
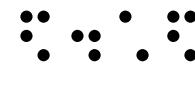
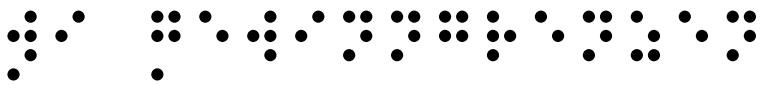
Sättigungsmenge  $x_S$  = Schnittstelle mit der x-Achse

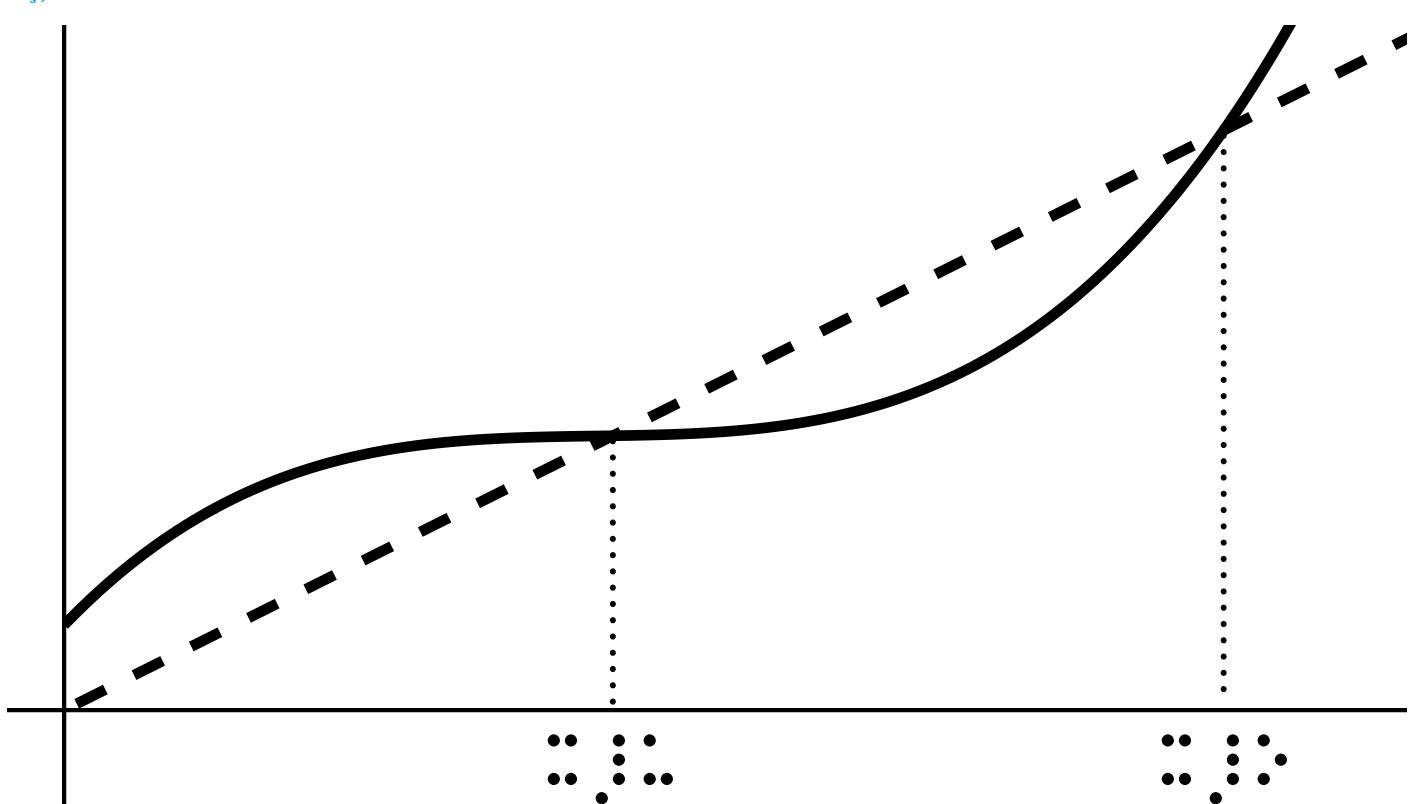
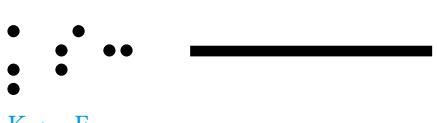
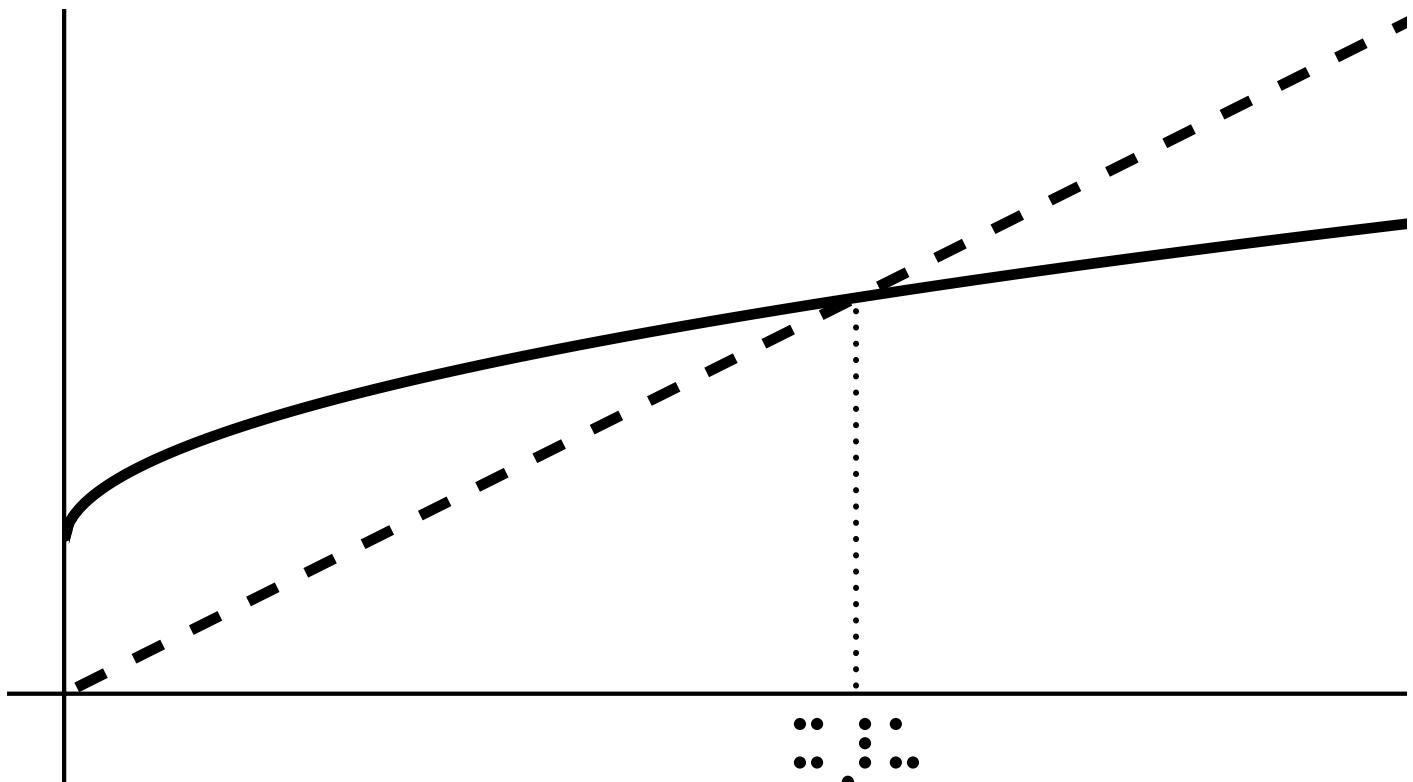
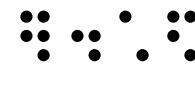
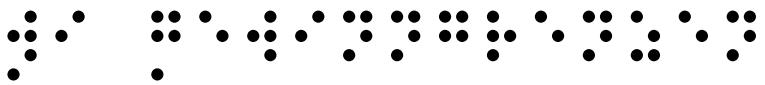


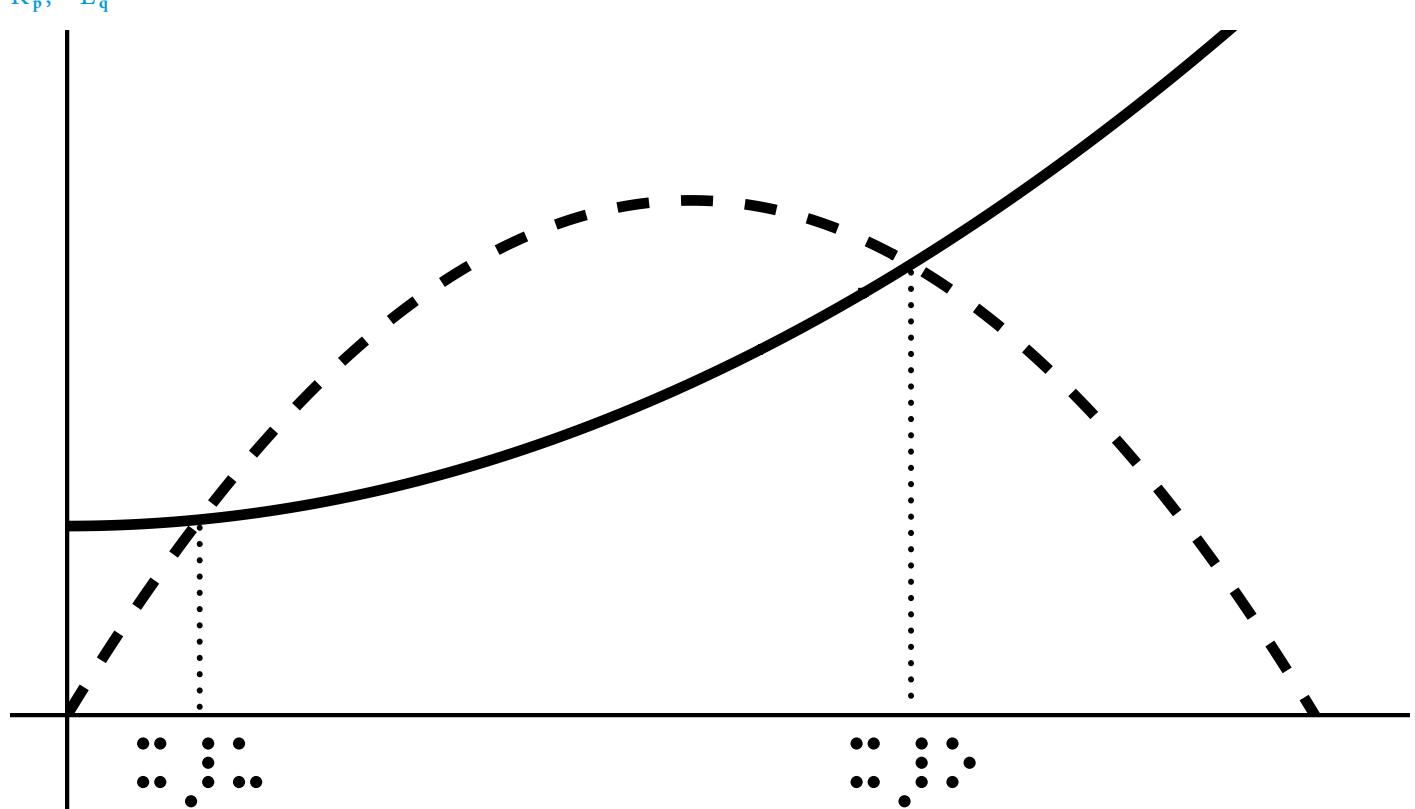
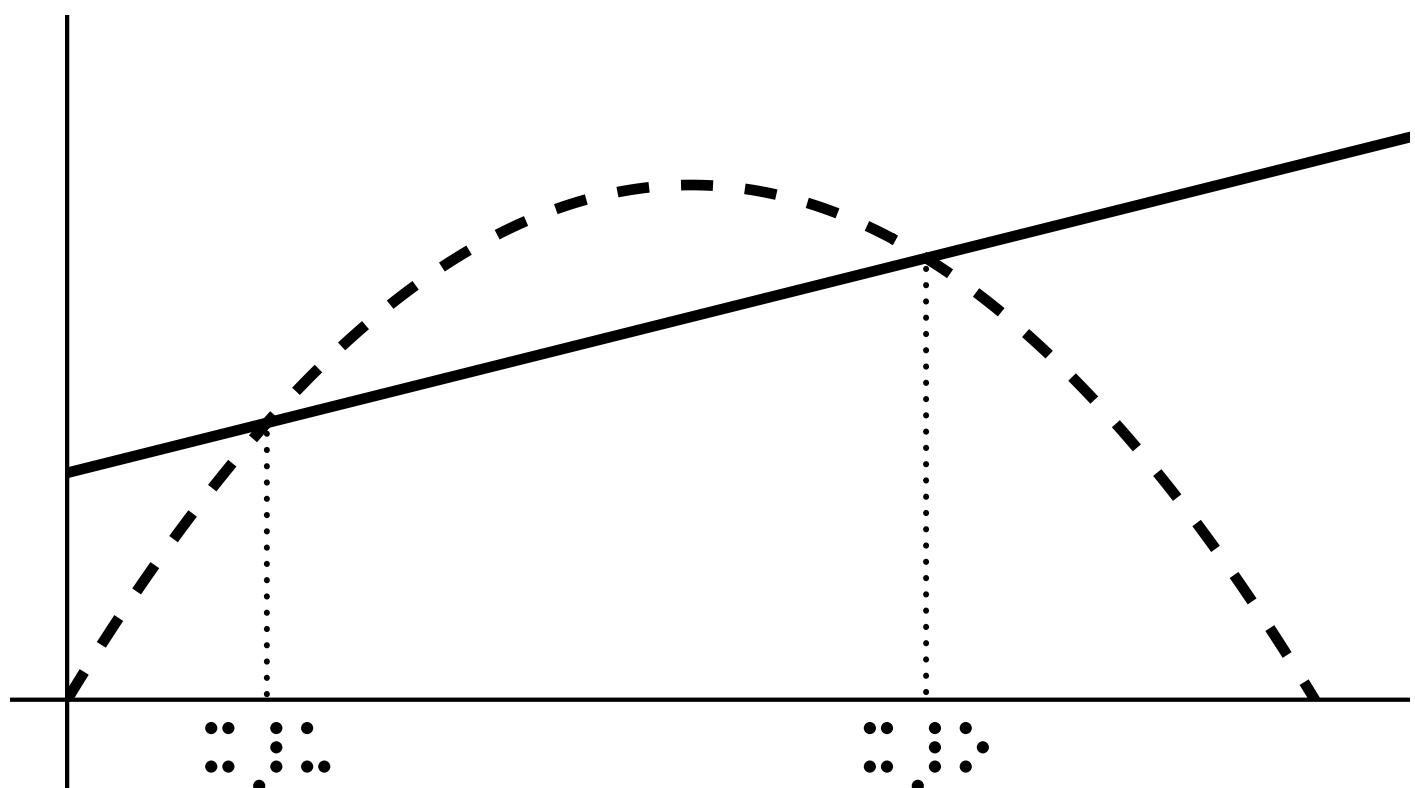
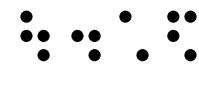
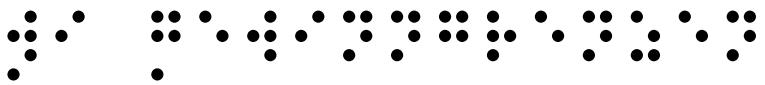


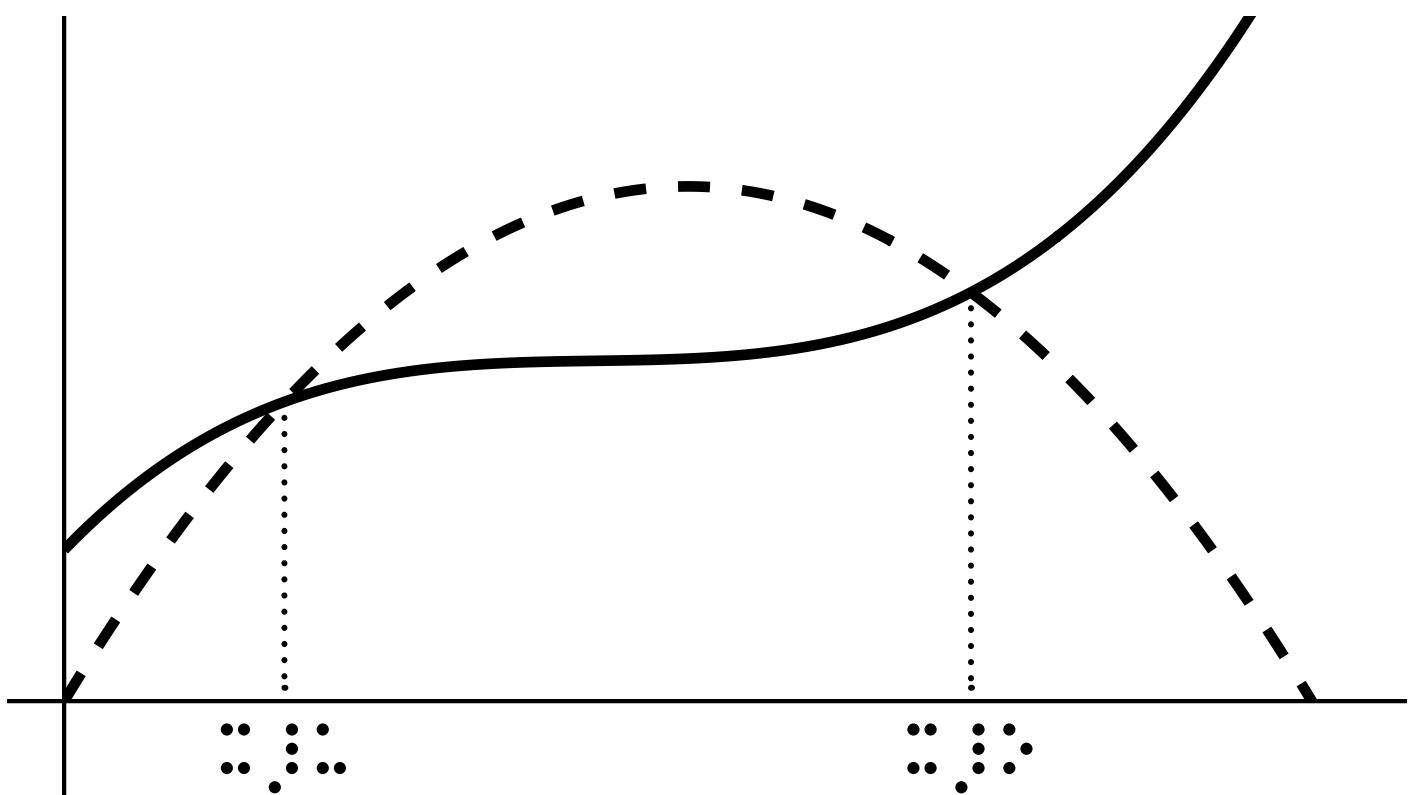
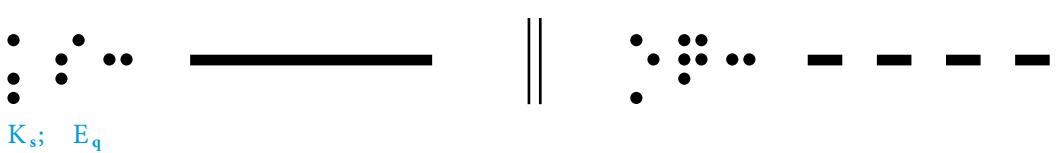
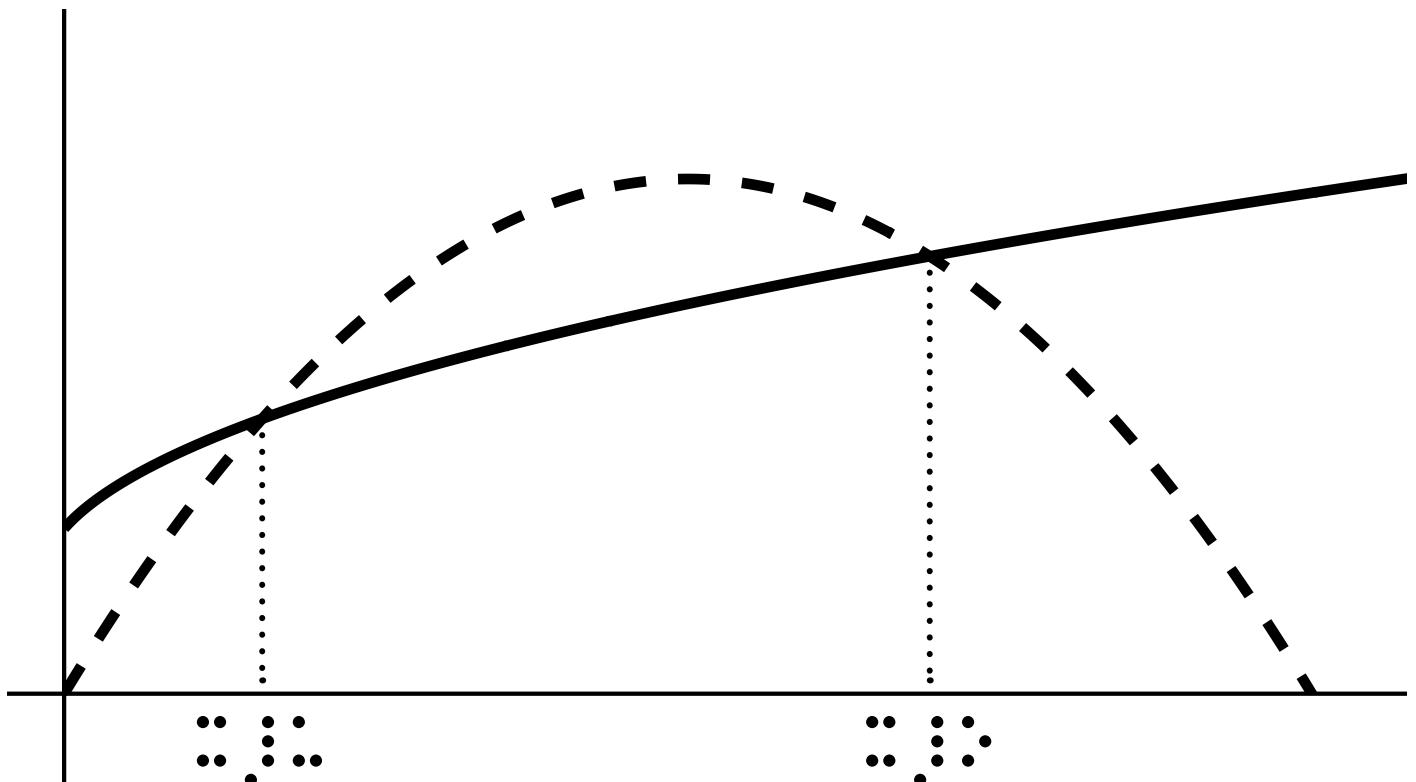
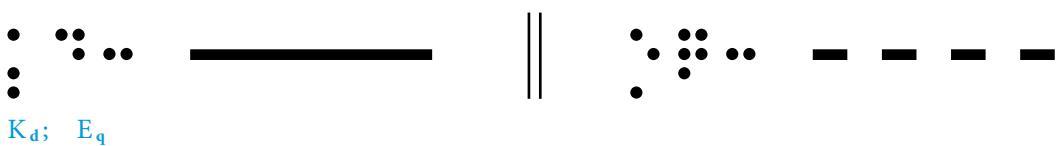
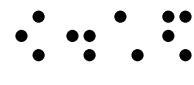
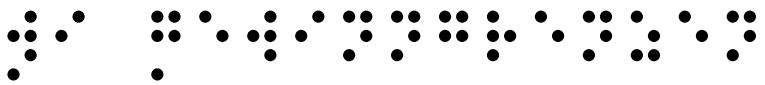
p; E

p(x); E<sub>q</sub>





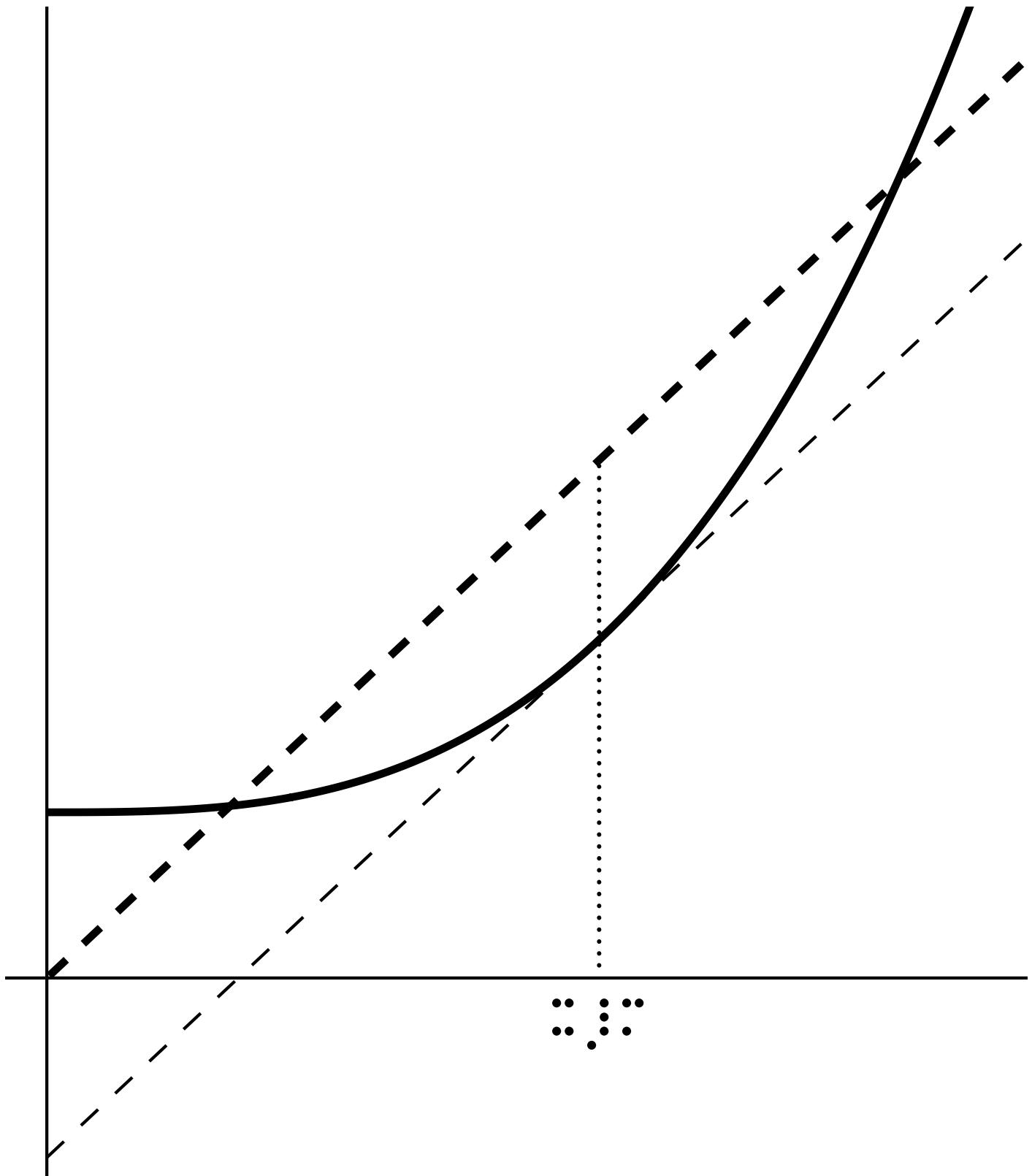




• • • • • • • • • • • • • • • • • • •

• • • • • • • • • • • • • • • • • • •

$K_p$ ;  $E = p^* x$ ;  $E_{\parallel}$



;

;

;

;

;

;

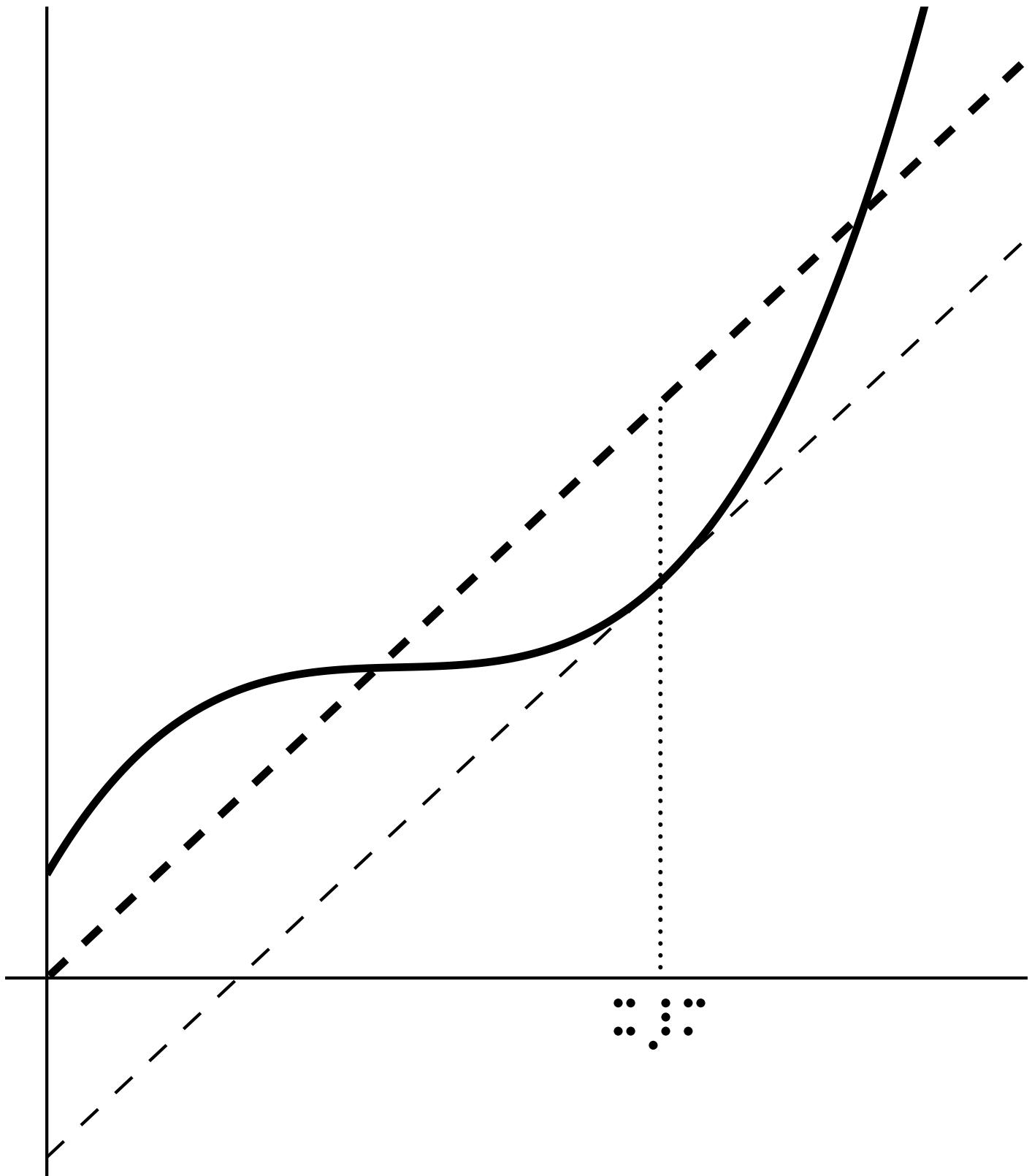
;

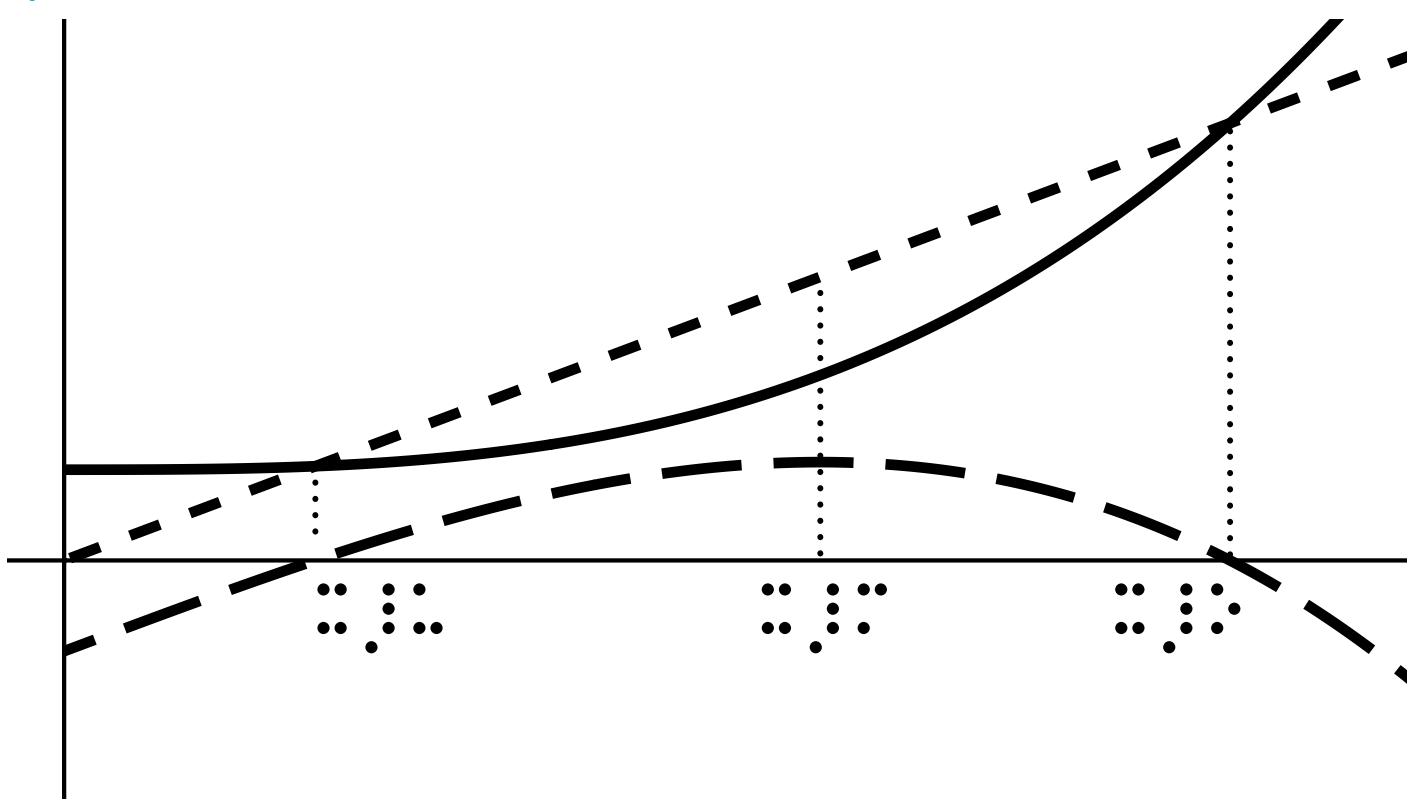
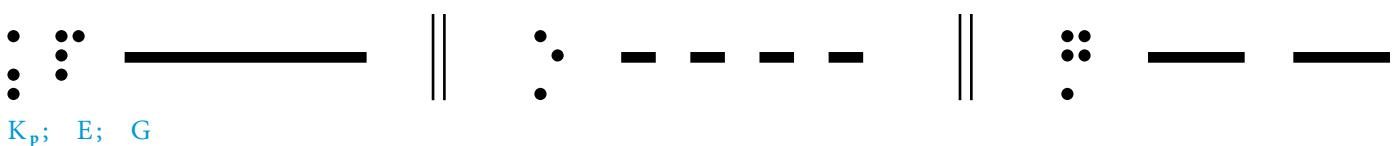
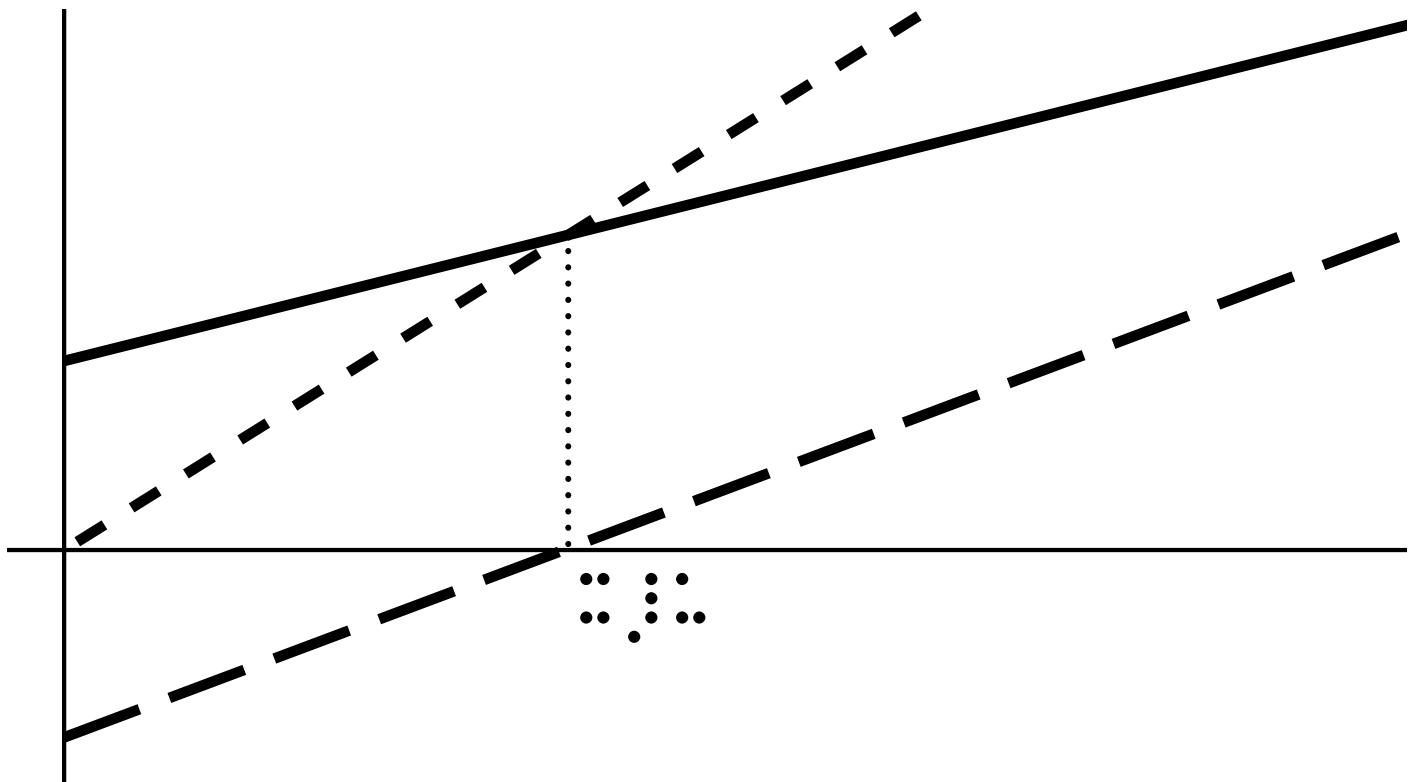
;

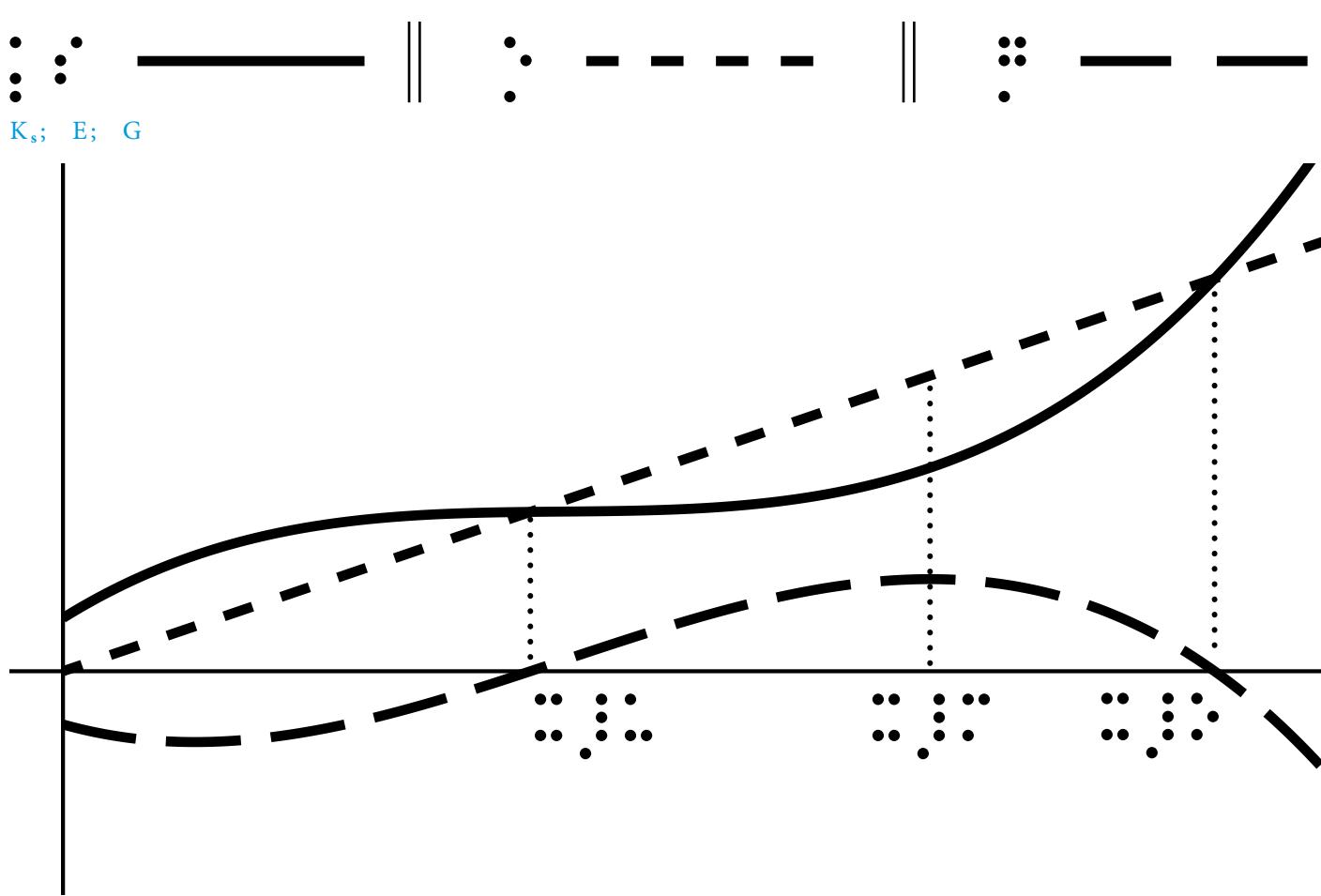
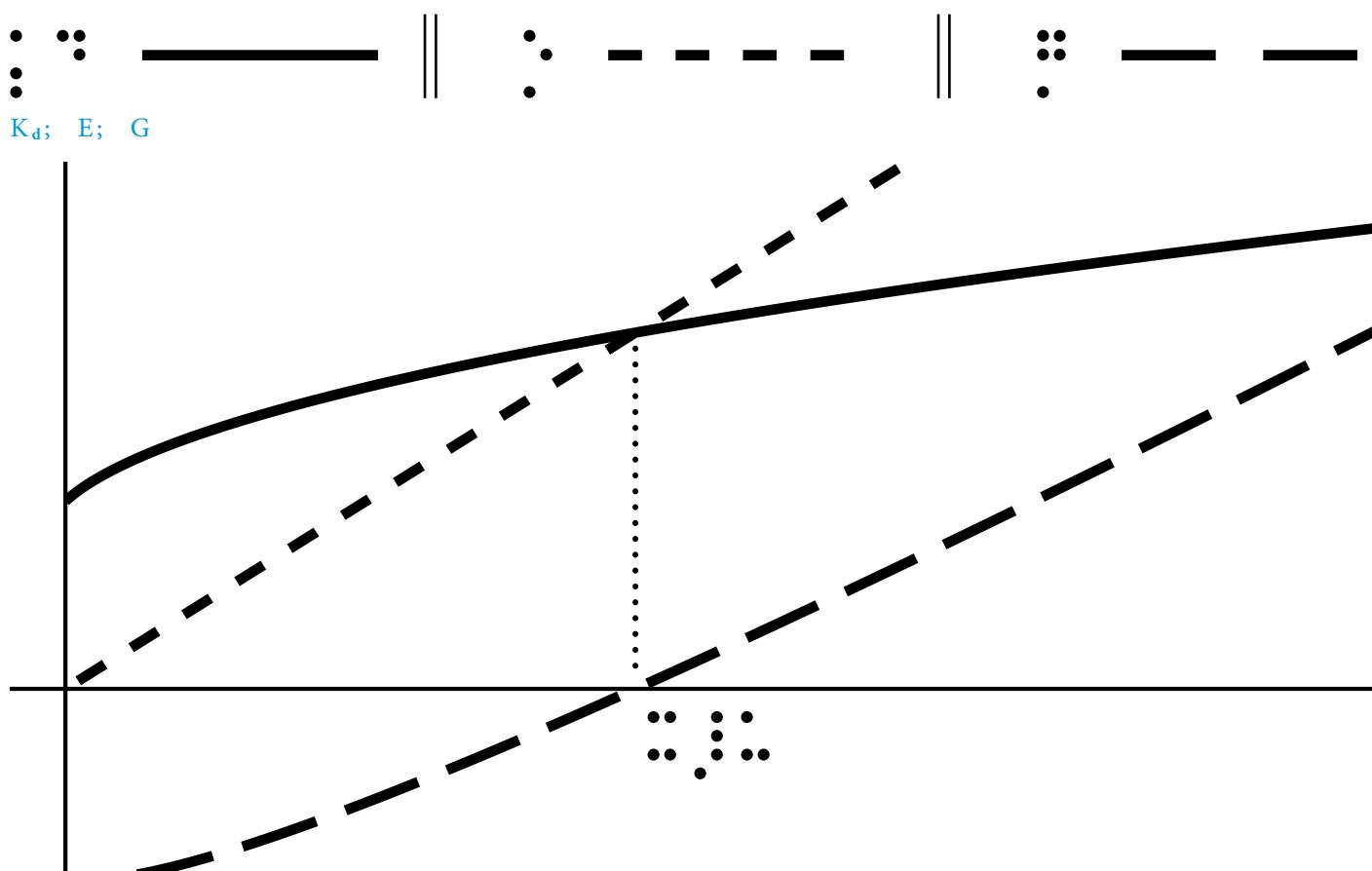
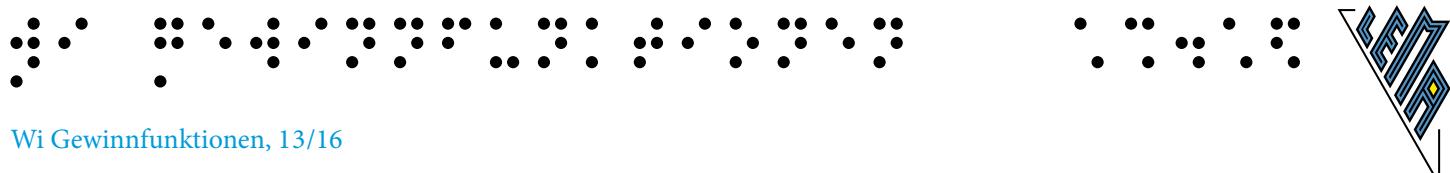
;

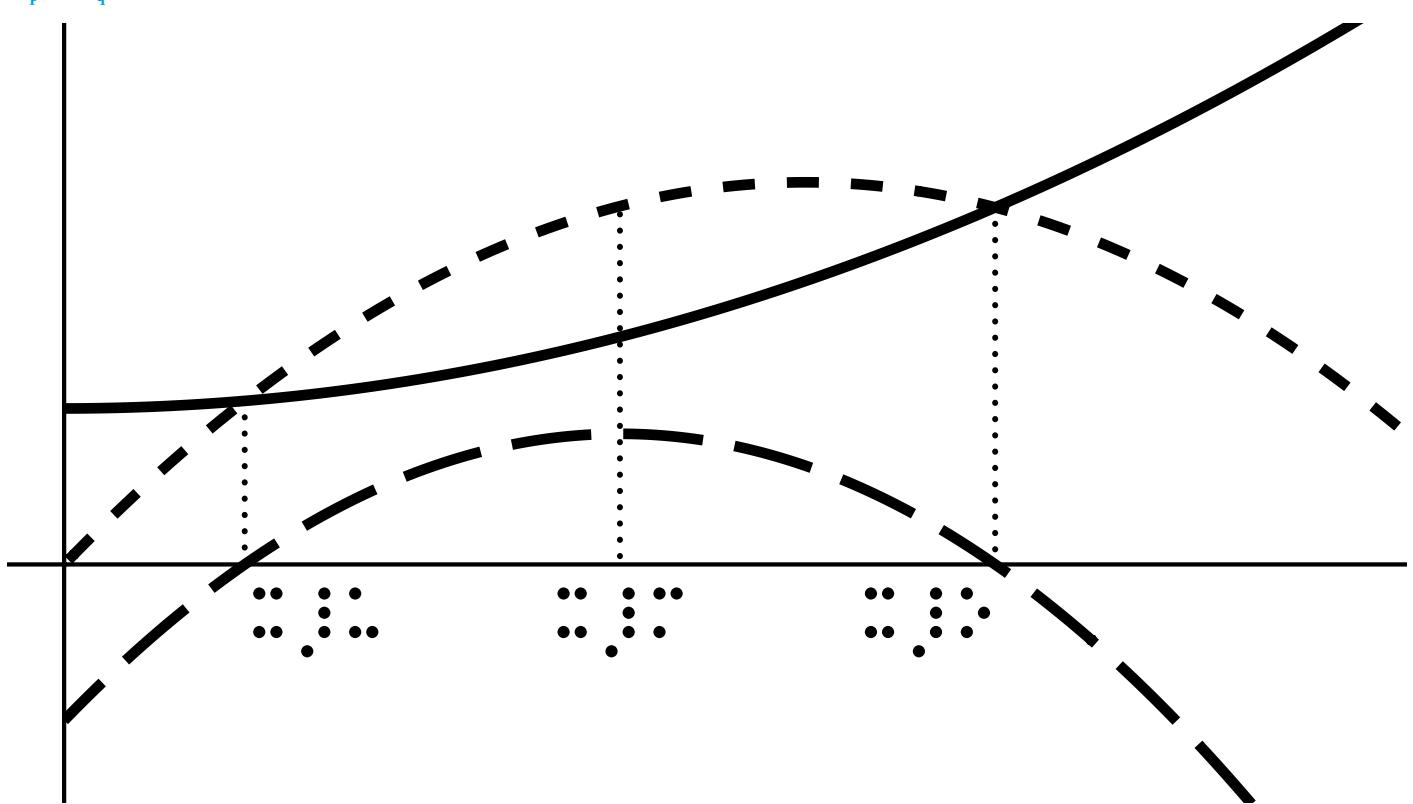
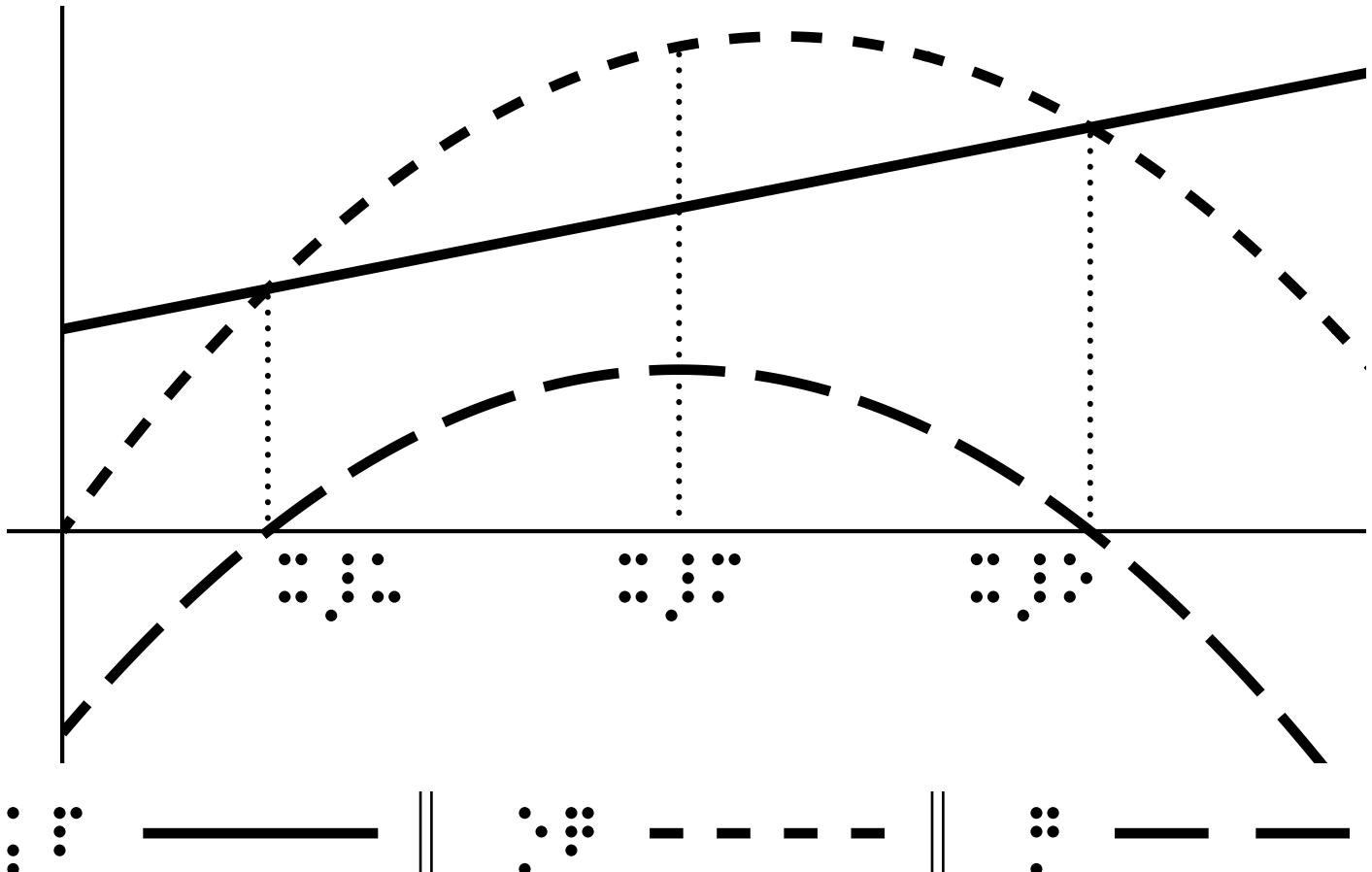
;

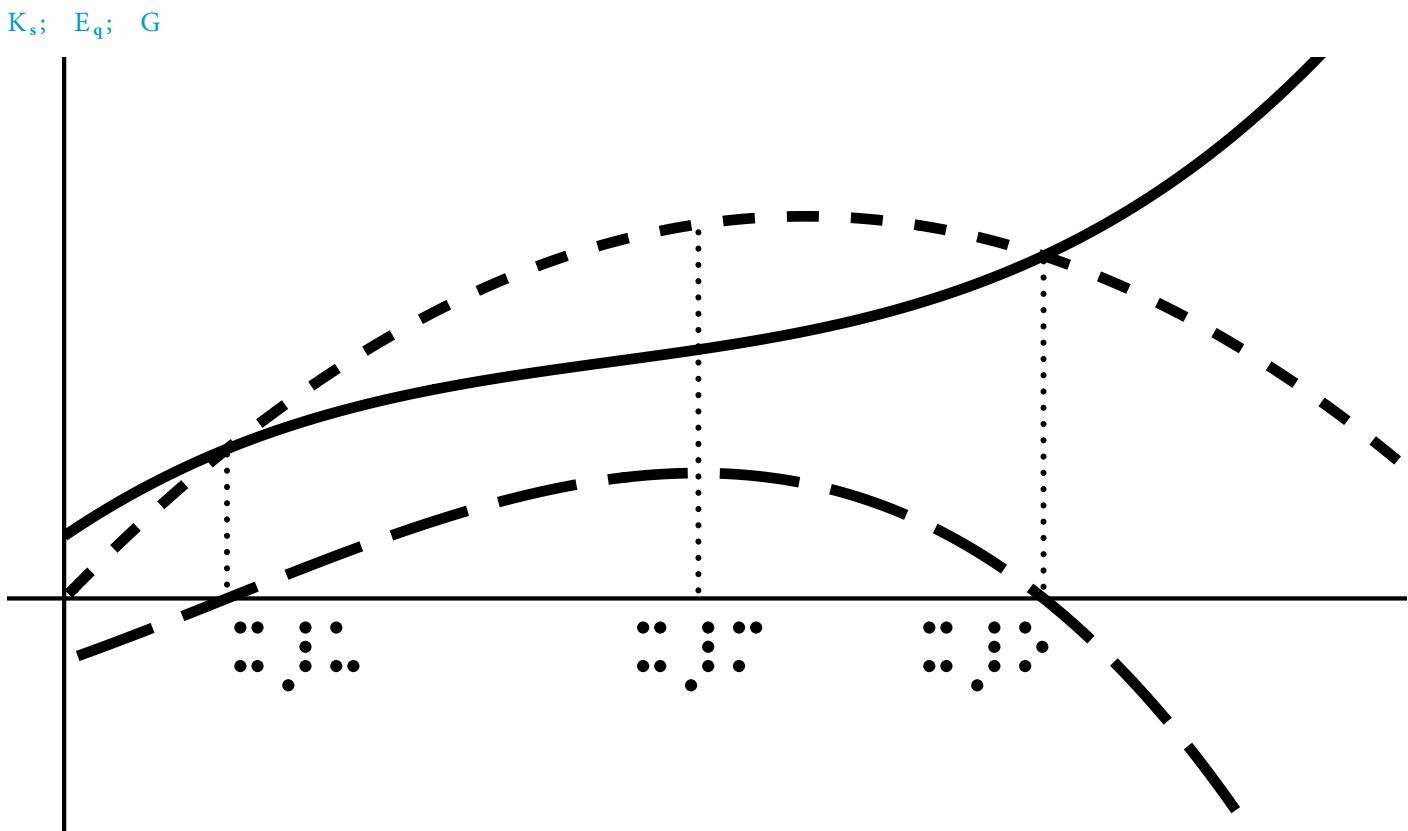
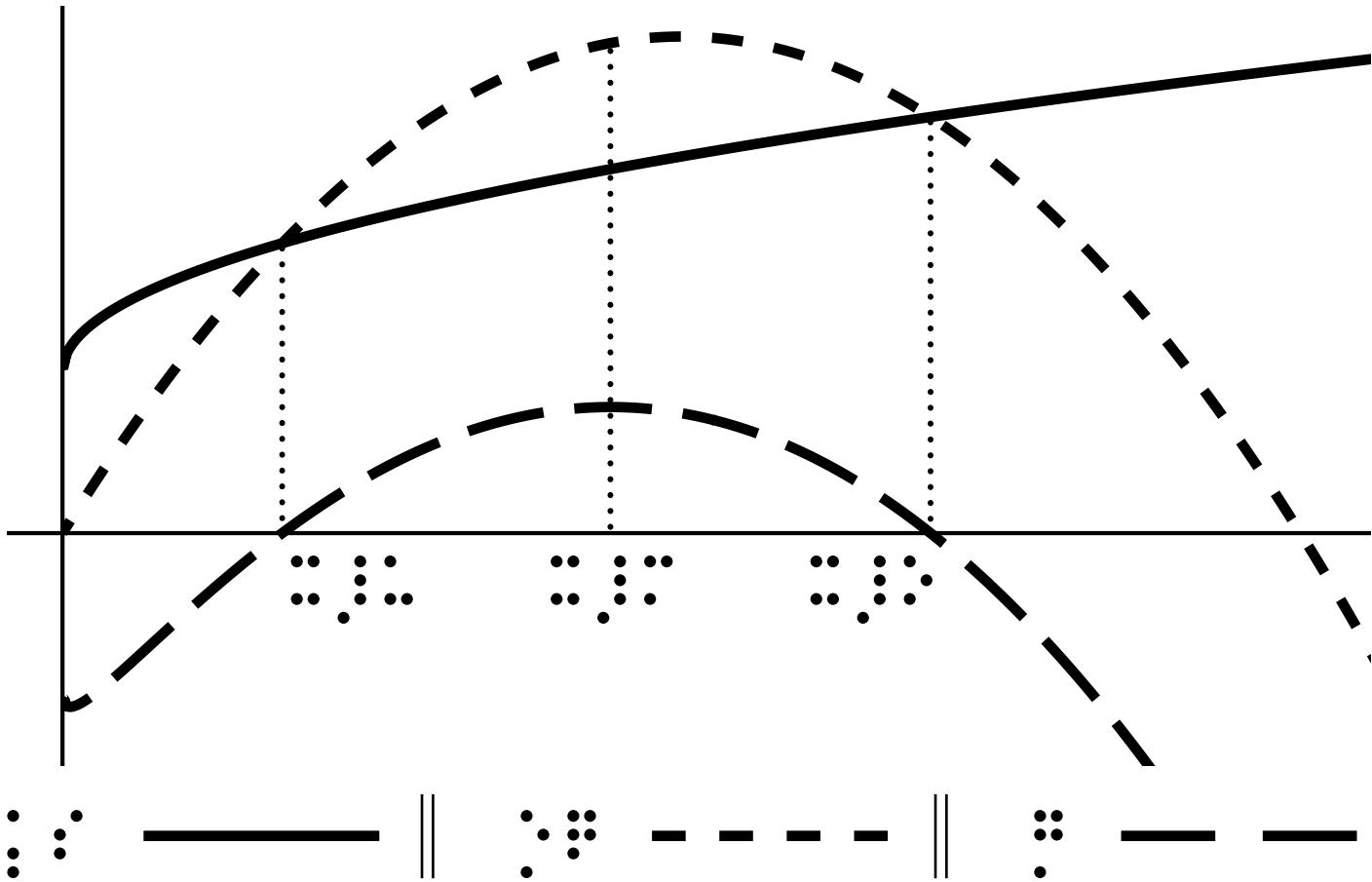
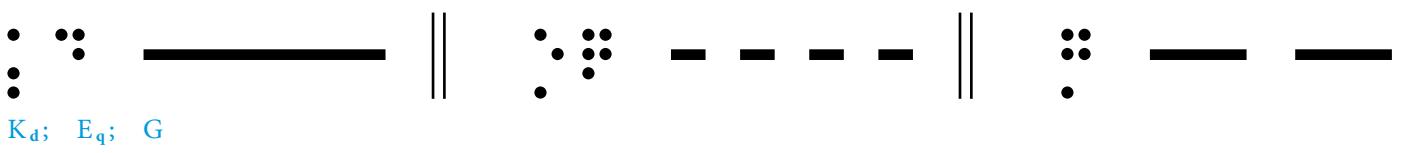
$K_s$ ;  $E = p^* x$ ;  $E_{\parallel}$











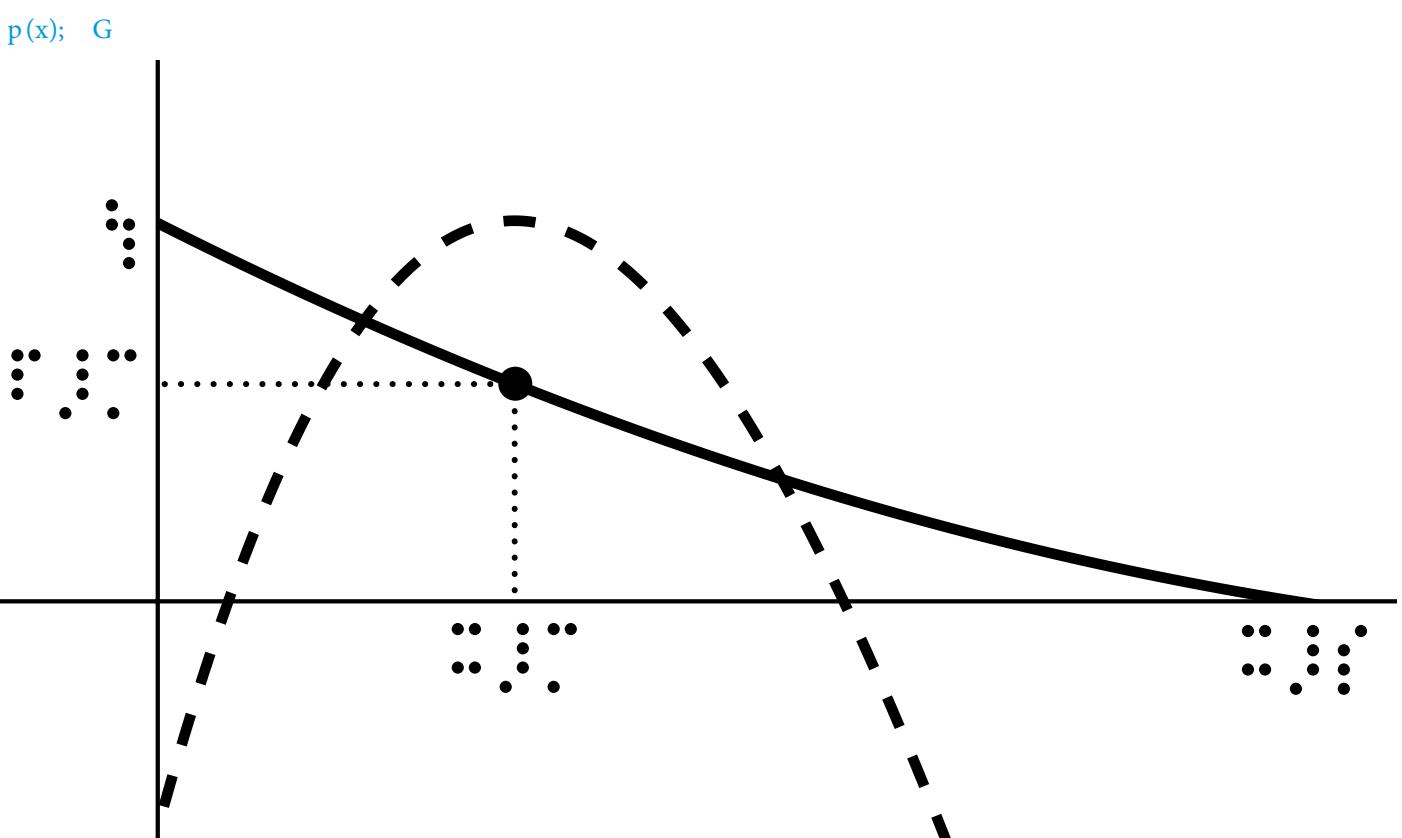
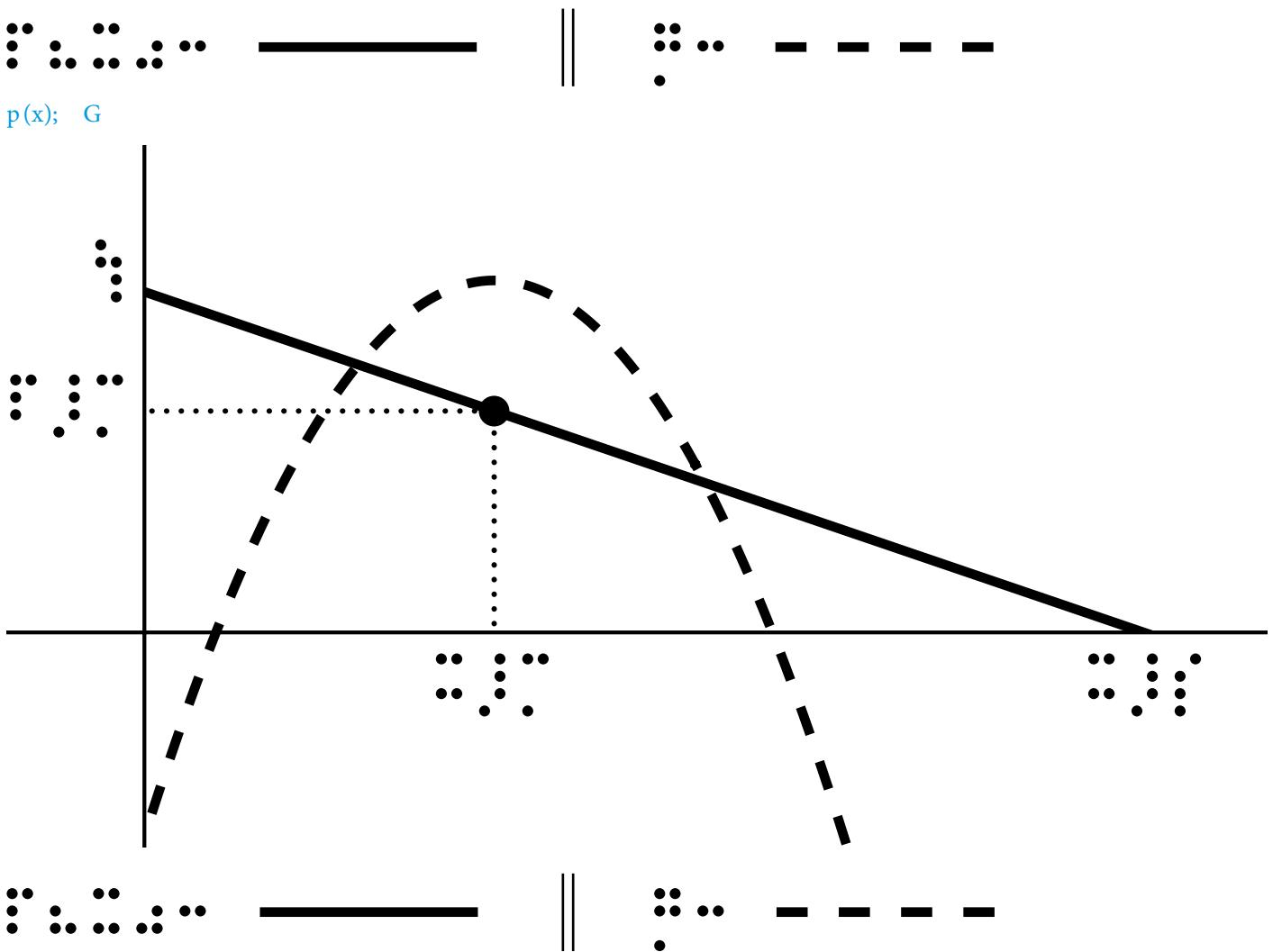
## Ergänzungen zu Seite 16

$C(x_C|p_C)$

Cournot'scher Punkt

**x\_C:** gewinnmaximale Produktionsmenge eines Monopolisten





# **Normalverteilung**

**Schulstufe 12**

Histogramm

Dichtefunktion  $f$

Verteilungsfunktion  $F$

standardisierte Dichtefunktion ' $\phi$

standardisierte Verteilungsfunktion ' $\Phi$



# Inhalt

- 0 Liste der Abkürzungen
- 1 Histogramm n =6
- 2 Histogramm n =10
- 3 Verteilungsfunktion F  
Integration der Dichtefunktion
- 4 Allgem. Dichtefunktion f  
Allgem. Verteilungsfunktion F  
 $F(a) = P(X \leq a)$
- 5 Allgem. Verteilungsfktn  
 $P(-\infty <= X <= \infty) = 1$
- 6 Allgem. Dichtefunktion 'ph  
 $P(X \leq 'my) = P(X \geq 'my) = 0,5$
- 7 Standard. Dichtefunktion 'ph  
 $'my = ^0,1 *'si = ^1 *z$
- 8 Standard. Dichtefunktion 'ph  
Skala mit 'my, Skala mit z,
- 9 Standard. Dichtefunktion 'ph  
 $'ph(z) = f('my + z *'si))$

**10** Stand. Verteilungsfkt 'Ph  
 $'Ph(z) = P(X \leq z)$

**11** Stand. Verteilungsfkt 'Ph  
 $1 - 'Ph(z) = 'Ph(-z)$

**12** Stand. Verteilungsfkt 'Ph  
 $2 * 'Ph(z) - 1$

**13** Stand. Verteilungsfkt 'Ph  
 $'Ph(z), z < 0$

**14** Stand. Verteilungsfkt 'Ph  
 $1 - 'Ph(z), z > 0$

**15** Stand. Verteilungsfkt 'Ph  
 $'Ph(z_2) - 'Ph(z_1), z > 0$

**16** Stand. Verteilungsfkt 'Ph  
 $'Ph(z_2) - 'Ph(z_1)$

**17** Stand. Verteilungsfkt 'Ph  
 $'Ph(z_2) - 'Ph(z_1), z < 0$

**18** Unterschiedliche Standardabweichungen







# Normalverteilung - Abkürzungen

## Index

**f:** (Wahrscheinlichkeits-) Dichtefunktion von X

**F:** Verteilungsfunktion von X  
 $F(a) = P(X \leq a)$

**P:** Wahrscheinlichkeit

**P(a\_i):** Wahrscheinlichkeit für  $a_i$

**X:** diskrete Zufallsvariable

**z:**  $(x - \mu)/\sigma$

**'my:** Erwartungswert

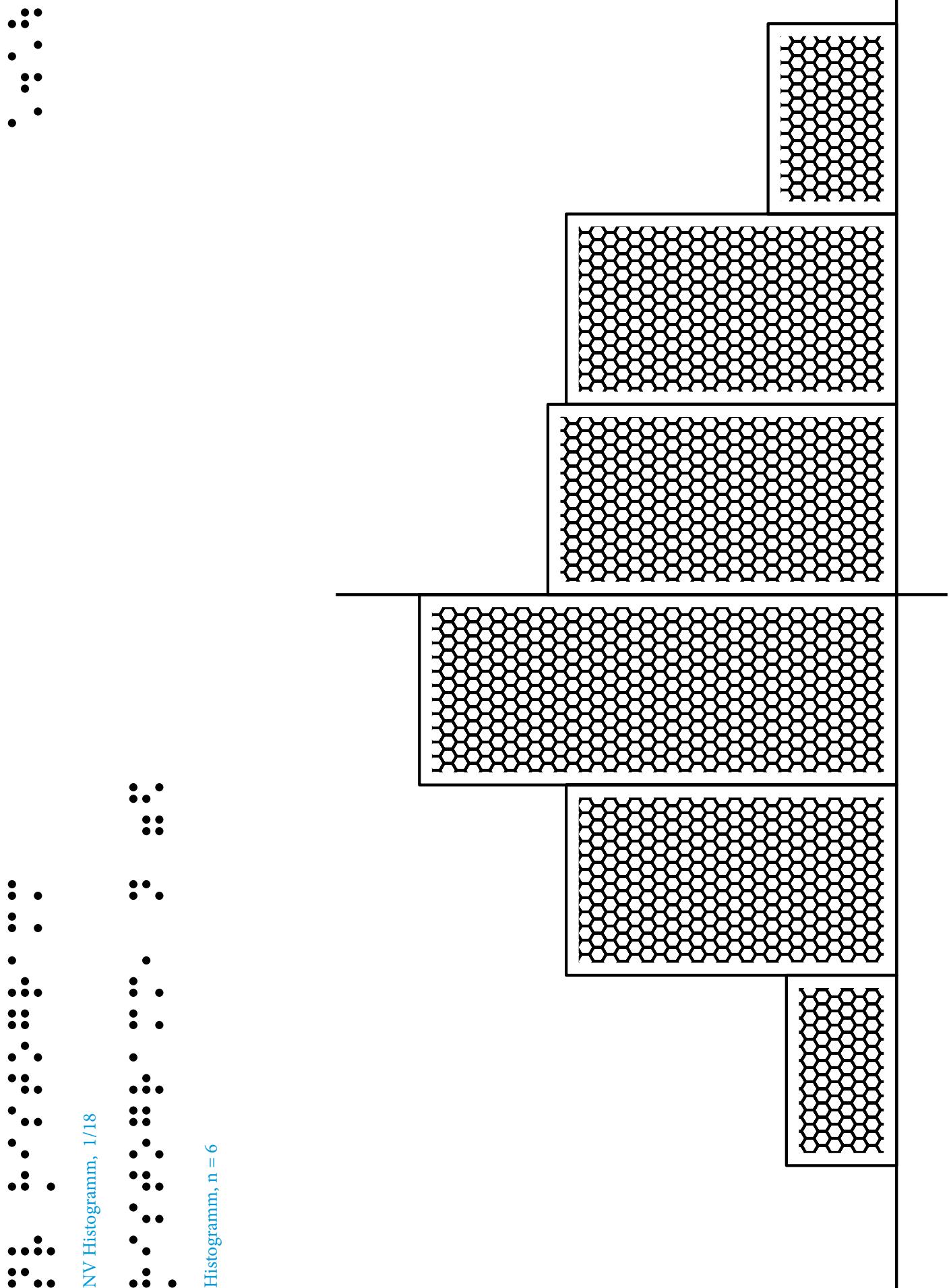
**'ph:** (Wahrscheinlichkeits-) Dichtefunktion der Standardnormalverteilung von X

**'Ph:** Verteilungsfunktion der Dichtefunktion der Standardnormalverteilung von X

**'si:** Standardabweichung



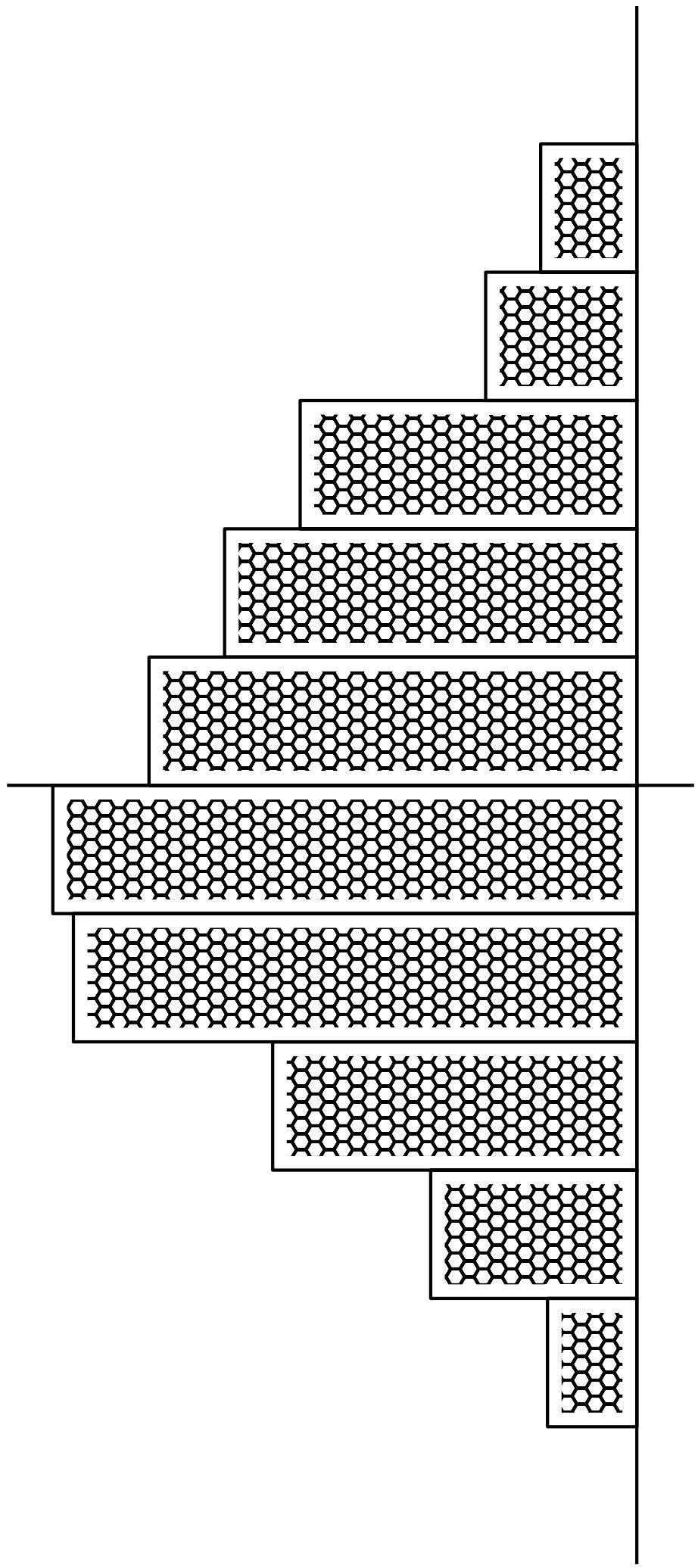


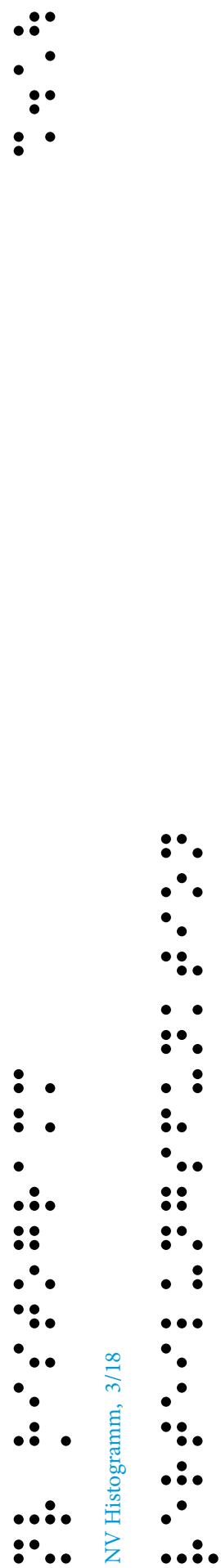




NV Histogramm, 2/18

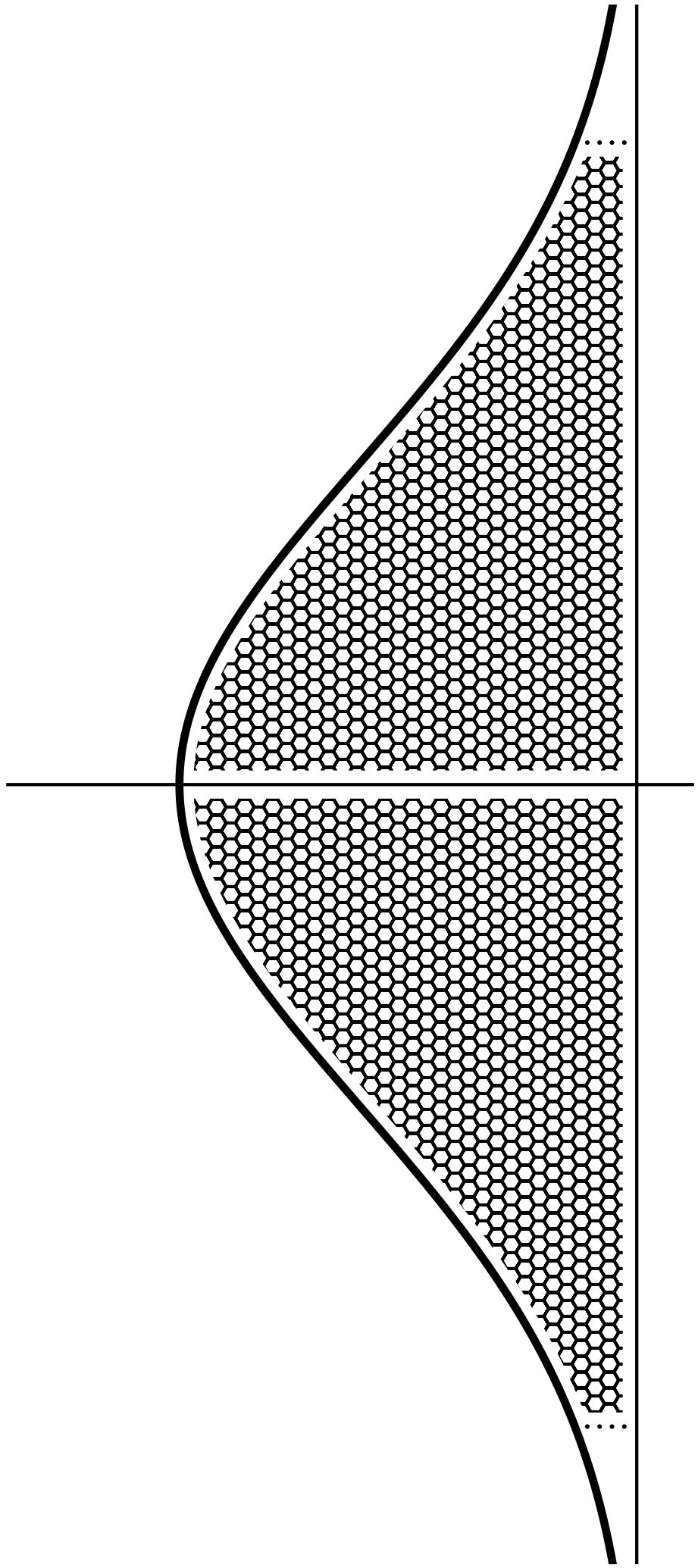
Histogramm,  $n = 10$

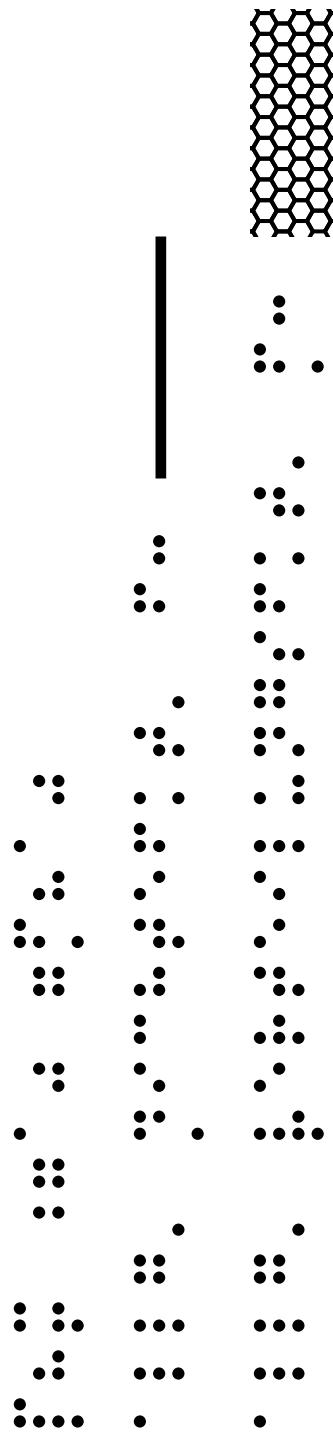
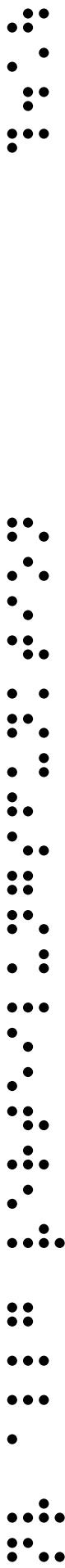




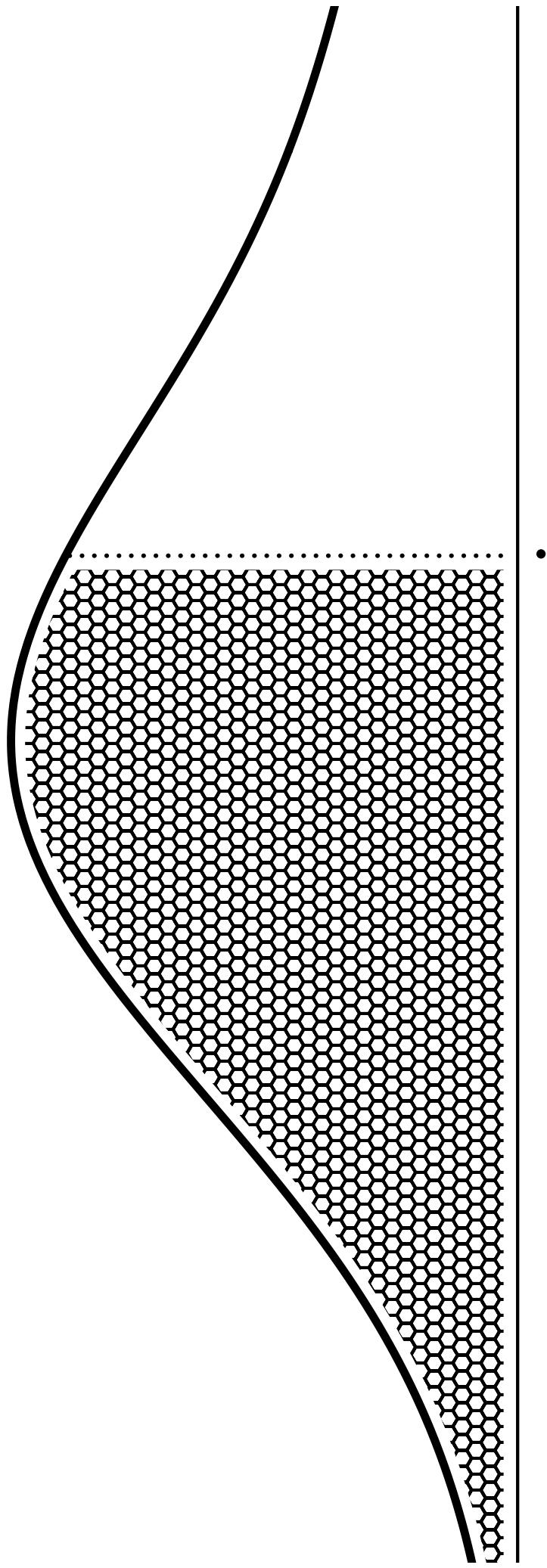
NV Histogramm, 3/18

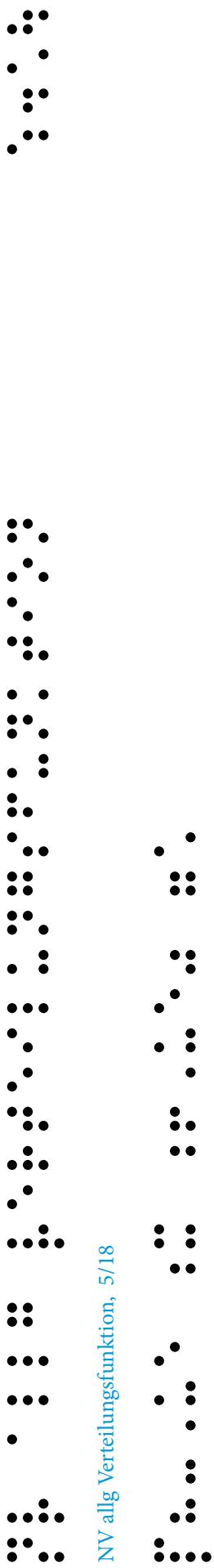
Verteilungsfunktion





$P(X \leq a) = F(a)$  allgemeine Dichtefunktion  $f$ , allgemeine Verteilungsfunktion  $F$





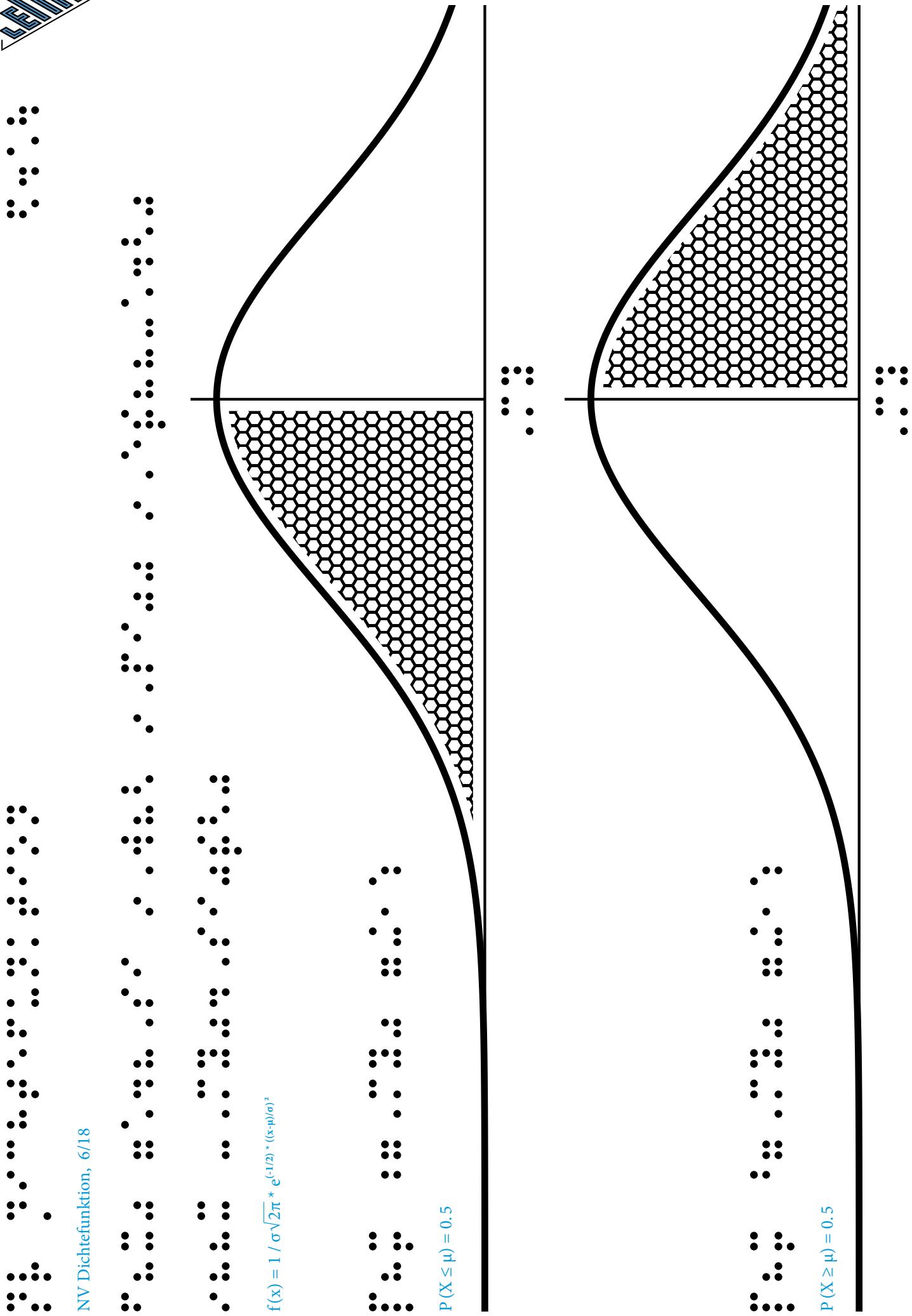
NV allg Verteilungsfunktion, 5/18

$$P(-\infty < x < +\infty) = 1$$

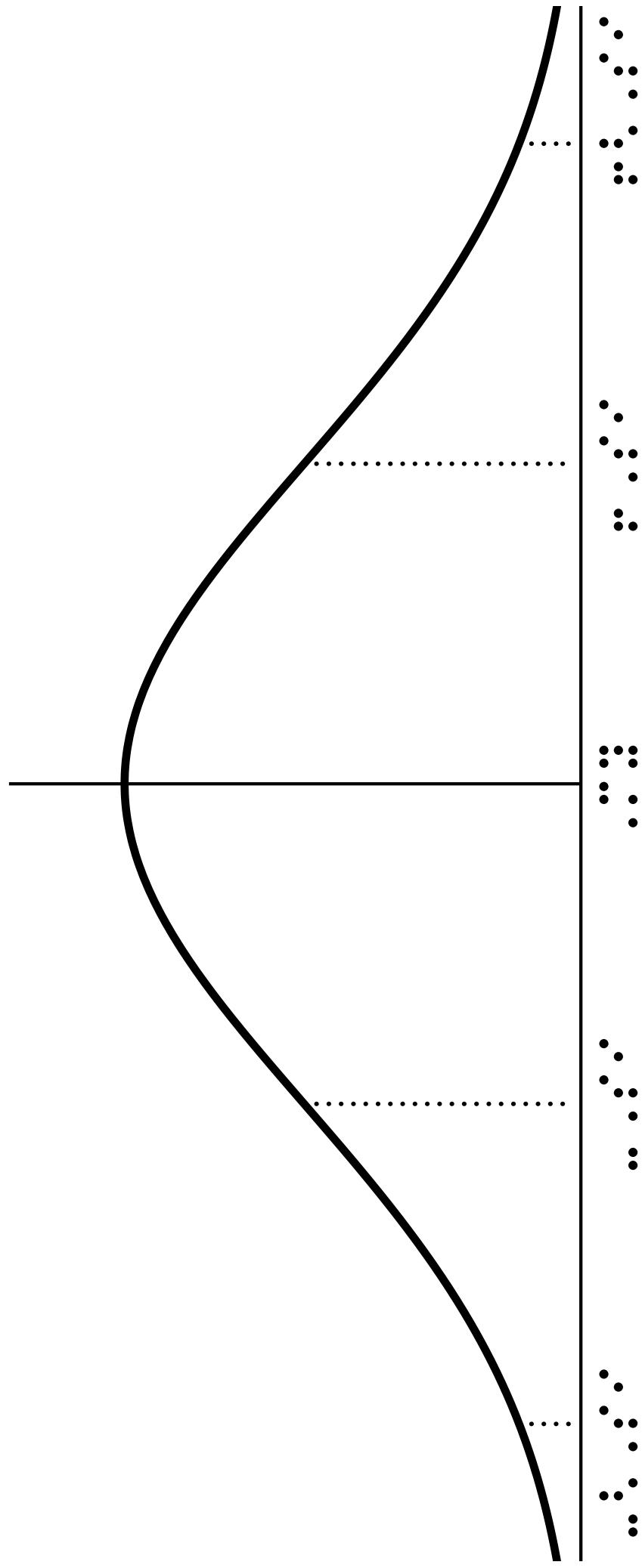
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} * e^{(-1/2) * ((x-\mu)/\sigma)^2}$$

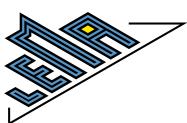
$$P(X \leq \mu) = 0.5$$

$$P(X \geq \mu) = 0.5$$

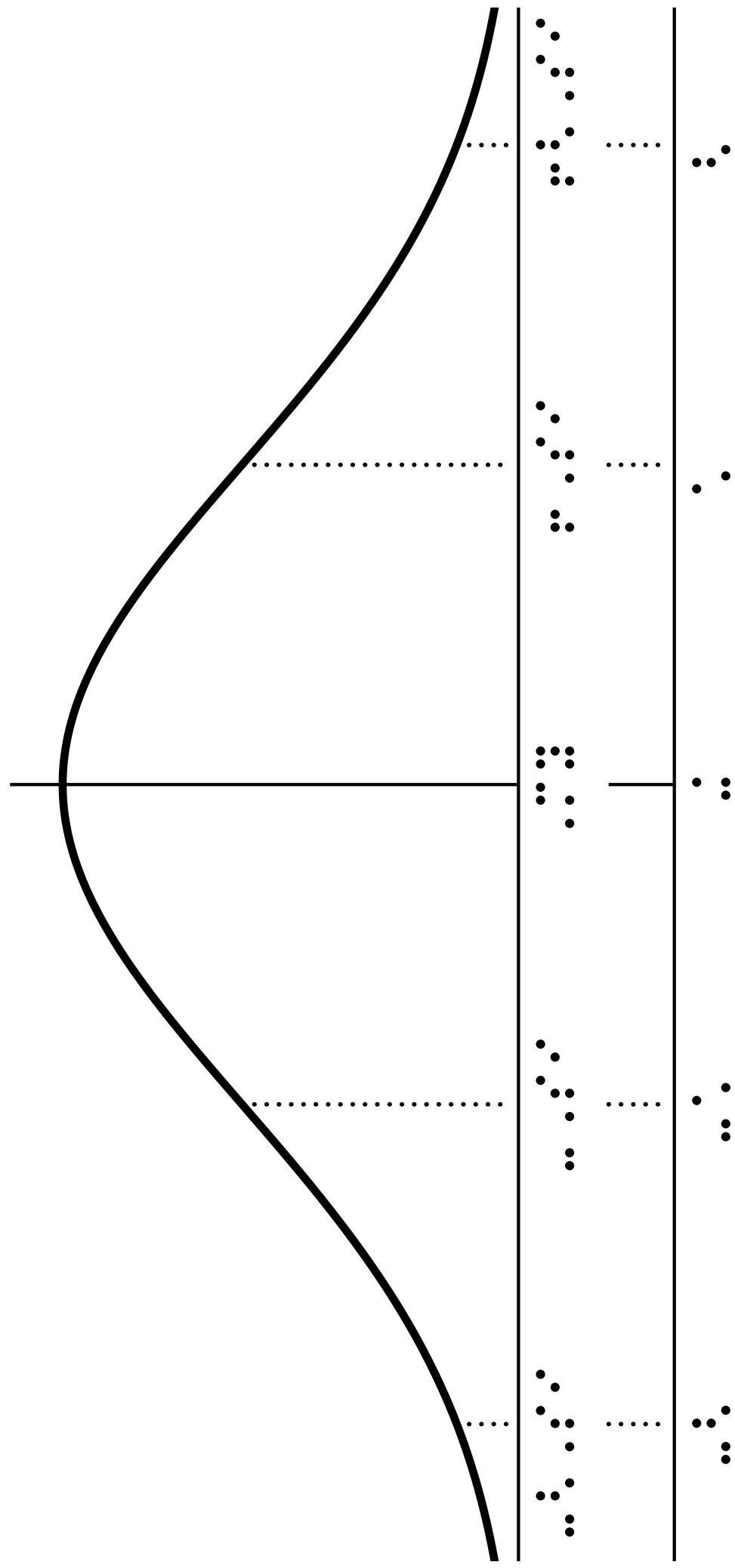


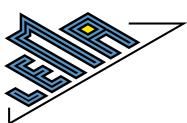
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} * e^{(-1/2) * ((x-\mu)/\sigma)^2}$$



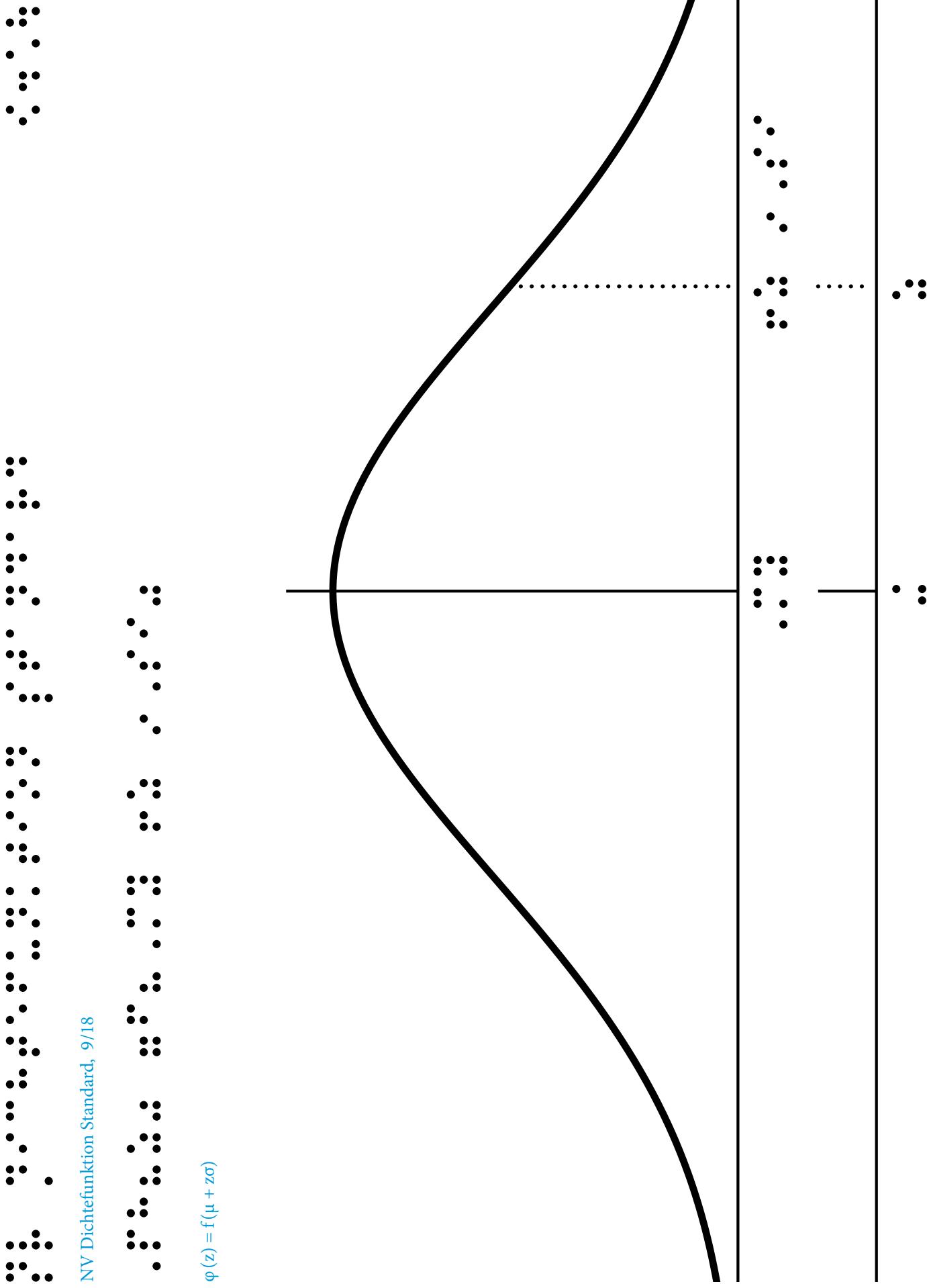


$$\varphi(z) = 1/\sqrt{2\pi} \cdot e^{-z^2/2}$$

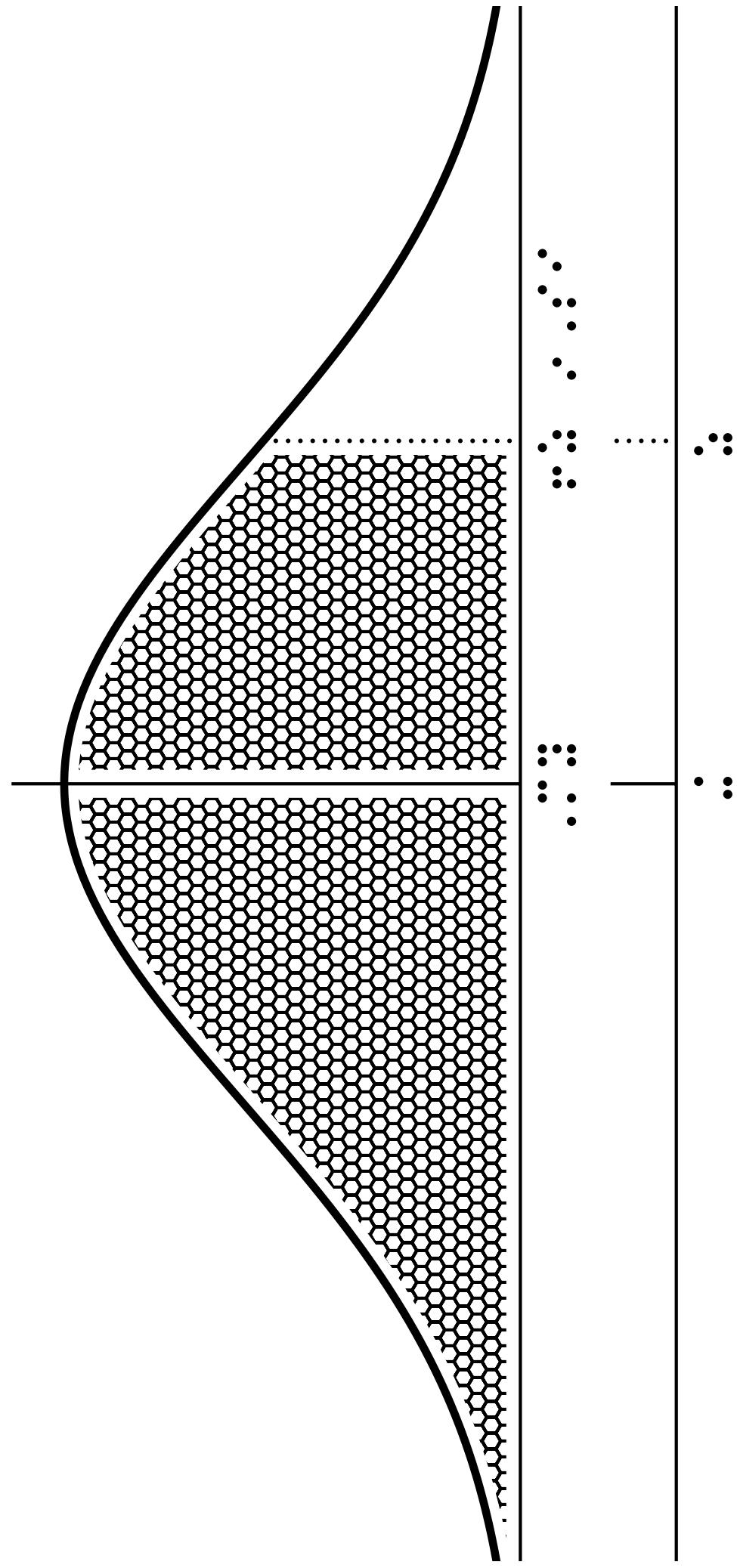


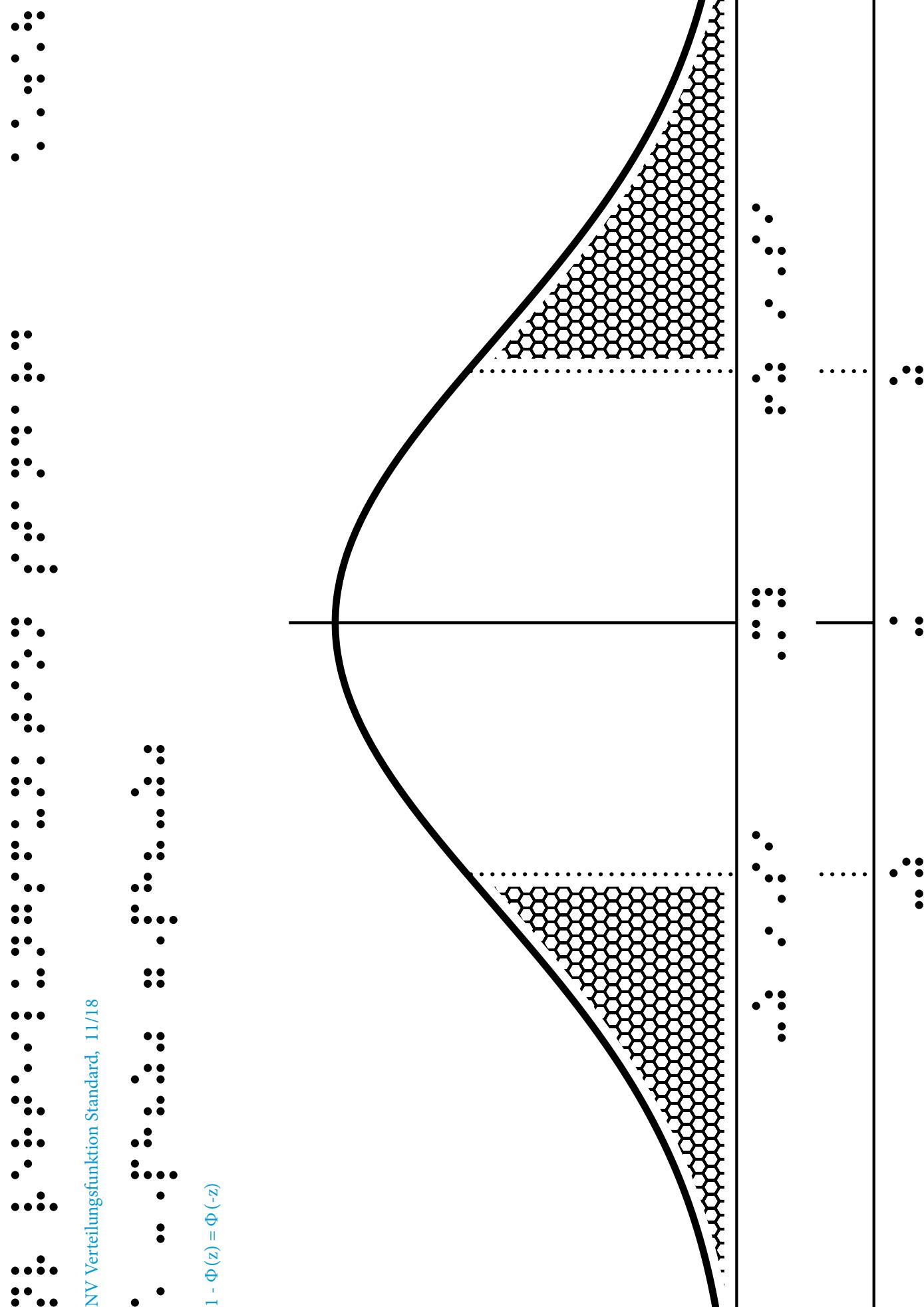


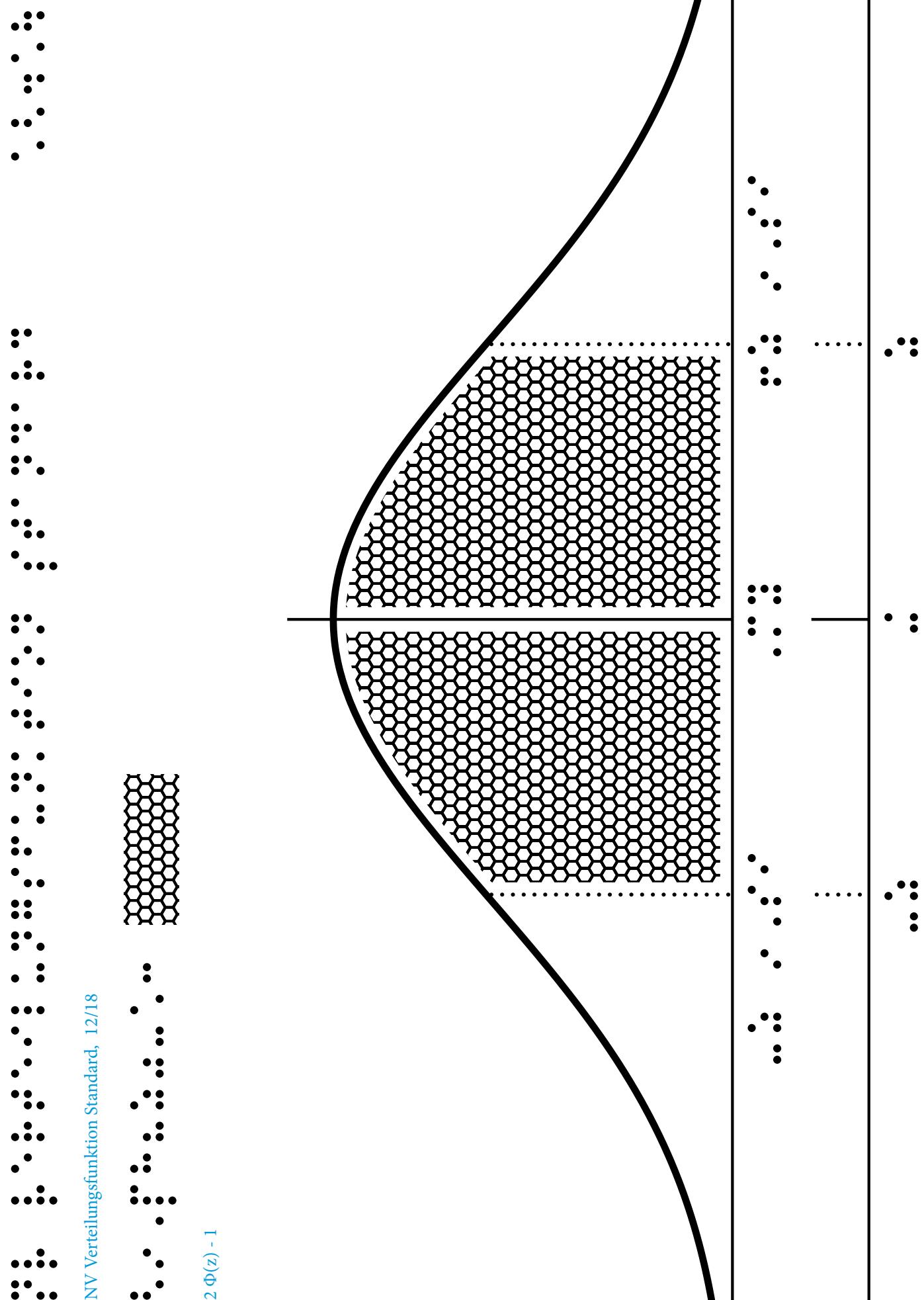
$$\varphi(z) = f(\mu + z\sigma)$$

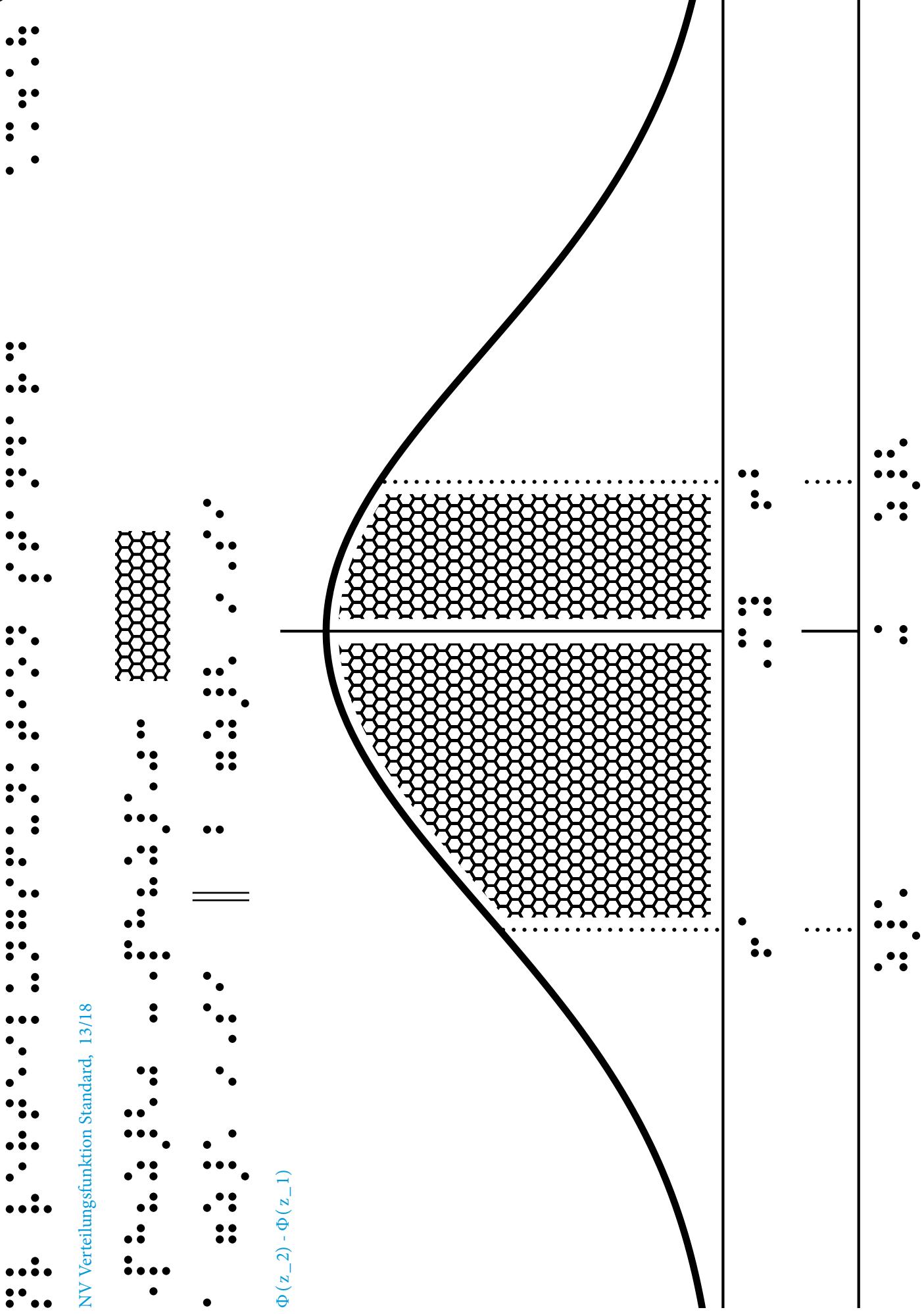


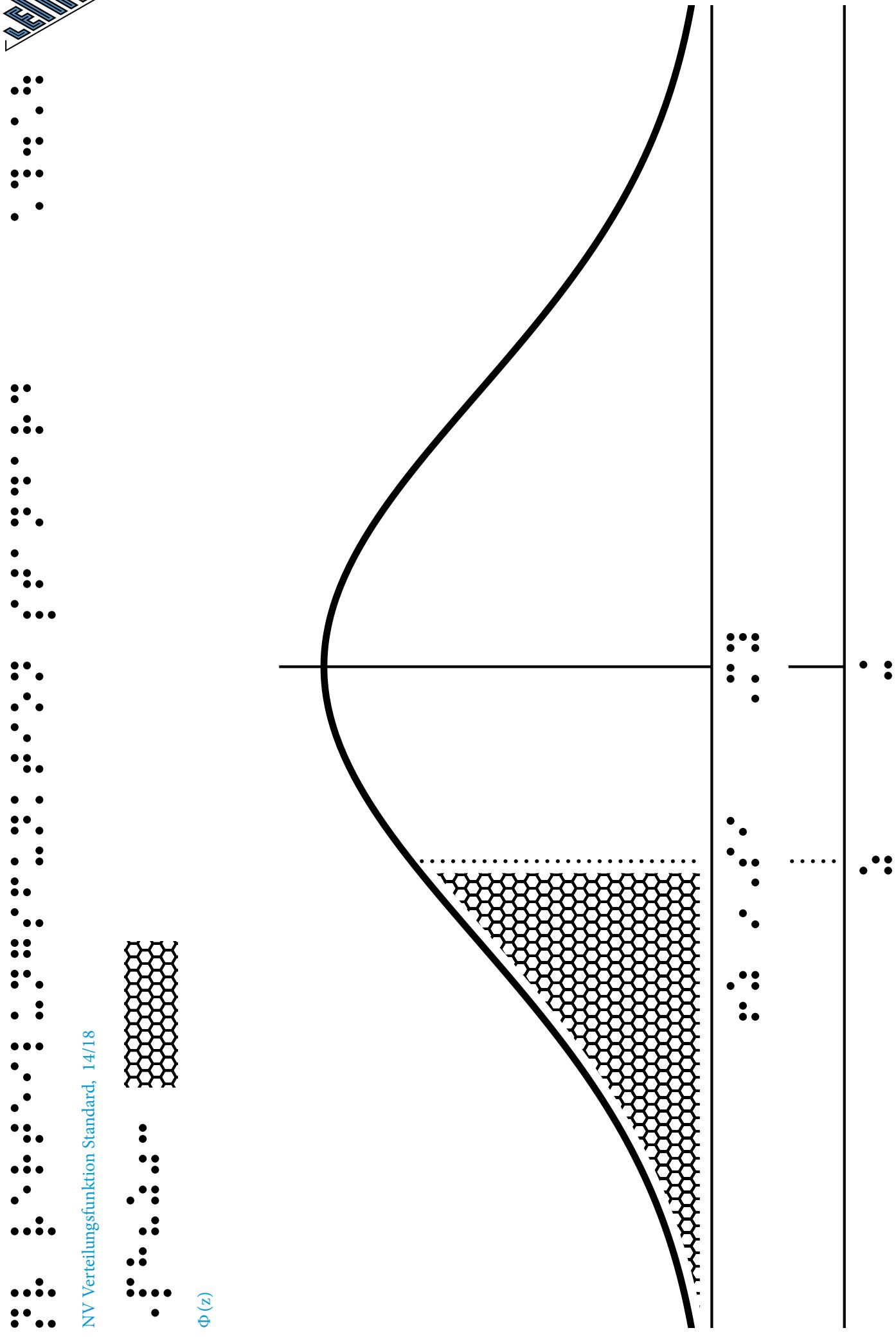
$$\Phi(z) = P(X \leq \mu + z\sigma)$$

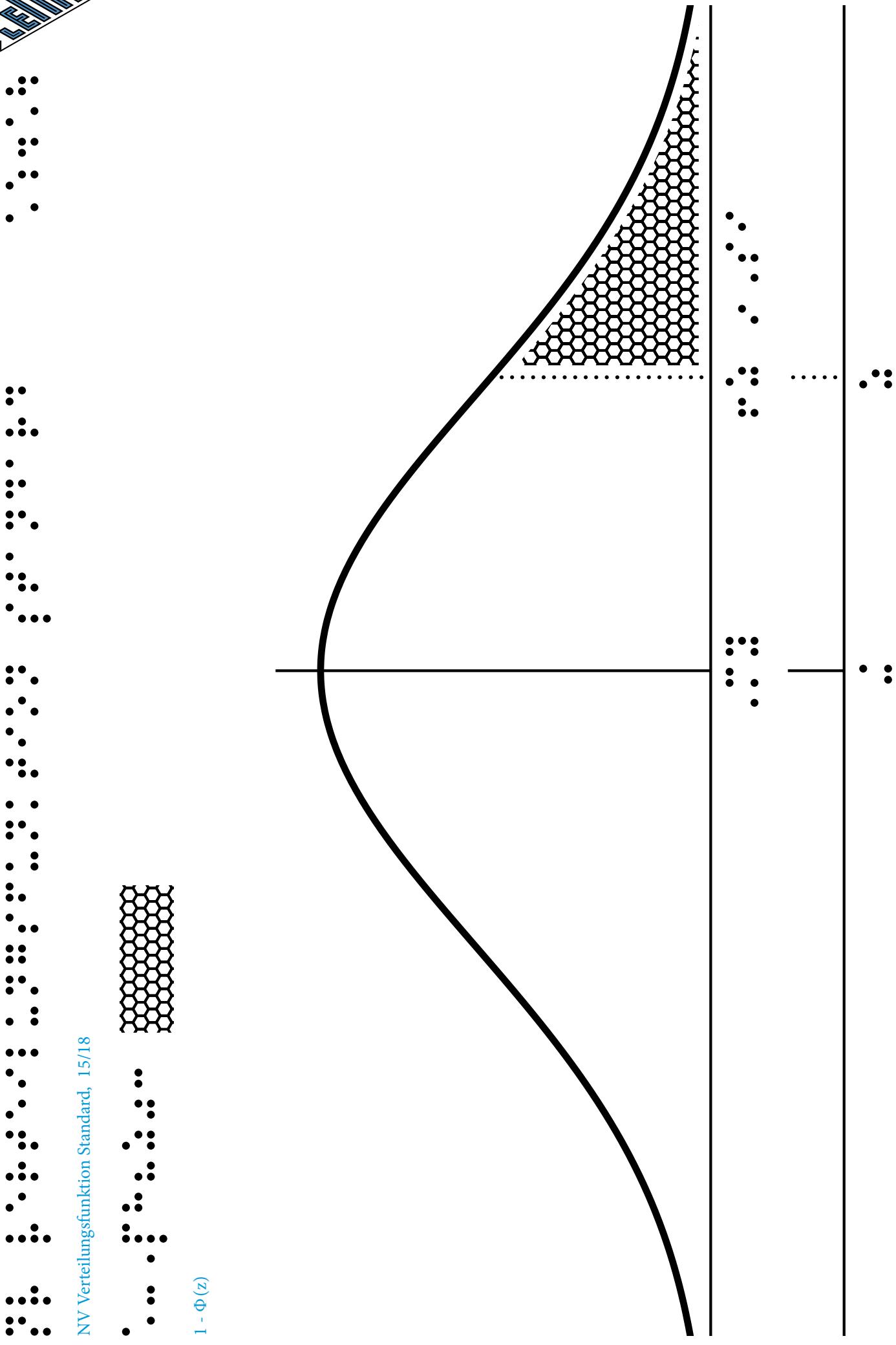


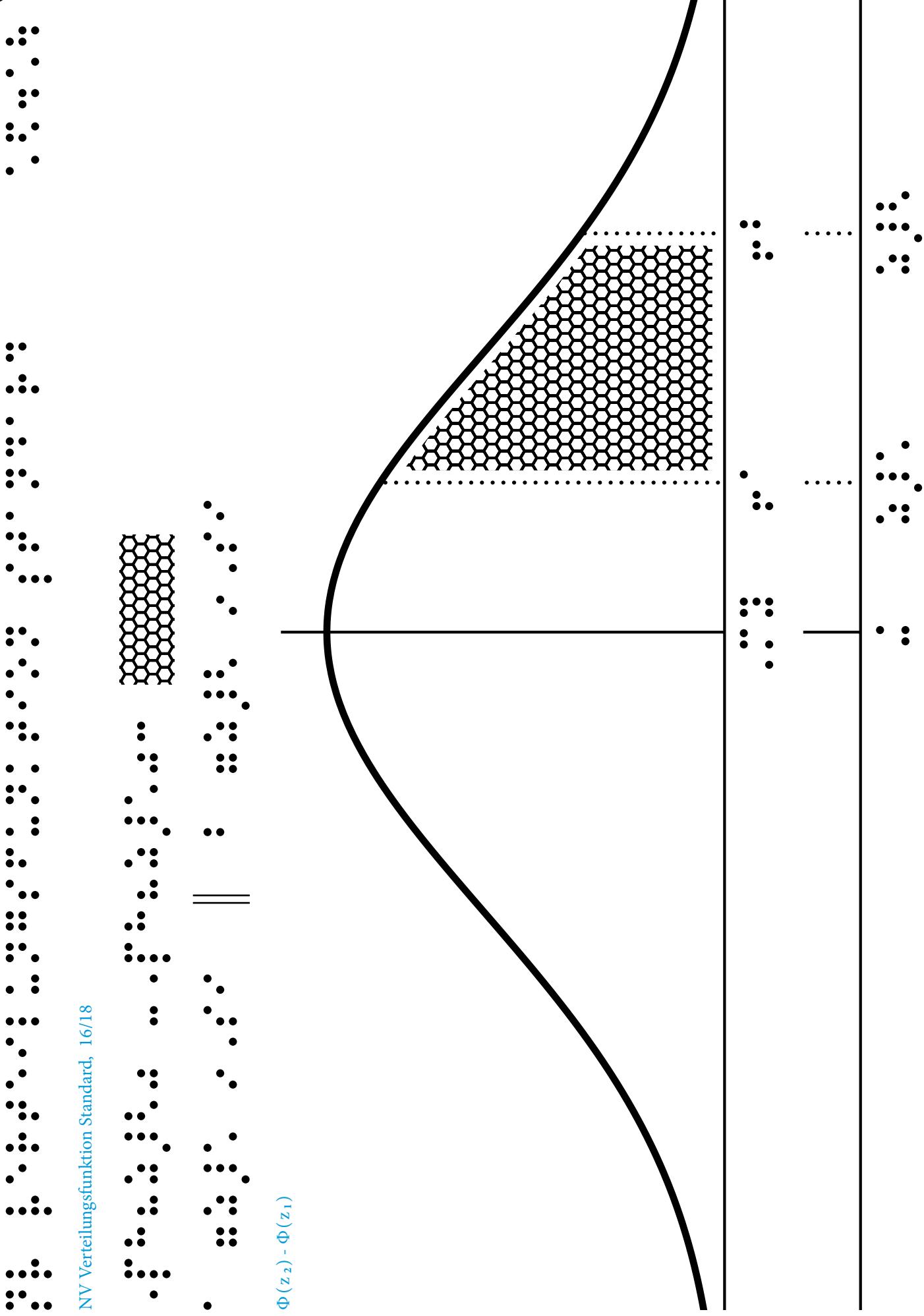


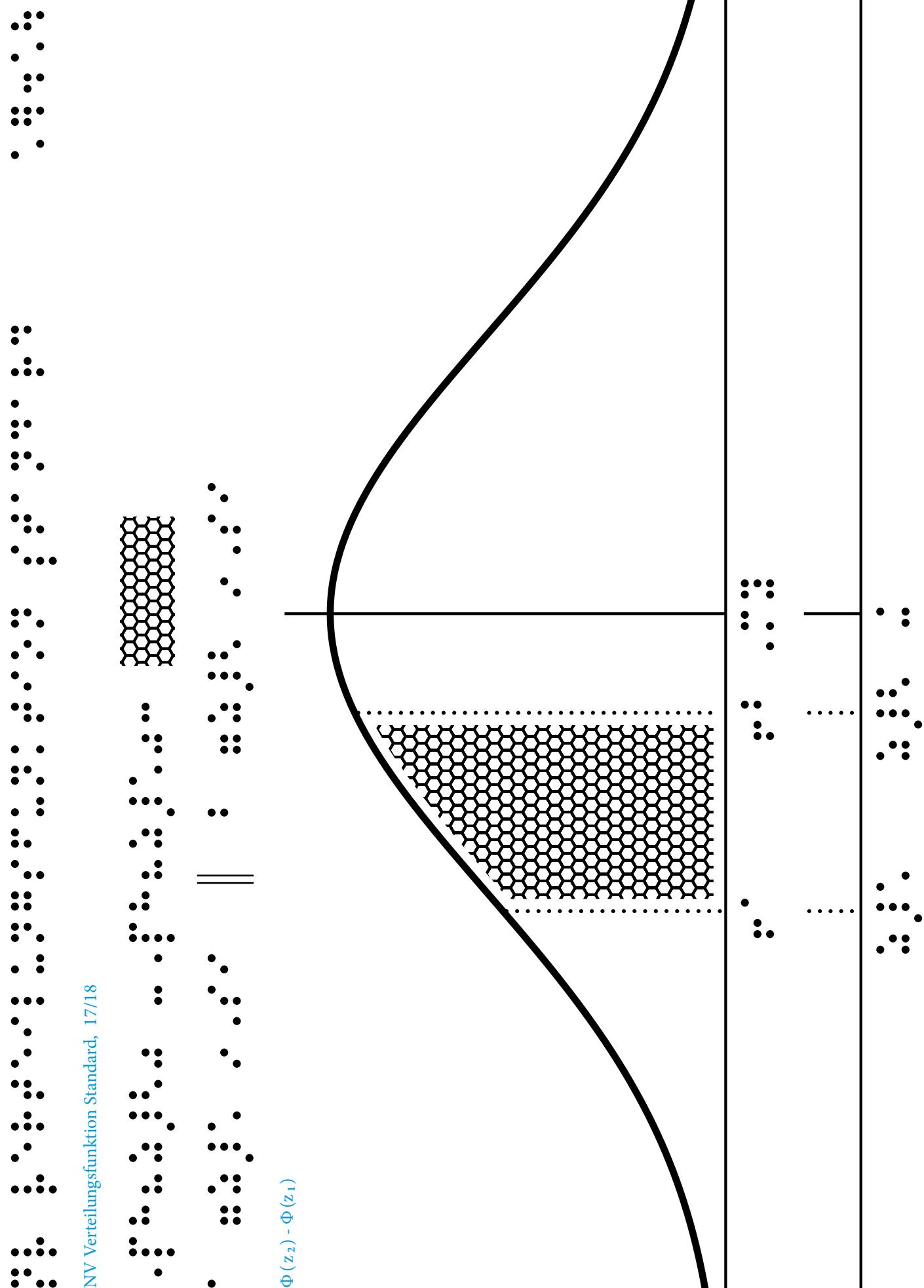


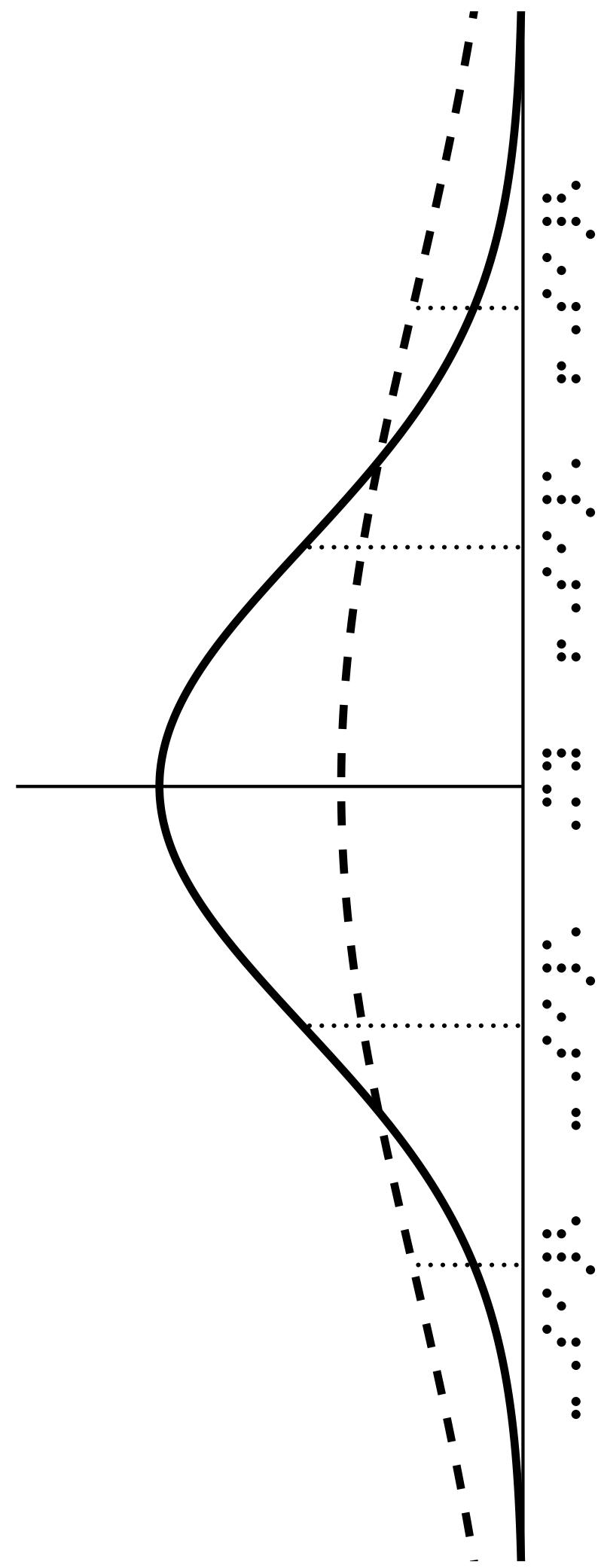
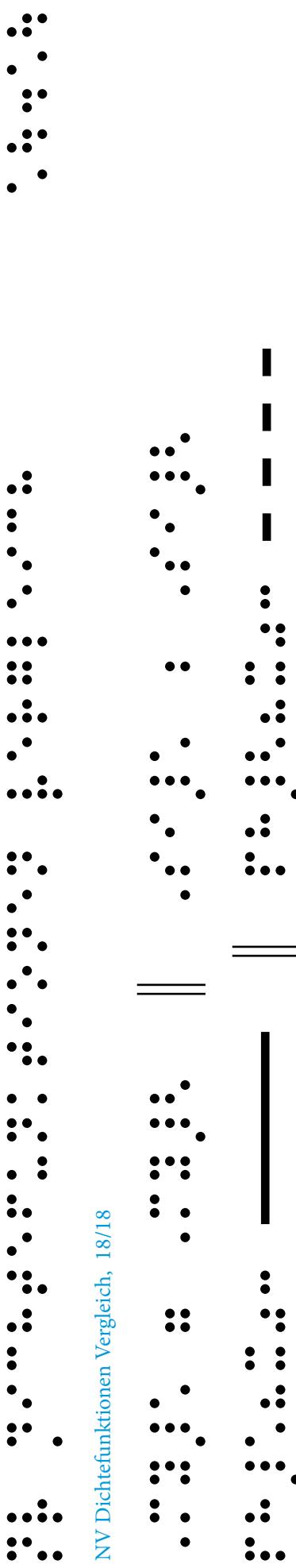






$\Phi(z_2) - \Phi(z_1)$ 





# **Reelle Funktionen**

## **Schulstufe 12**

Typische Funktionen (TF)

Potenzfunktionen

Polynomfunktionen

Exponentialfunktionen

Vergleichen von Funktionen (VF)

Änderungsmaße (ÄM)



# Inhalt

## Typische reelle Funktionen

### 1 Potenzfunktionen

$a \cdot x^r$  mit  $a \neq 0, r \in \mathbb{Z}$

### 2 Polynomfunktionen

Grad 1 Geraden

### 3 Polynomfunktionen

Grad 2 Parabeln

### 4 Polynomfunktionen

Grad 3 mit „Entartungen“

### 5 Polynomfunktionen Grad 4

### 6 Polynomfunktionen

Grad 4 mit „Entartungen“

### 7 Exponentialfunktionen $a^x$

$a > 1; 0 < a < 1$

## Vergleichen von reellen Funktionen

### 8 Mult: $a \cdot f(x)$

$a > 1; 0 < a < 1;$

### 9 Mult: $a \cdot f(x)$

$a = -1; a < -1; -1 < a < 0;$

**10** Mult:  $f(b^x)$

$b > 1; 0 < b < 1;$

**11** Add:  $f(x) + d$

$d > 0; d < 0;$

**12** Add:  $f(x + c)$

$c > 0; c < 0$

## Änderungsmaße

**13**  $f(x) = k^x + d$

$$f(x + 1) = f(x) + k$$

$$f(x + h) = f(x) + k^h$$

**14**  $f(x) = k^x + d$

## Steigungsdreiecke

**15**  $f(x) = c \cdot a^x$

$$f(x + 1) = c \cdot a^{(x + 1)} = c \cdot a^x \cdot a$$

$$f(x + h) = c \cdot a^{(x + h)} = c \cdot a^x \cdot a^h$$

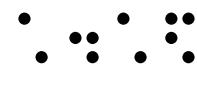


The image consists of a uniform grid of small, dark, irregular shapes. These shapes are arranged in a regular, rectangular pattern across the entire frame. The individual shapes are roughly circular or oval in orientation but lack a consistent size or a clear internal structure, giving them a stylized, abstract appearance.





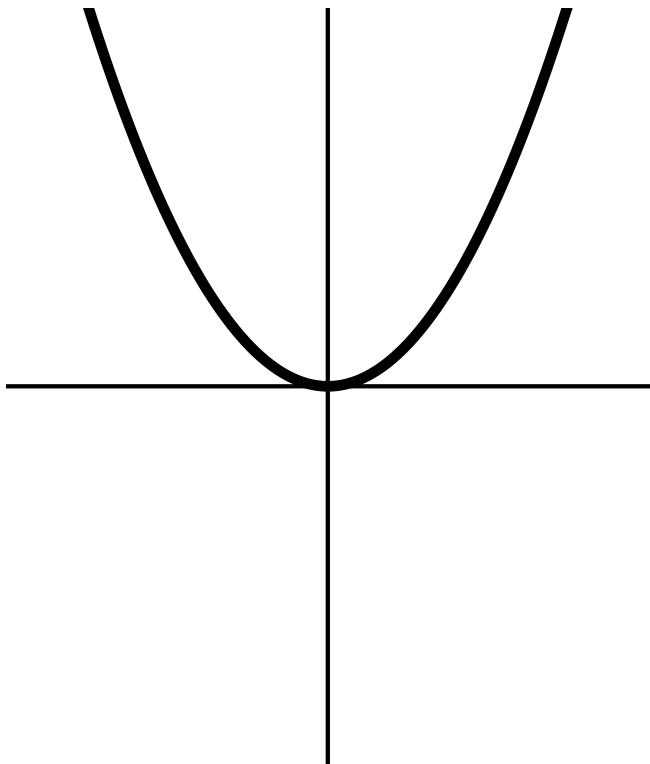
# RF typische Potenzfunktionen



RF typische Potenzfunktionen, 1/16

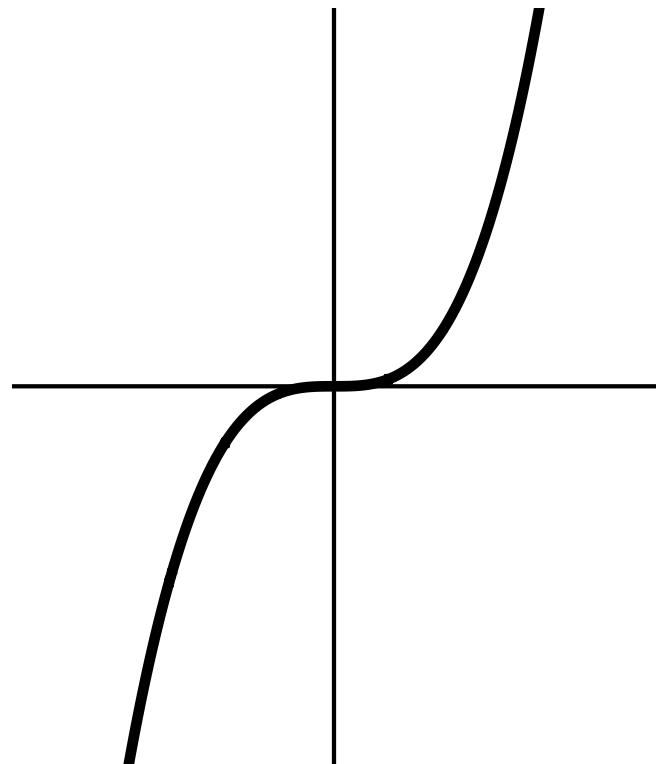
Funktionsgraphen:

$$f(x) = x^2$$



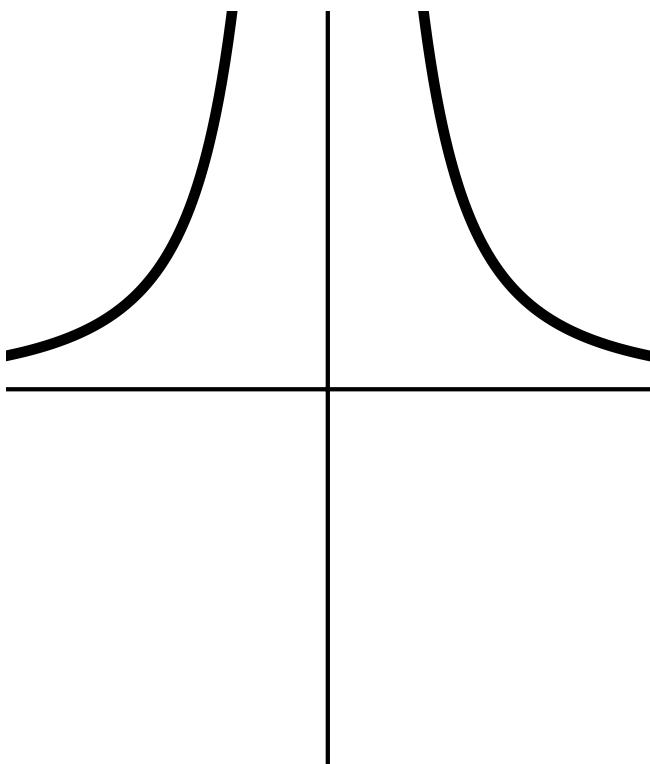
Funktionsgraphen:

$$f(x) = x^3$$



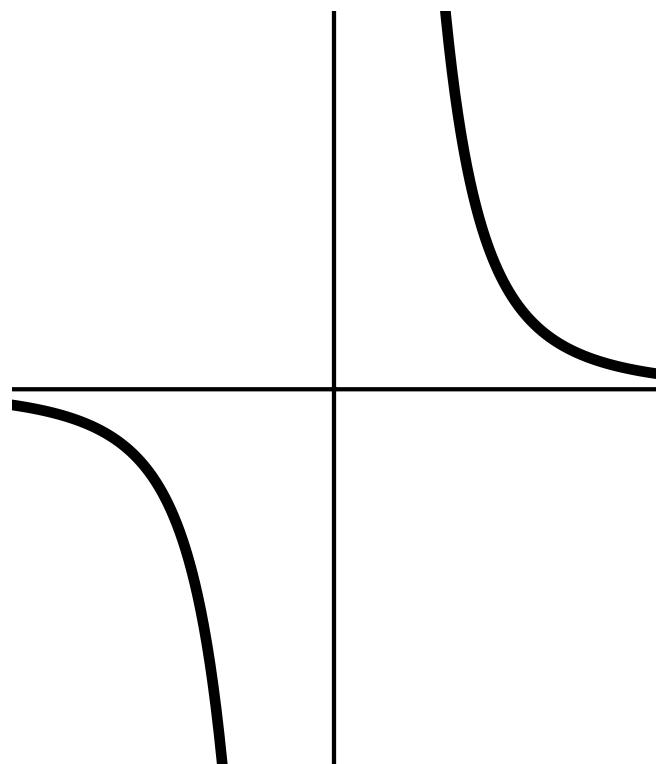
Funktionsgraphen:

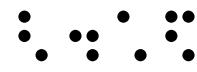
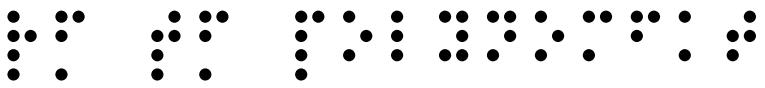
$$f(x) = 1/x^2$$



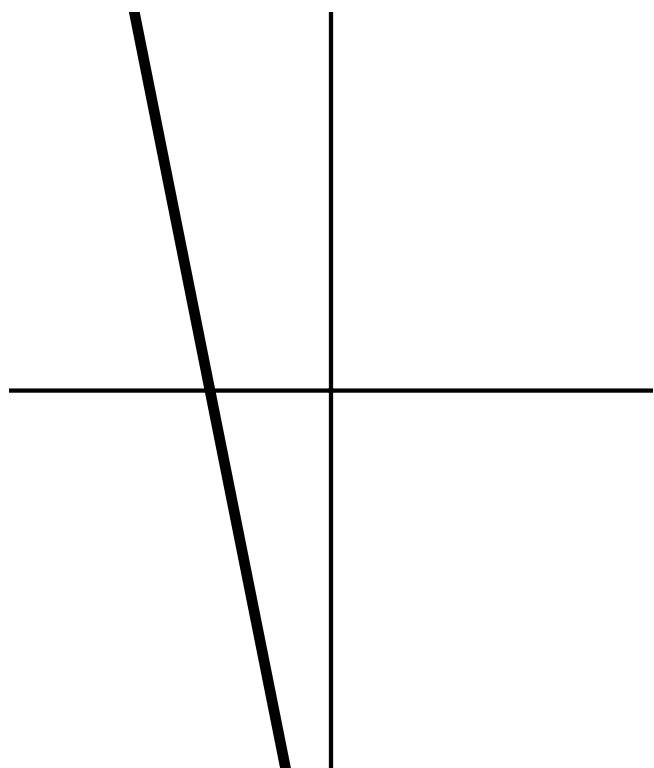
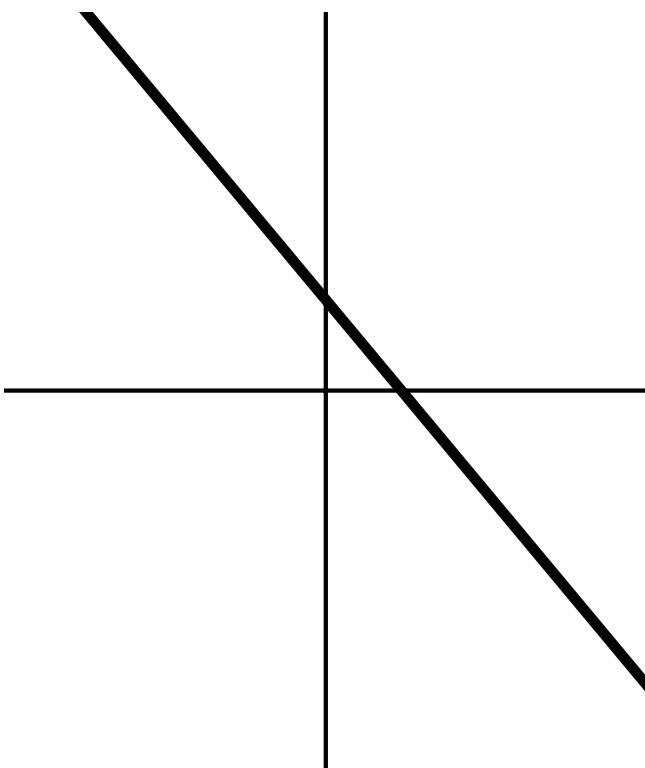
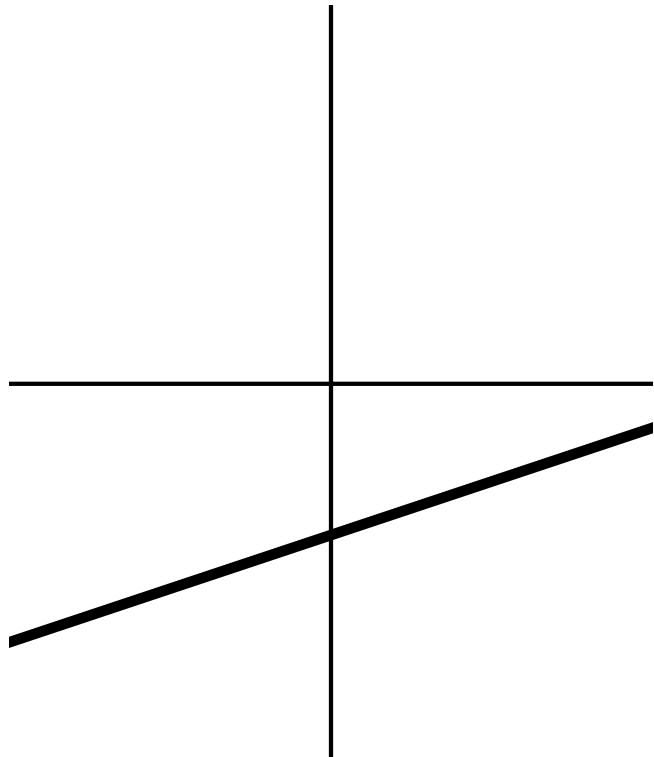
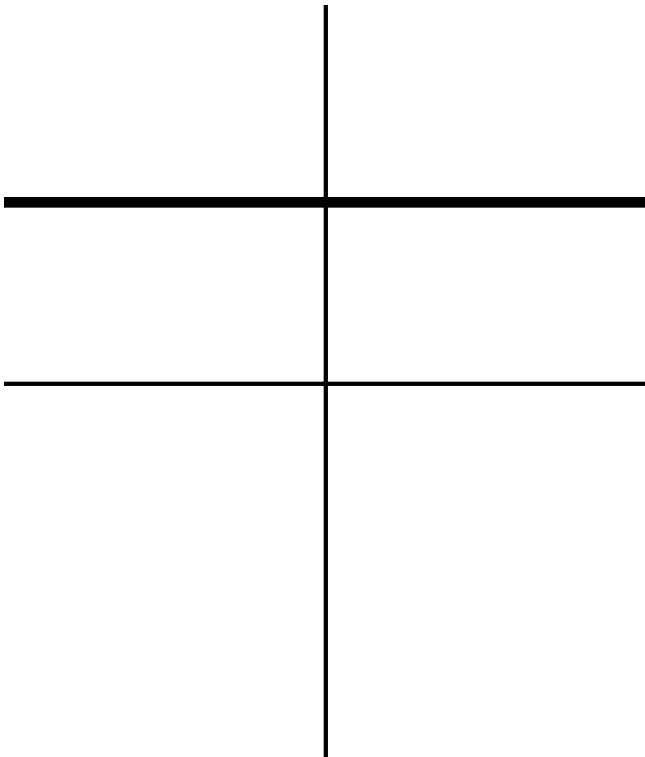
Funktionsgraphen:

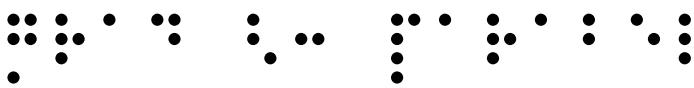
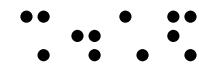
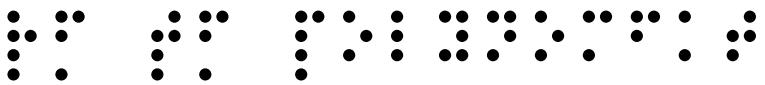
$$f(x) = 1/x^3$$



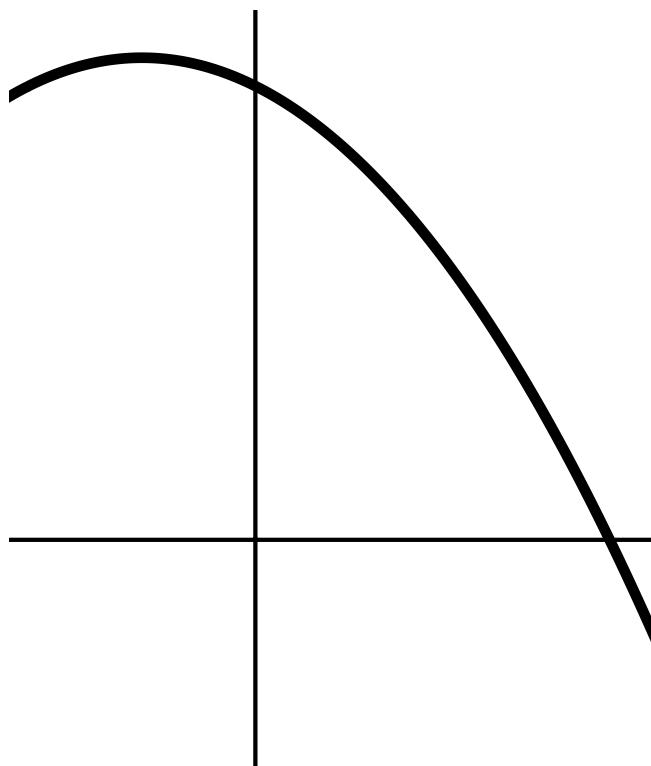
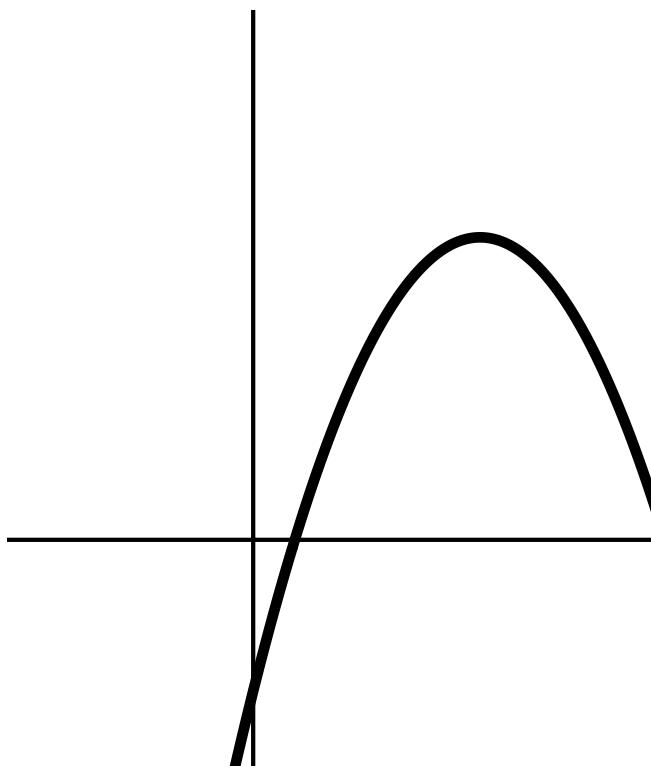
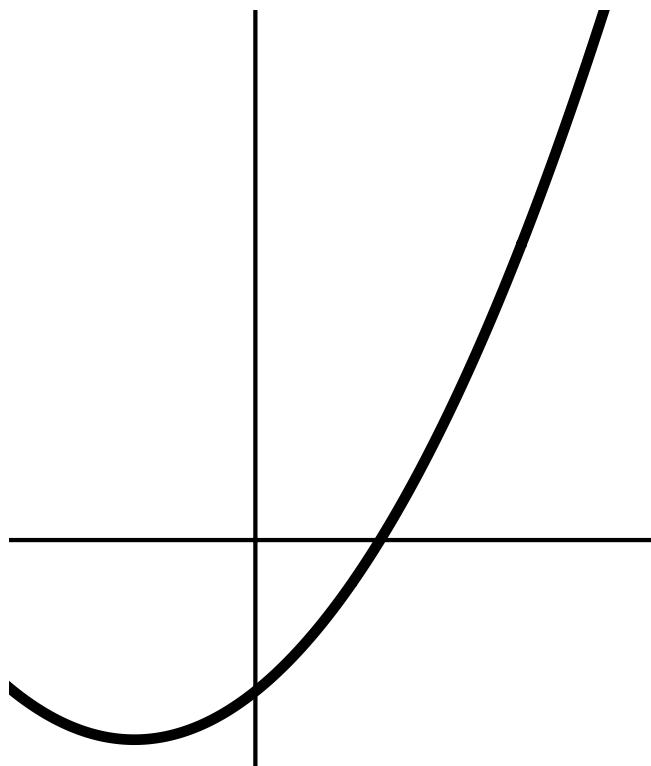
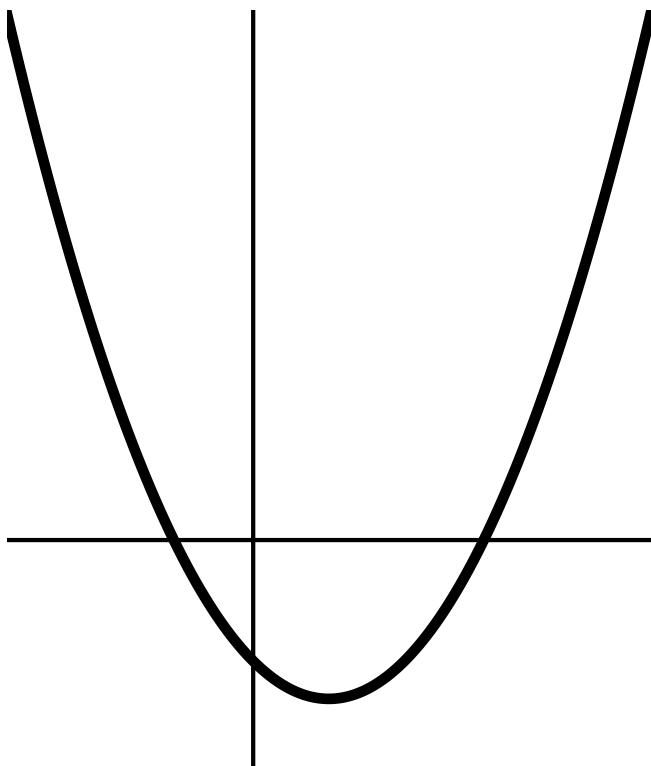


Grad 1: Gerade



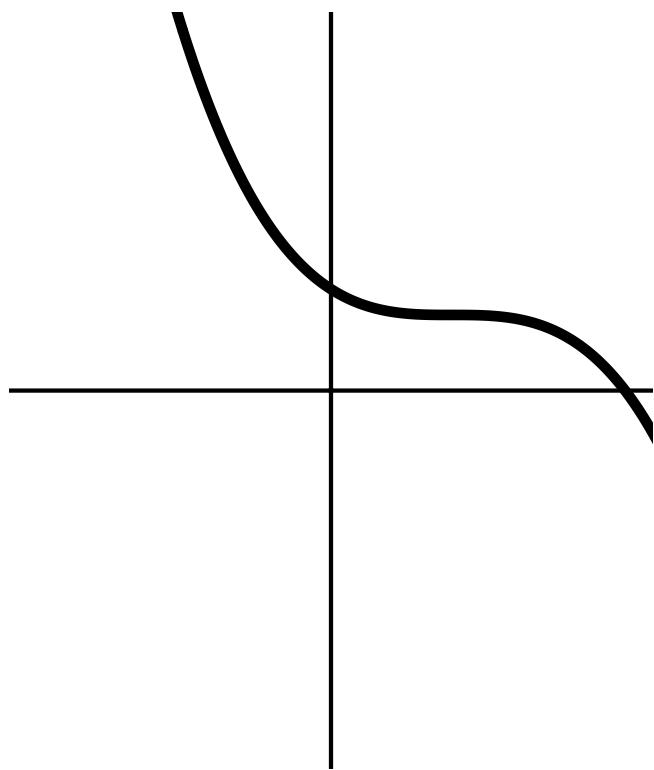
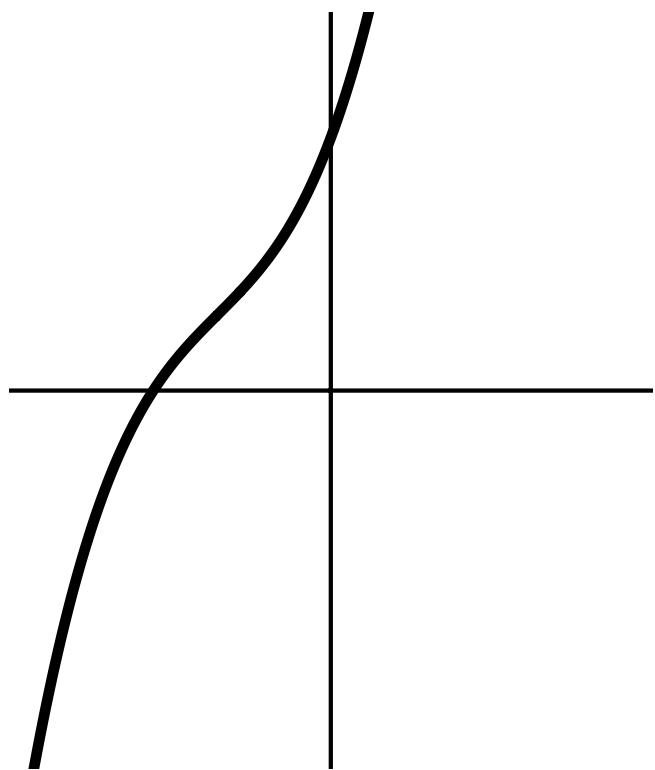
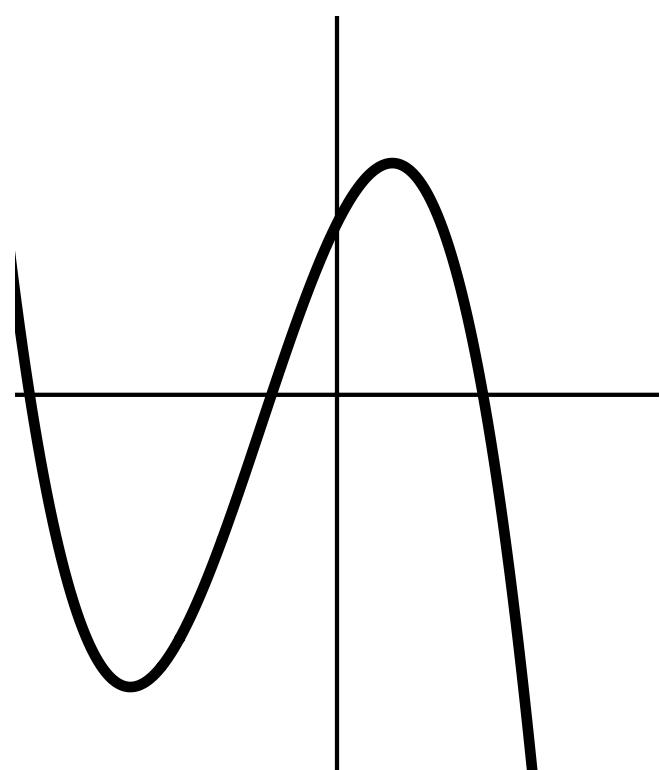
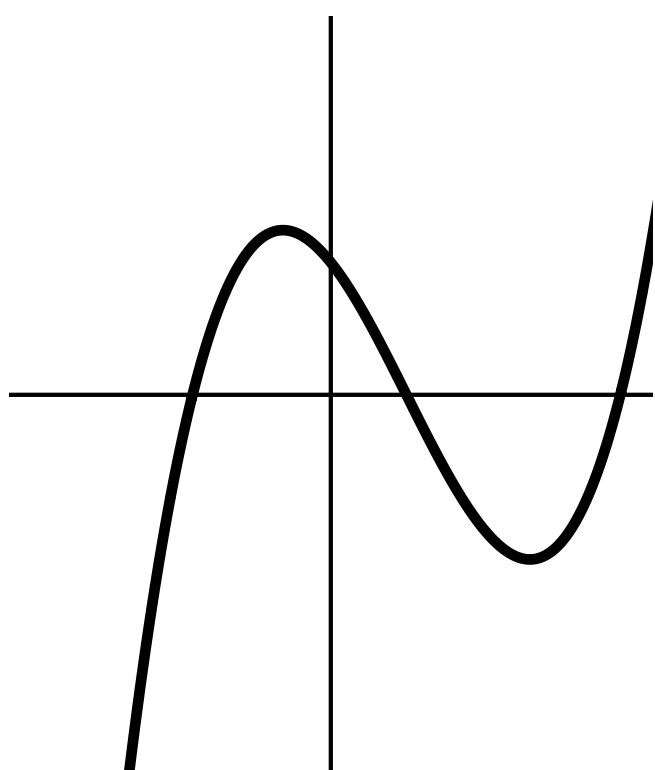


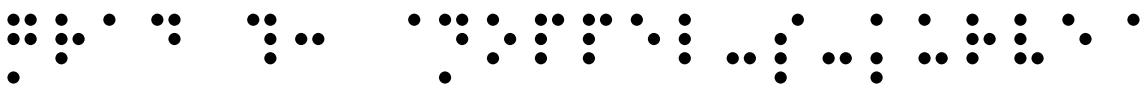
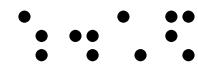
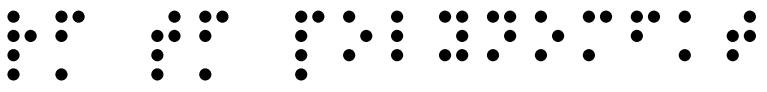
Grad 2: Parabel



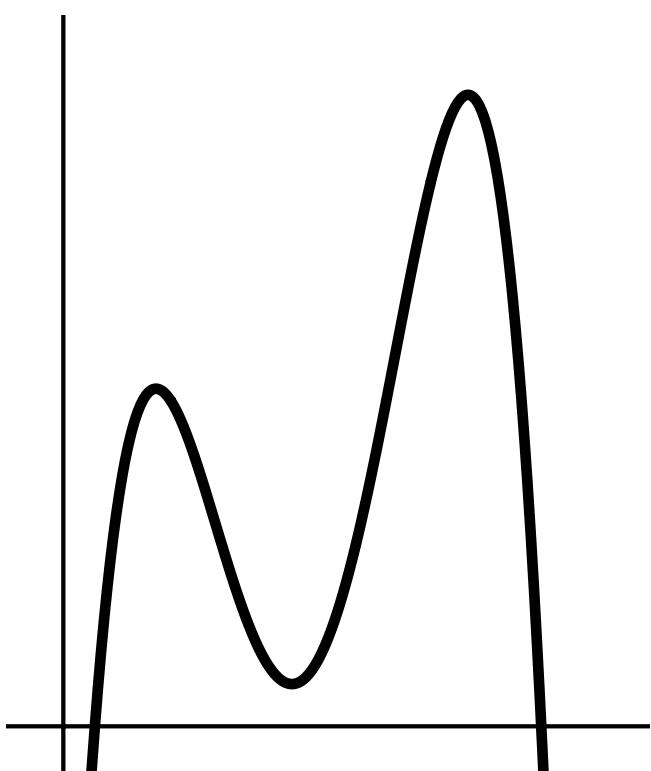
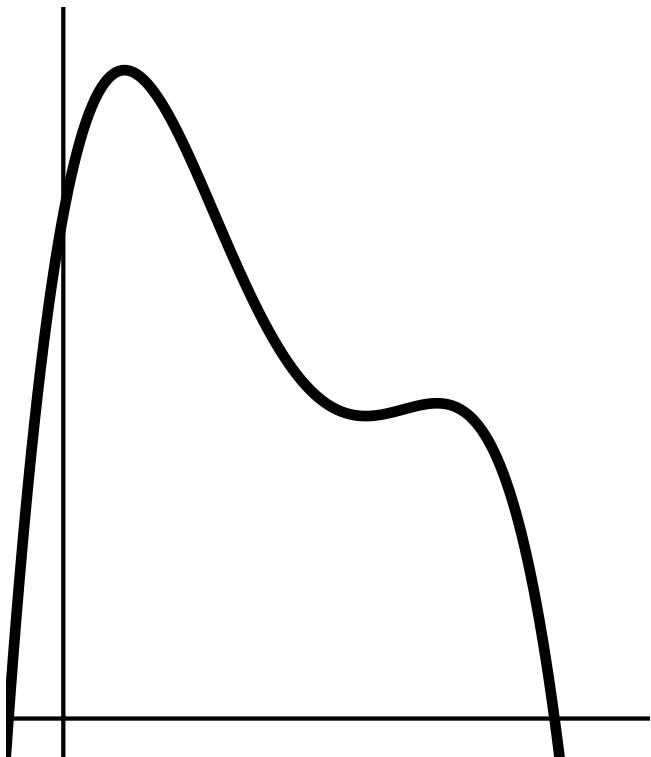
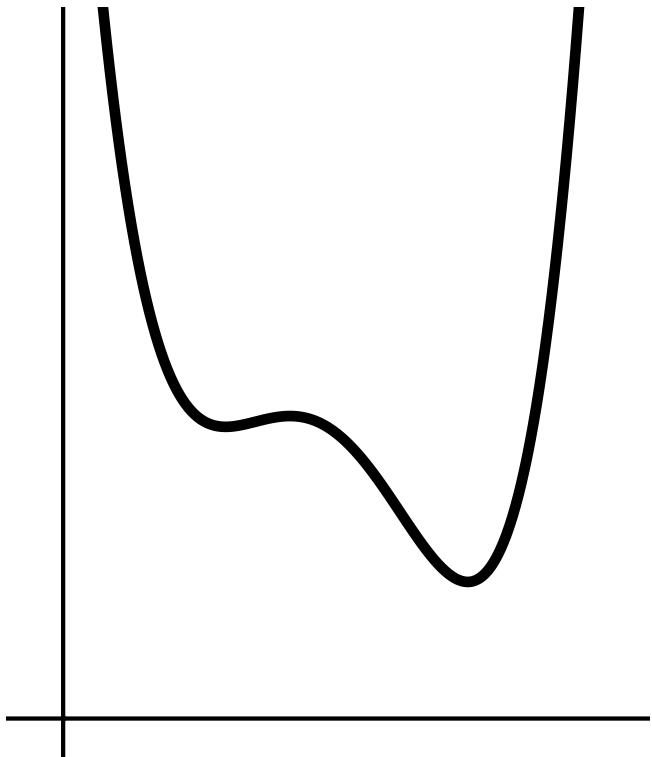
RF typische Polynomfunktionen, 4/16

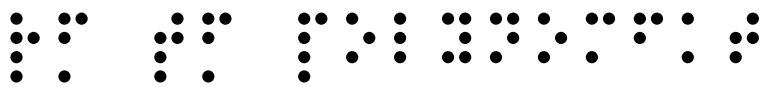
Polynomfunktionen  
Grad 3: „S-Kurve“ mit „Entartungen“





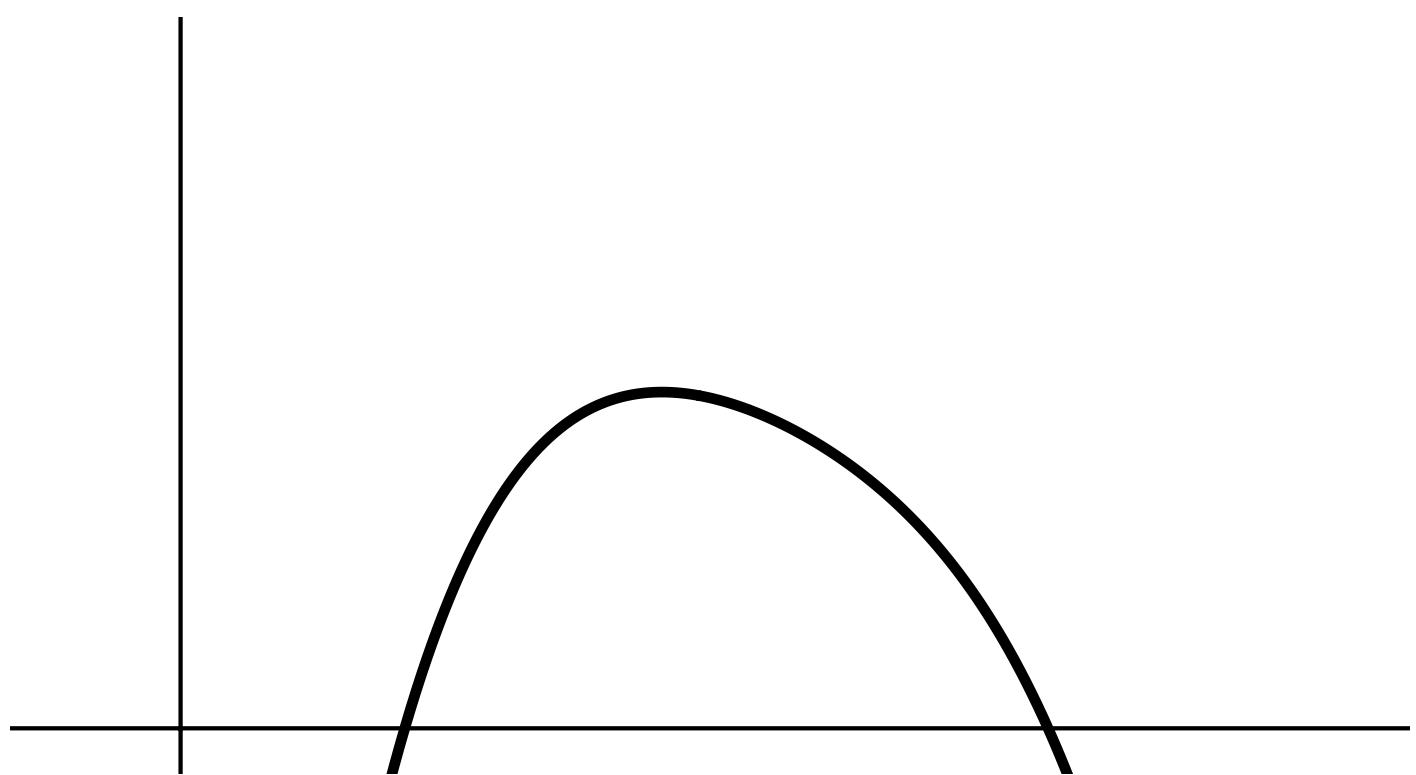
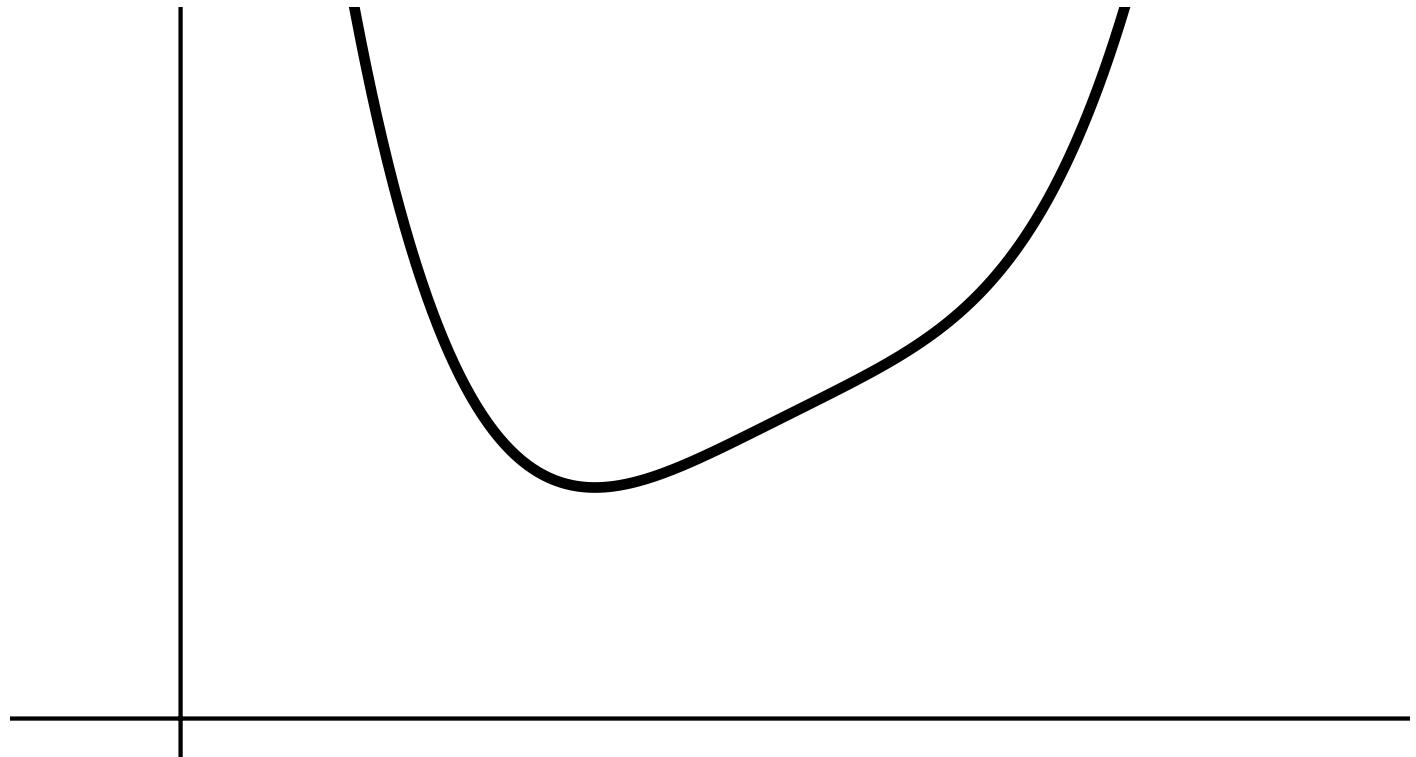
Grad 4: „Doppel-S-Kurve“





The image shows a horizontal sequence of Braille characters. Each character is a 2x3 grid of dots. The sequence reads from left to right as follows: a dot in the top-left position, a dot in the middle-left position, a dot in the bottom-left position, a dot in the top-middle position, a dot in the middle-middle position, a dot in the bottom-middle position, a dot in the top-right position, a dot in the middle-right position, and a dot in the bottom-right position. This pattern repeats three times across the page.

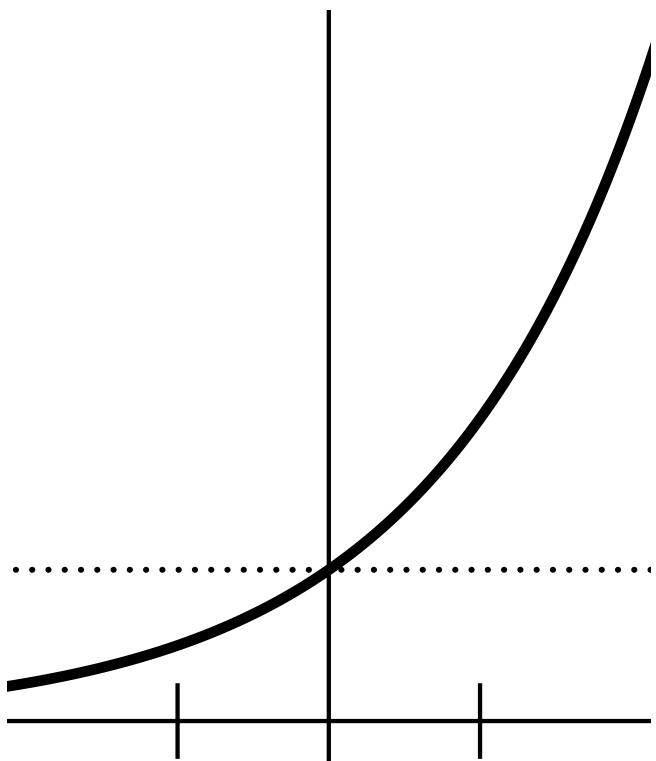
#### Grad 4: „Doppel-S-Kurve“ mit „Entartungen“





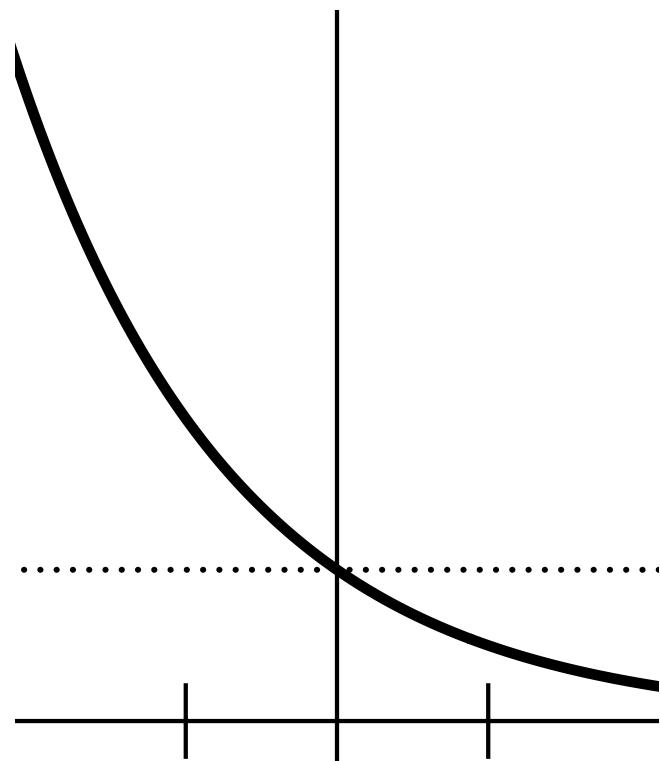
Funktionsgraphen

$$f(x) = 2^x$$



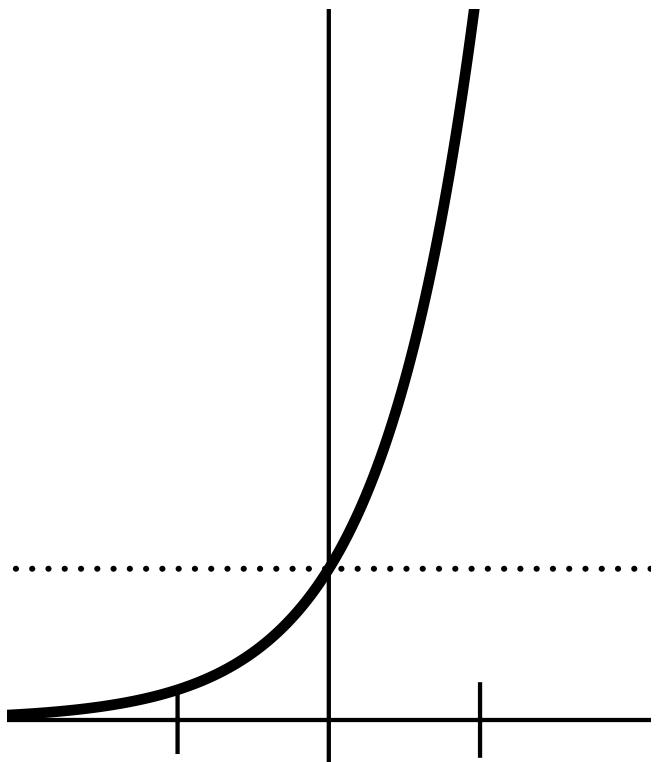
Funktionsgraphen

$$f(x) = (1/2)^x$$



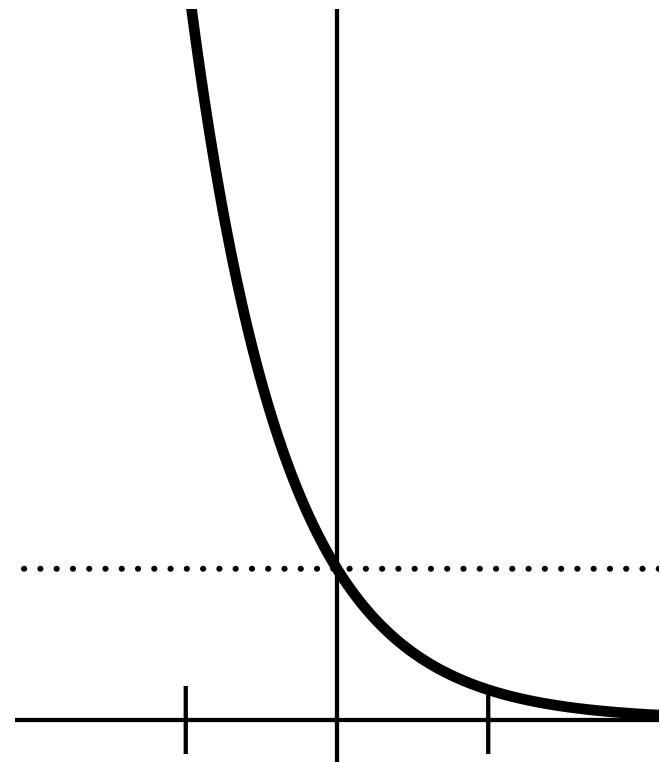
Funktionsgraphen

$$f(x) = 5^x$$

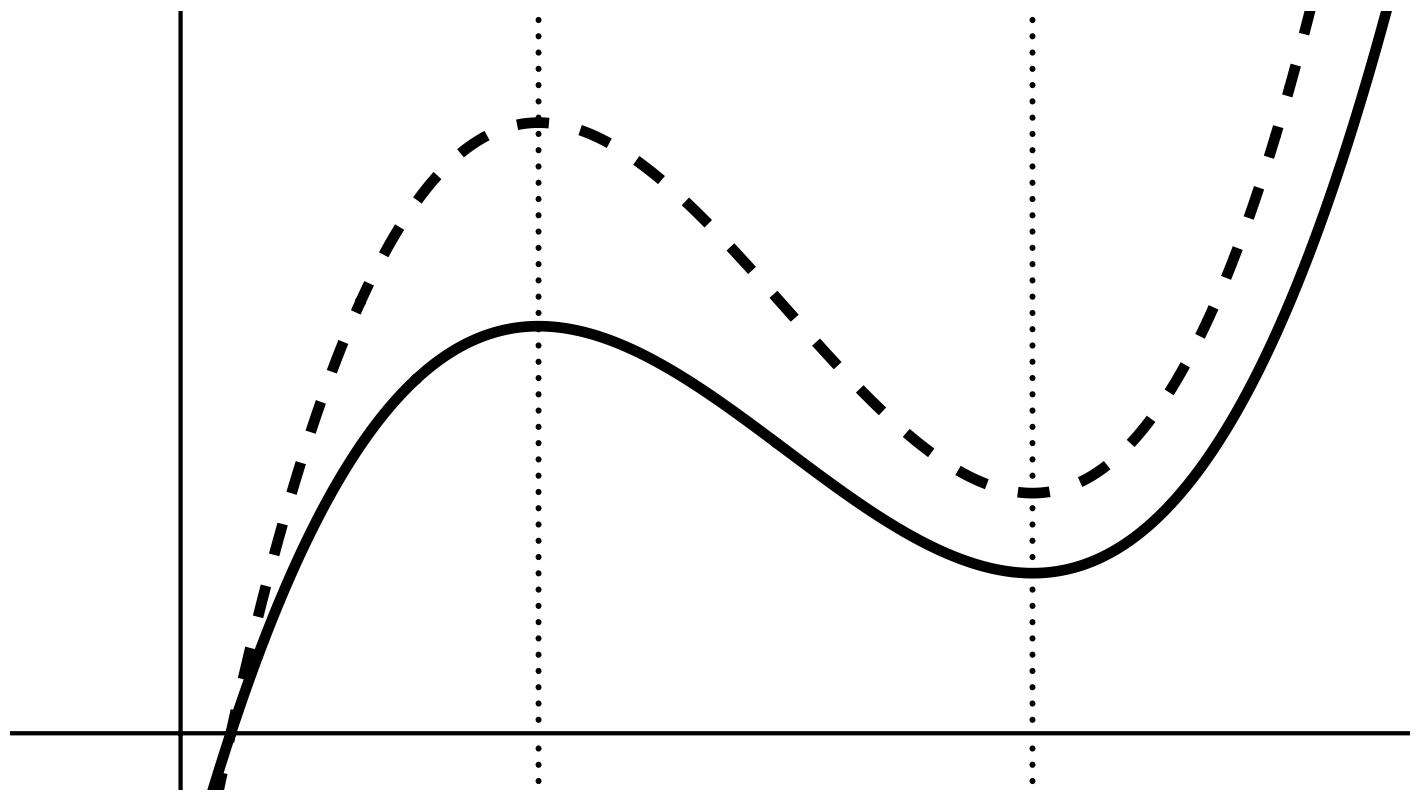


Funktionsgraphen

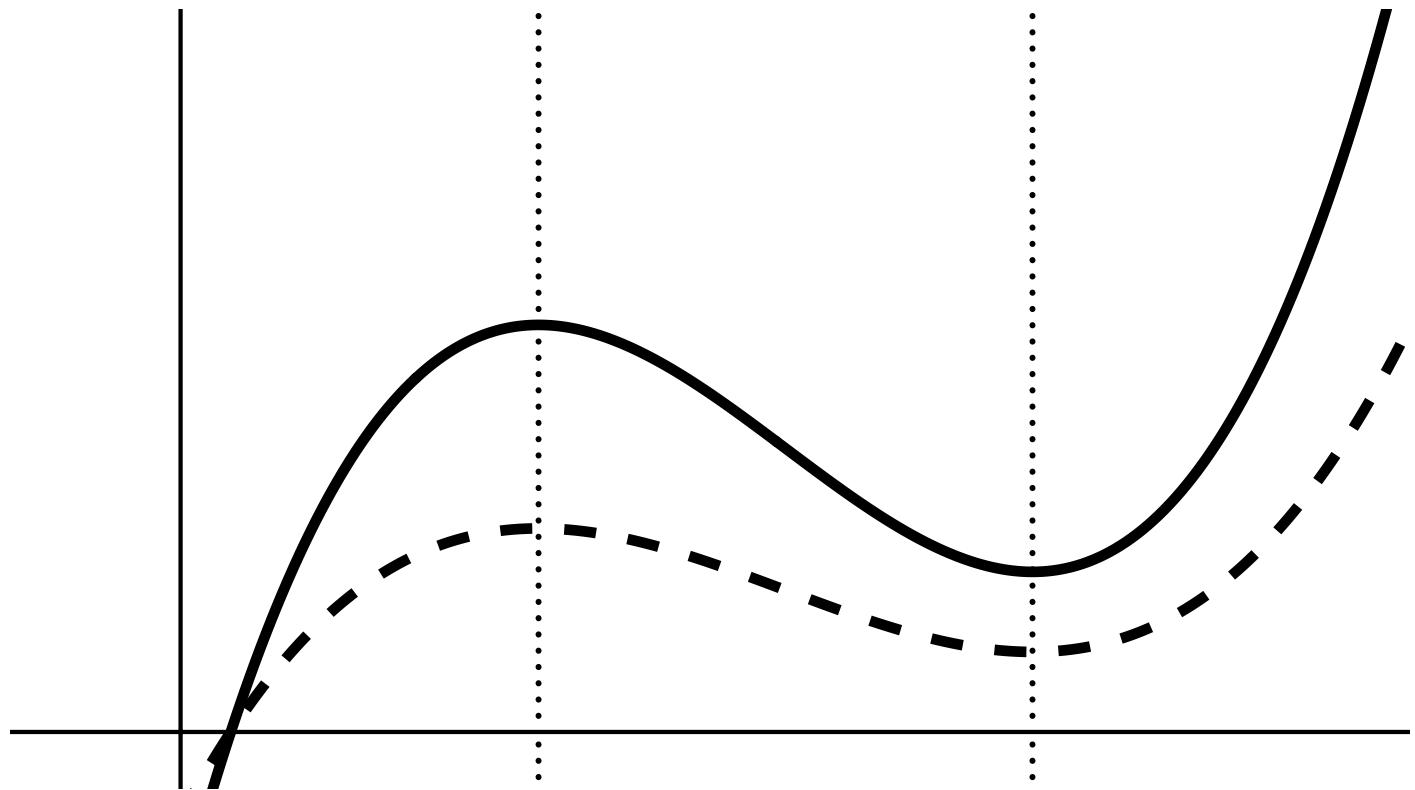
$$f(x) = (1/5)^x$$



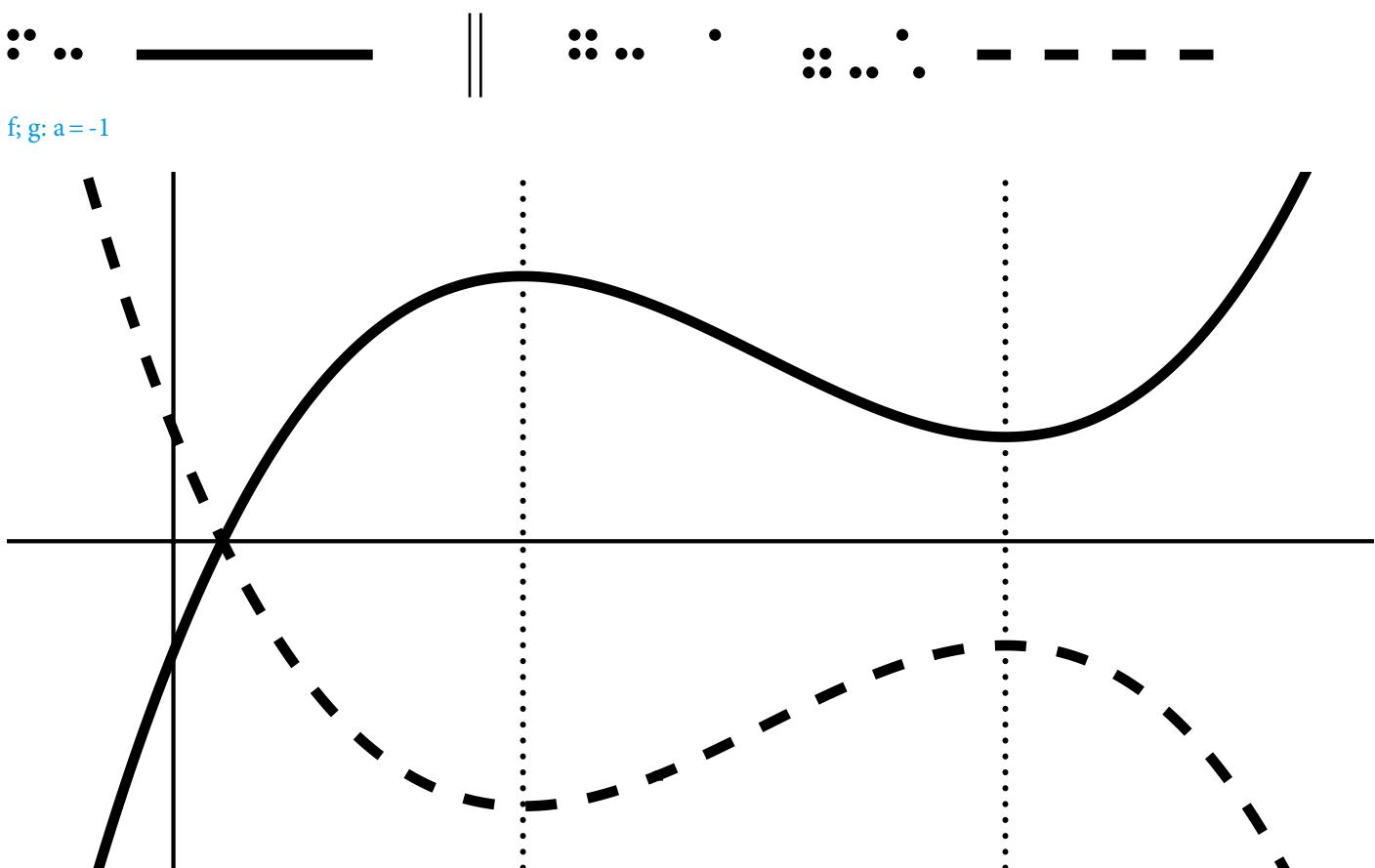
• .. — || • .. • .. - - -

f; g:  $a > 1$ 

• .. — || • .. .. ; ; ; - - -

f; h:  $0 < a < 1$ 

RF Vergleichen von Funktionen, Multiplikation:  $a \cdot f(x)$ , 9/16

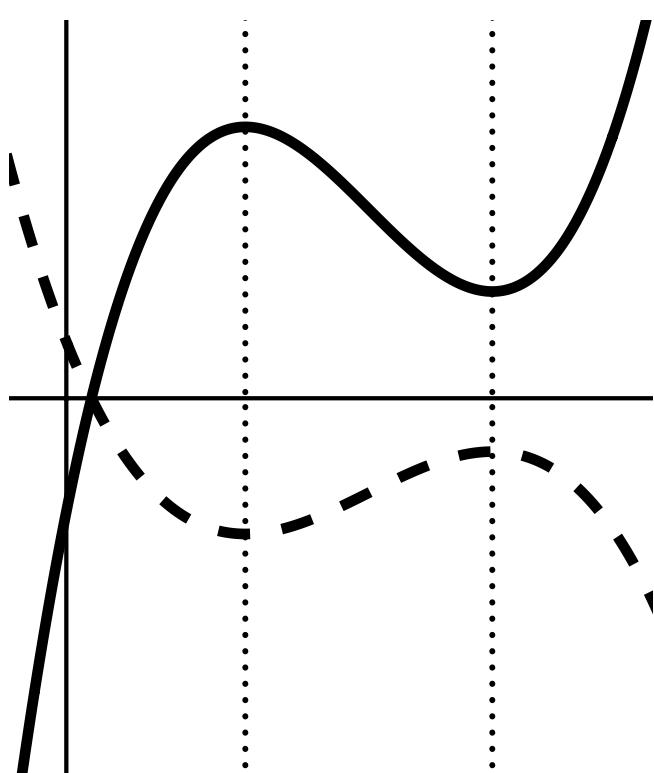
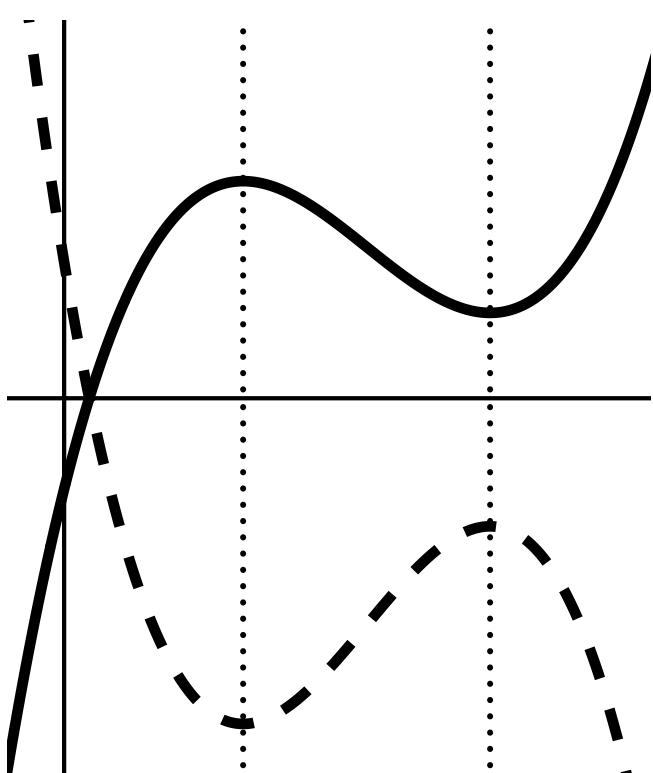


$\vdots \dots \cdot \vdots \dots \vdots \dots$

$f; h: a < -1$

$\vdots \dots \cdot \vdots \dots \vdots \dots$

$f; i: -1 < a < 0$

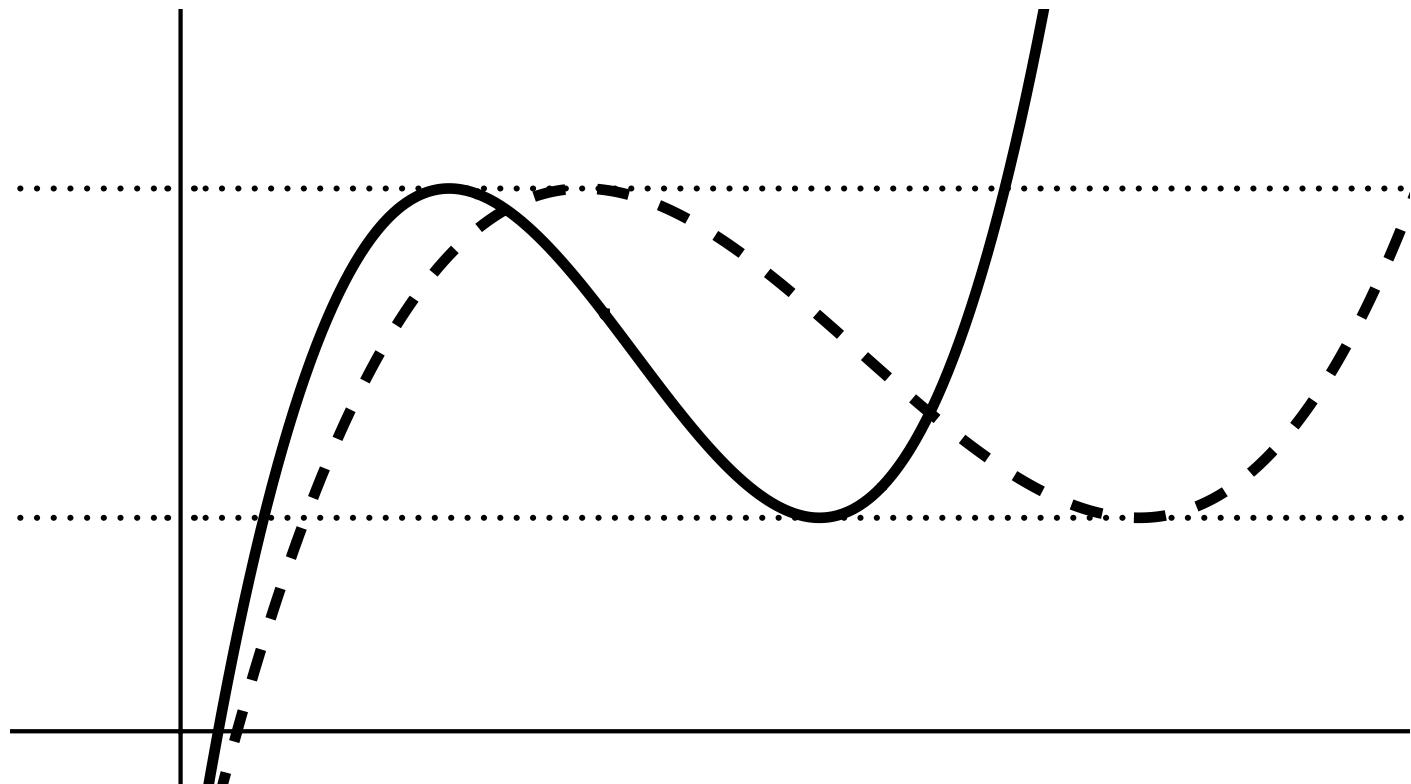


# RF Vergleichen von Funktionen, Multiplikation: $f(b \cdot x)$ , 10/16

Diagramm zur Multiplikation von Funktionen:

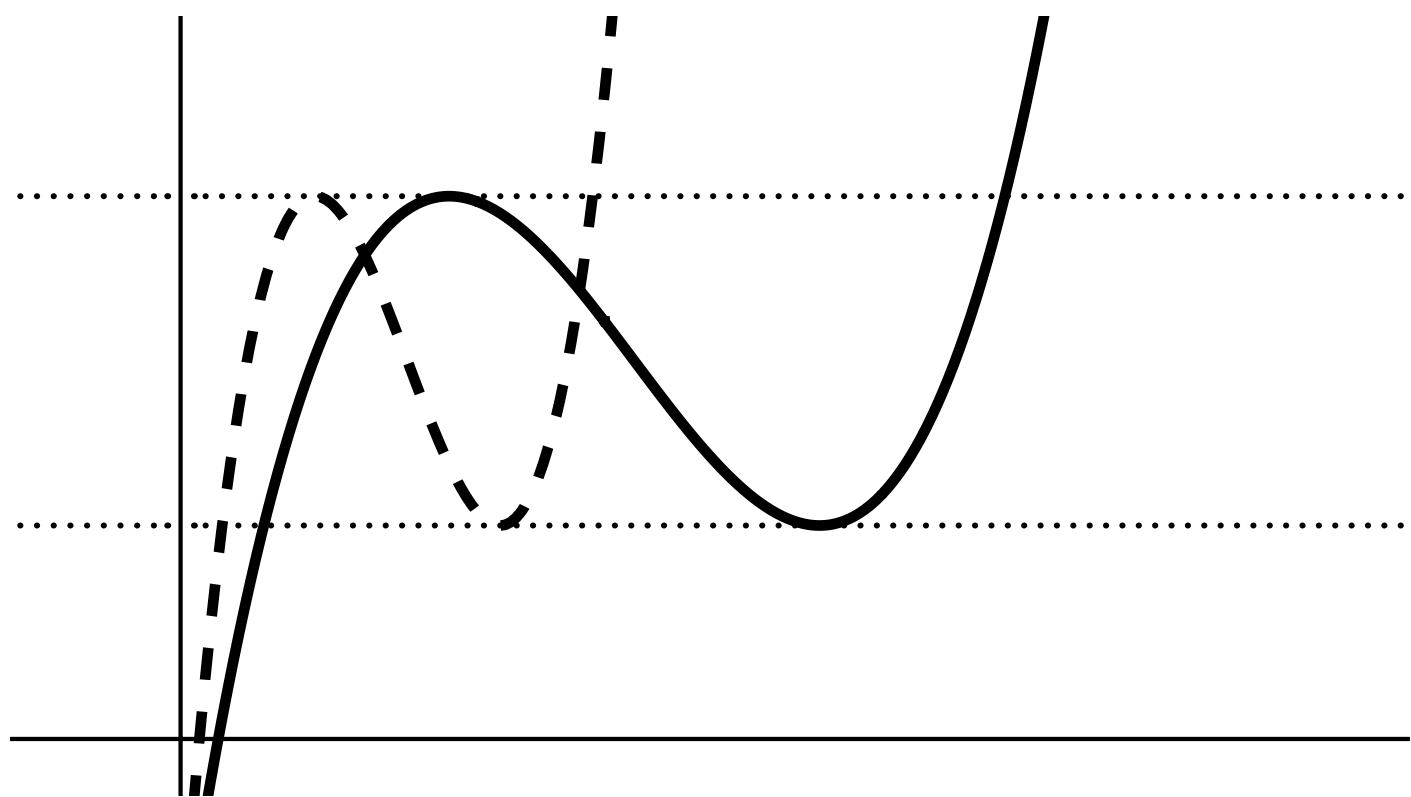
Oben:  $f$  (durchgezogene Linie) und  $g$  (gestrichelte Linie). Die Achsen sind horizontal beschriftet mit Punkten (..), einem vertikalen Strich (|) und einem horizontalen Strich (—).

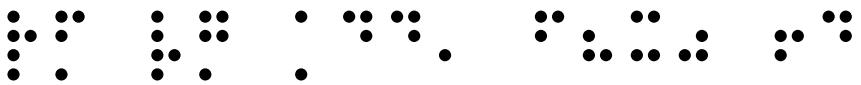
$f; g: b > 1$



Unten:  $f$  (durchgezogene Linie) und  $h$  (gestrichelte Linie). Die Achsen sind horizontal beschriftet mit Punkten (..), einem vertikalen Strich (|) und einem horizontalen Strich (—).

$f; h: 0 < b < 1$





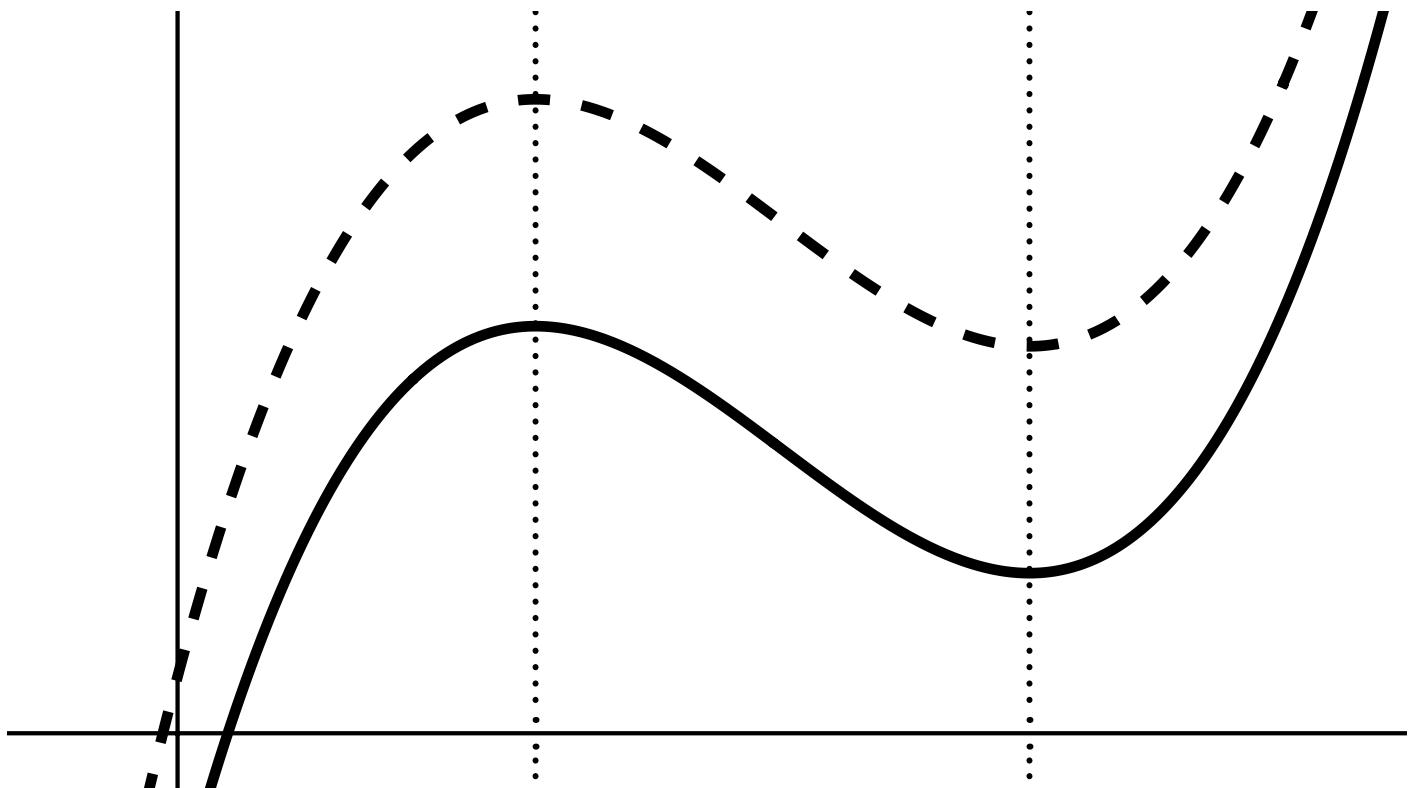
.. —

||

.. ..

.. ..

$f; g; d > 0$



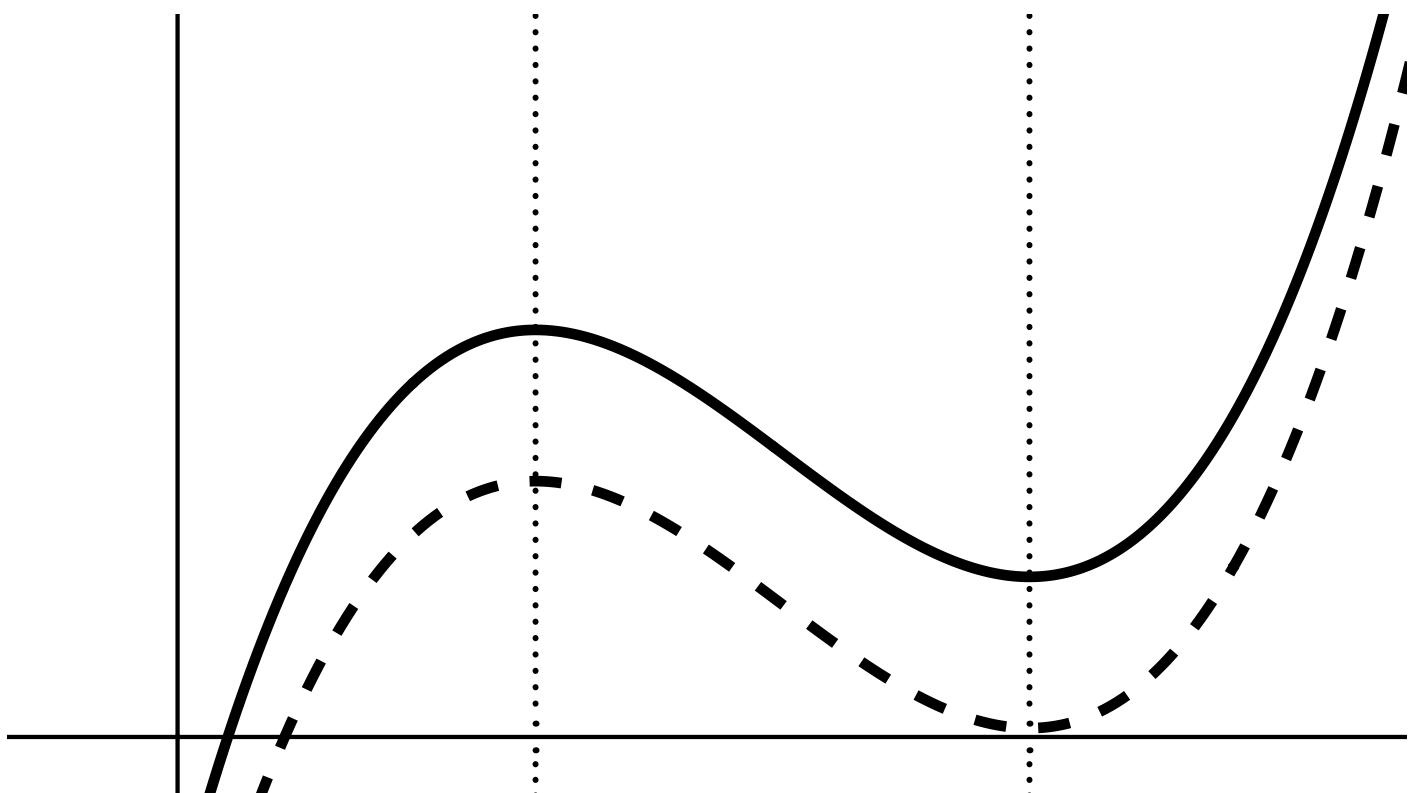
.. —

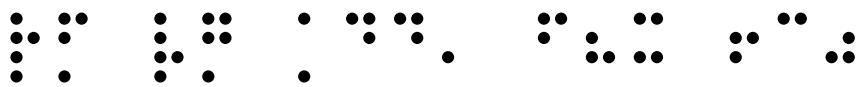
||

.. ..

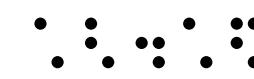
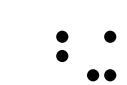
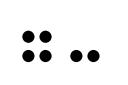
.. ..

$f; h; d < 0$

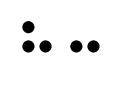
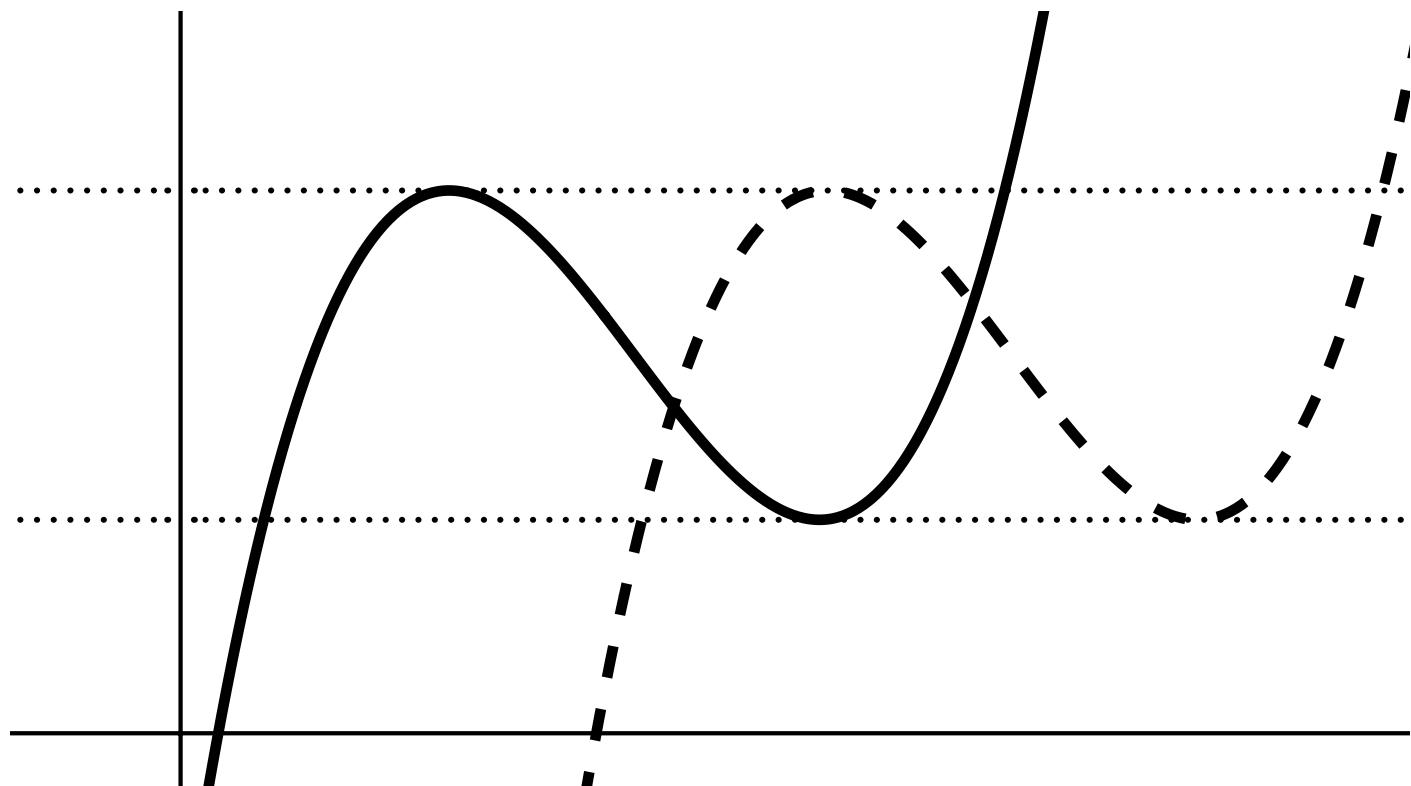




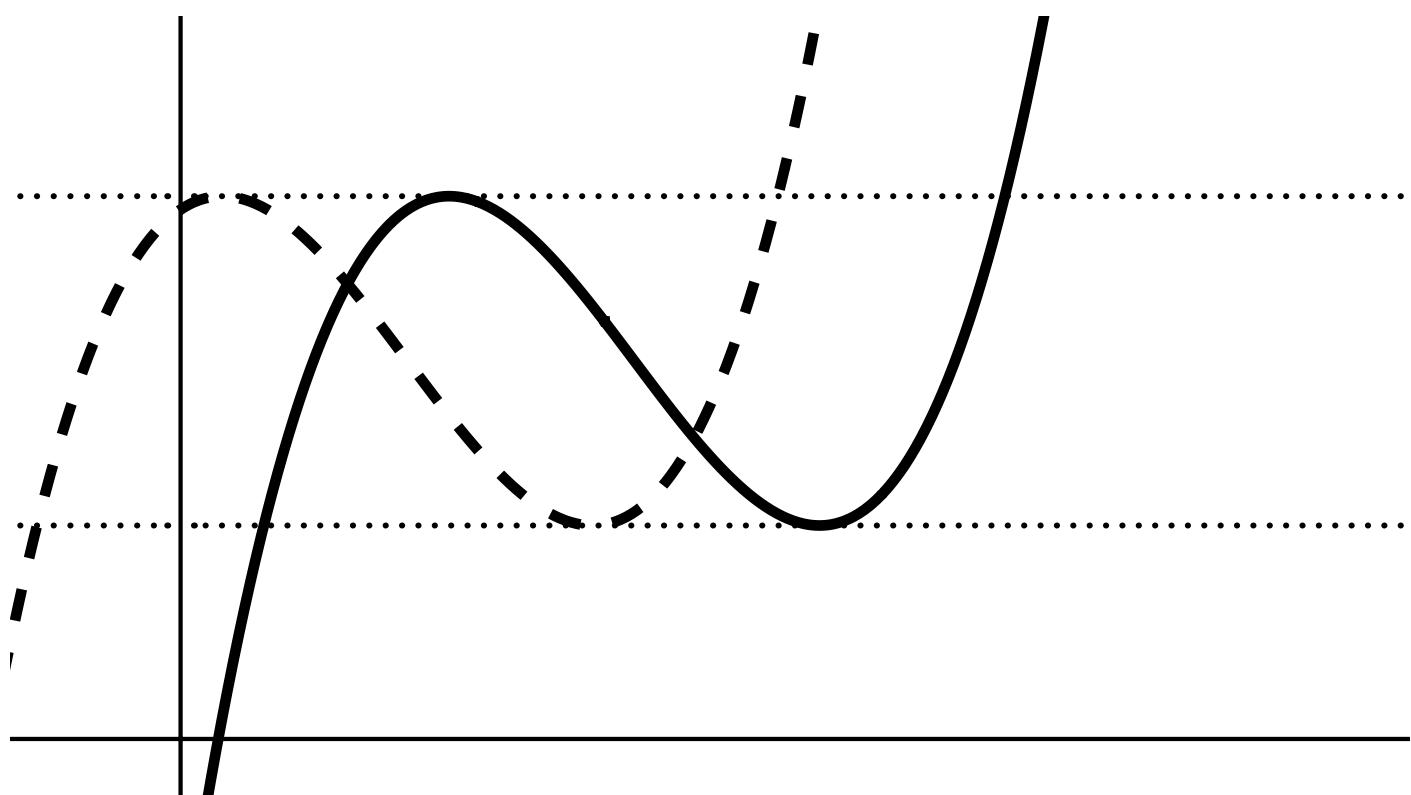
RF Vergleichen von Funktionen, Addition:  $f(x+c)$ , 12/16

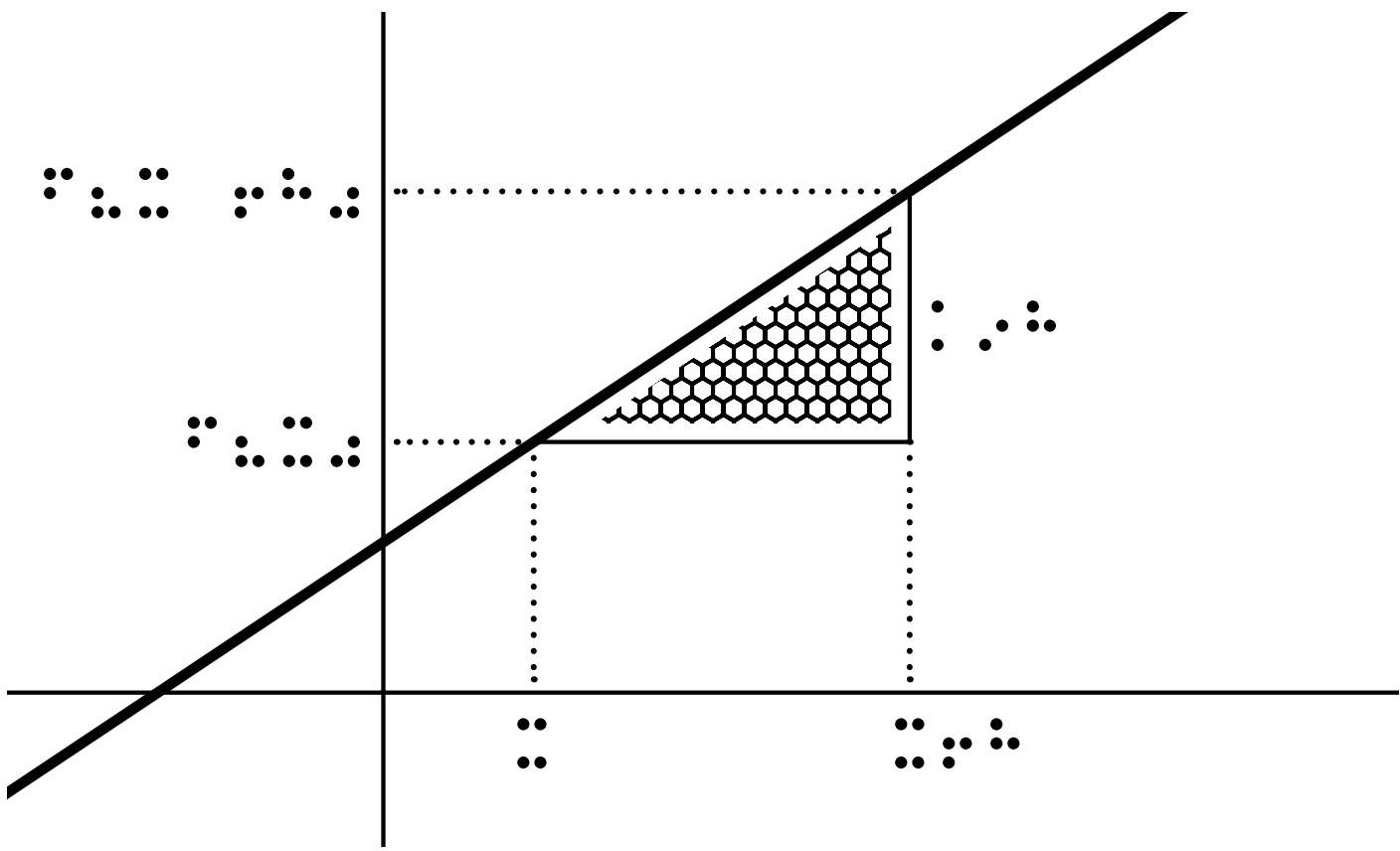
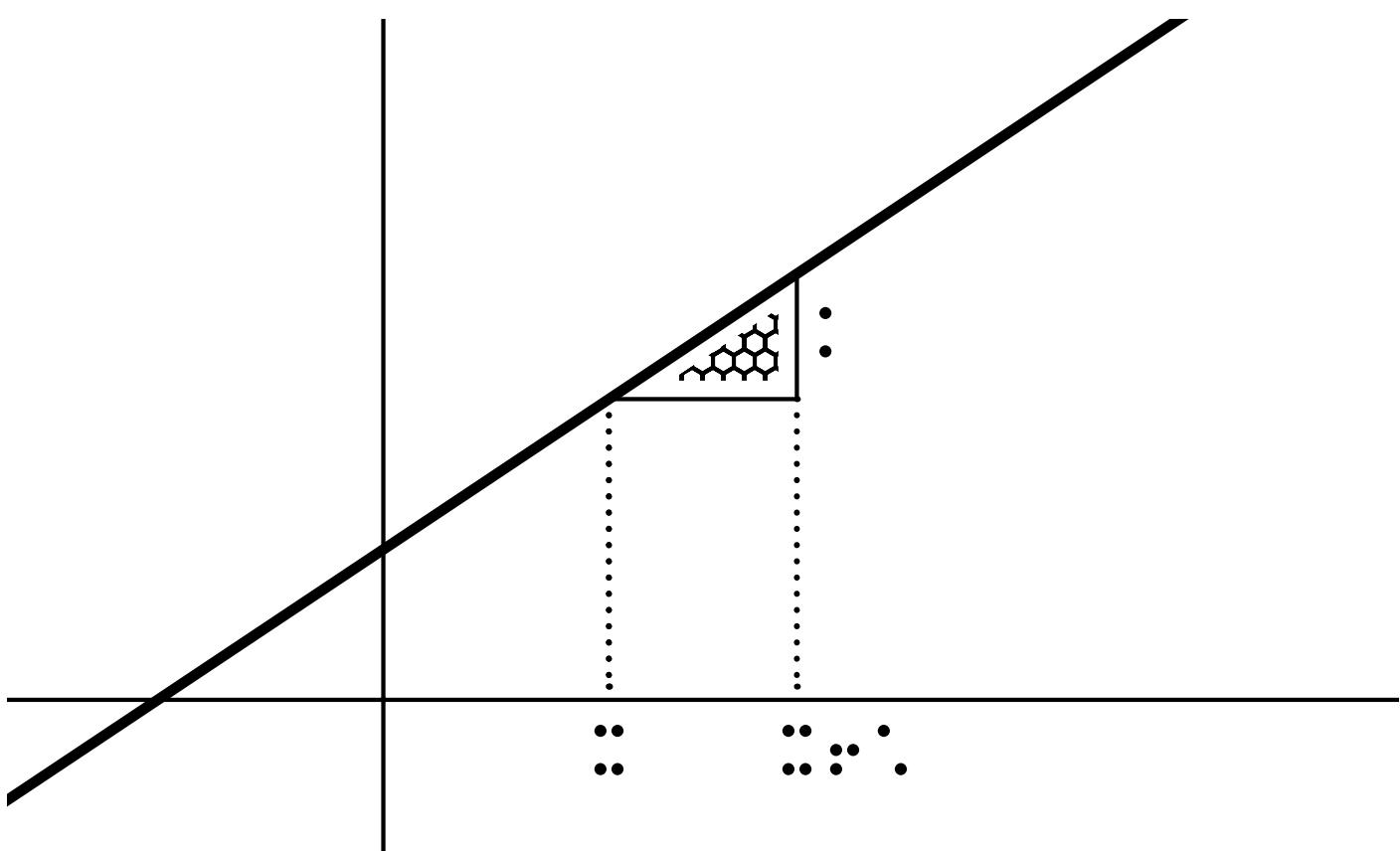


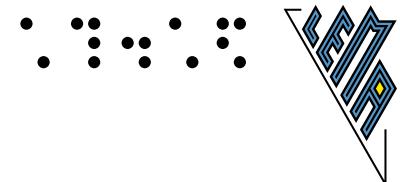
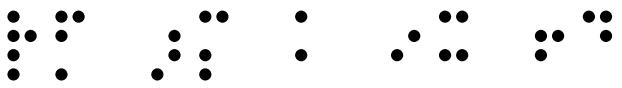
$f; g; c > 0$



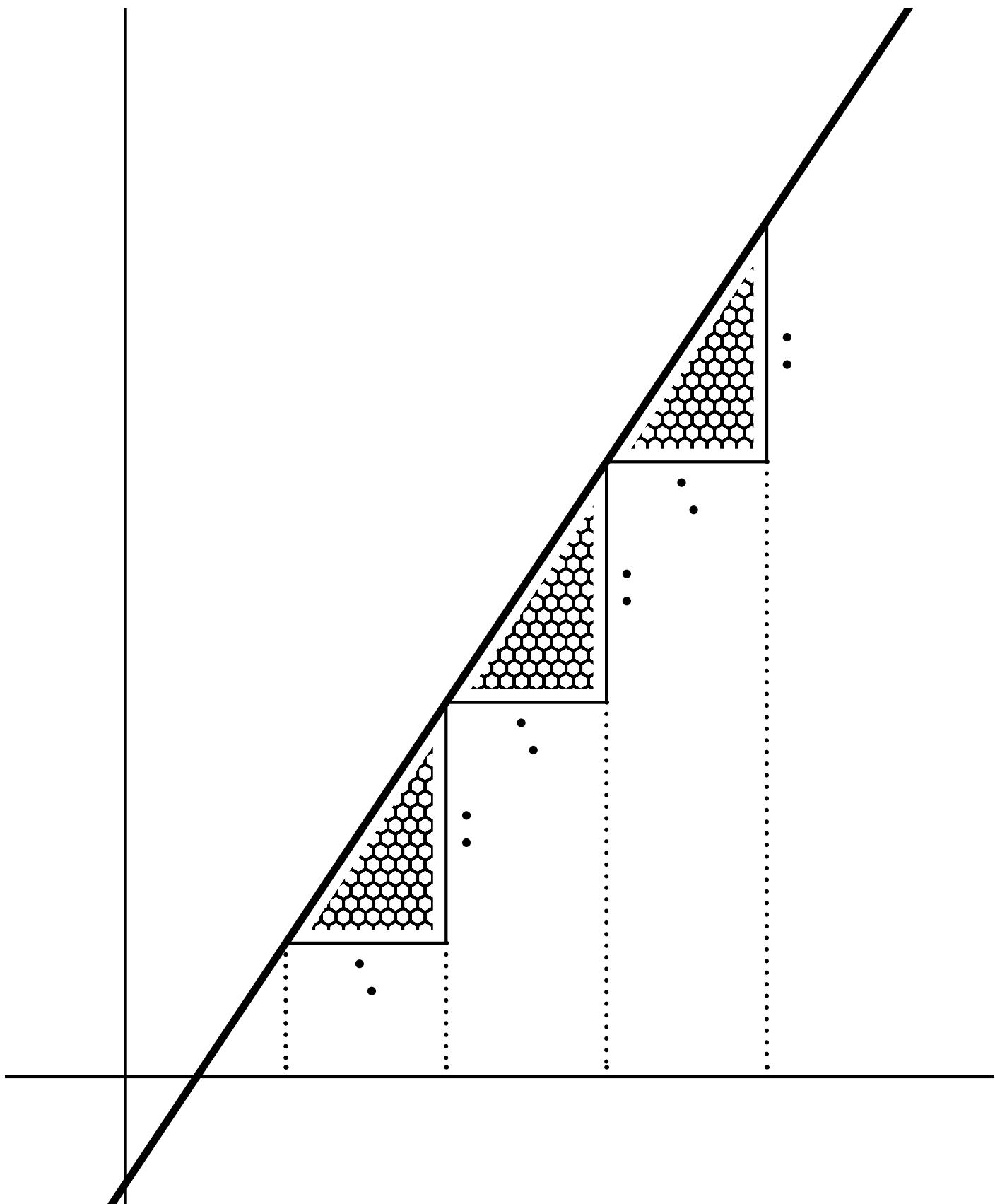
$f; h; c < 0$



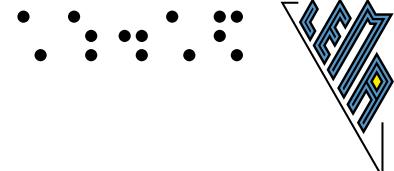




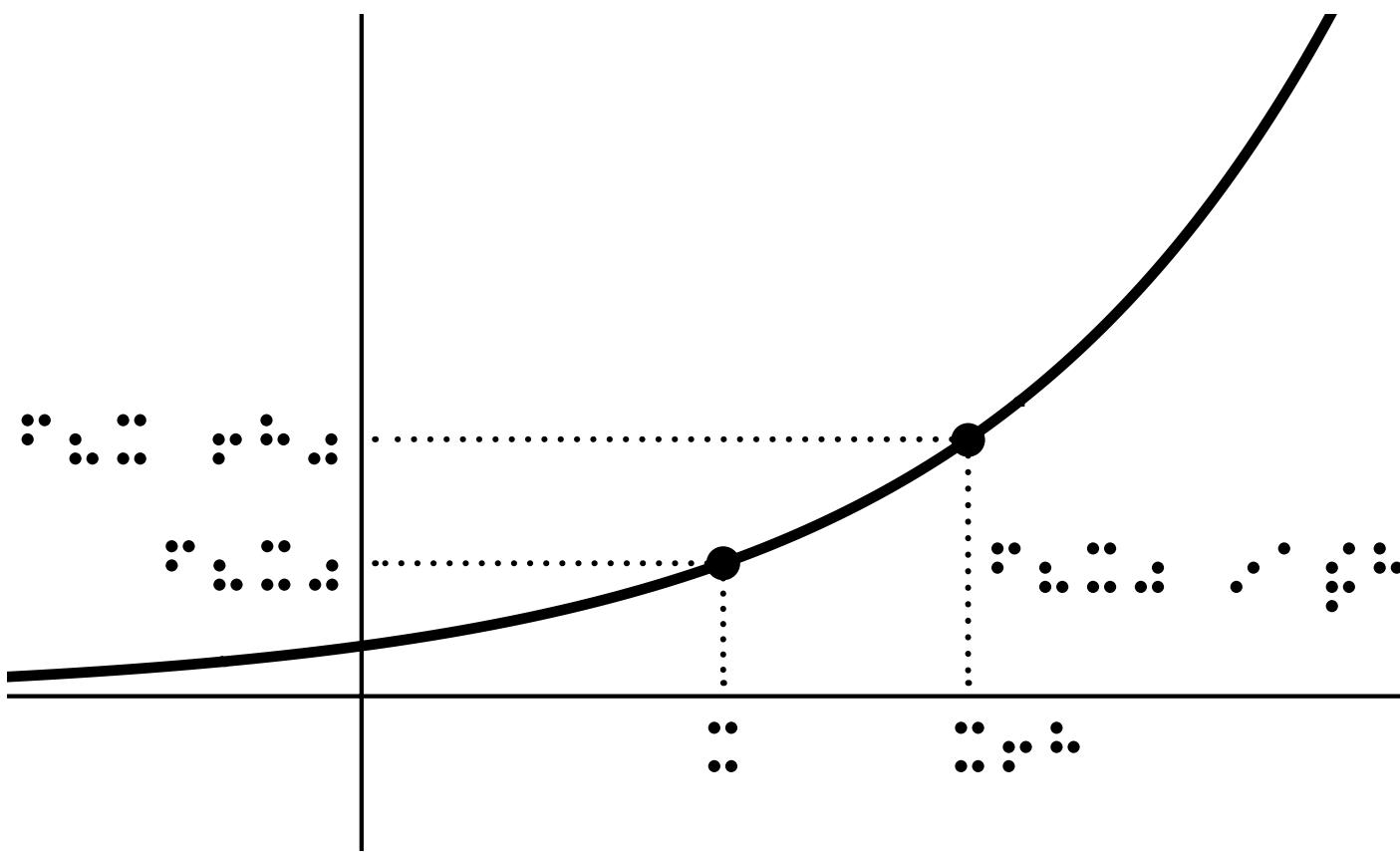
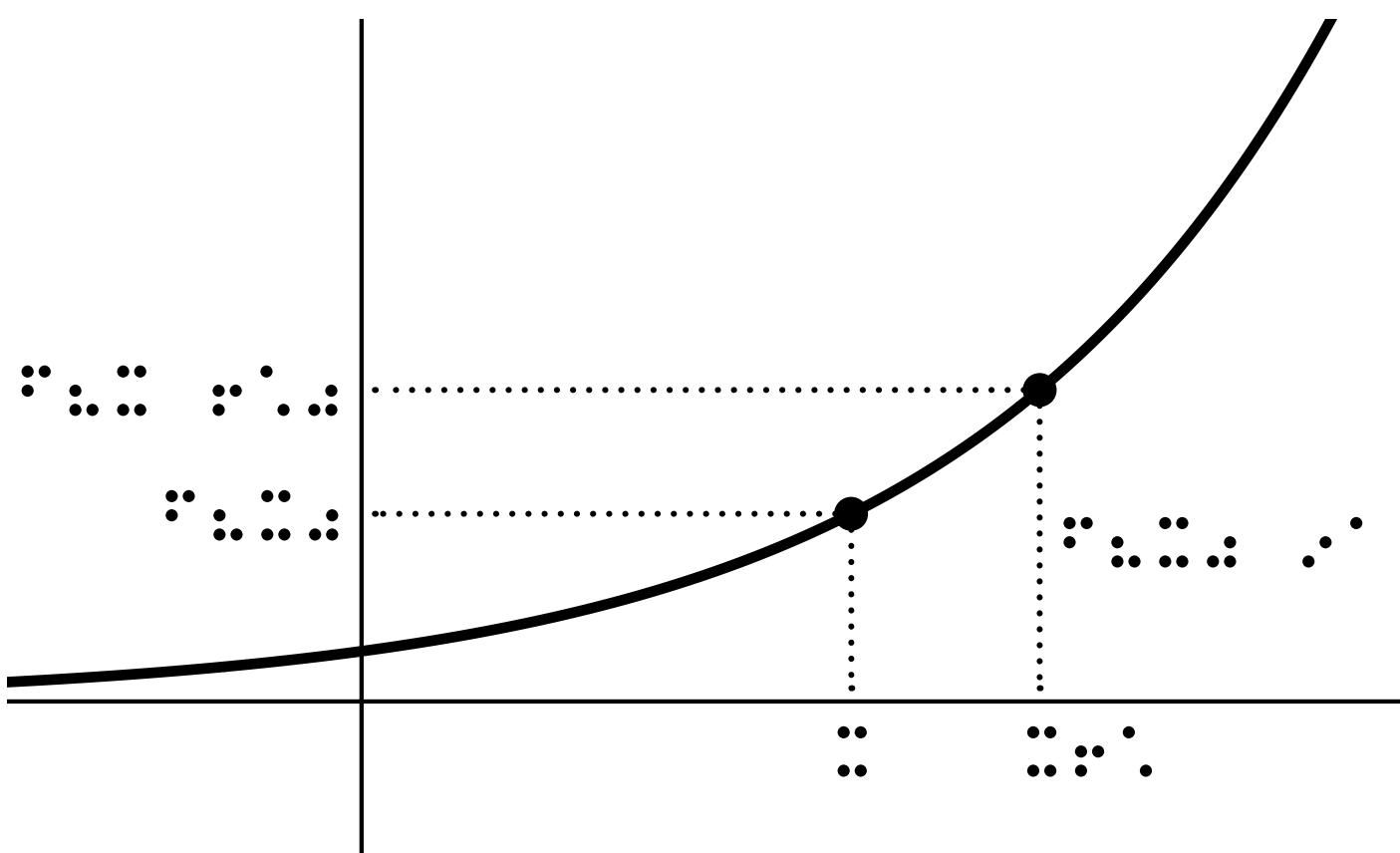
RF Änderungsmaße:  $k \cdot x + d$ , 14/16

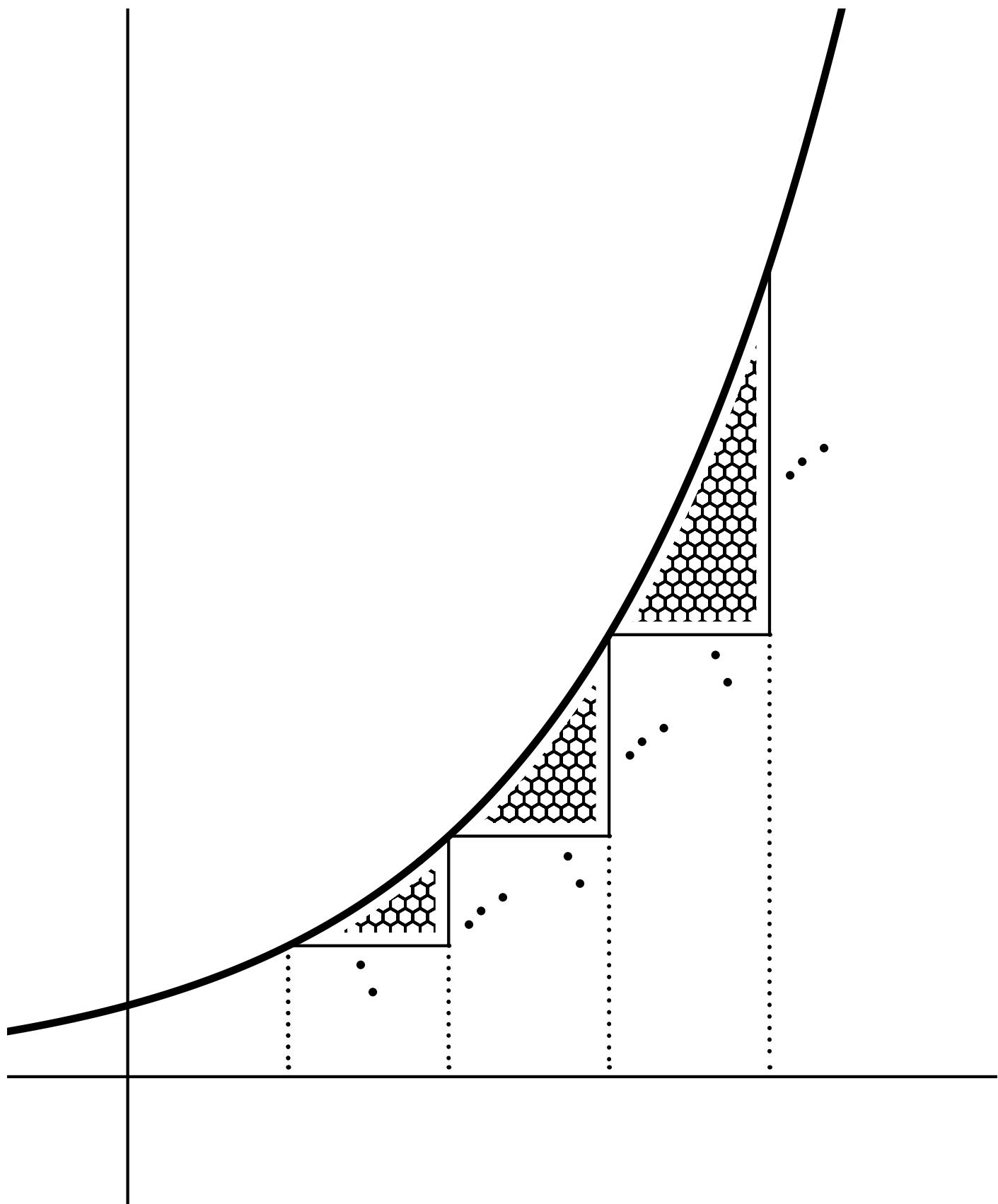


IT AG



RF Änderungsmaße:  $c^*a^x$ , 15/16







## **Impressum**

Verlag: Braille-Zentrum in Zusammenarbeit mit der Abteilung für Inklusion und Lehrmittel (LMZ) am Bundes-Blindeninstitut Wien

Autor: Elisabeth Stanetty

Grafiken: Alexander Steiner, Angela Prendl

Erstellungsjahr: 2021

2. Auflage