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Blindeninstitut
Wien

LeMa
Lehr- und Lernmaterialien

Vergrößerte und aufbereitete Versionen
ausgewählter Grafiken aus dem Schulbuch

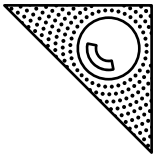
Angewandte Mathematik HAK 4



Grafiken: Tomáš Batha

LEMA - Lehr- und Lernmaterialien

Hergestellt von der Abteilung für Inklusion und Lehrmittel am Bundes-Blindeninstitut Wien,
Wittelsbachstraße 5, 1020 Wien, Tel.: 01/728 08 66-405, 406, lmz@bbi.at, www.bbi.at

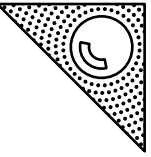


Angewandte Mathematik

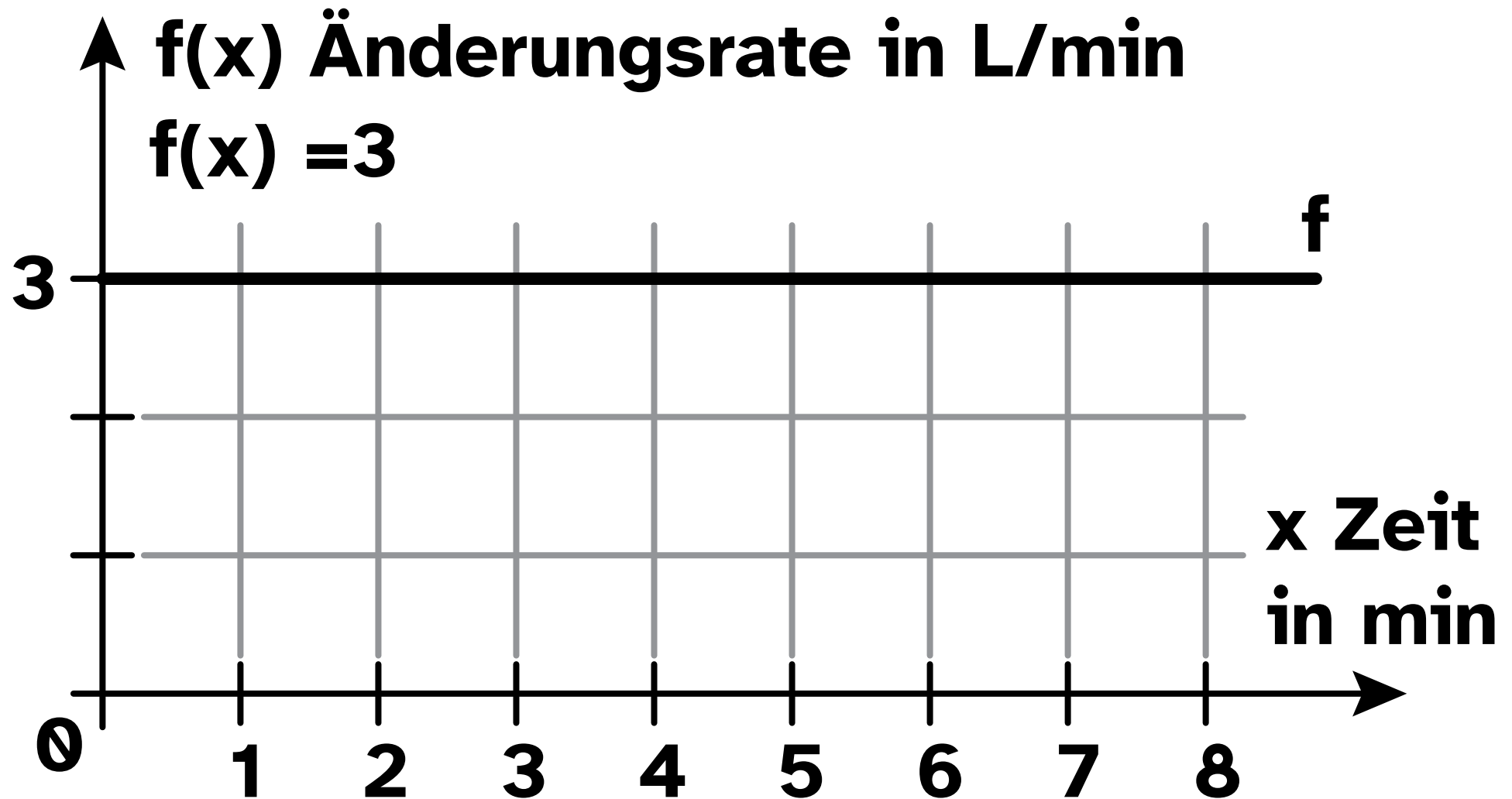
HAK Band 4

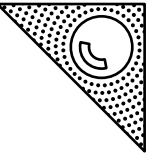
5.

Integralrechnung

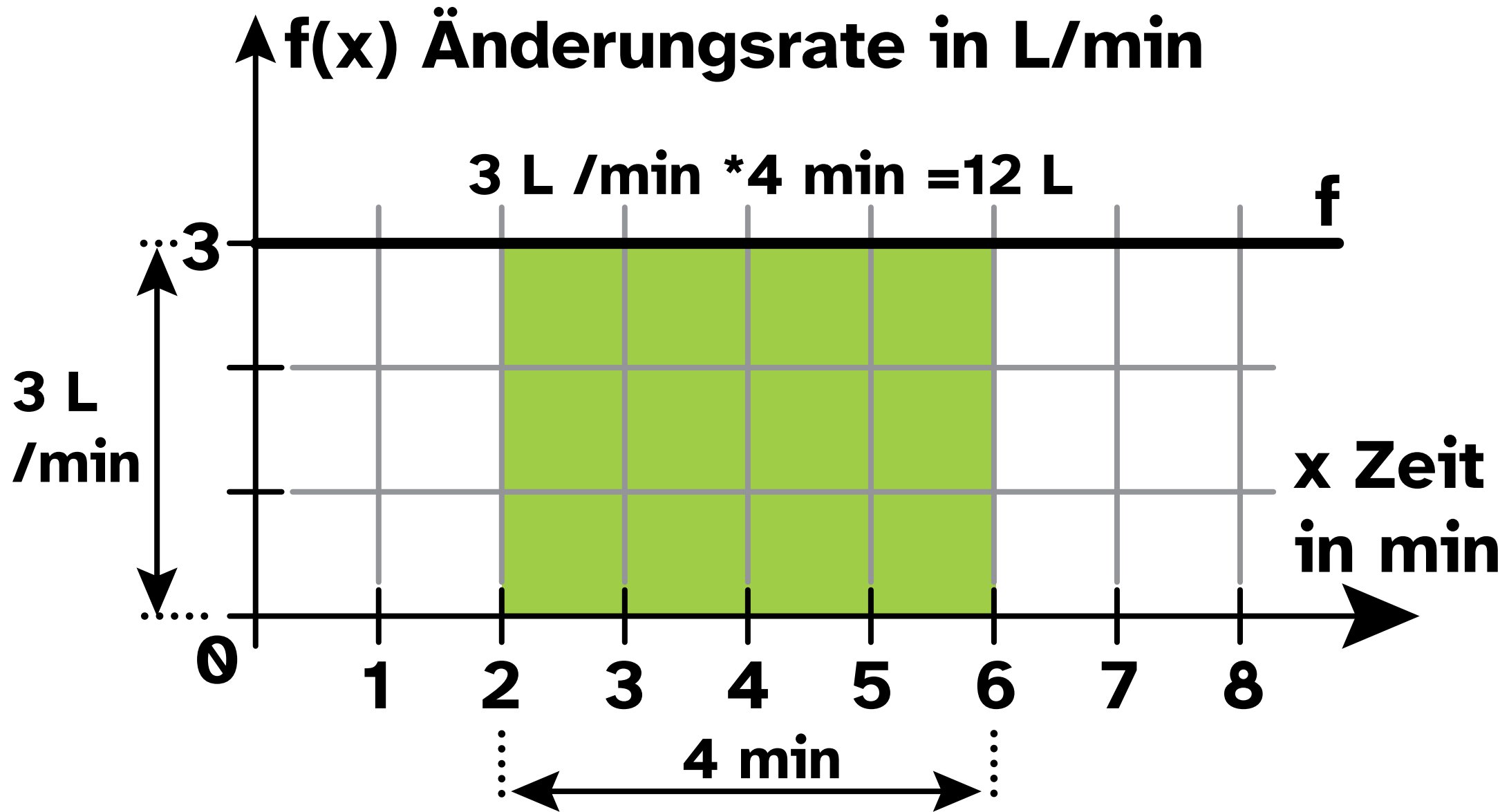


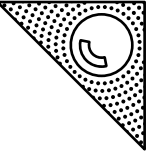
Beispiel mit konstanter Zuflussrate:



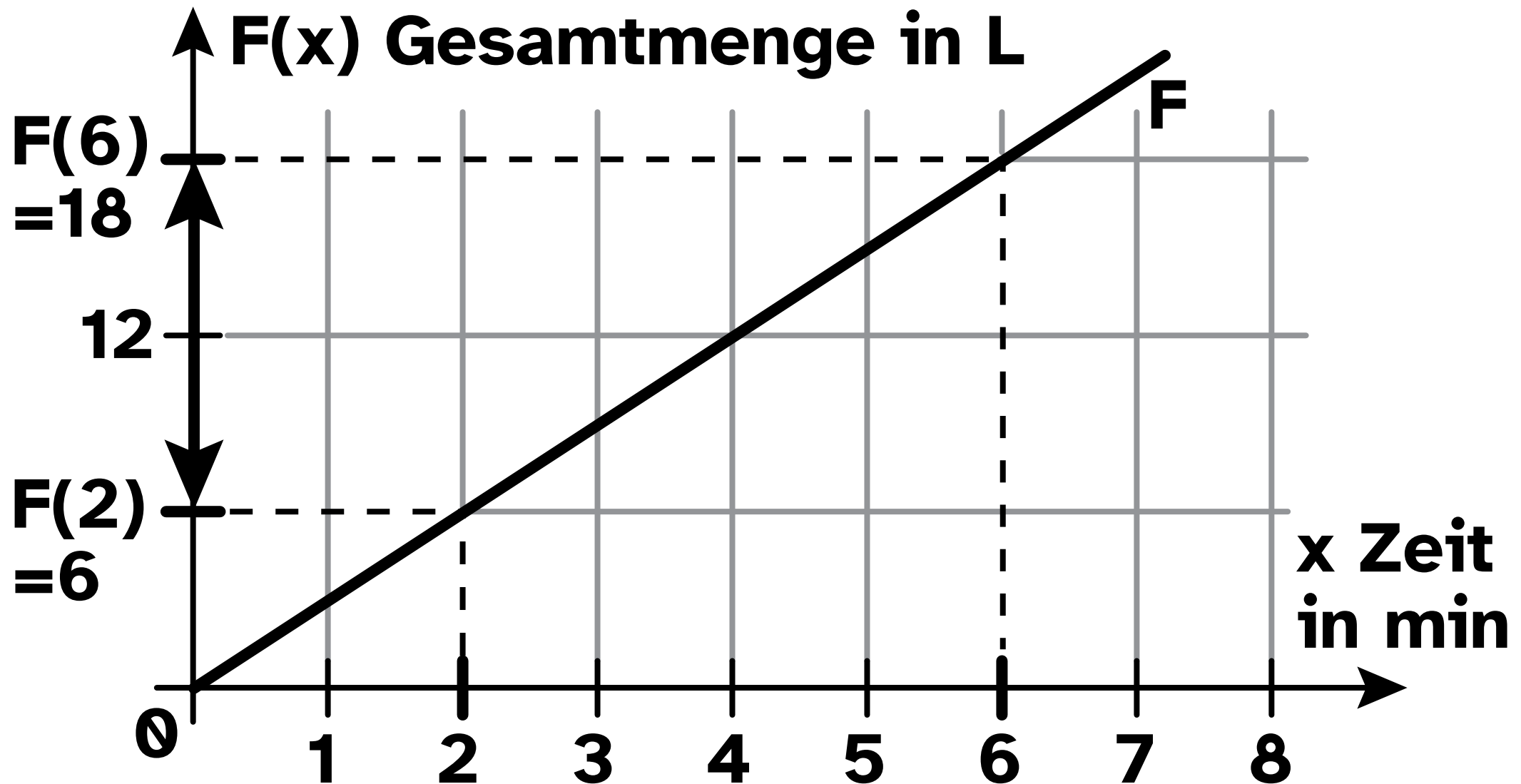


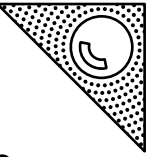
Zuflussmenge als Flächeninhalt:



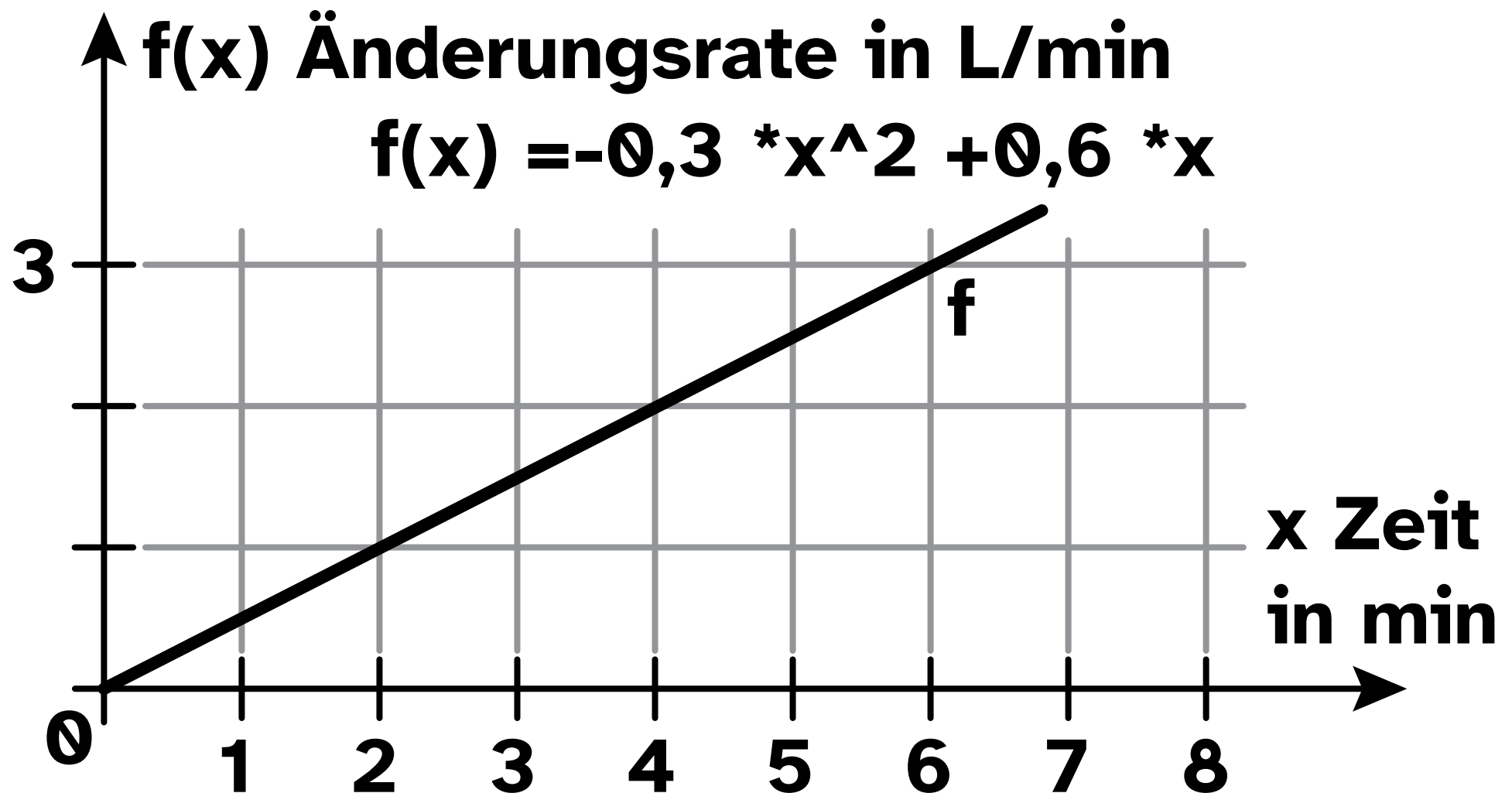


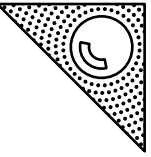
Zuflussmenge mithilfe der Bestandsfunktion



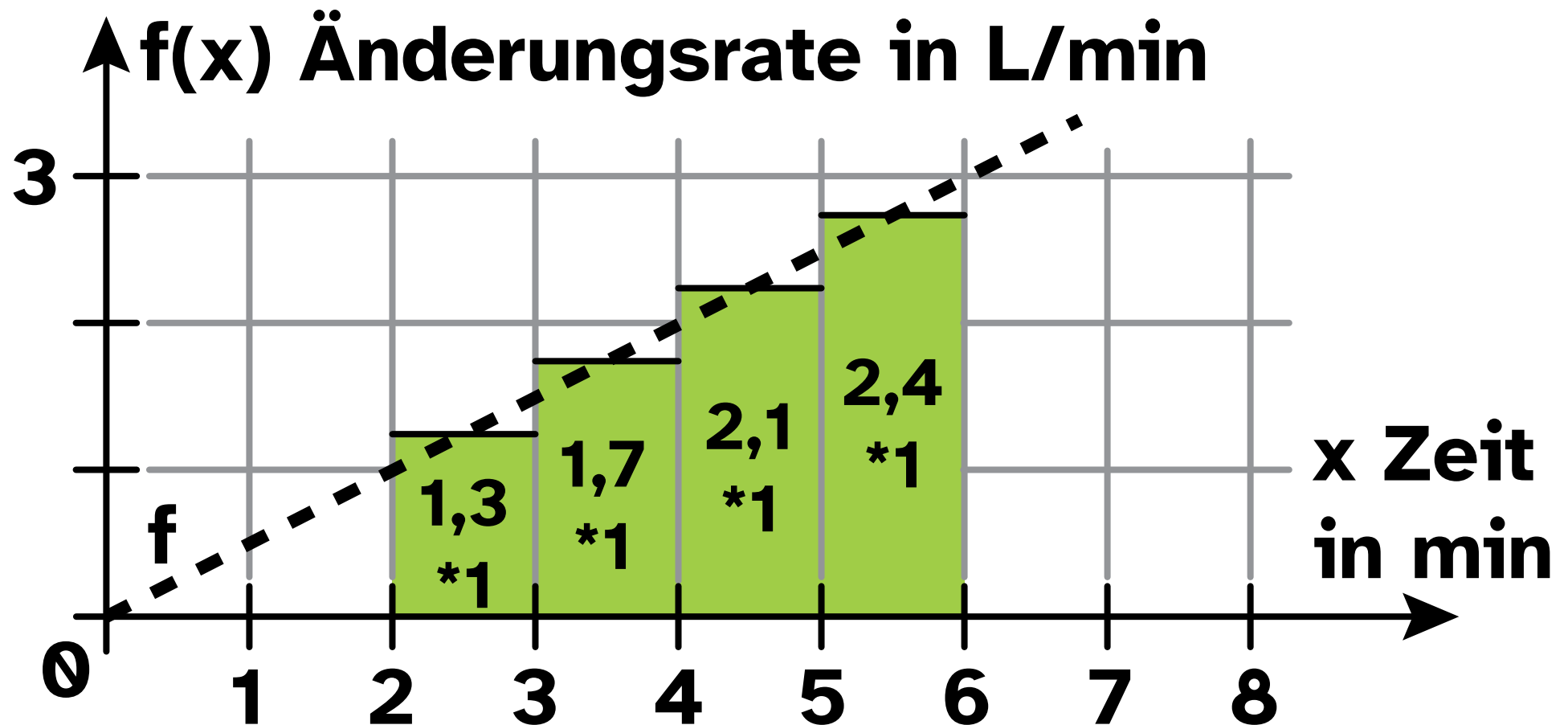


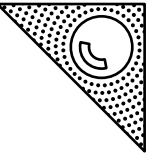
Beispiel mit nicht konstanter Zuflussrate:



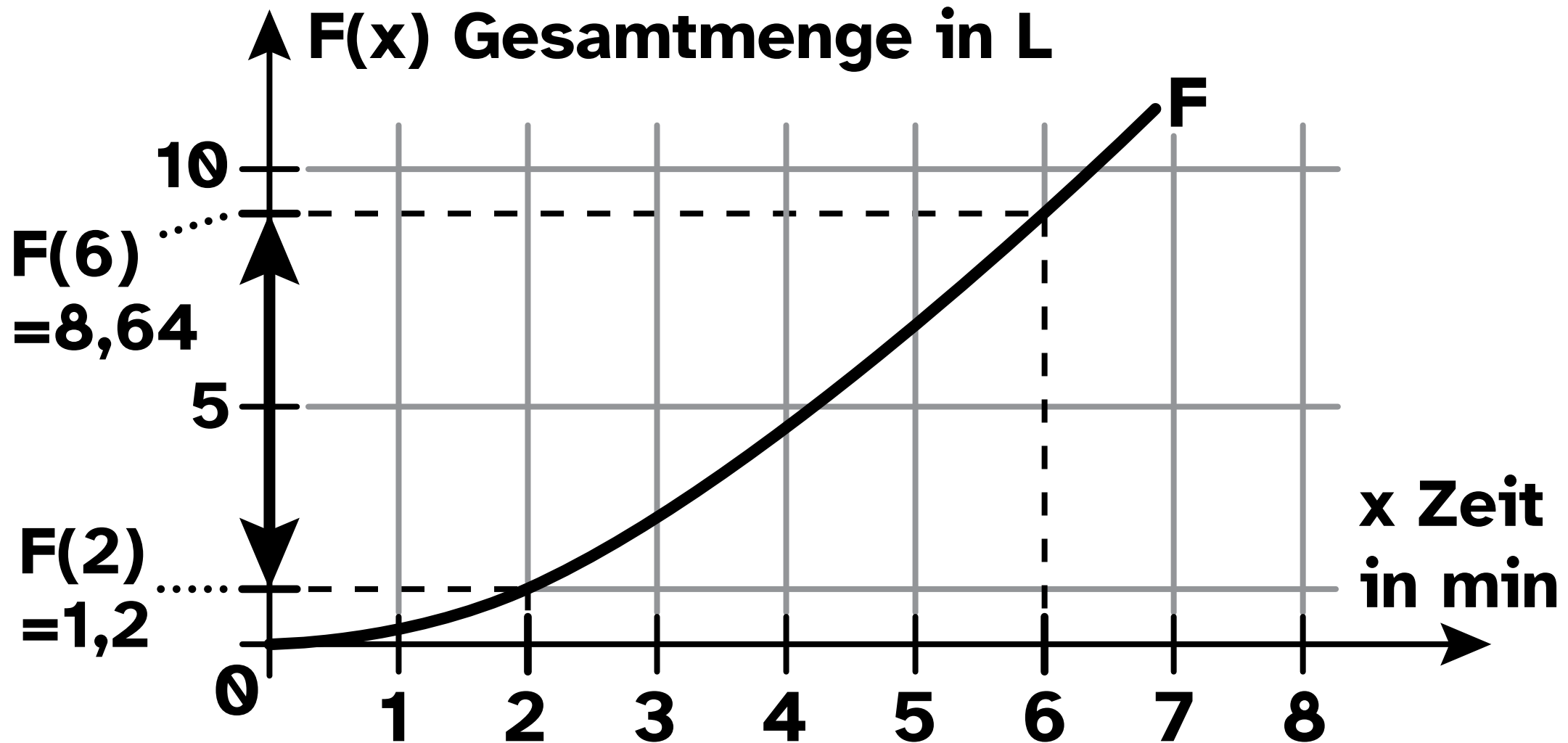


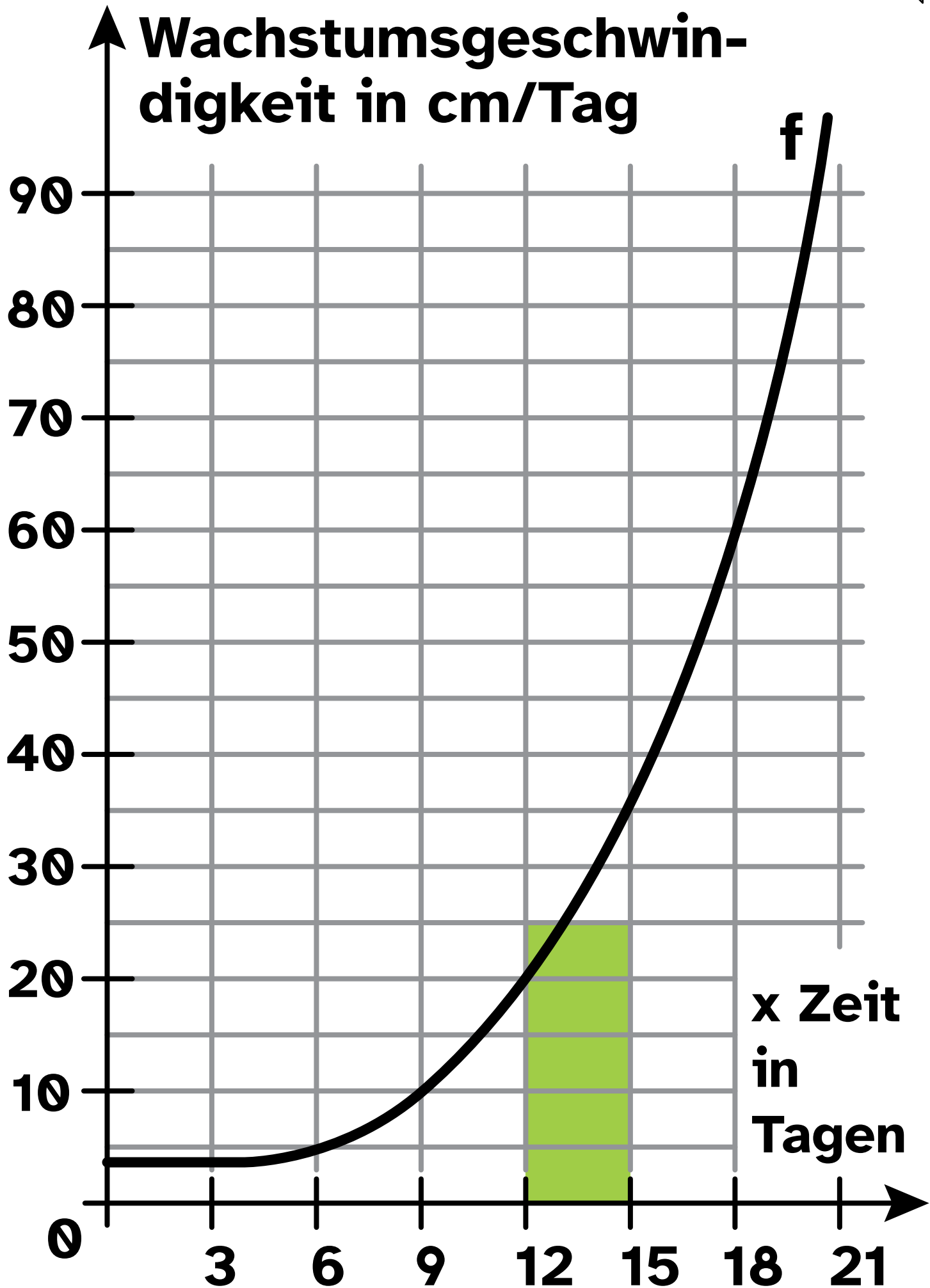
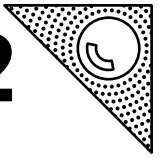
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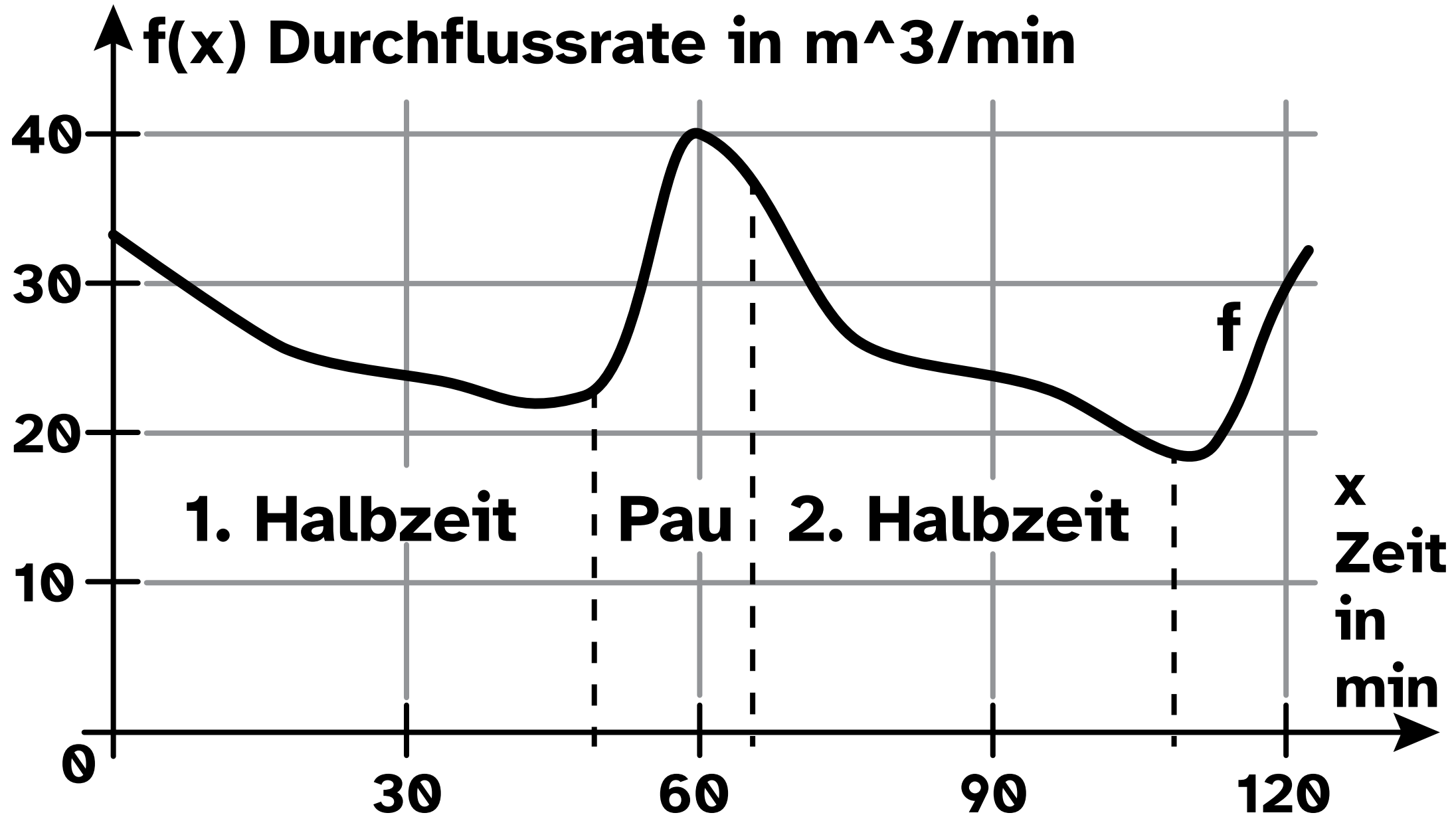
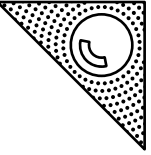


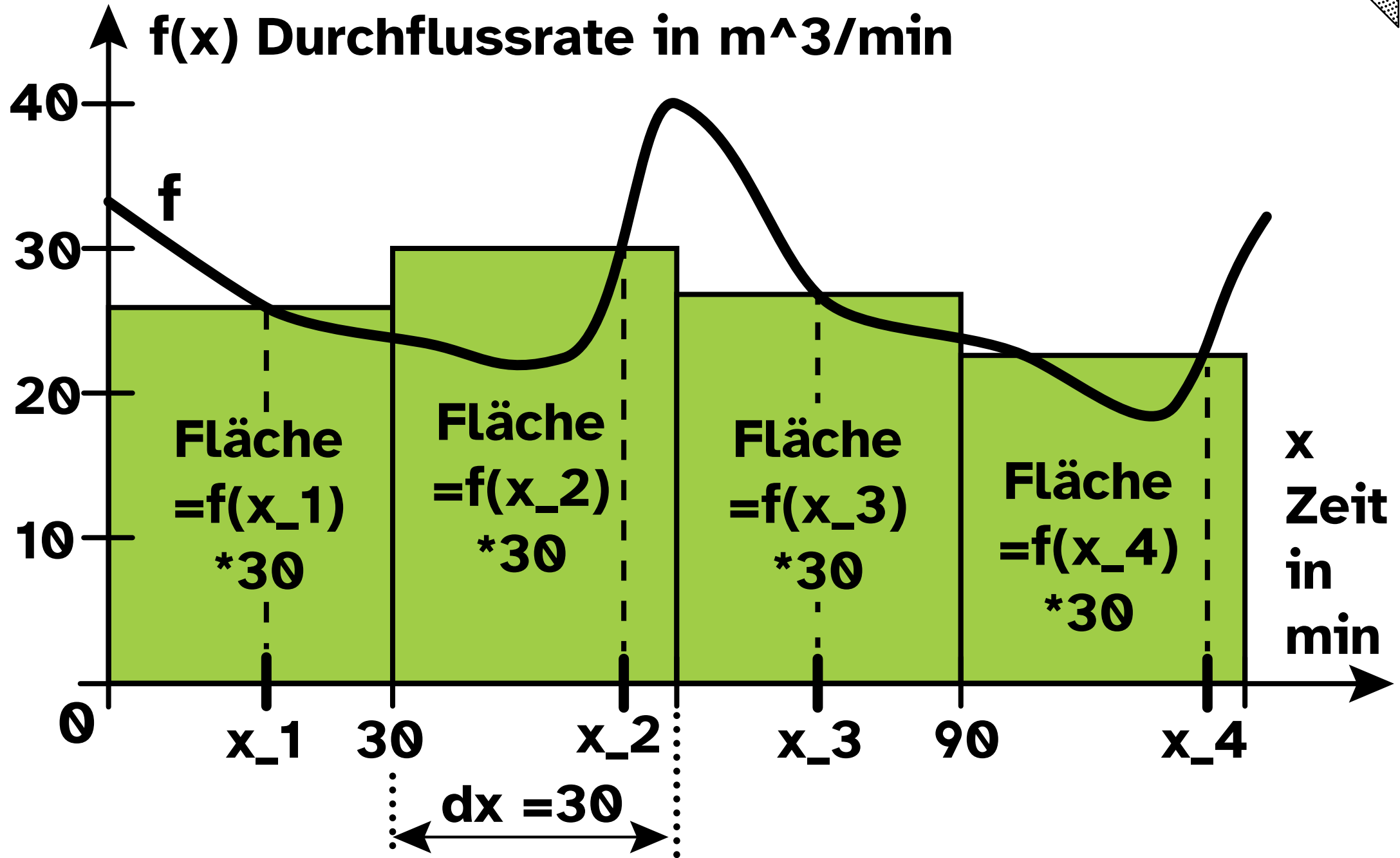
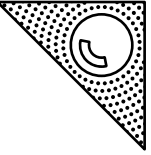


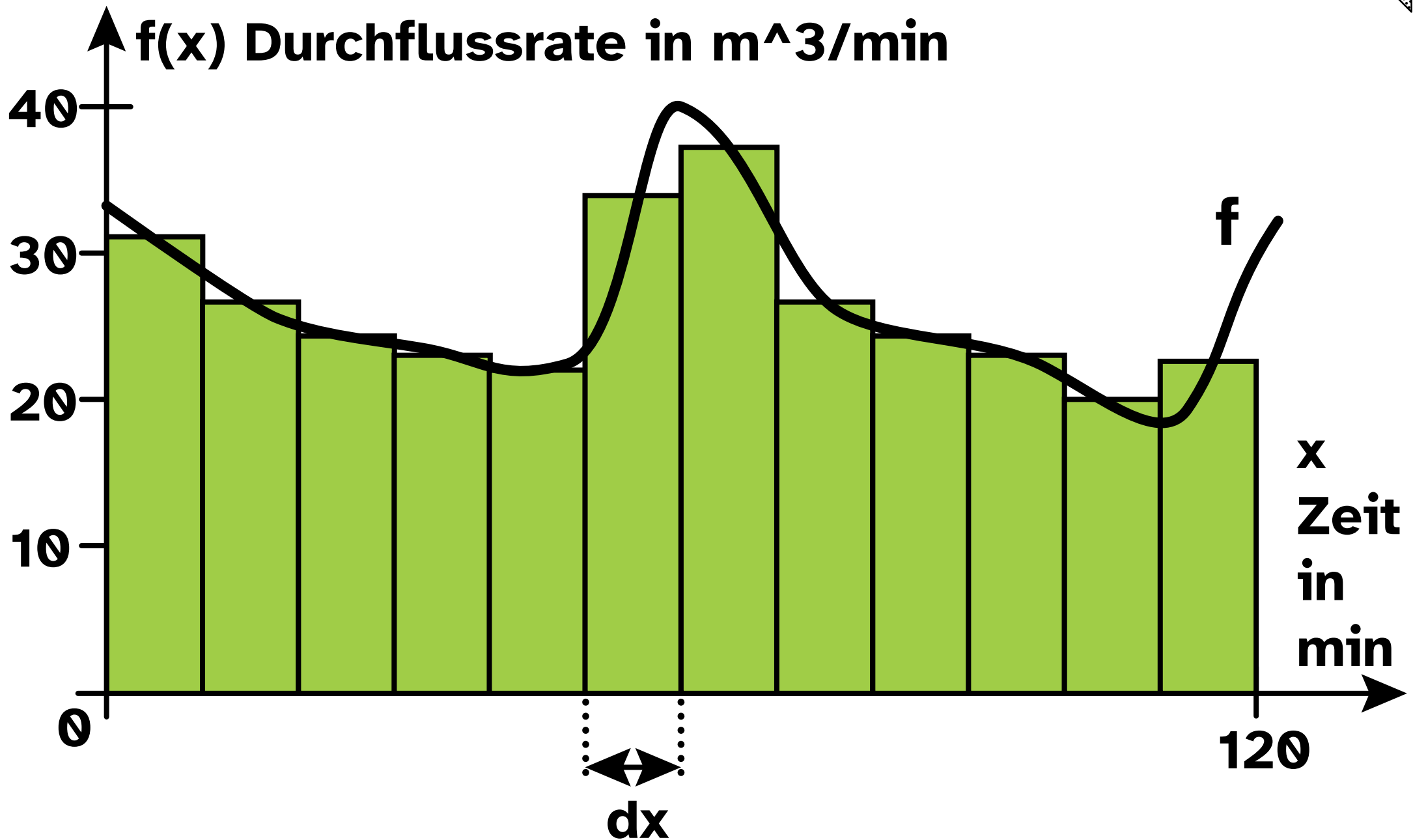
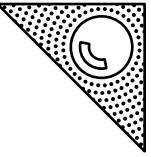
Zuflussmenge mithilfe der Bestandsfunktion

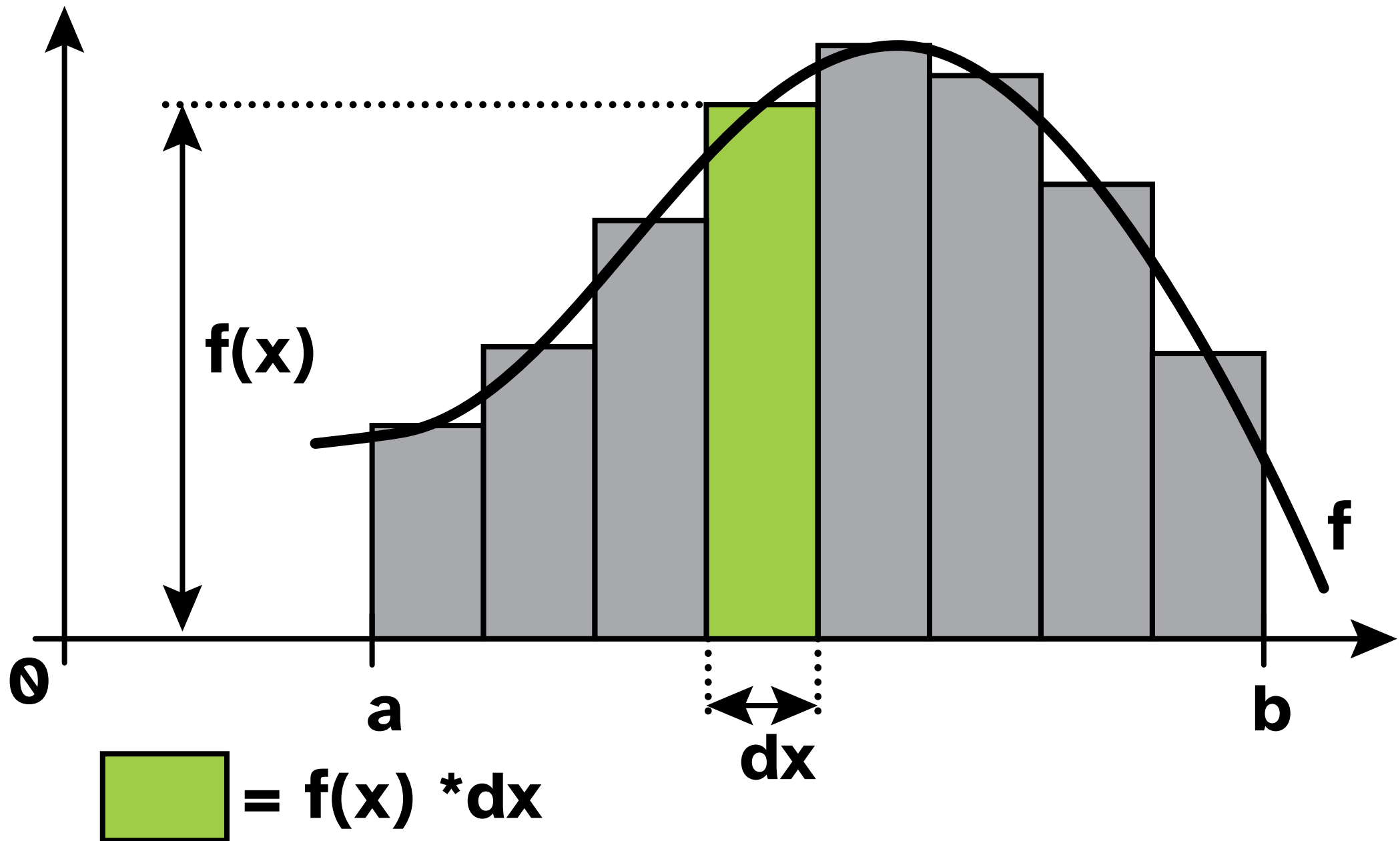
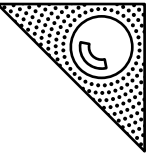


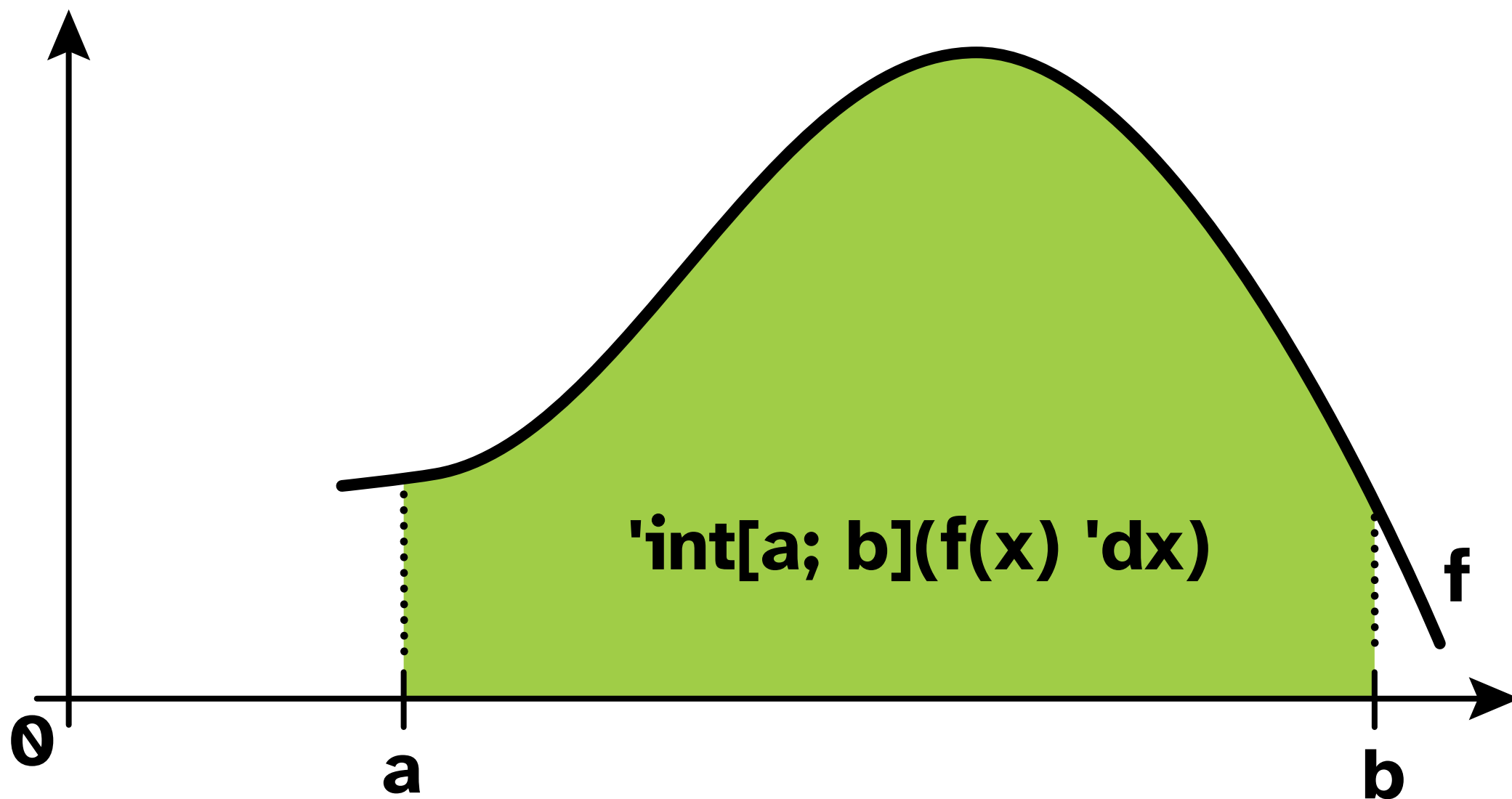
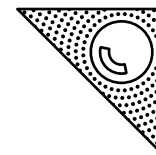


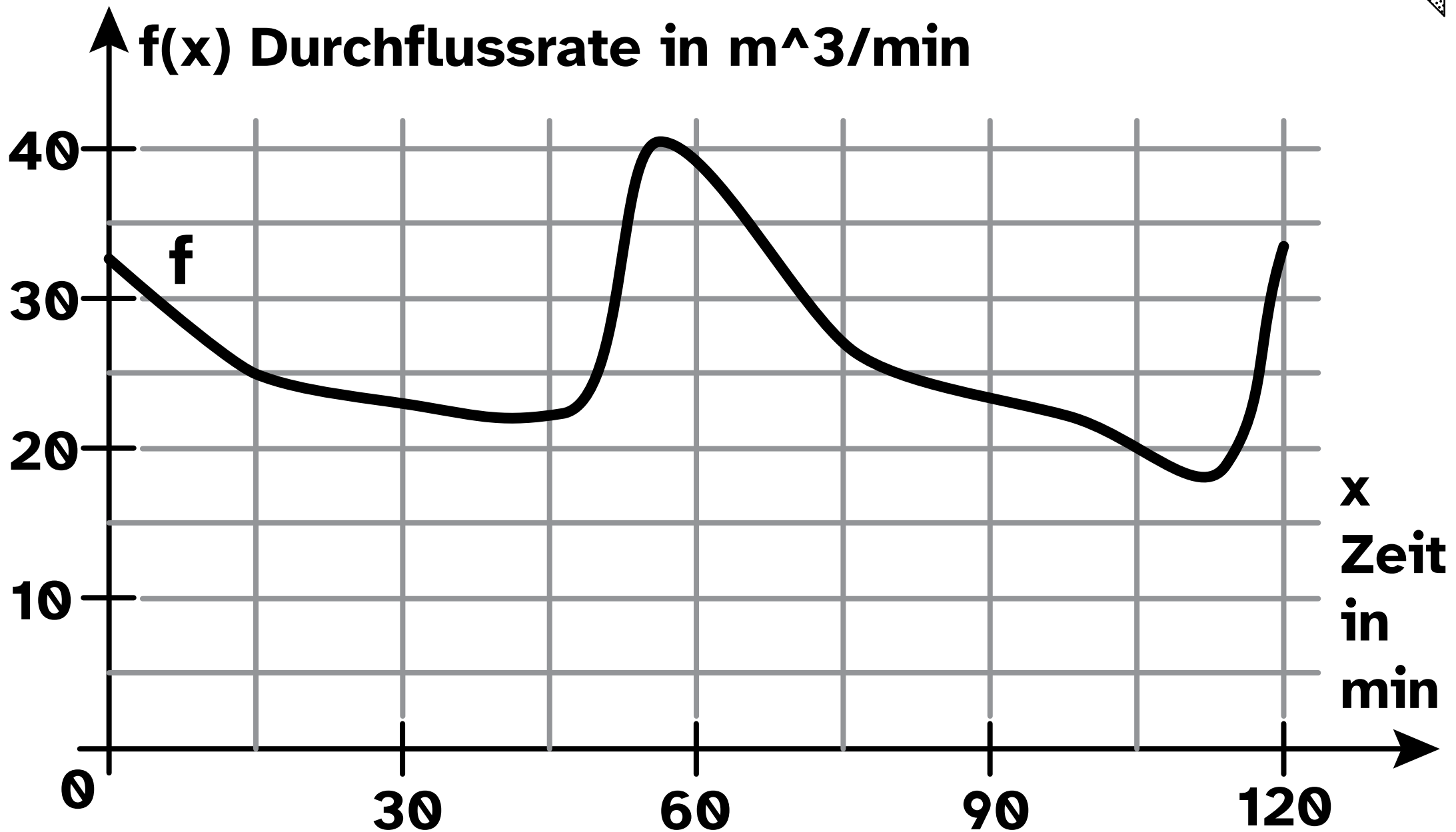
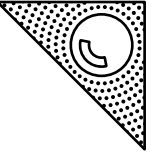


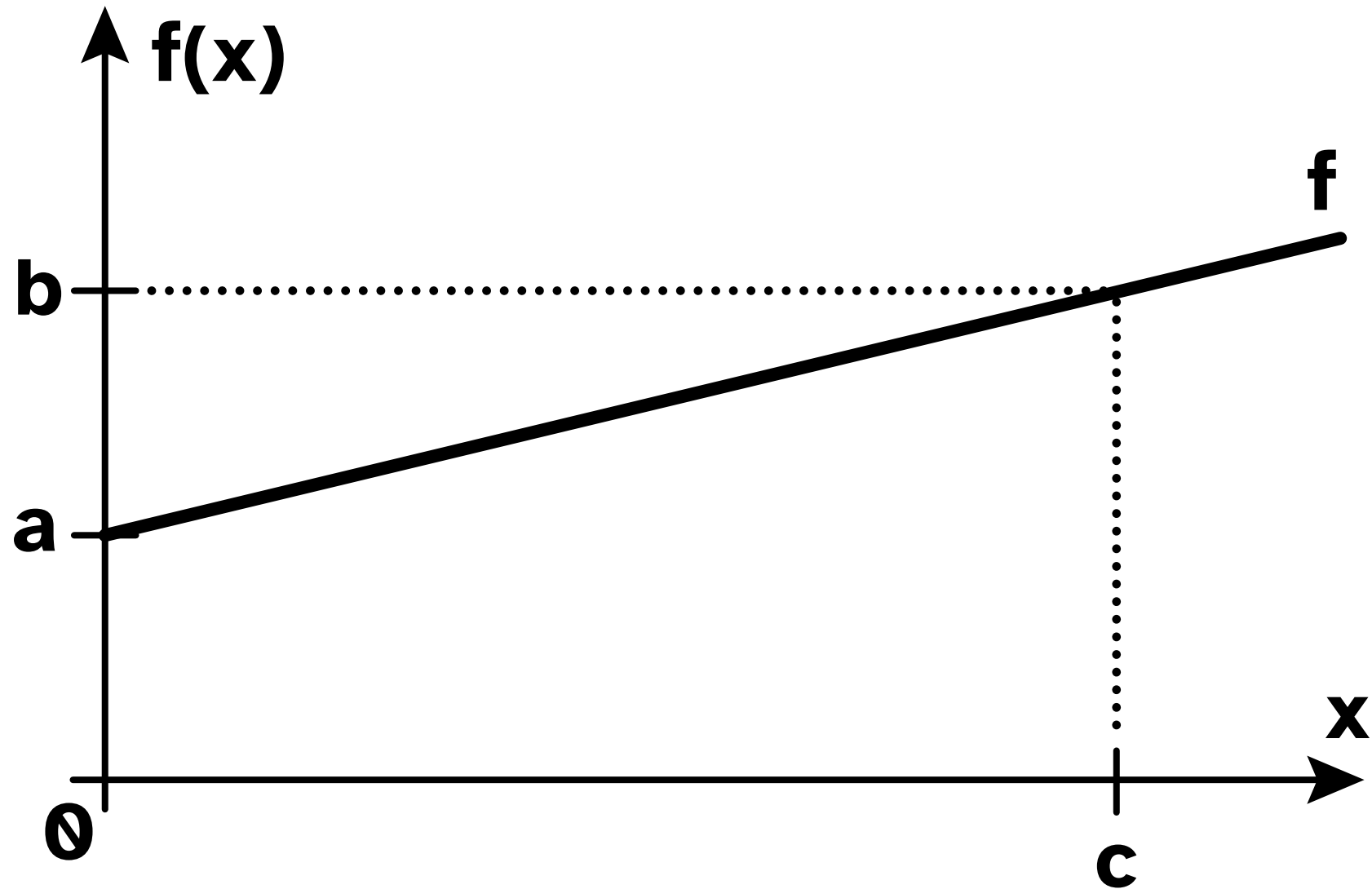
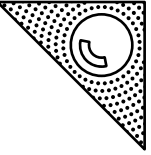


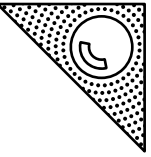




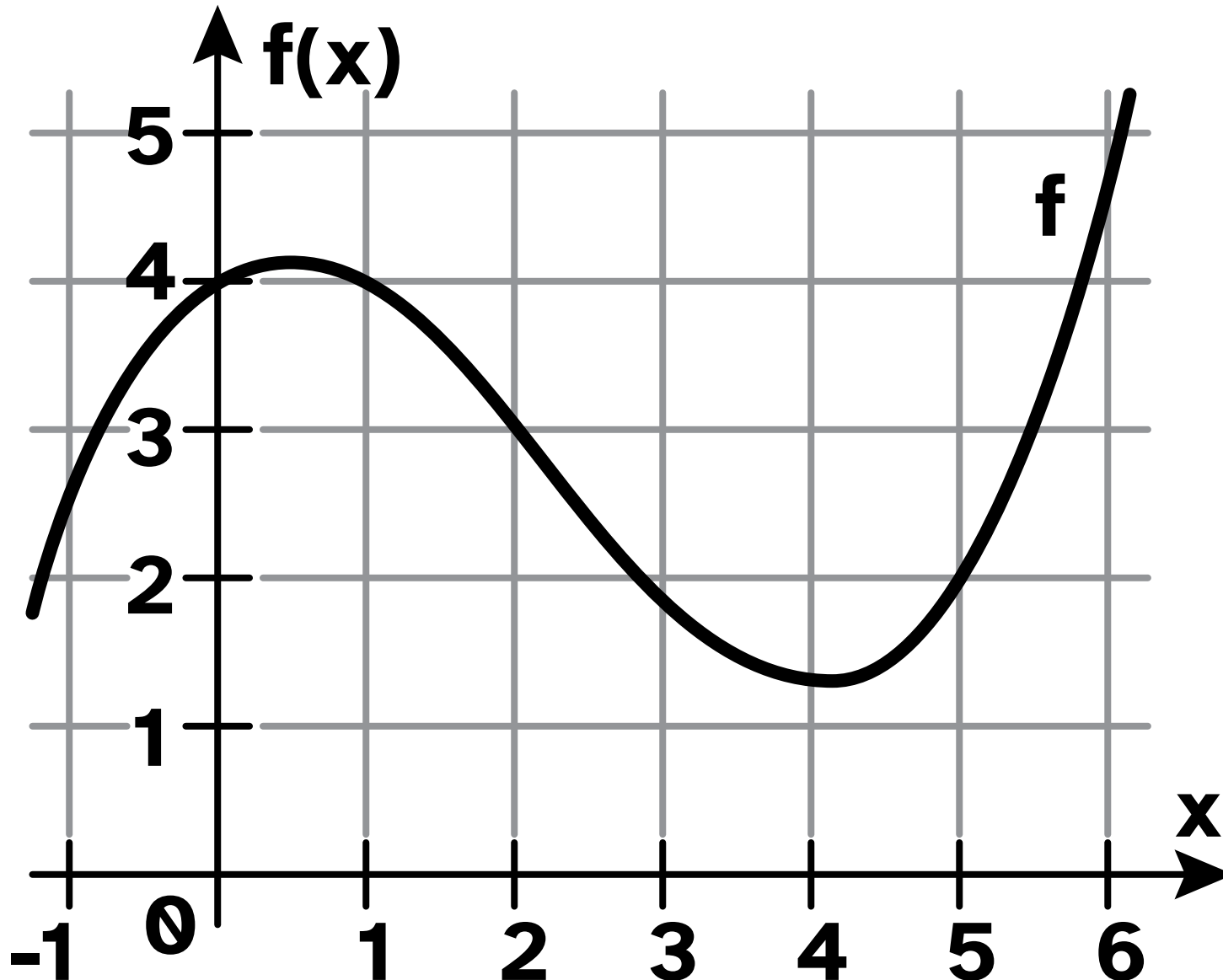


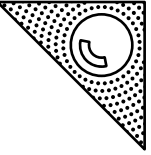




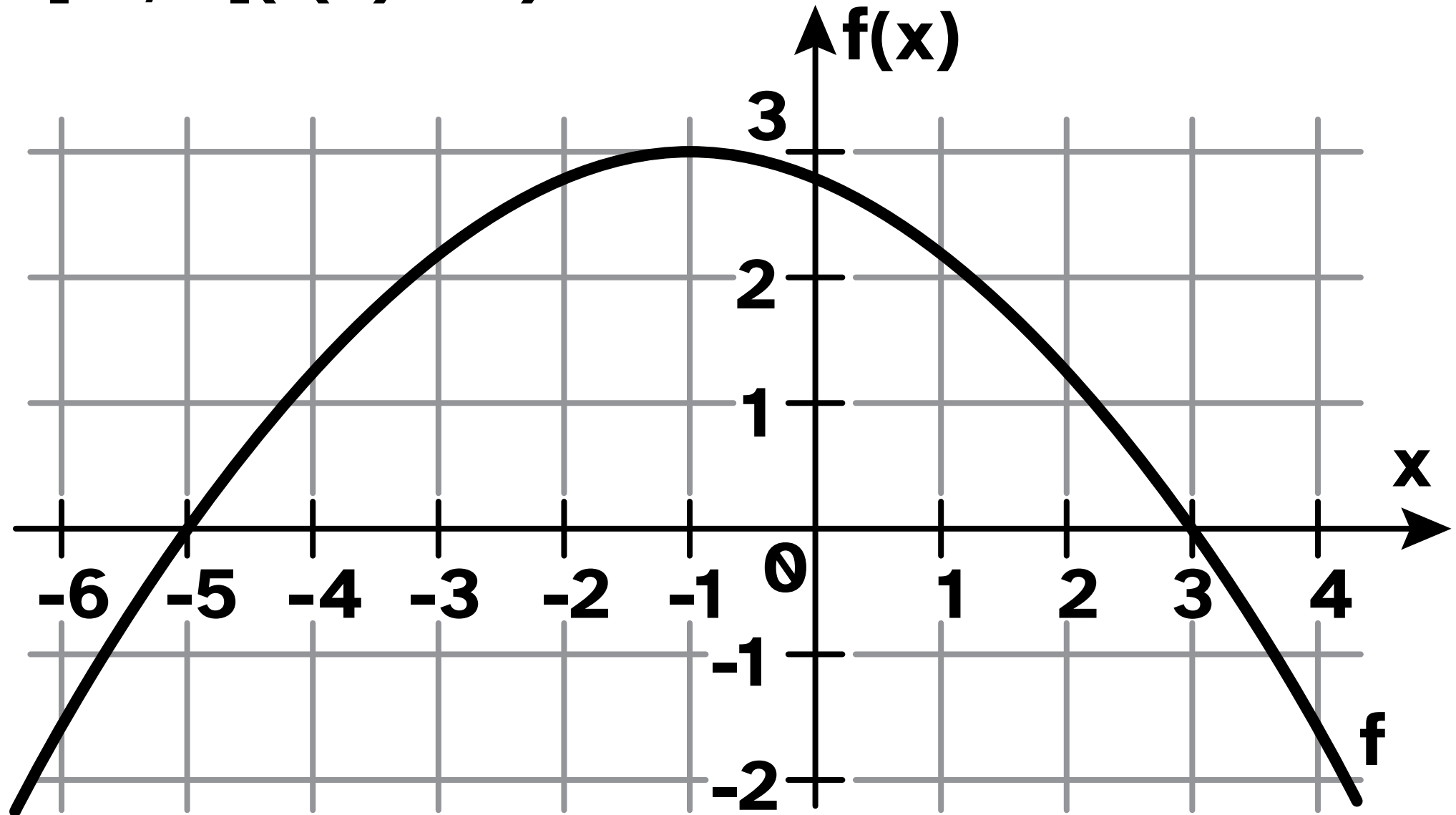


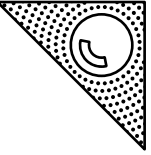
$\int_0^5 f(x) dx$



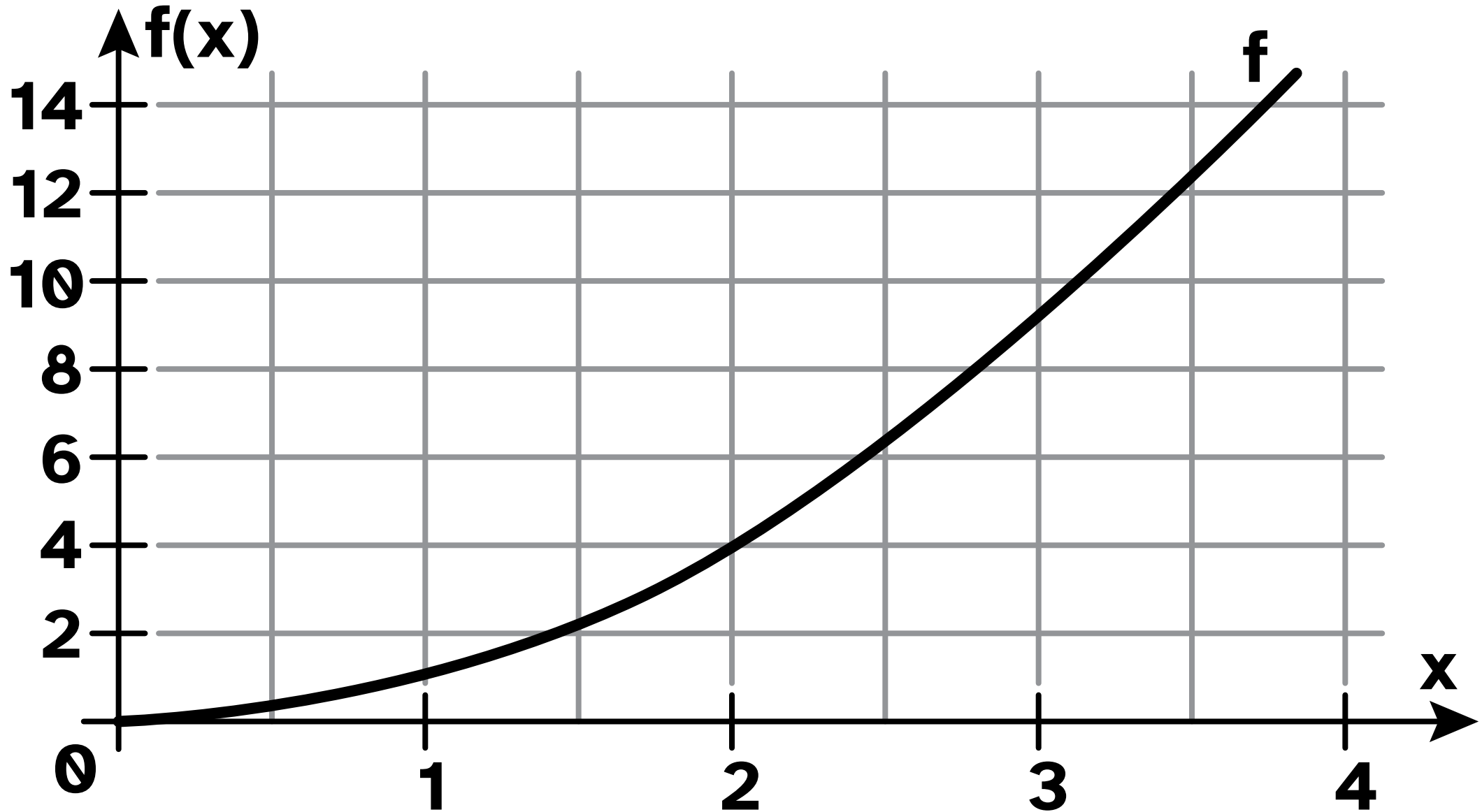


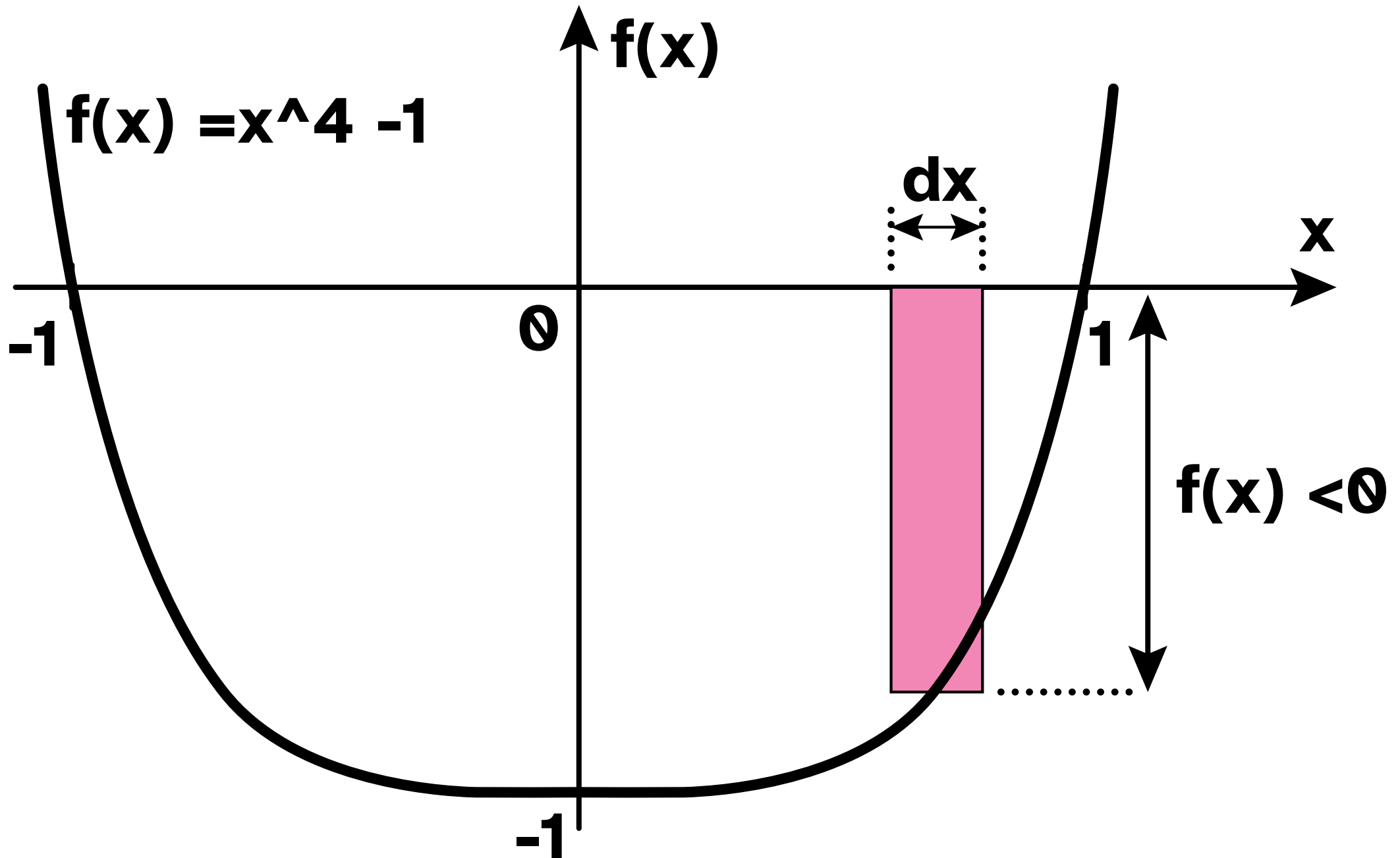
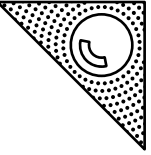
$\int_{-2}^3 f(x) dx$

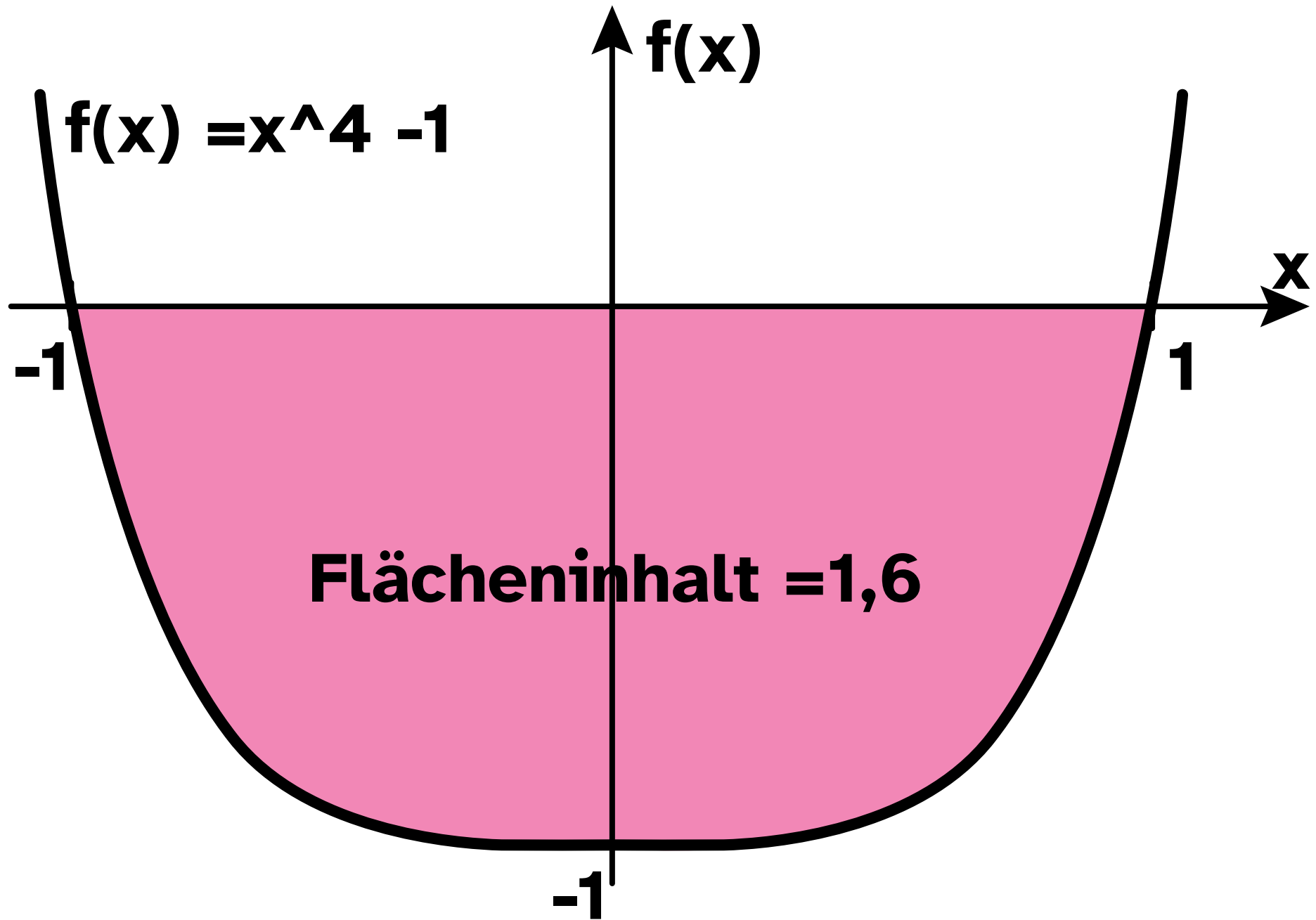
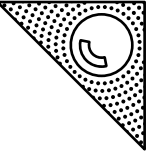


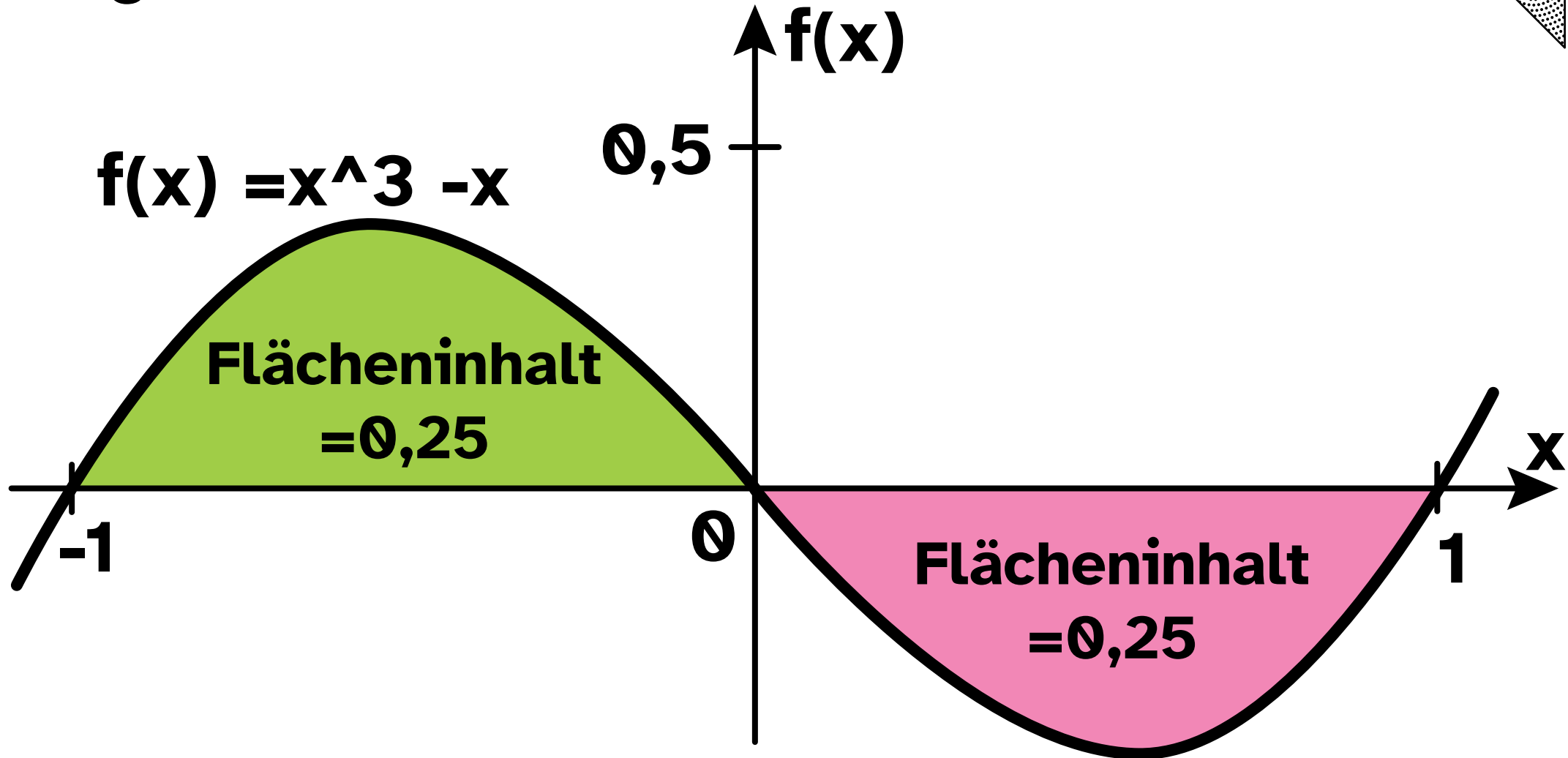
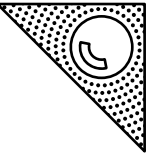


$\int_1^3 f(x) dx$

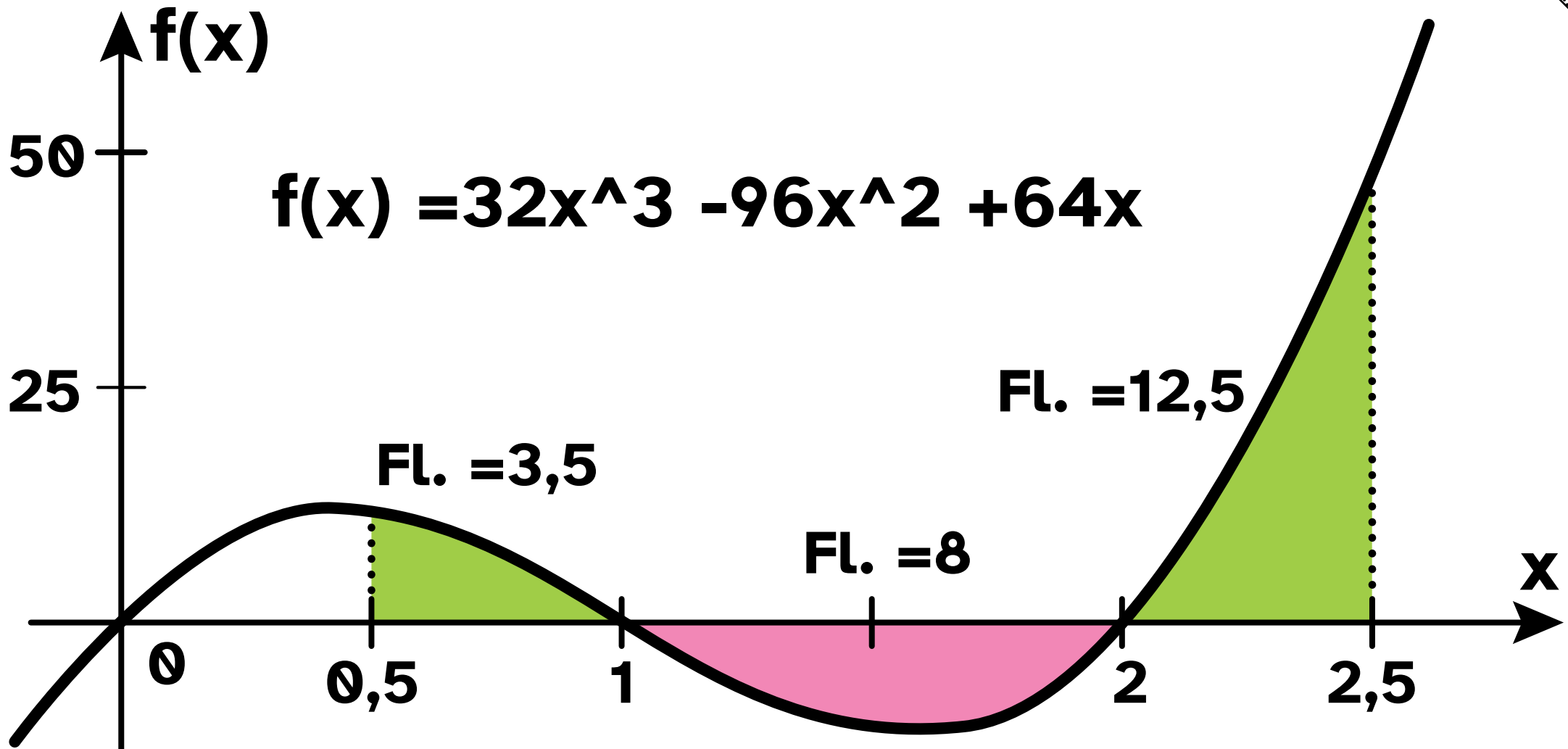
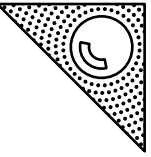




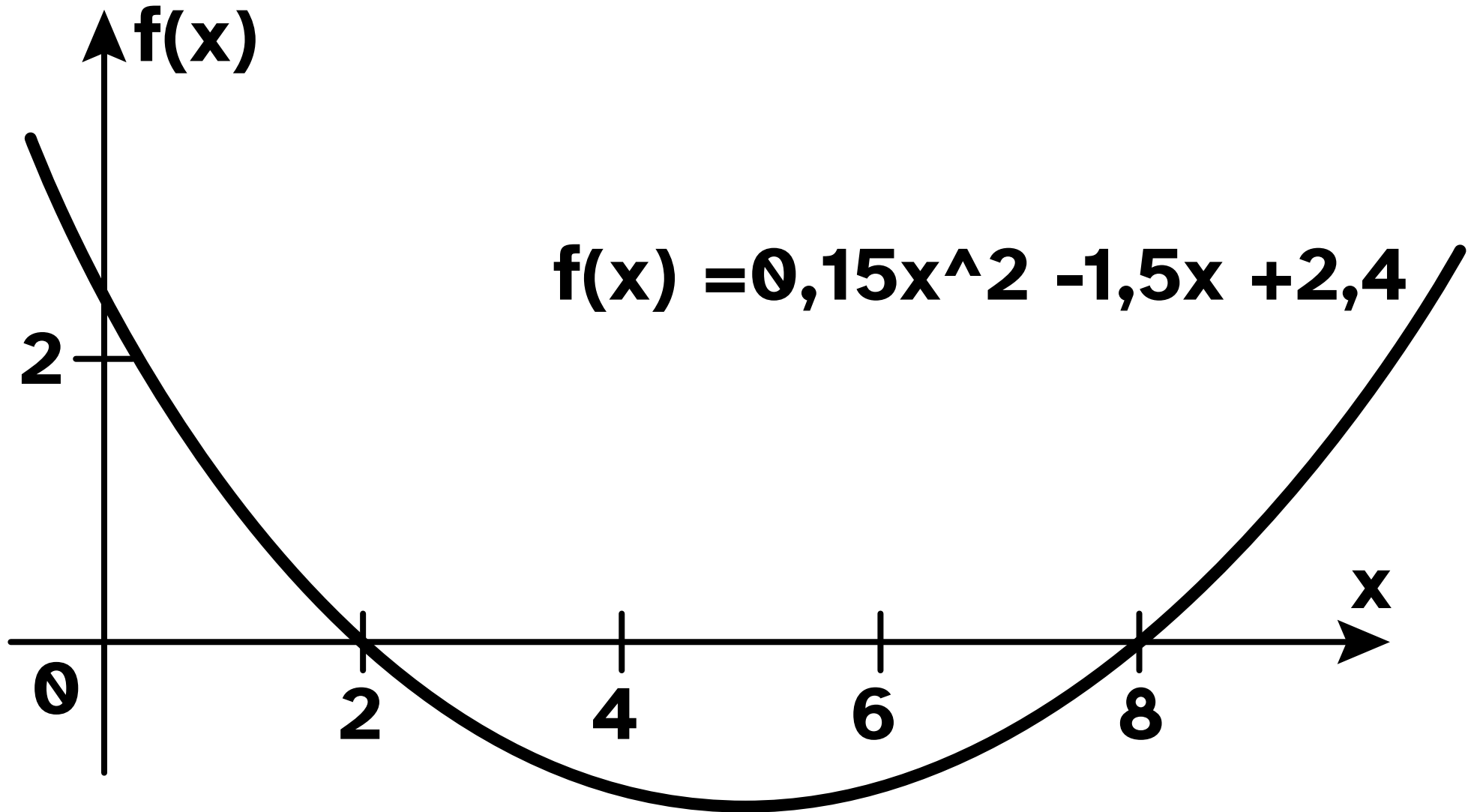
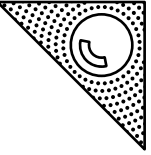


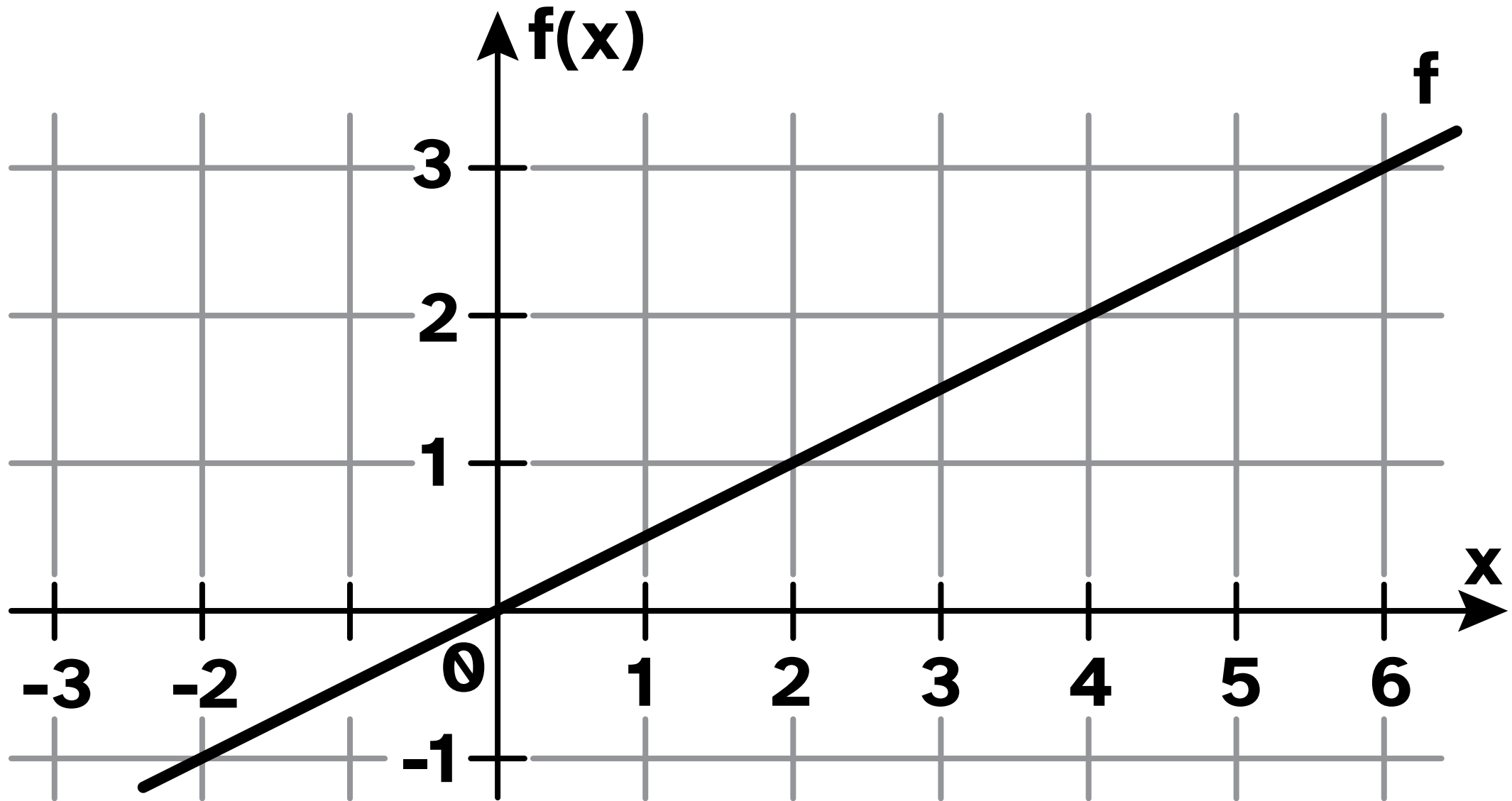
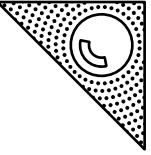


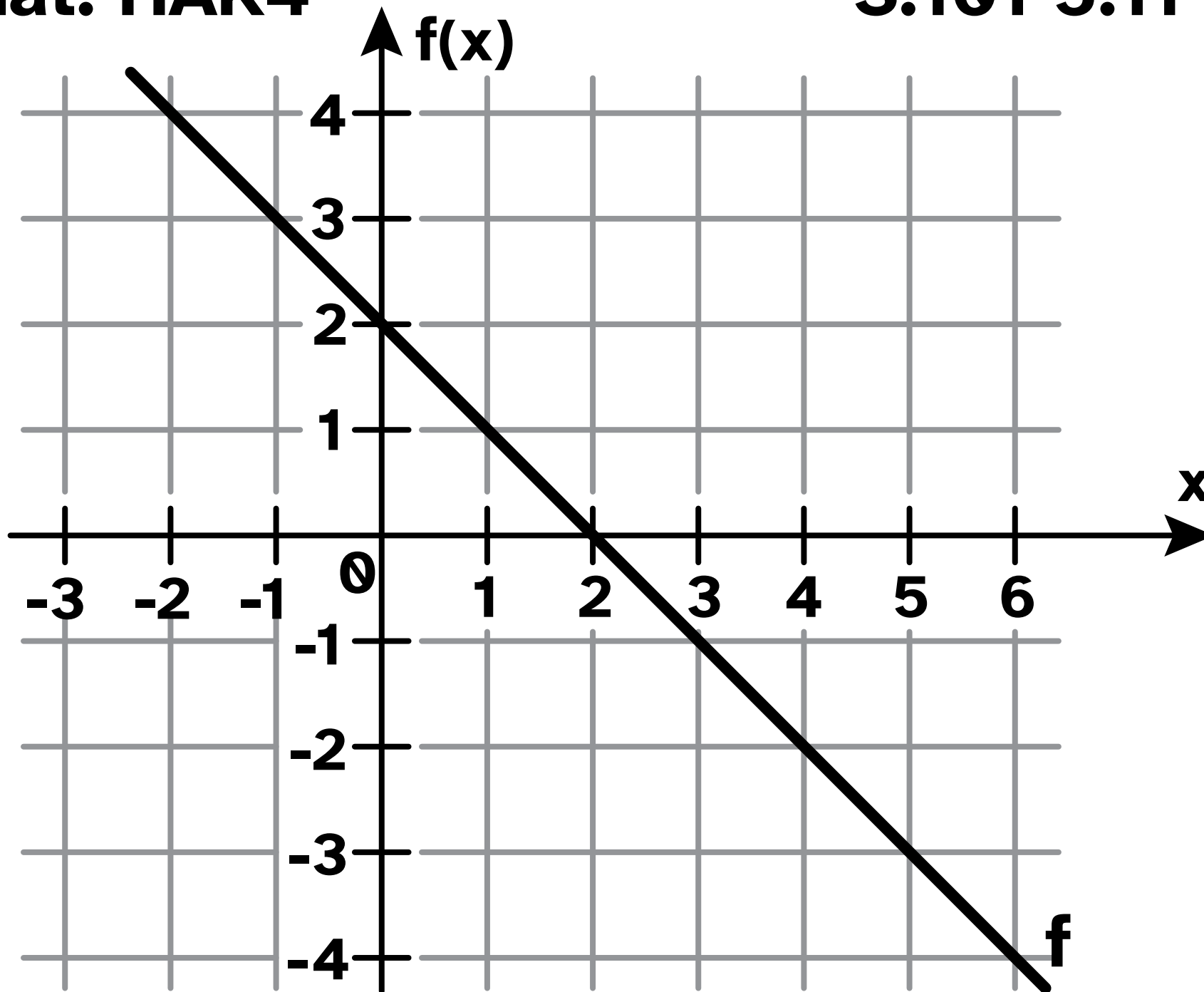
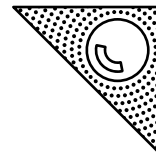
$$\int_{-1}^1 (f(x)) \, dx = 0,25 - 0,25 = 0$$

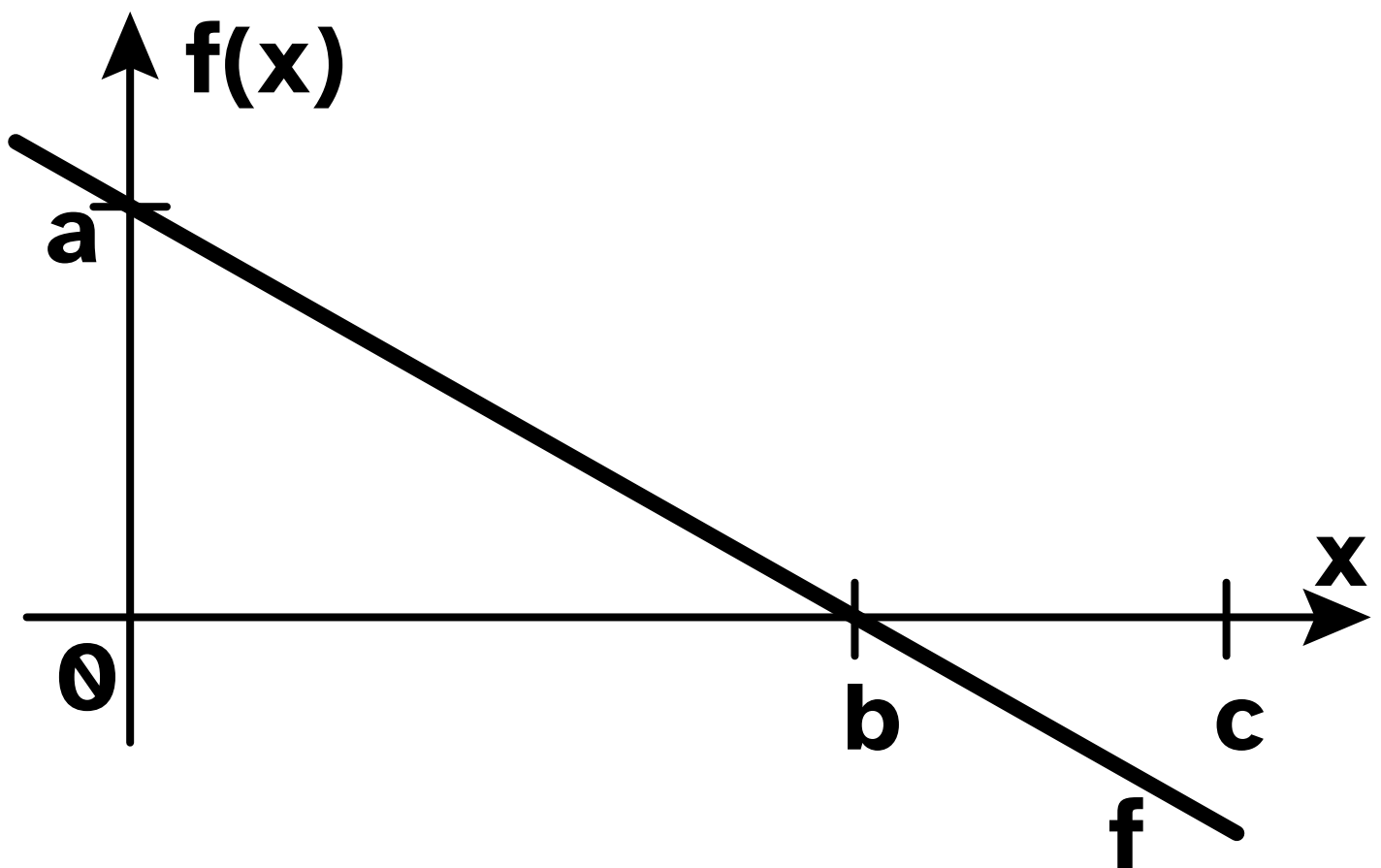
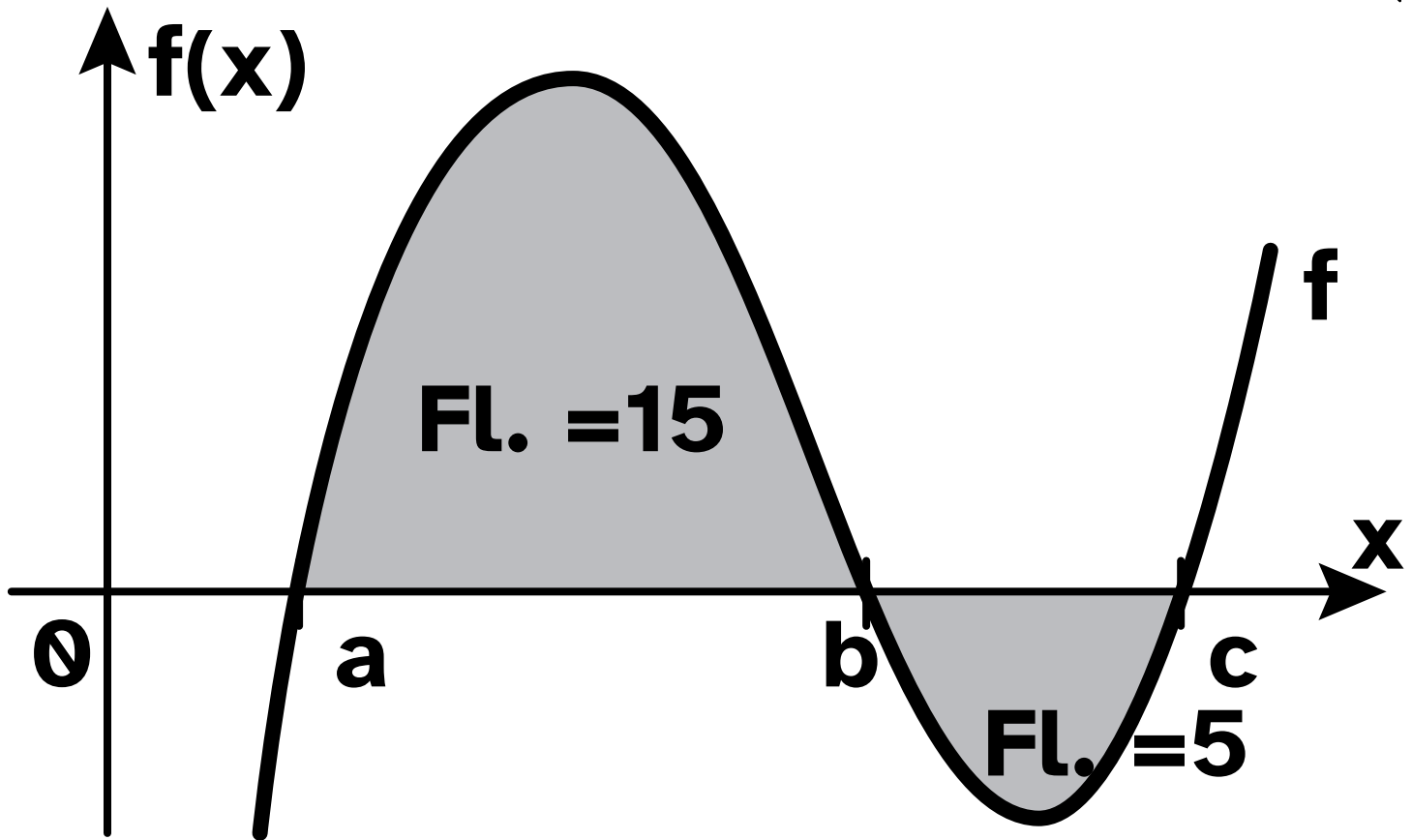
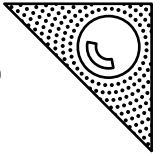


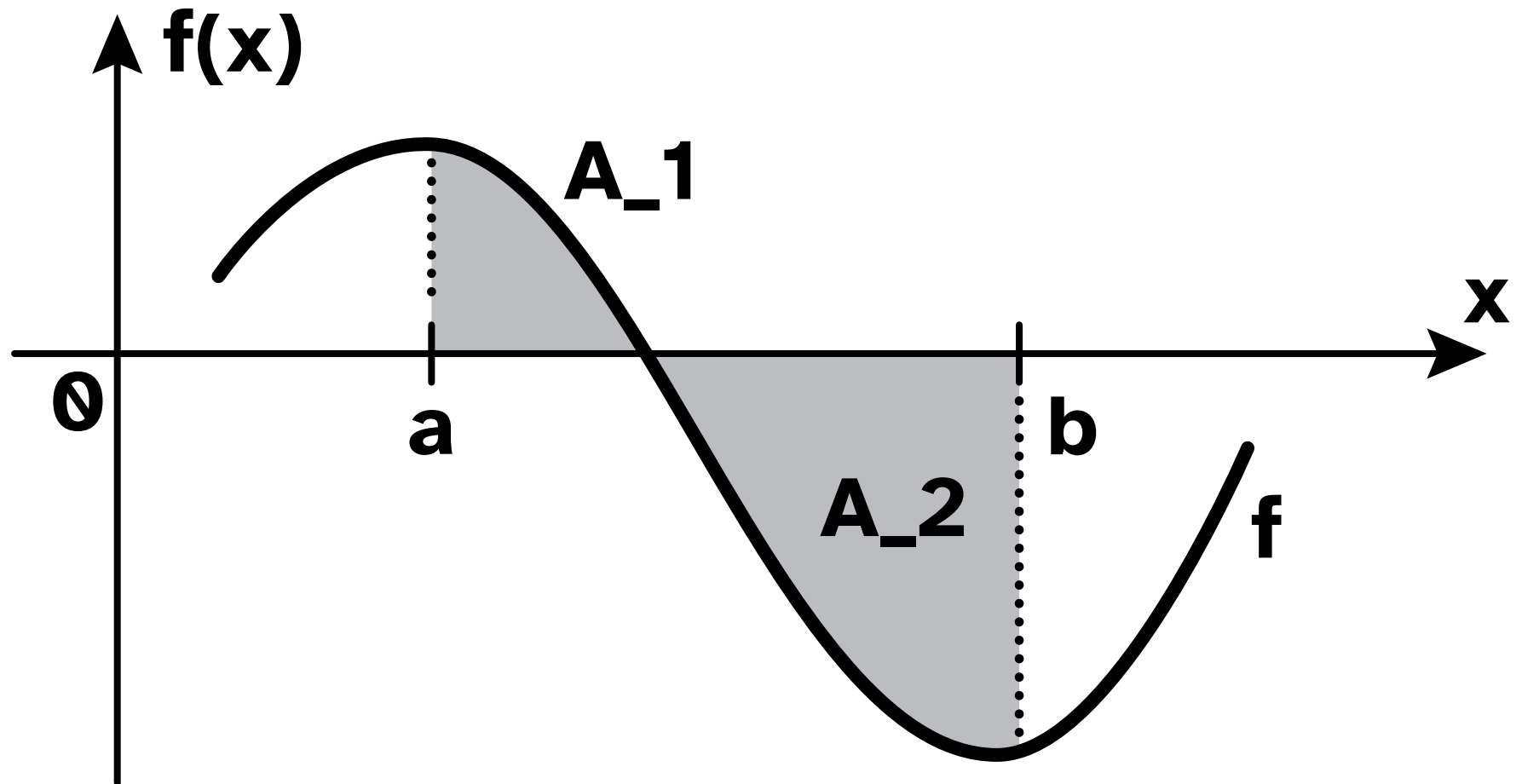
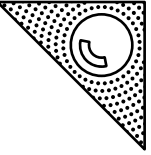
$$\int_{0,5}^{2,5} f(x) \, dx = 3,5 - 8 + 12,5 = 8$$

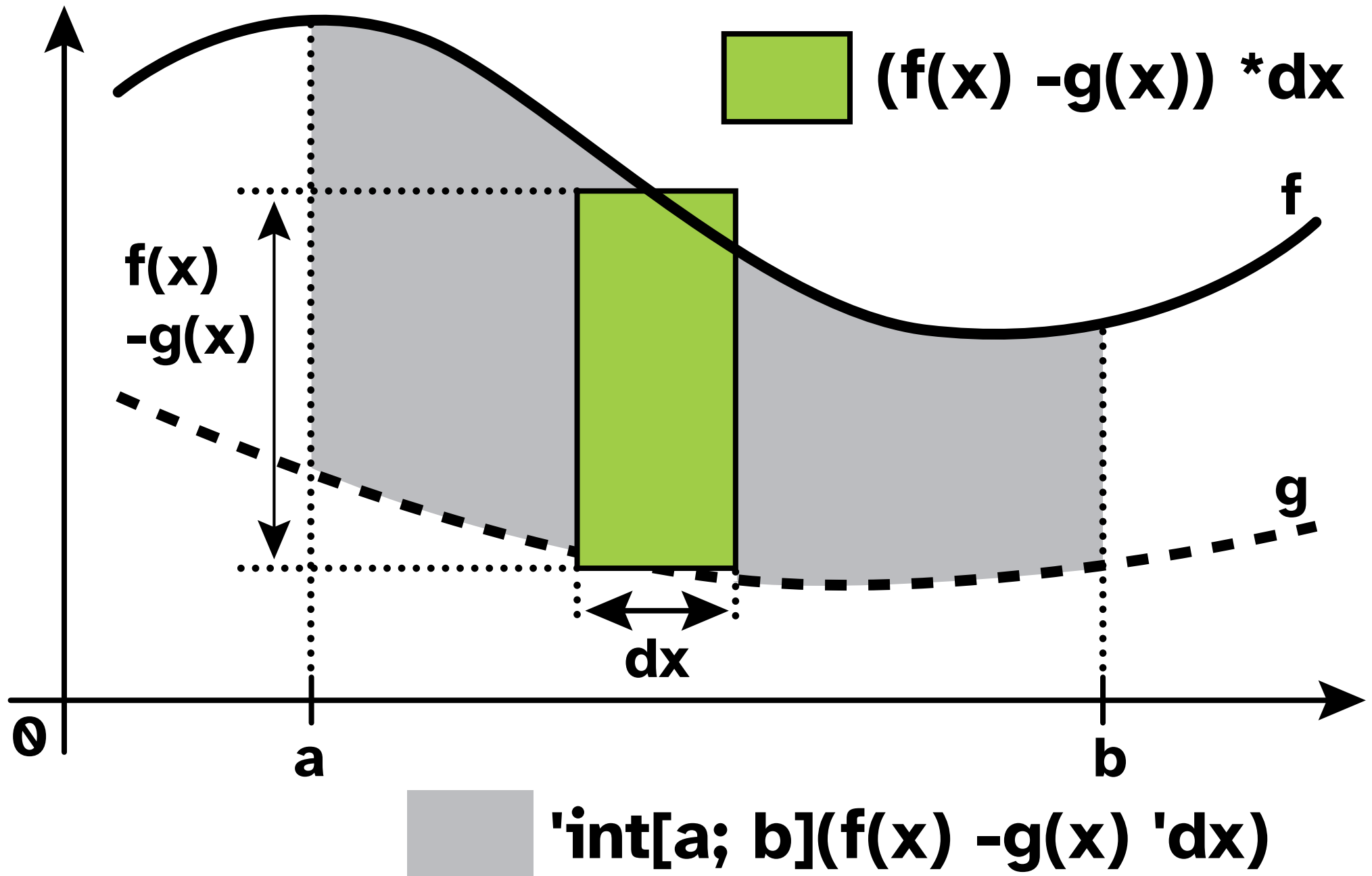
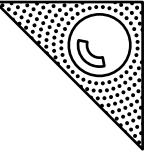


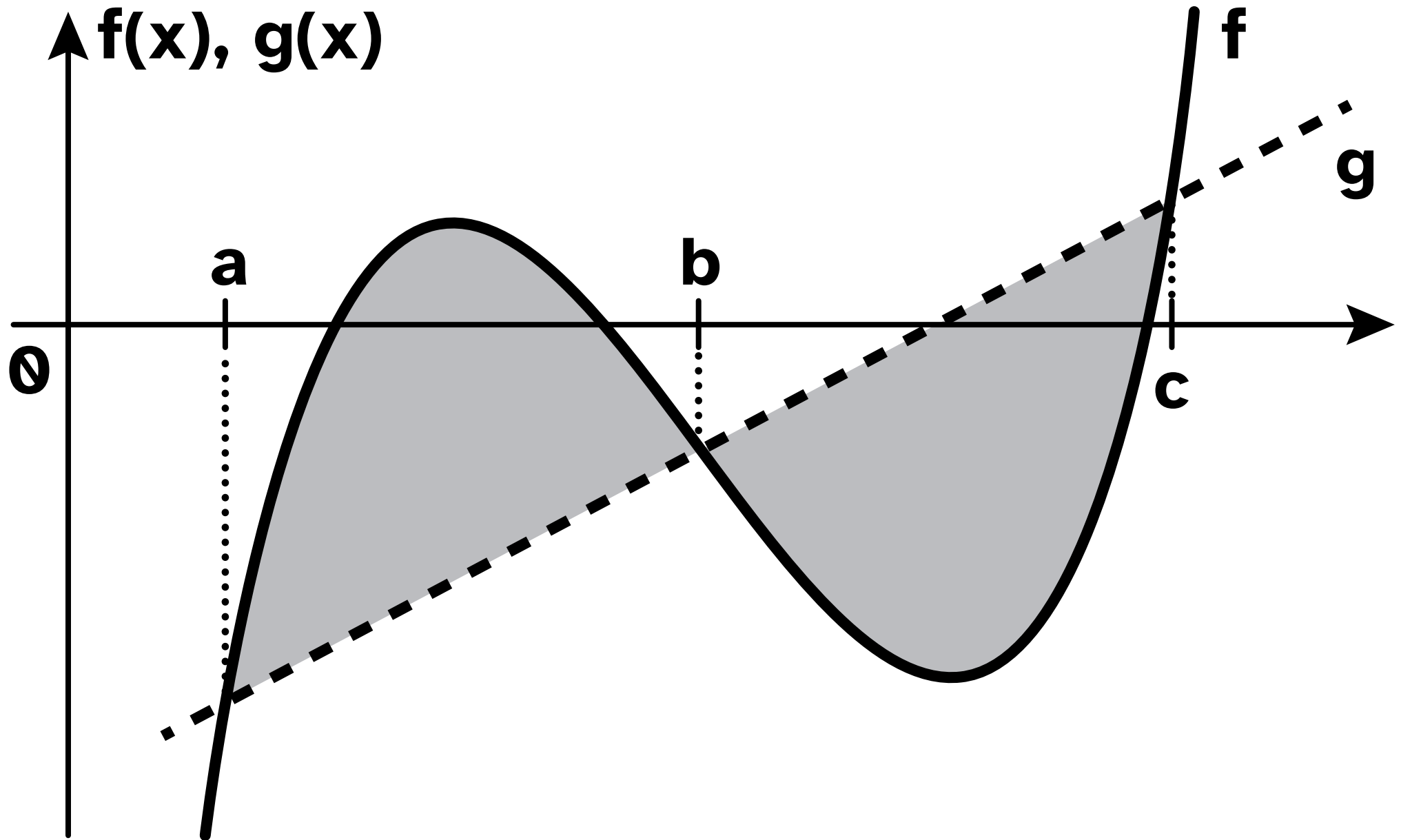
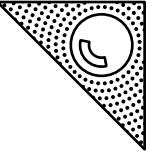


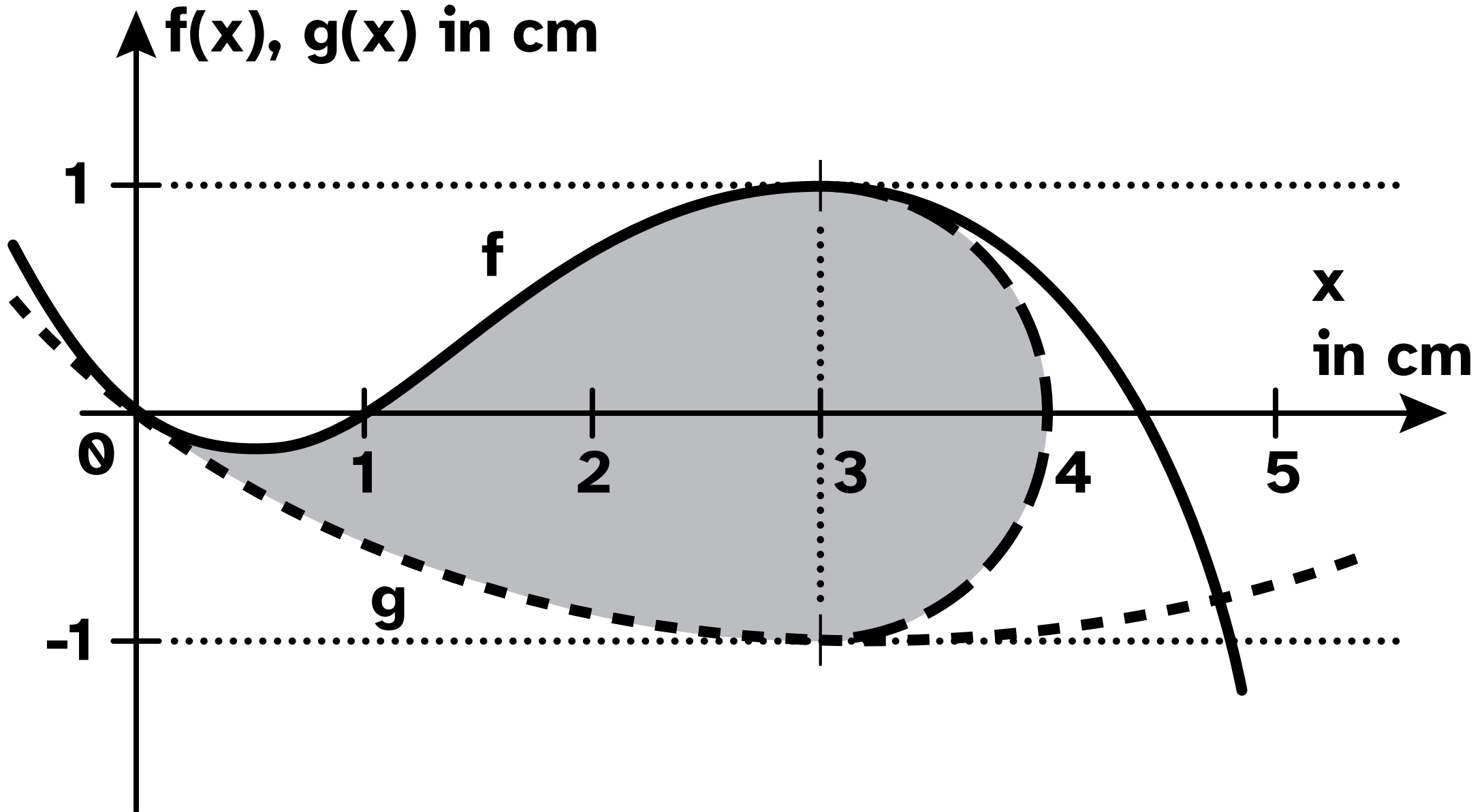
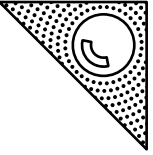


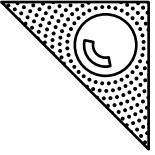




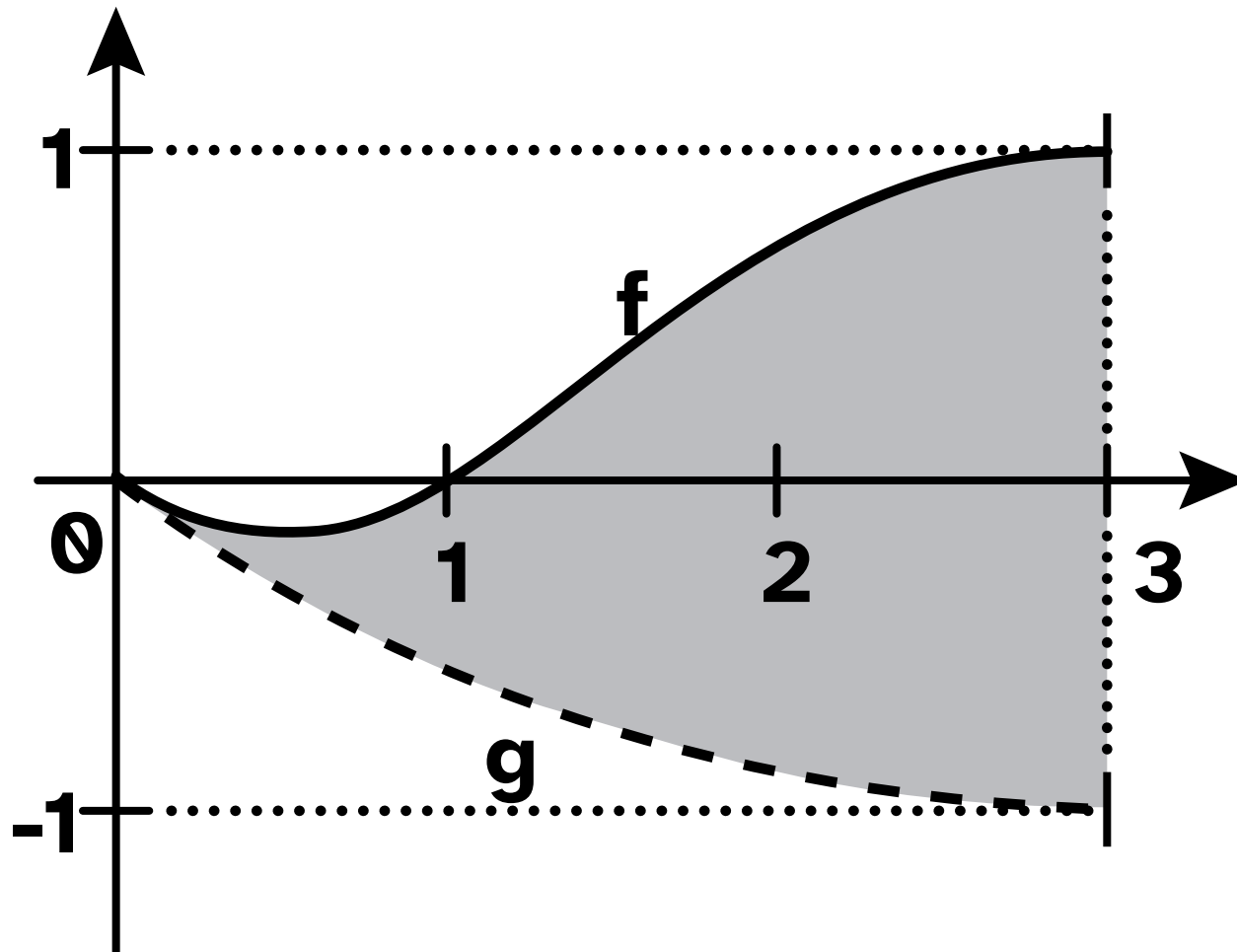






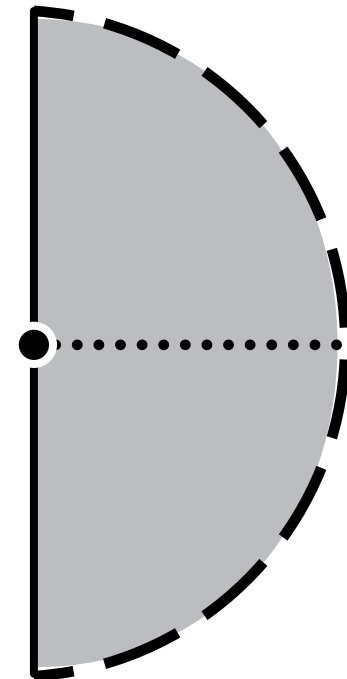


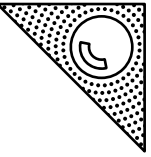
Teilfläche 1:



Teilfläche 2:

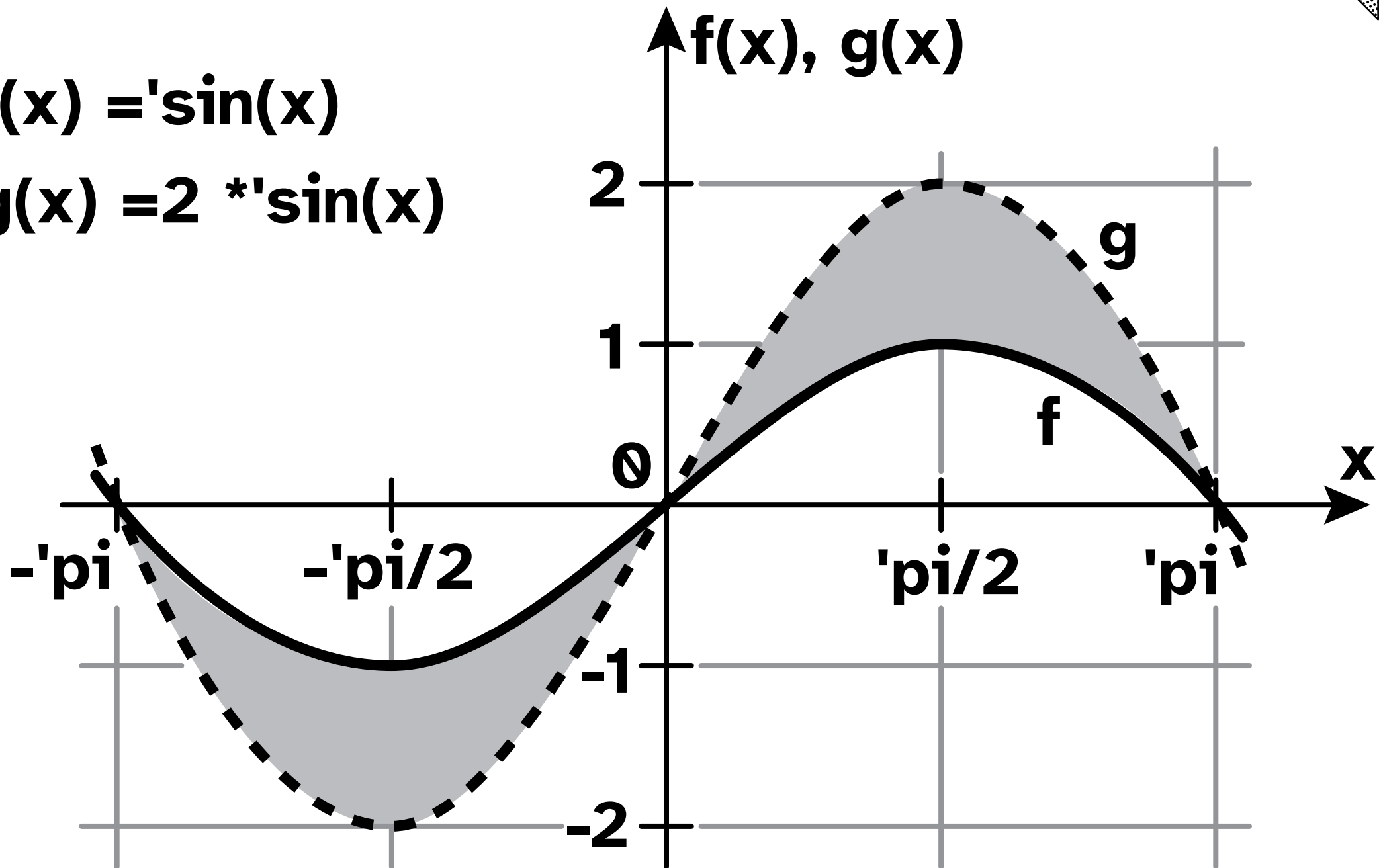
$r = 1$

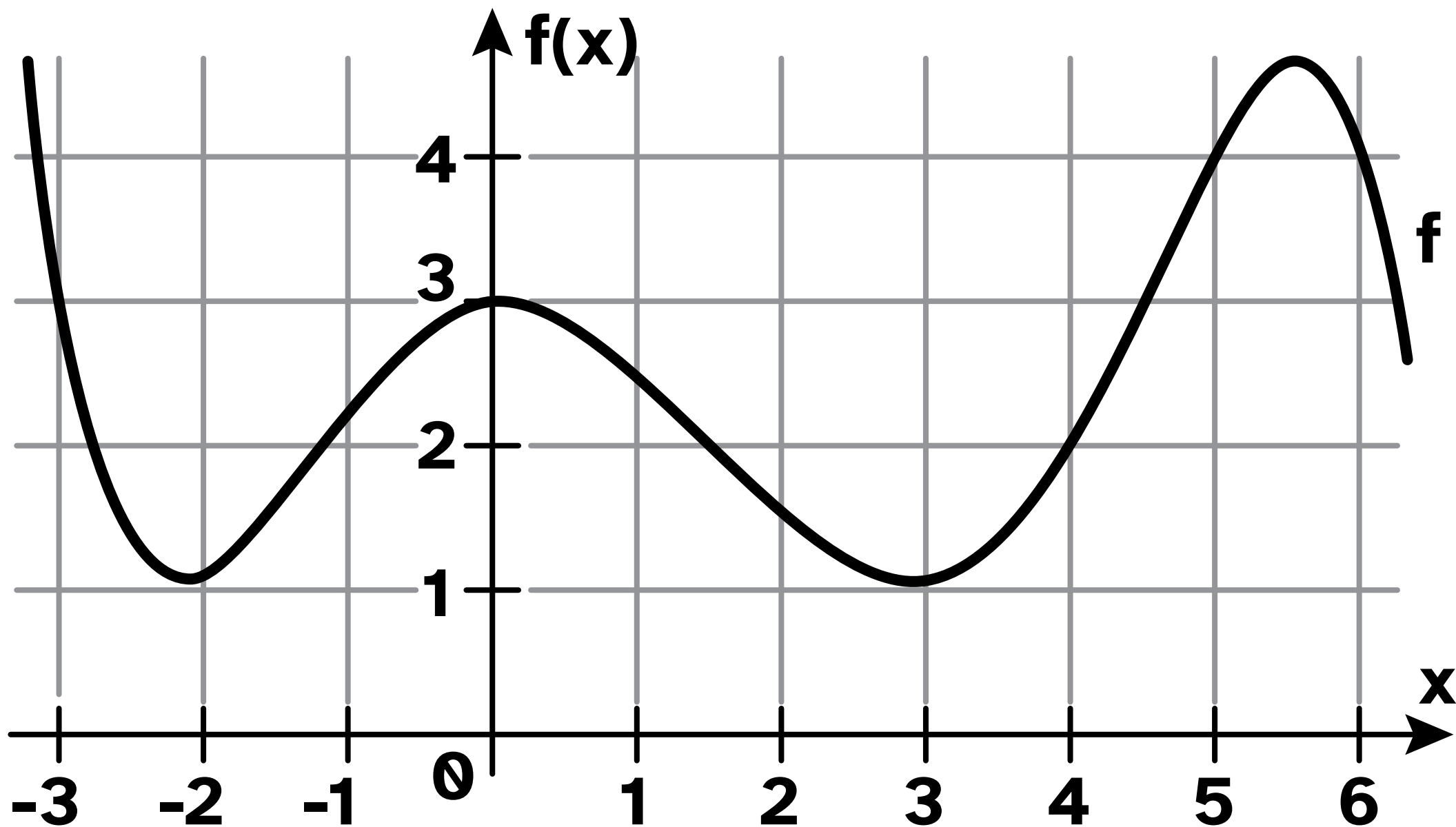
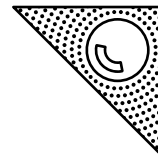


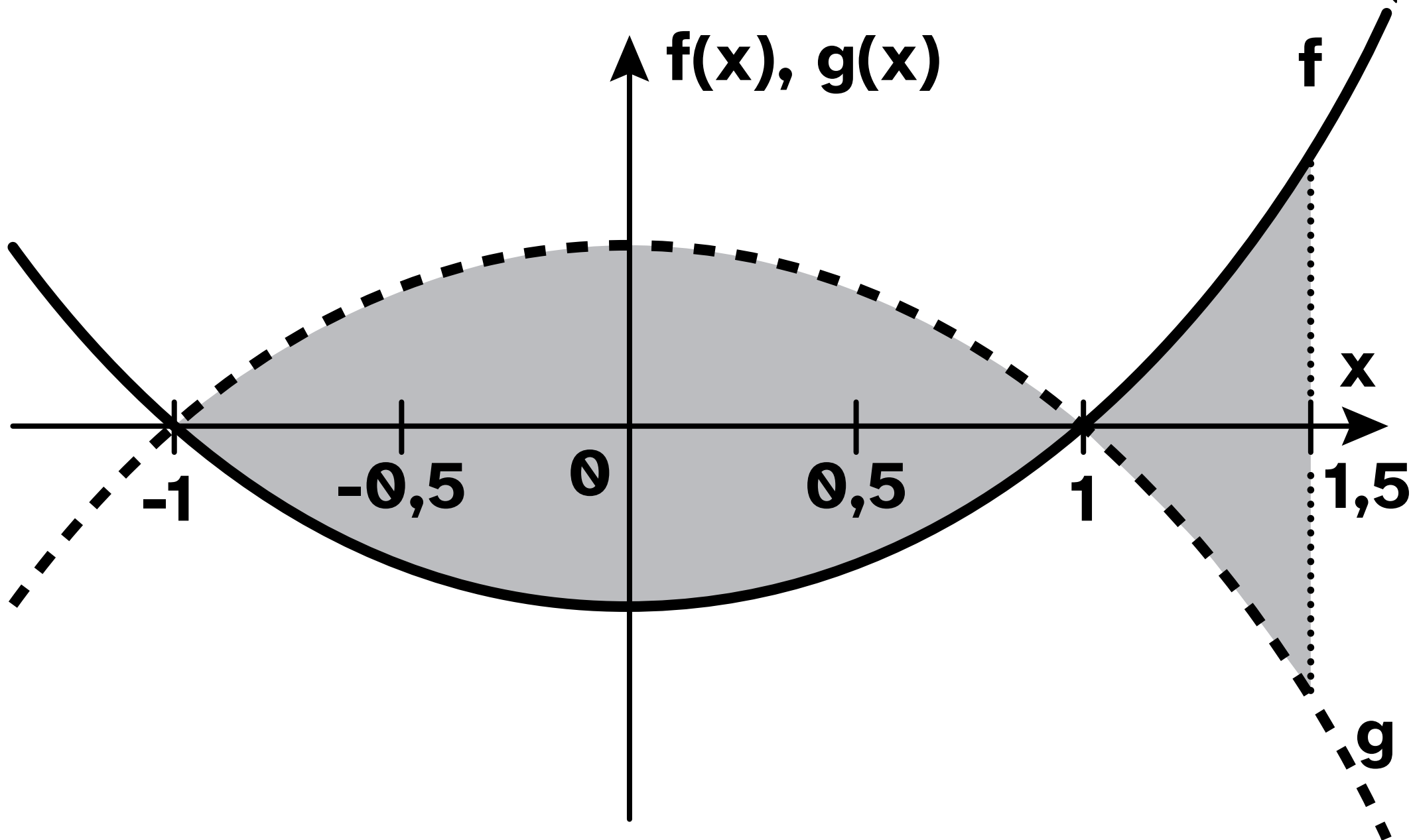
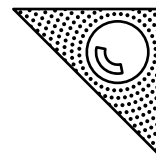


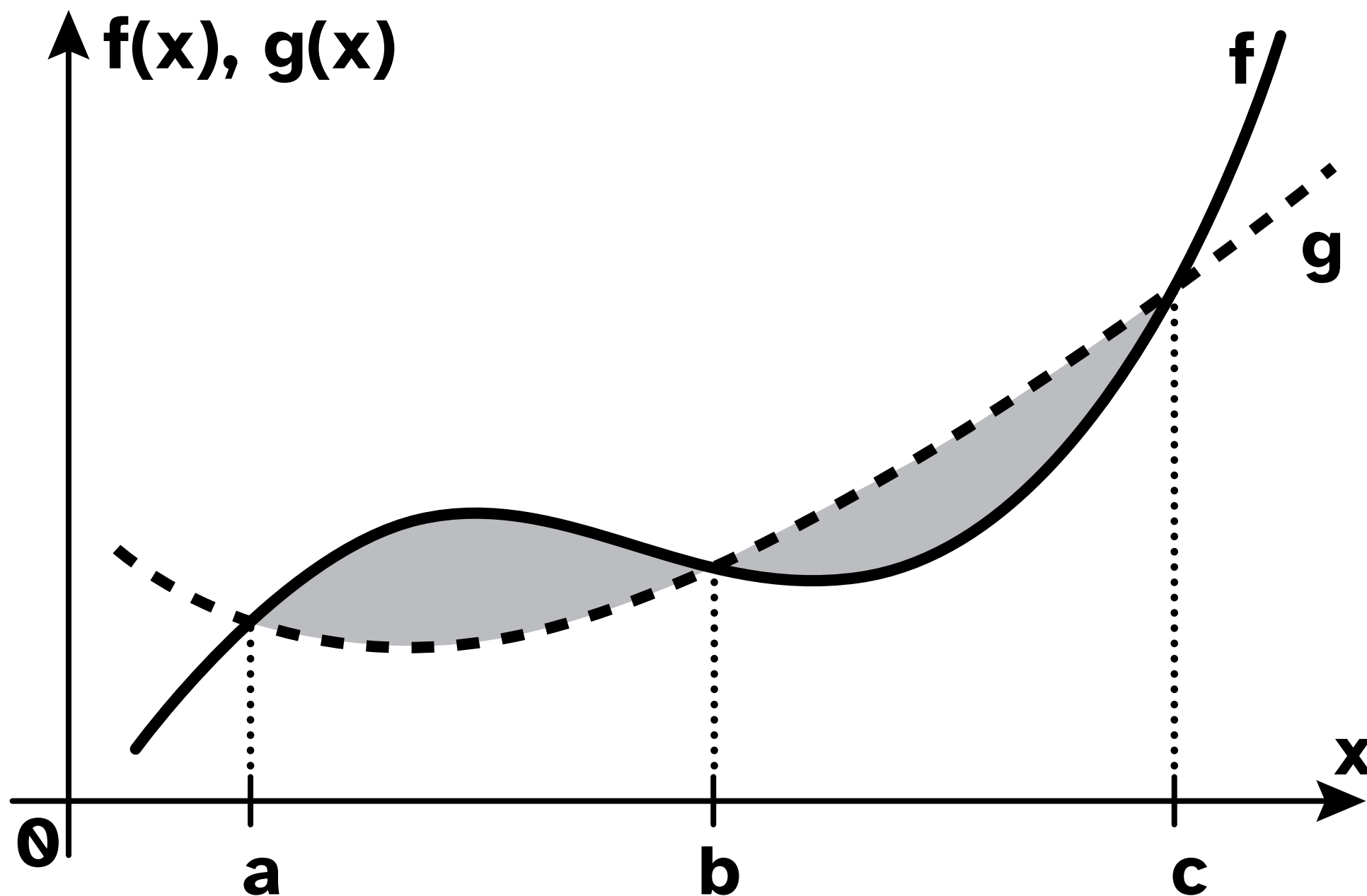
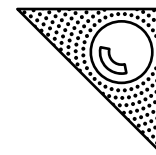
$$f(x) = \sin(x)$$

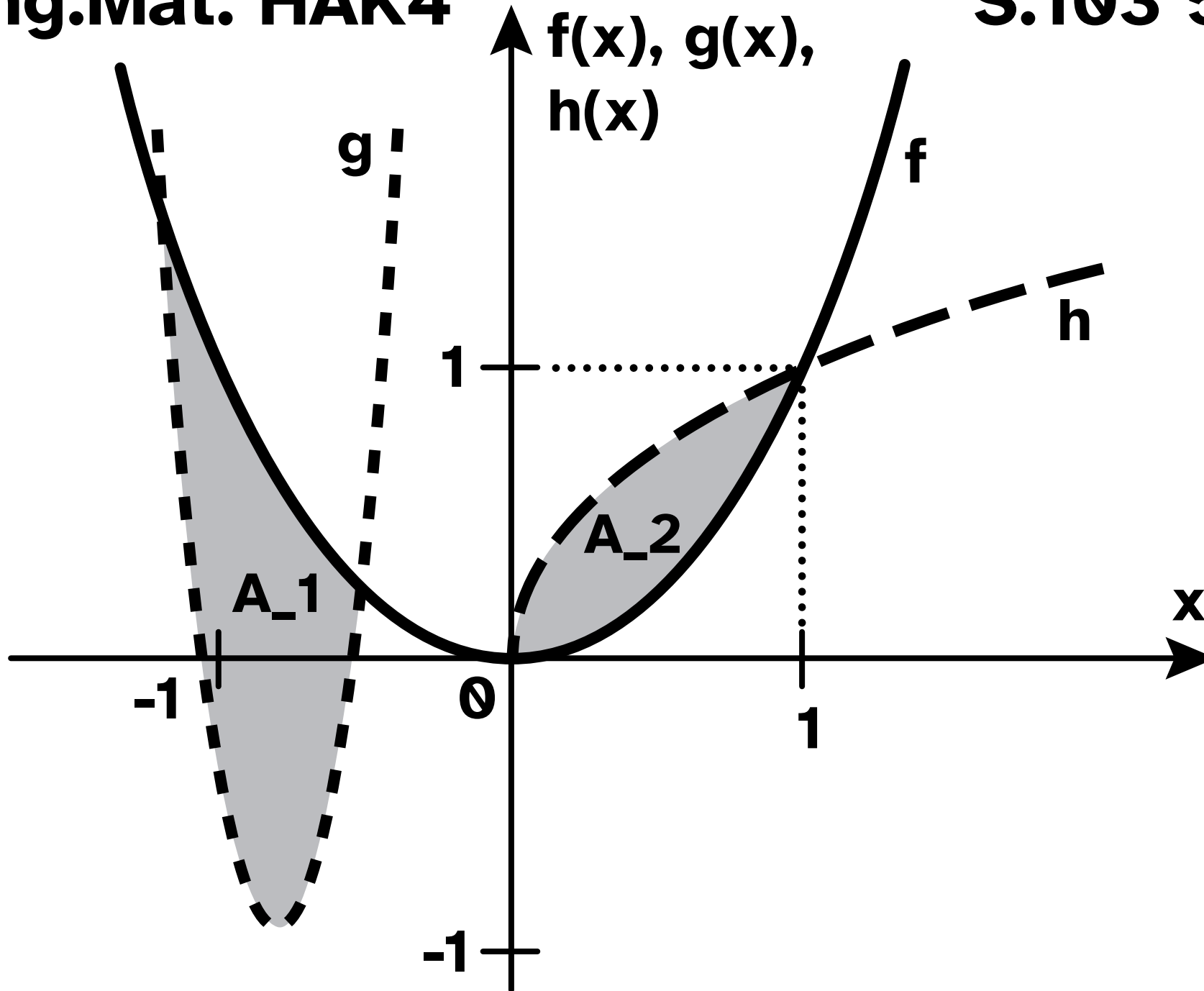
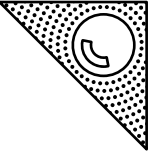
$$g(x) = 2 \cdot \sin(x)$$

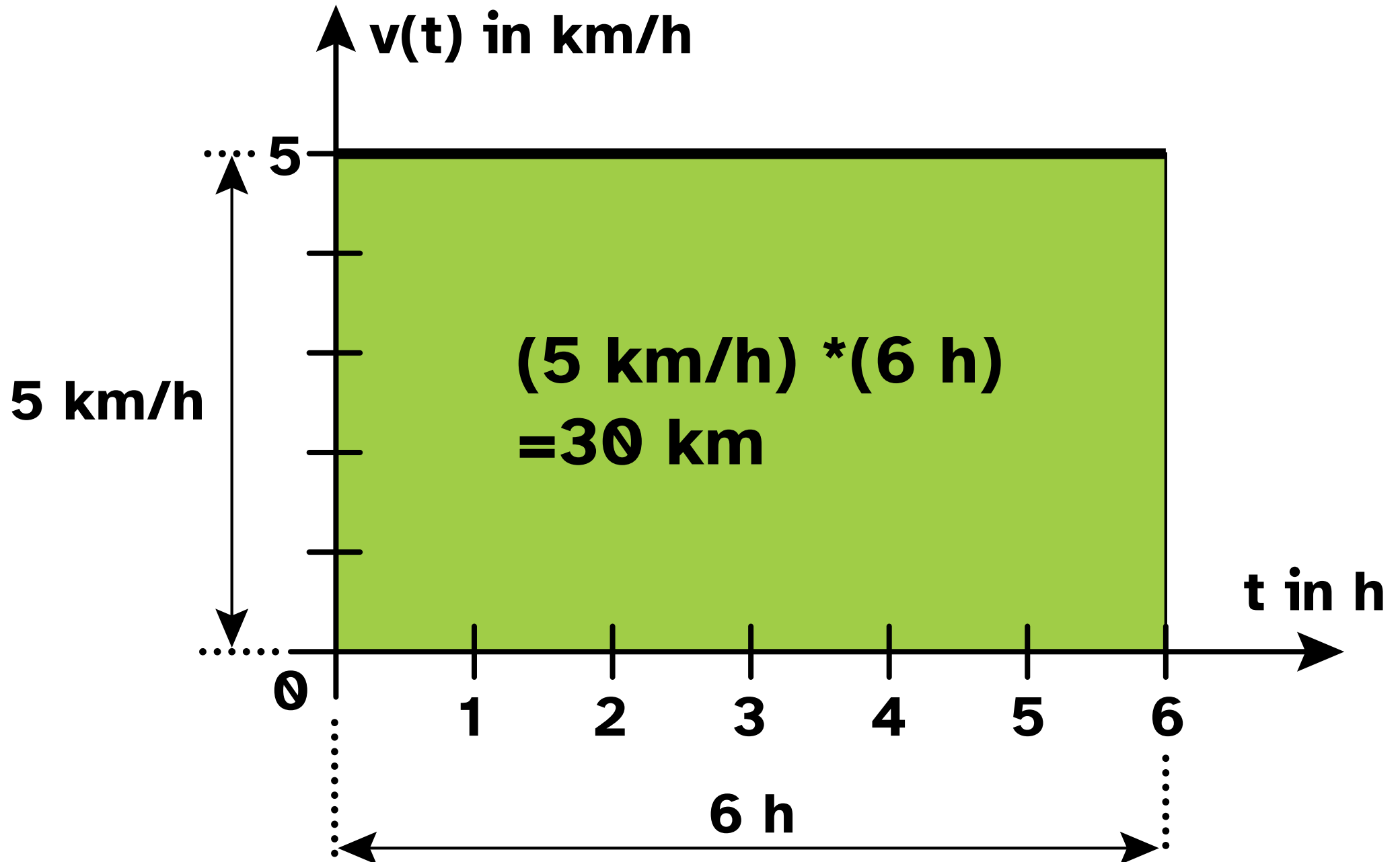
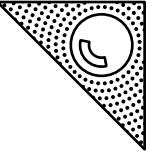


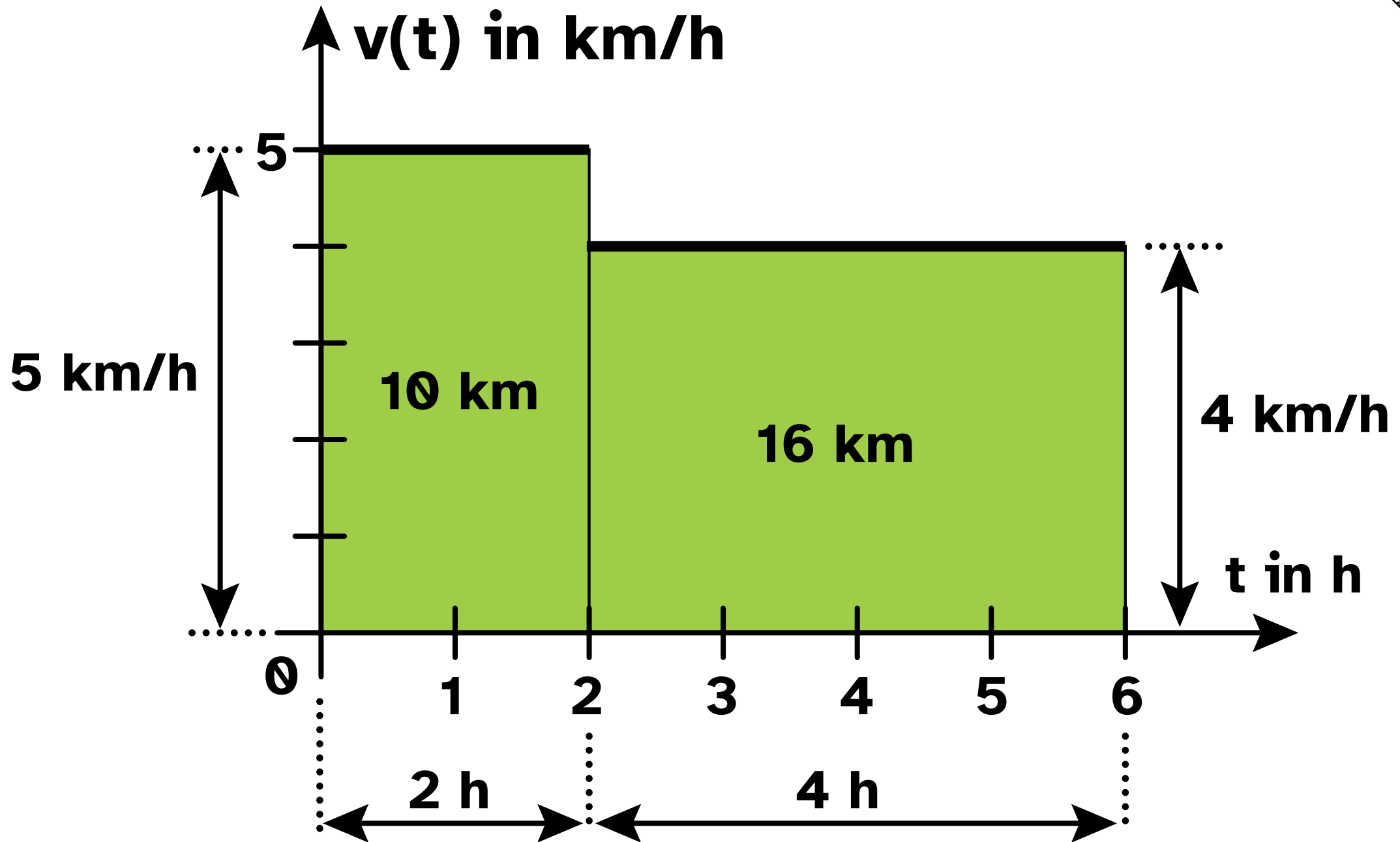
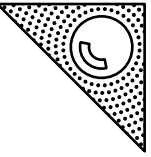


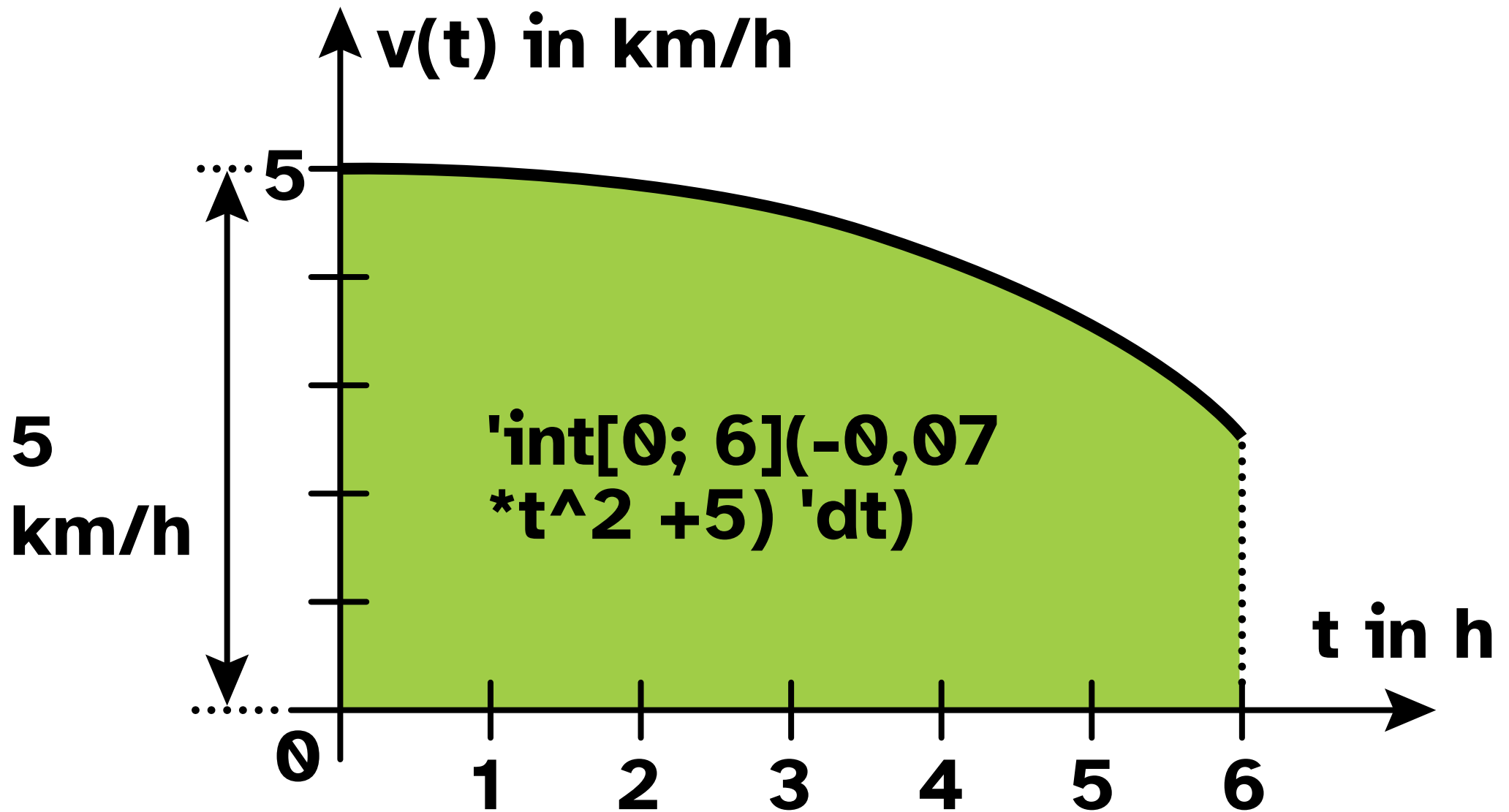
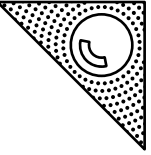


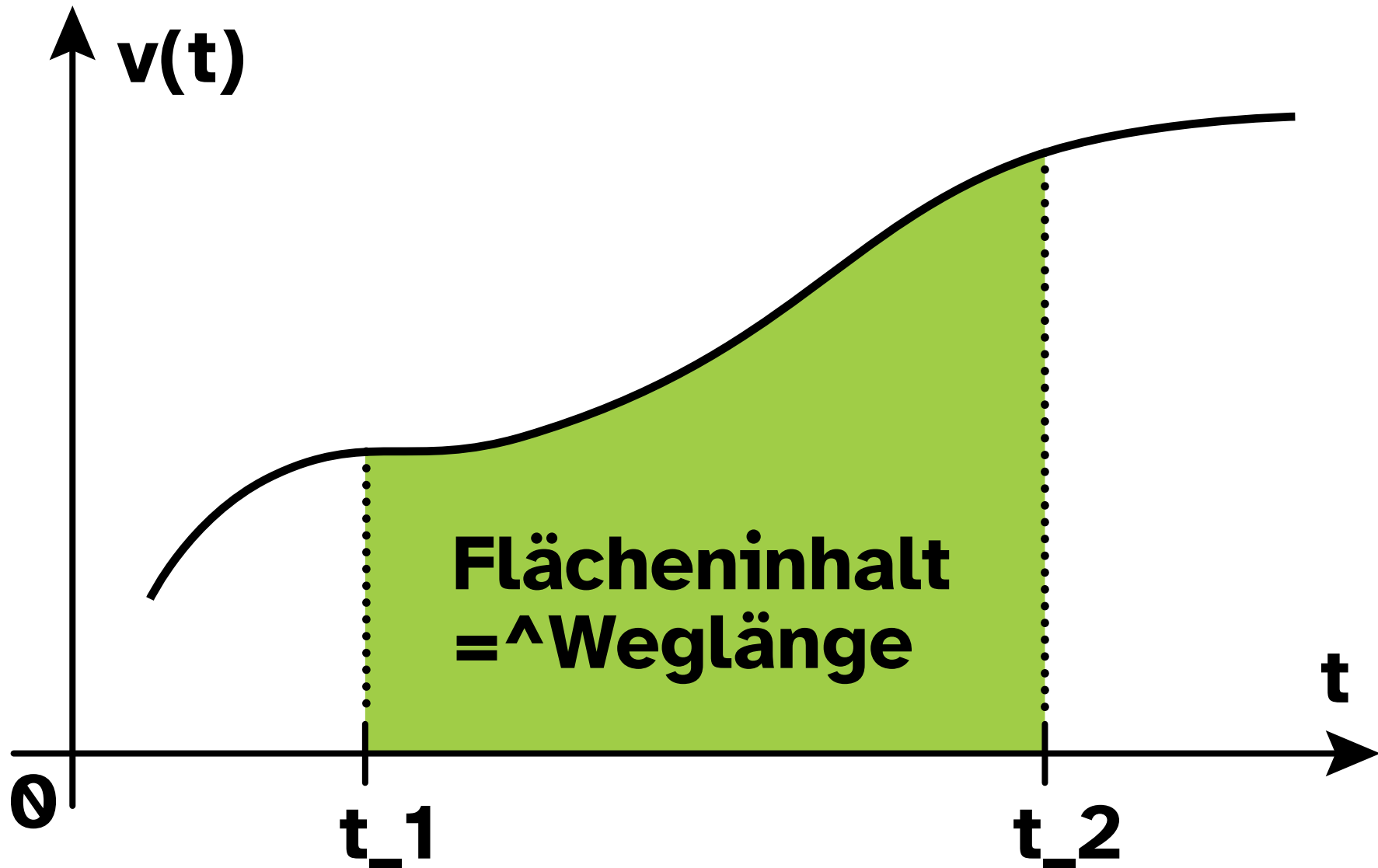
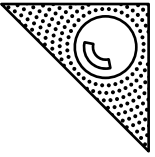


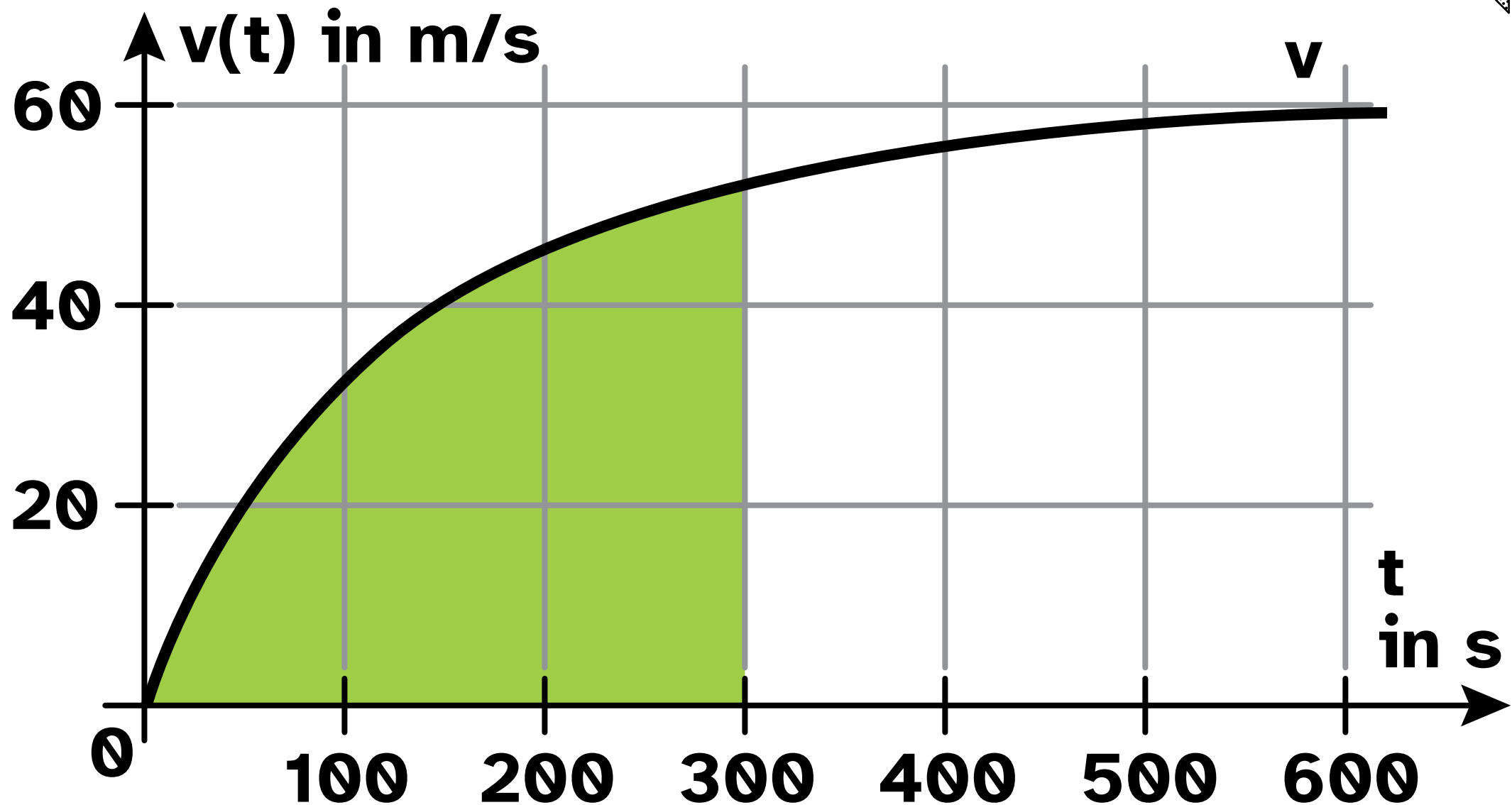
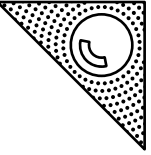




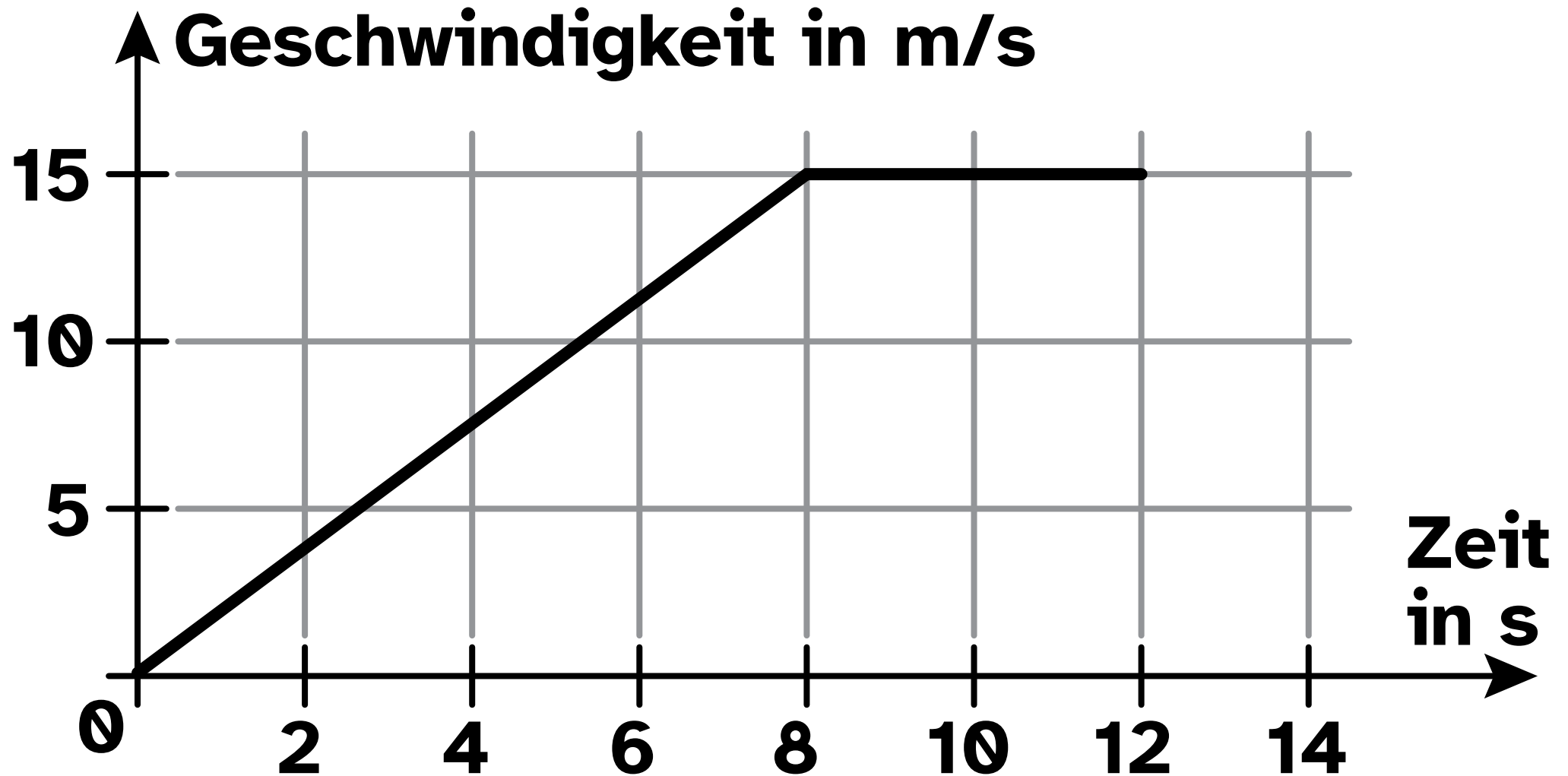
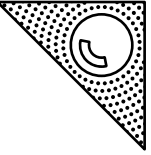


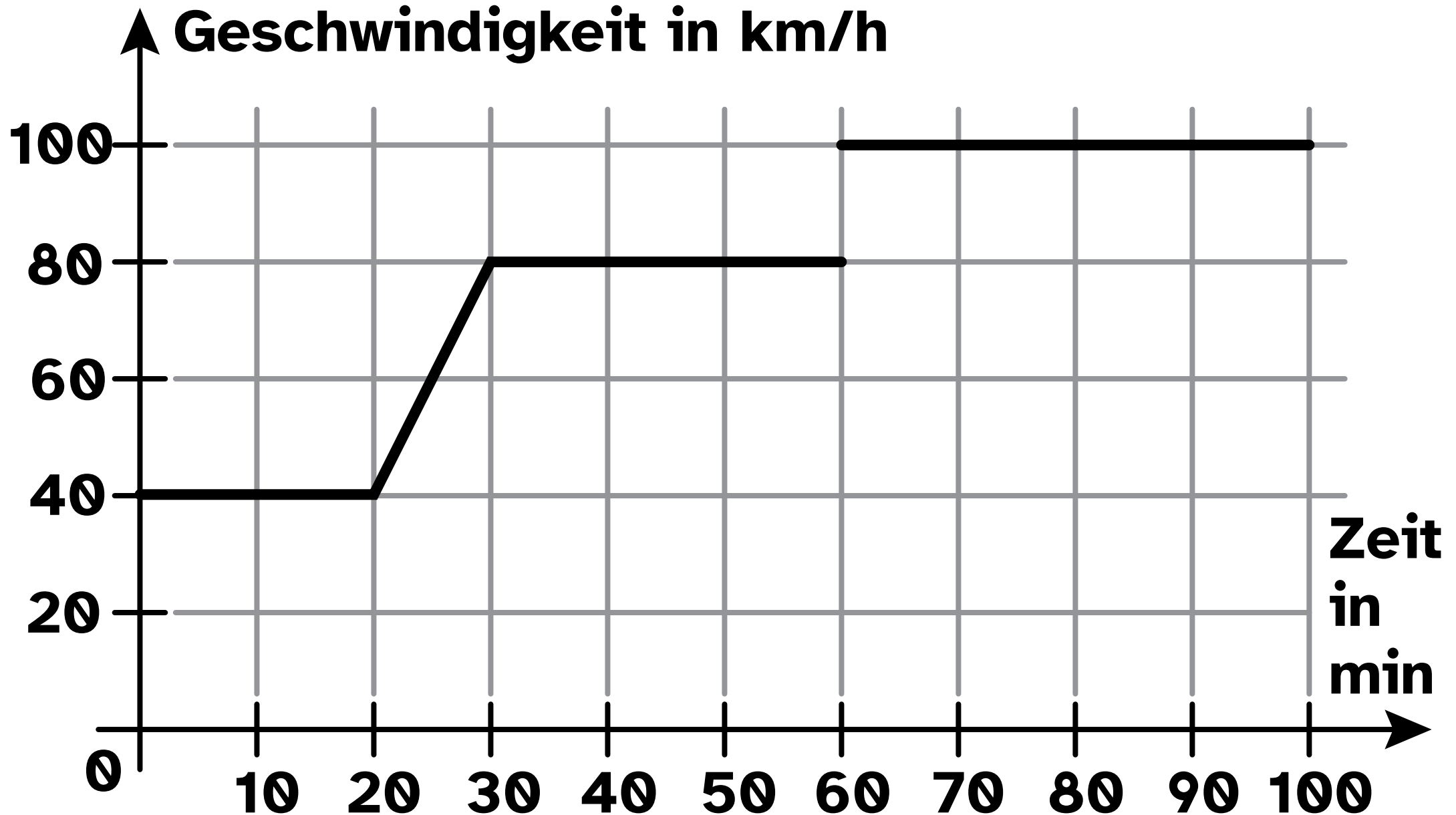
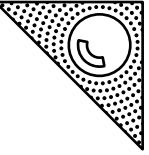


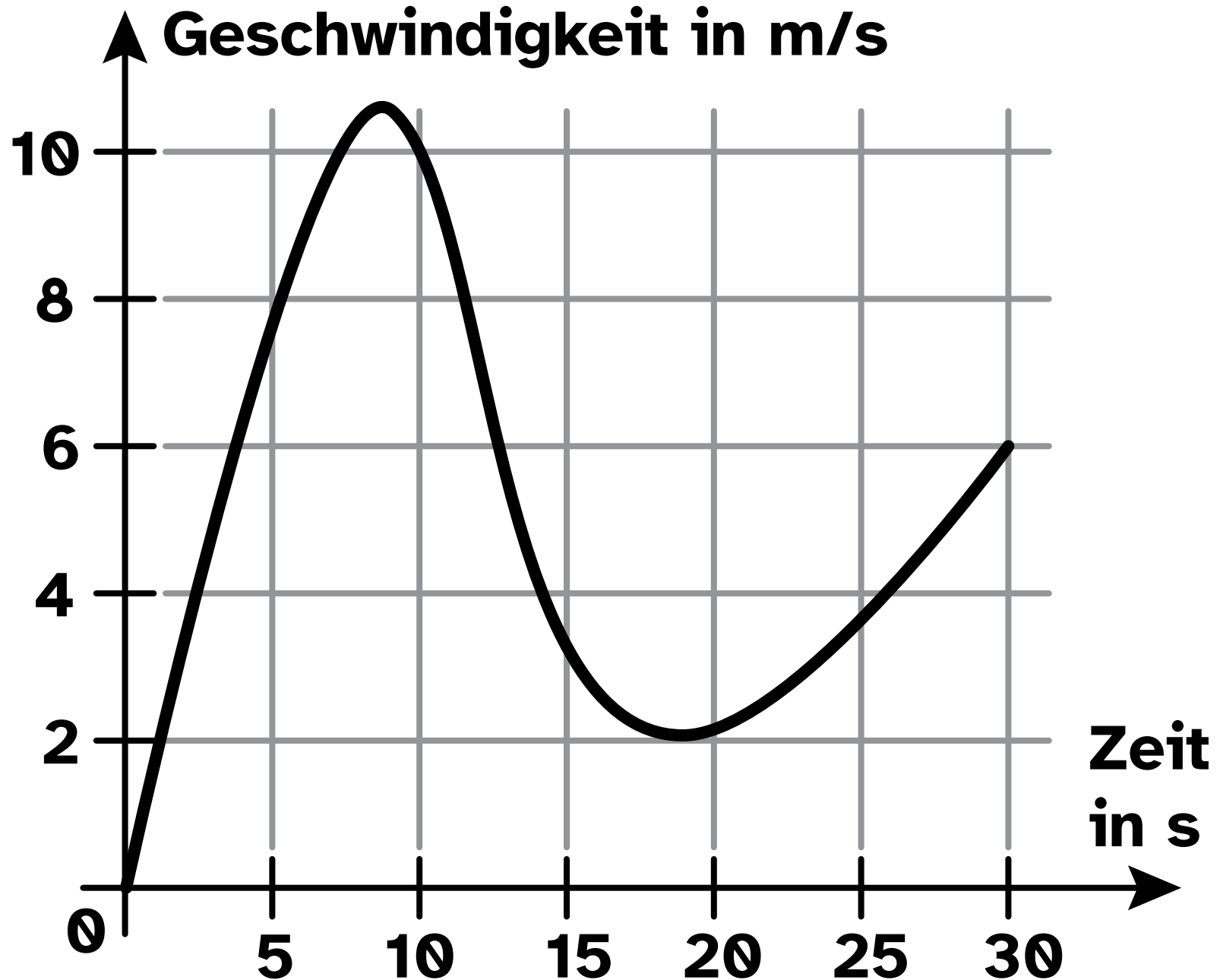
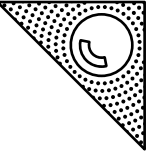


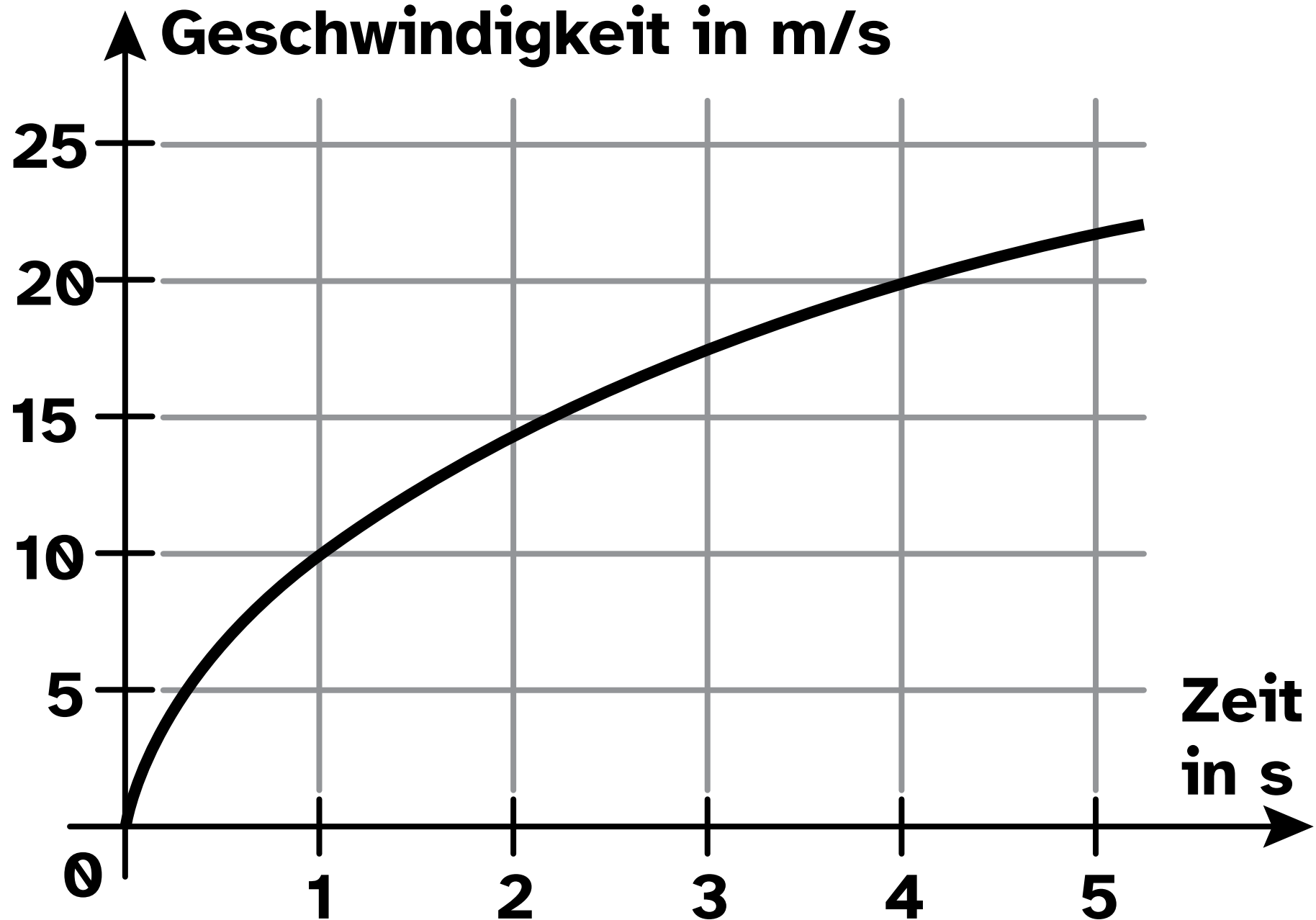
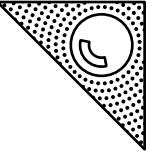


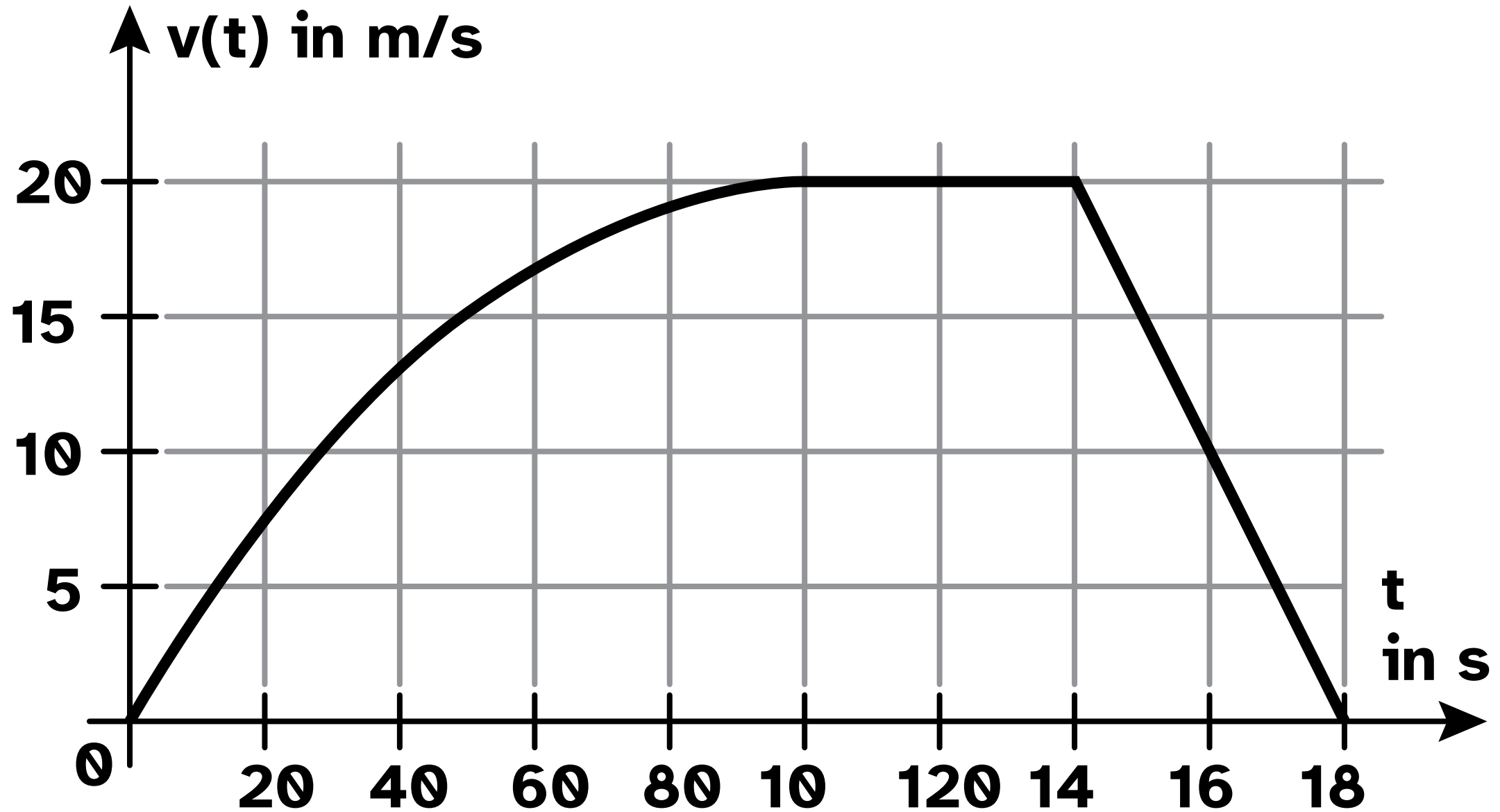
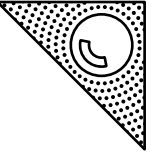
 $\int_0^{300} v(t) \, dt$

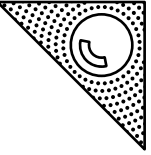












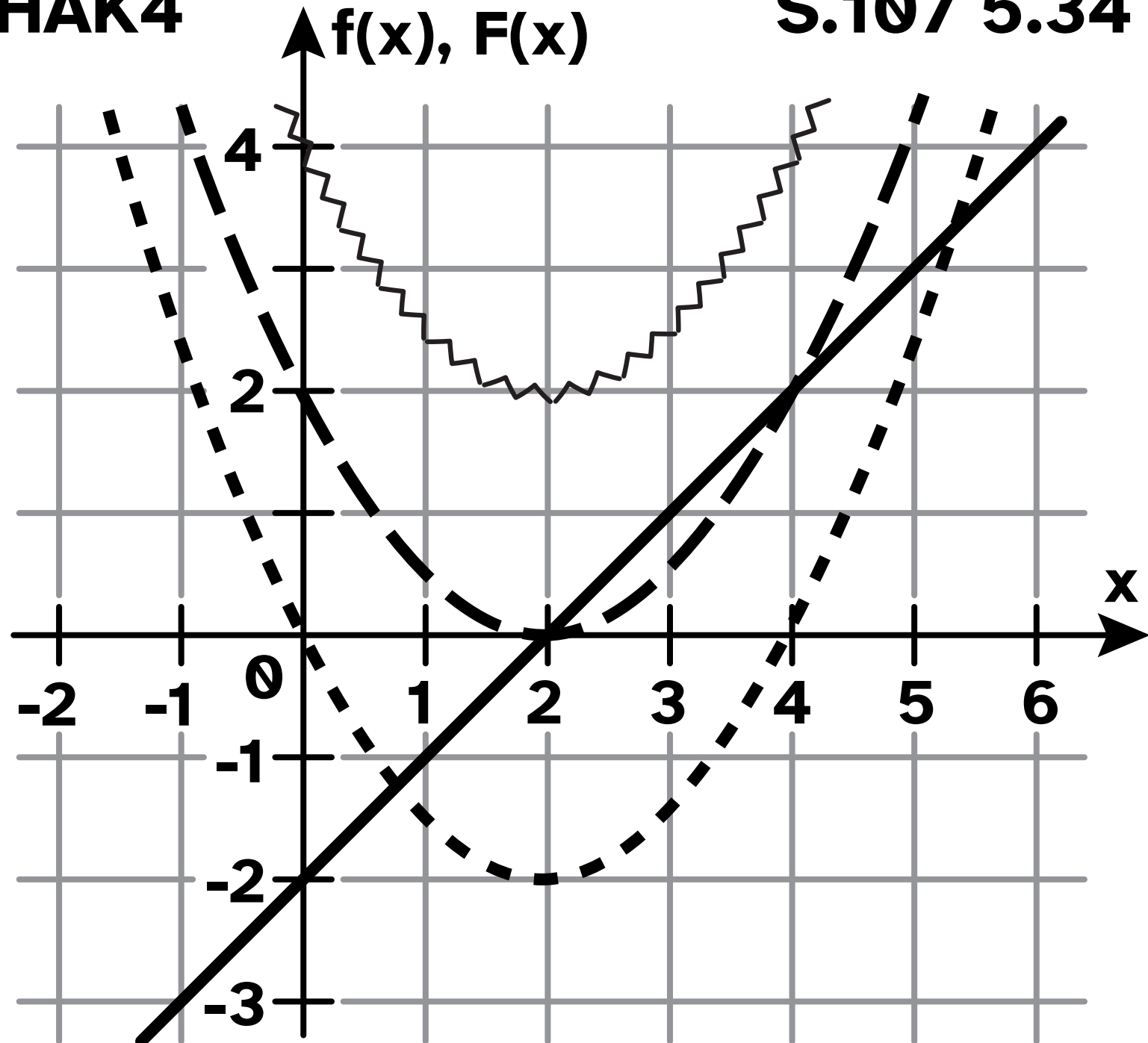
Legende:

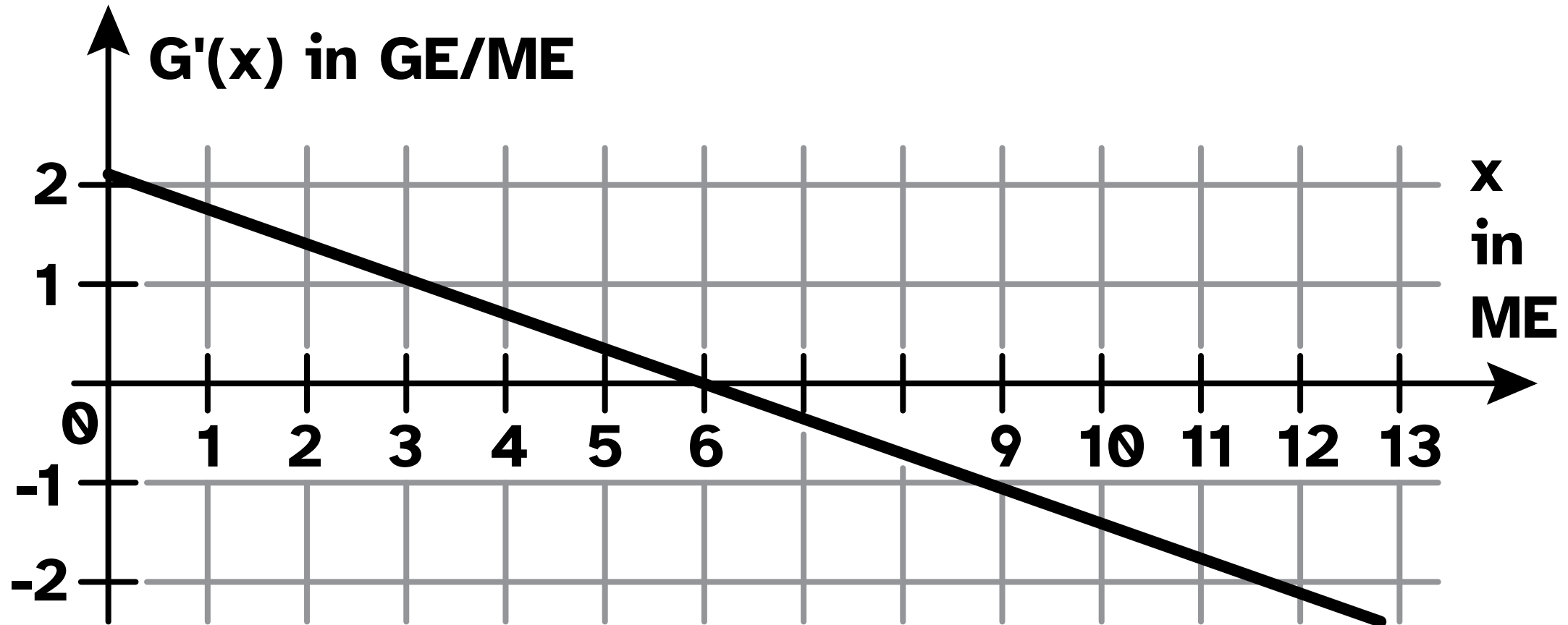
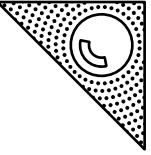
— f

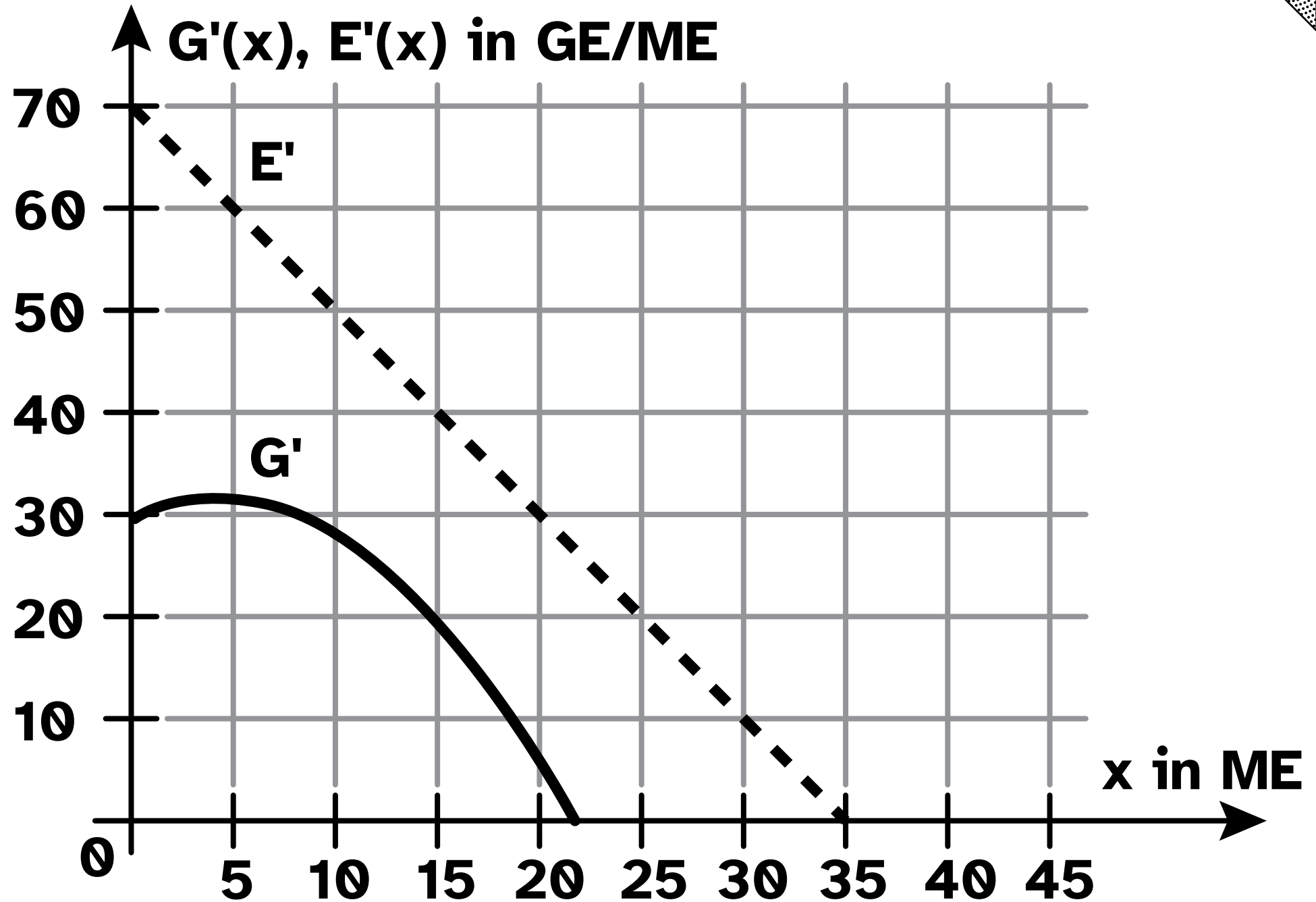
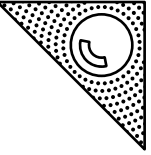
- - - F_1

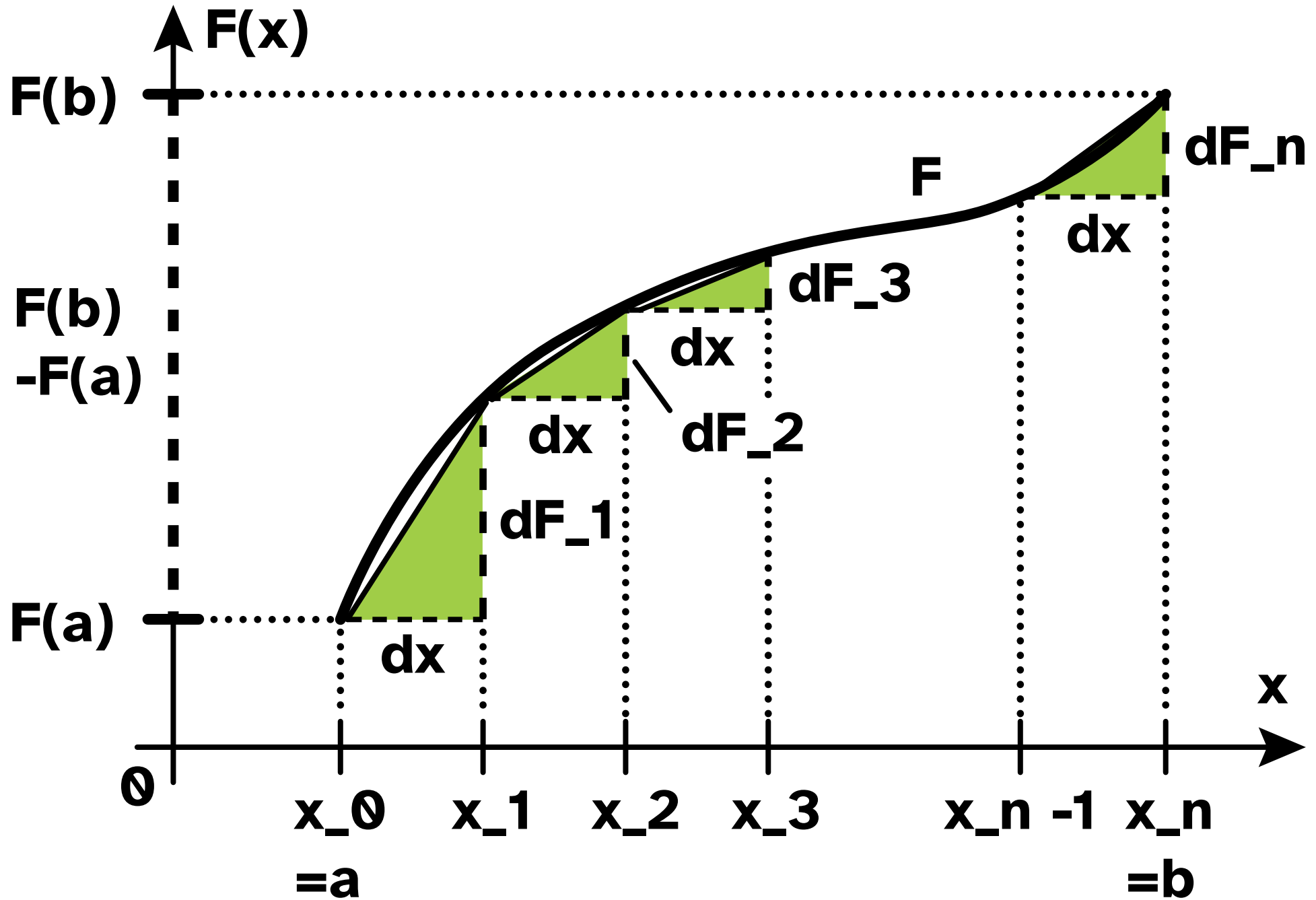
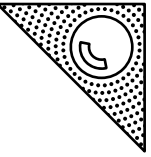
- - - F_2

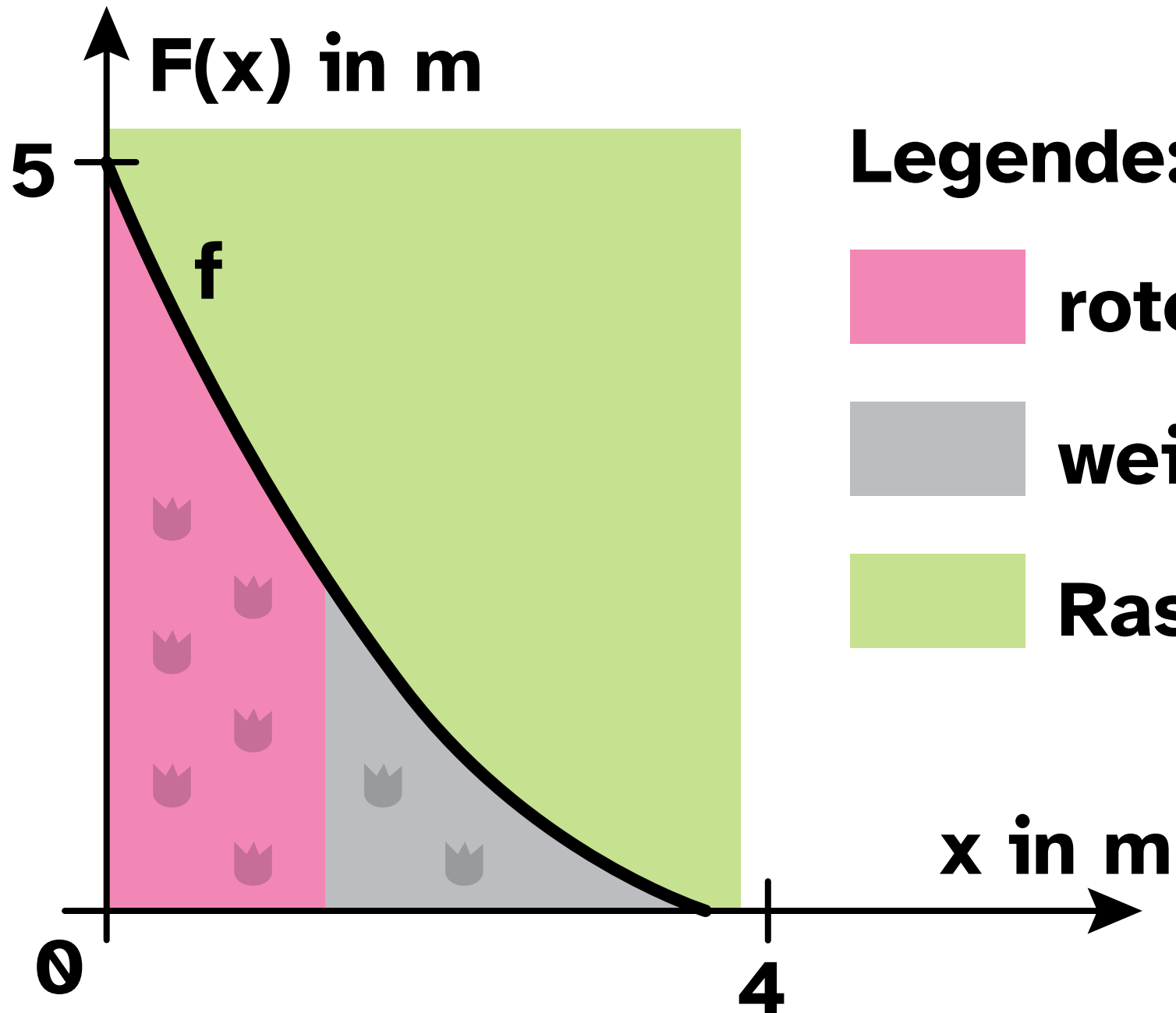
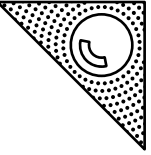
~~~~~  $F_3$











**Legende:**



**rote Tulpen**

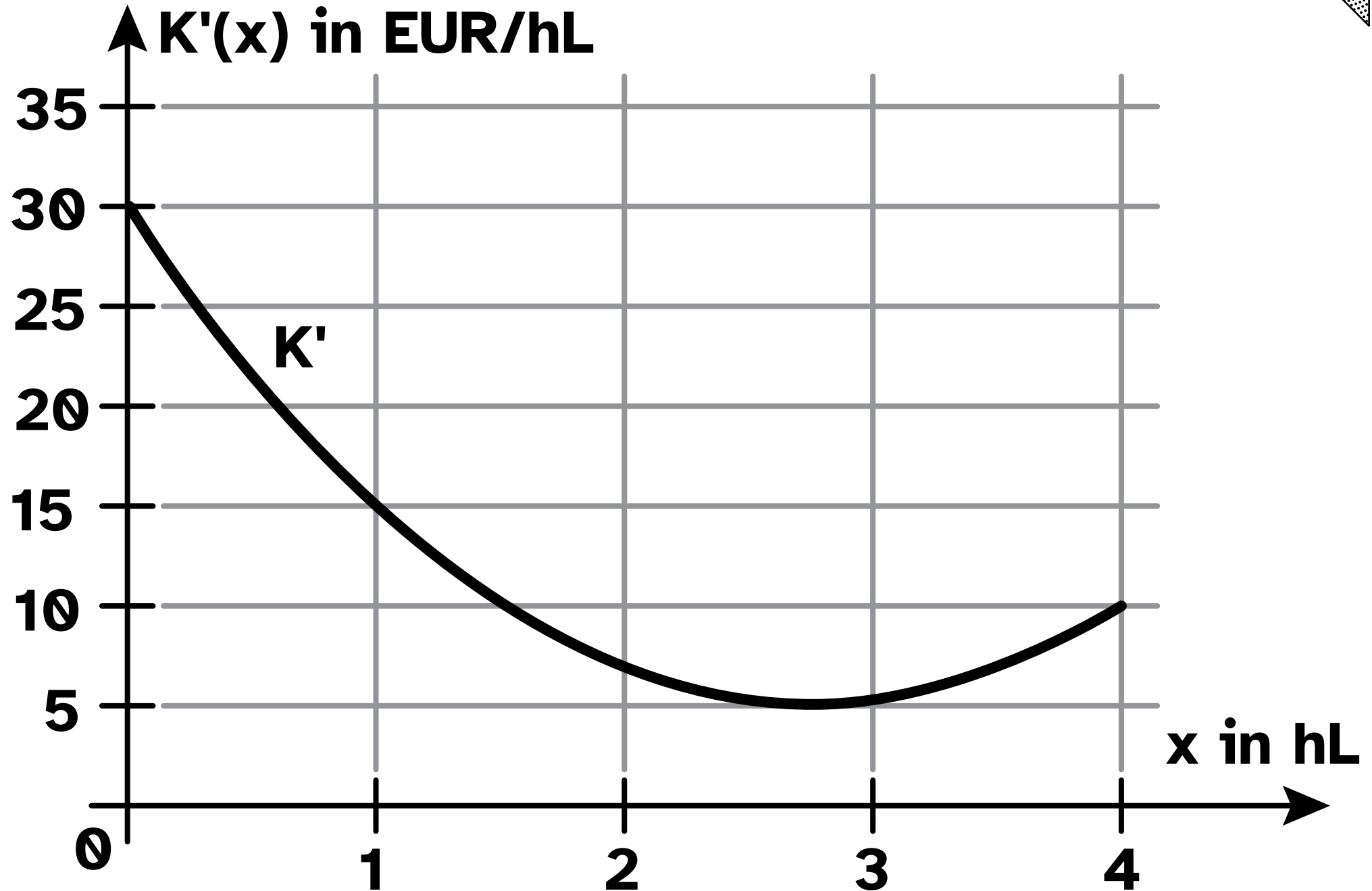
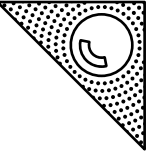


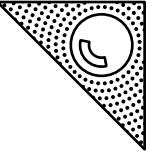
**weiße Tulpen**



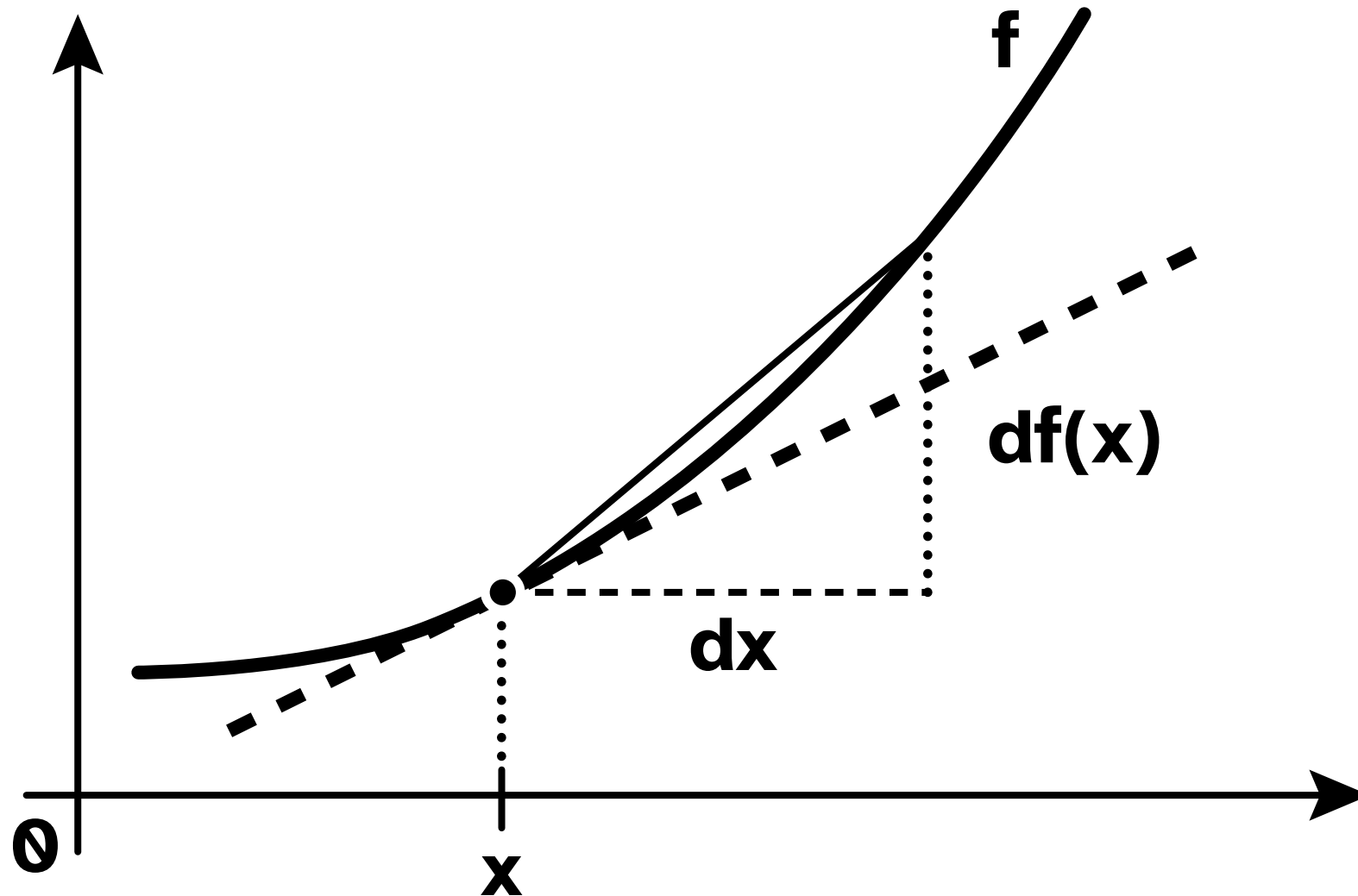
**Rasen**

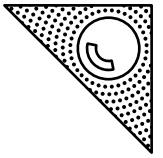






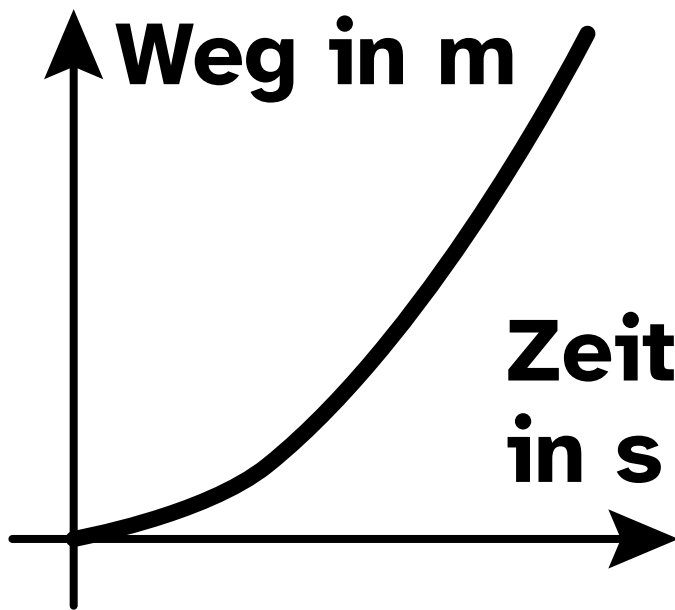
### Differentialrechnung - Grundvorstellung: Steigung an der Stelle $x$





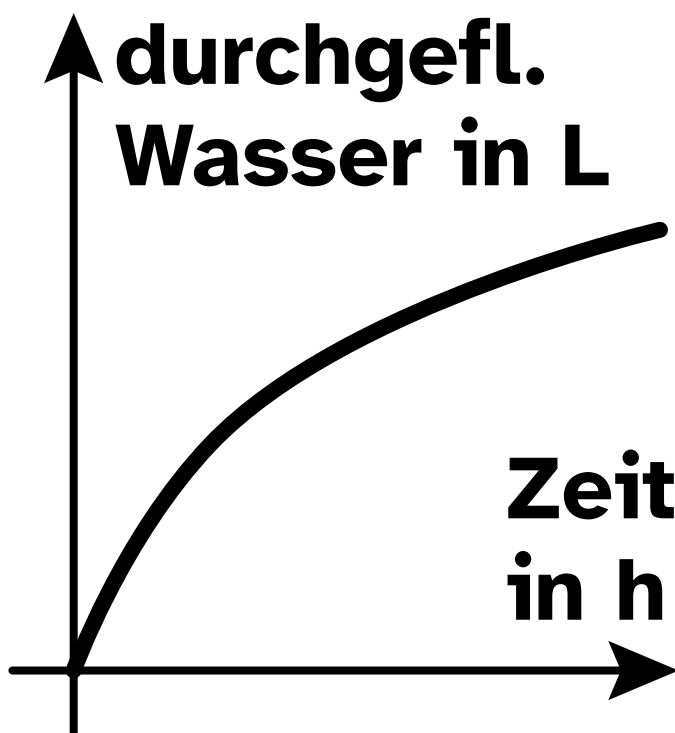
# Differentialrechnung - Anwendungsbeispiele:

## 1) Weg -> Geschwindigkeit

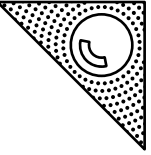


Weg in m  
/Zeit in s  
=Geschwindigkeit  
in m/s

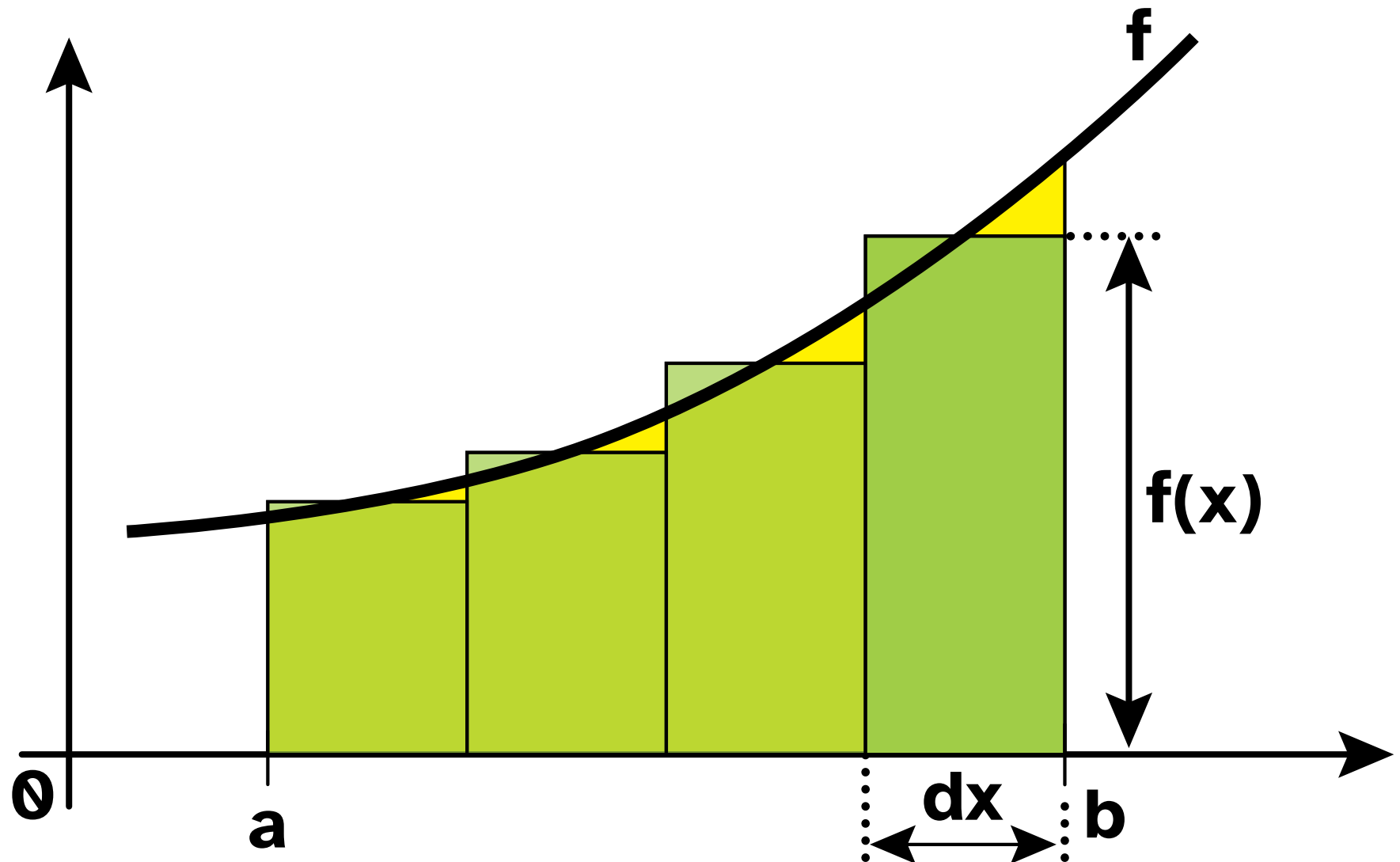
## 2) Bestand -> Änderungsrate

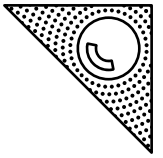


durchgefl.  
Wasser Volumen in L /Zeit  
in h =Durch-  
flussrate in  
L/h



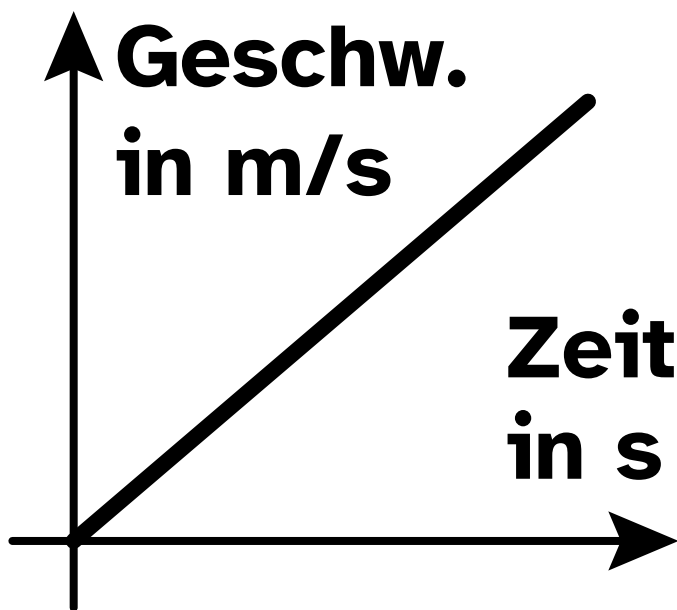
### Integralrechnung - Grundvorstellung: Flächeninhalt im Intervall $[a; b]$





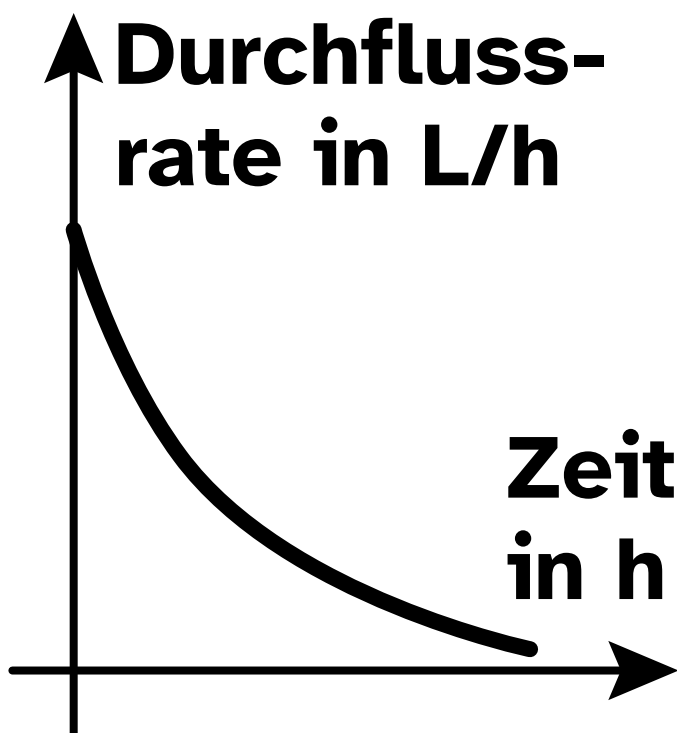
# Integralrechnung - Anwendungsbeispiele:

## 1) Geschwindigkeit -> Weg

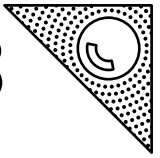


**Geschwindigkeit \* Zeit in  
 $\text{m/s} * \text{s} = \text{Weg}$   
in m**

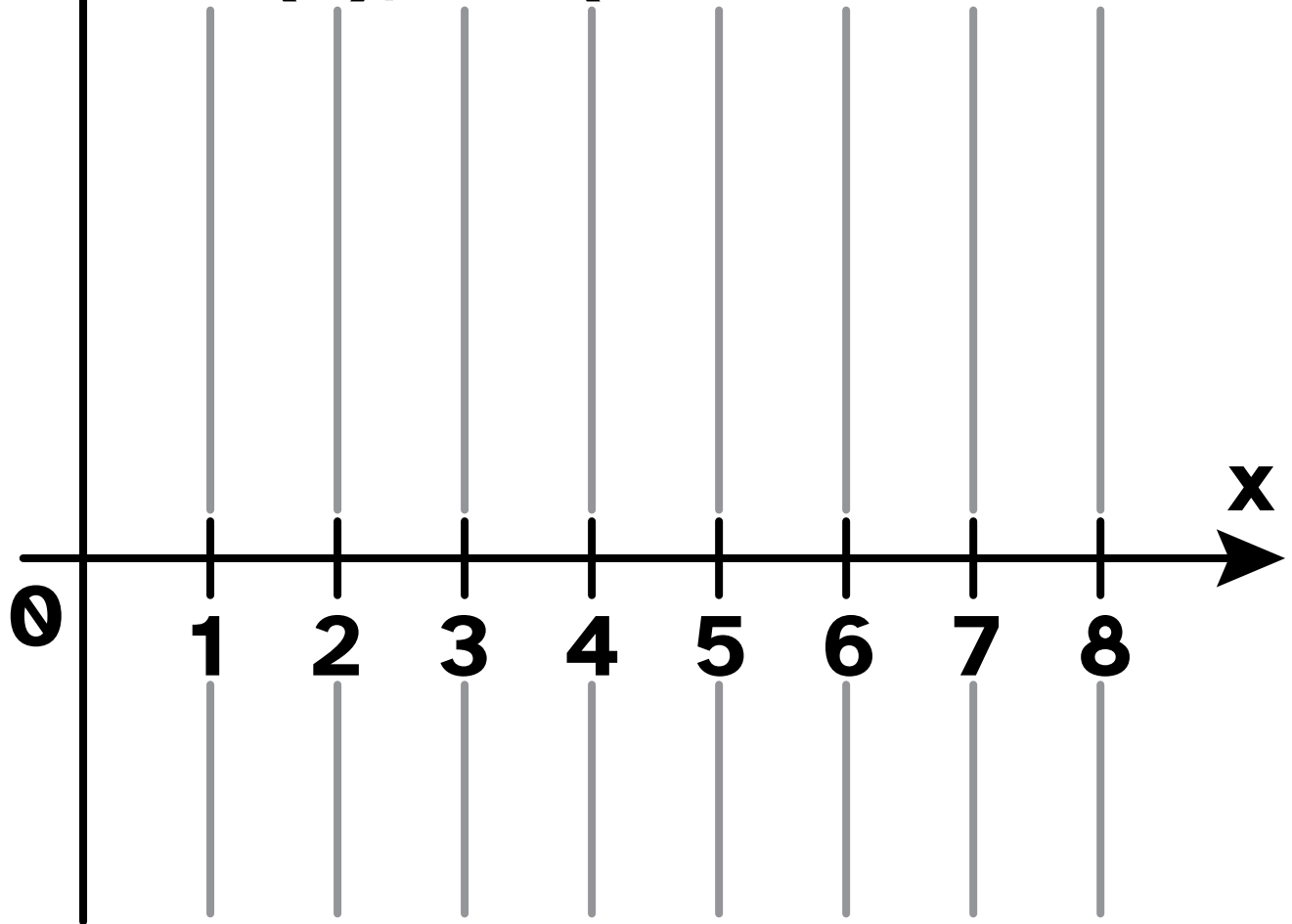
## 2) Änderungsrate -> Bestand



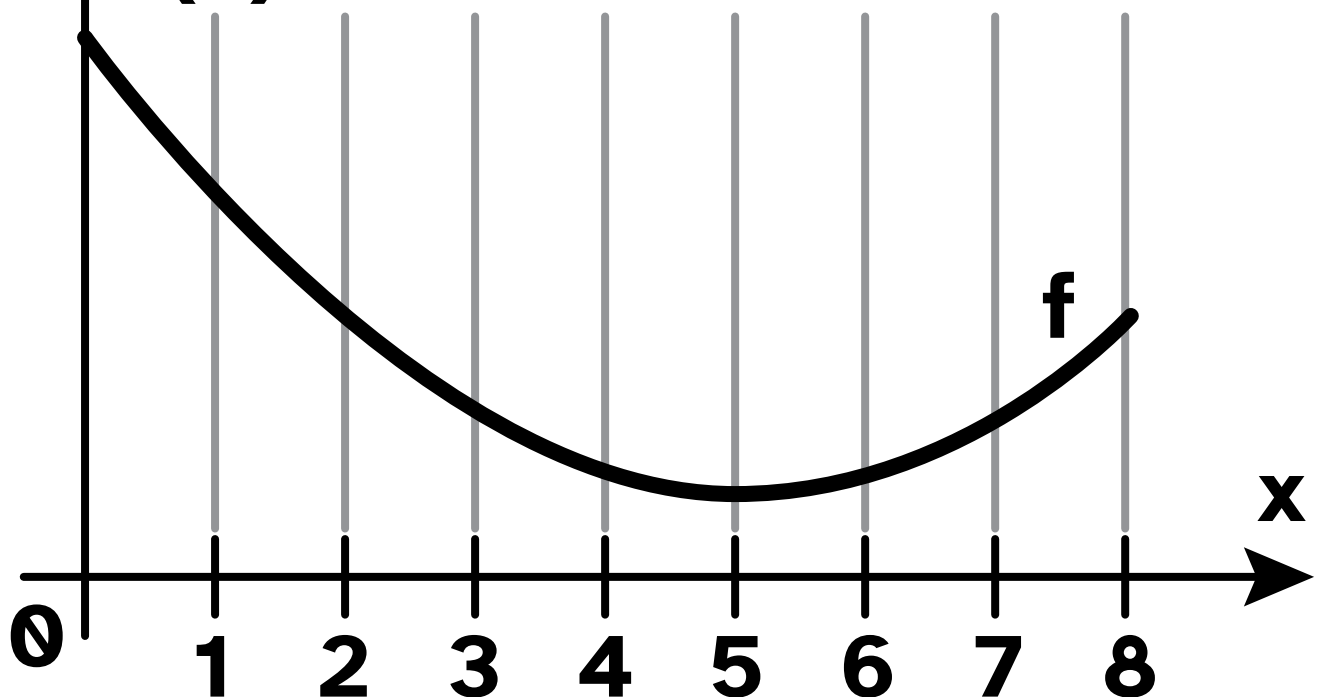
**Durchflussrate  
\* Zeit in  $\text{L/h} * \text{h}$   
= durchgeflossenes Volumen  
in L**

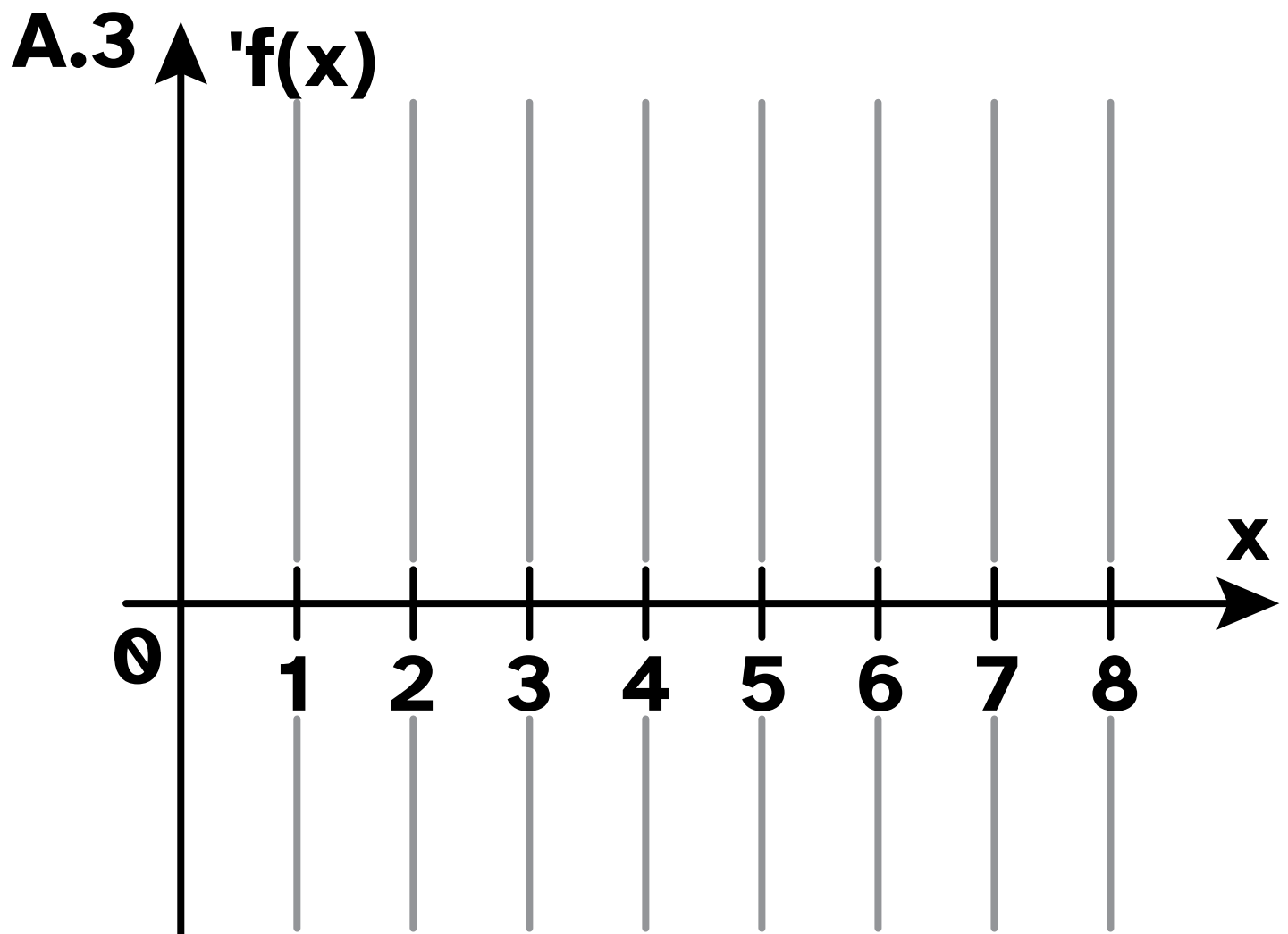
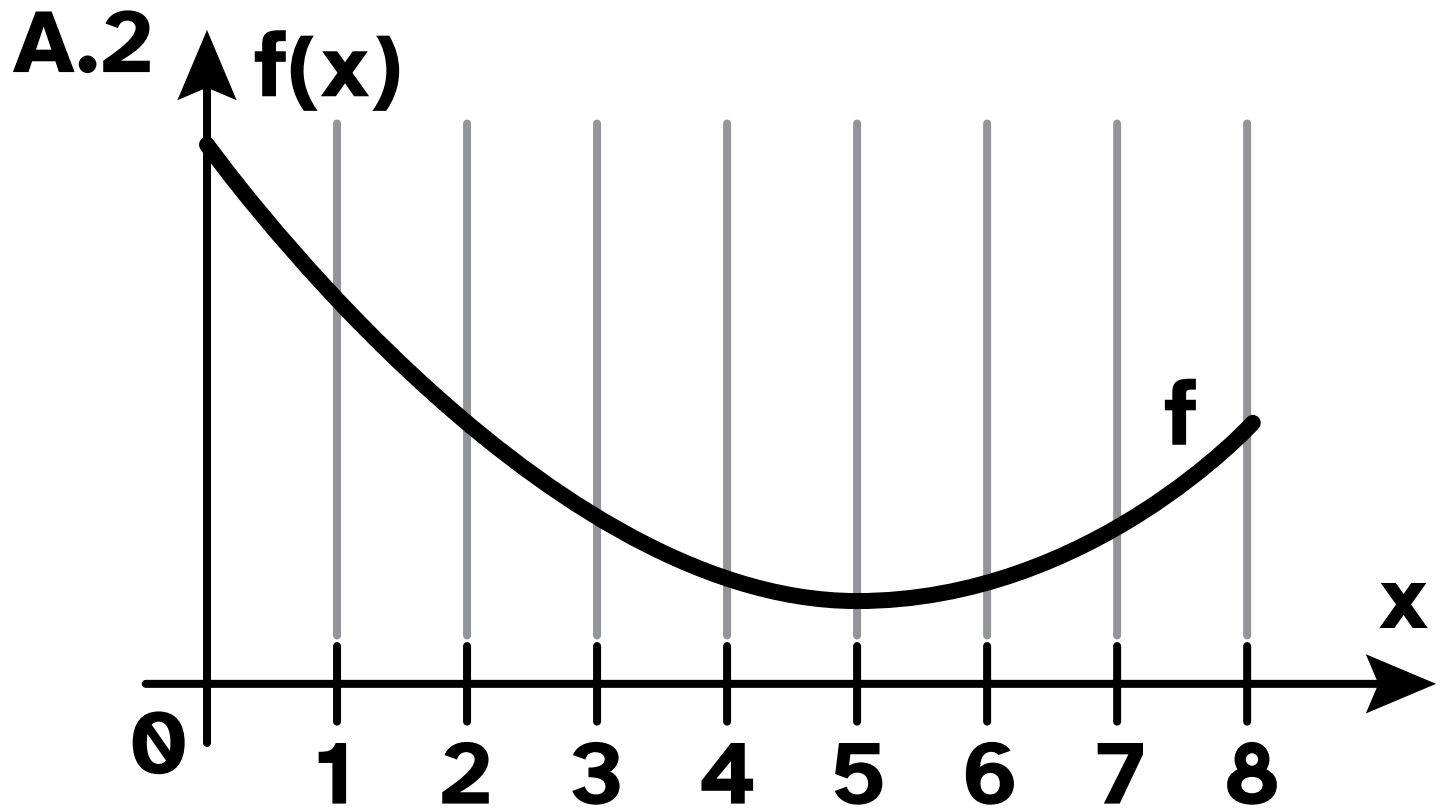
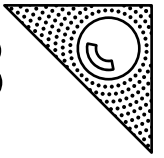


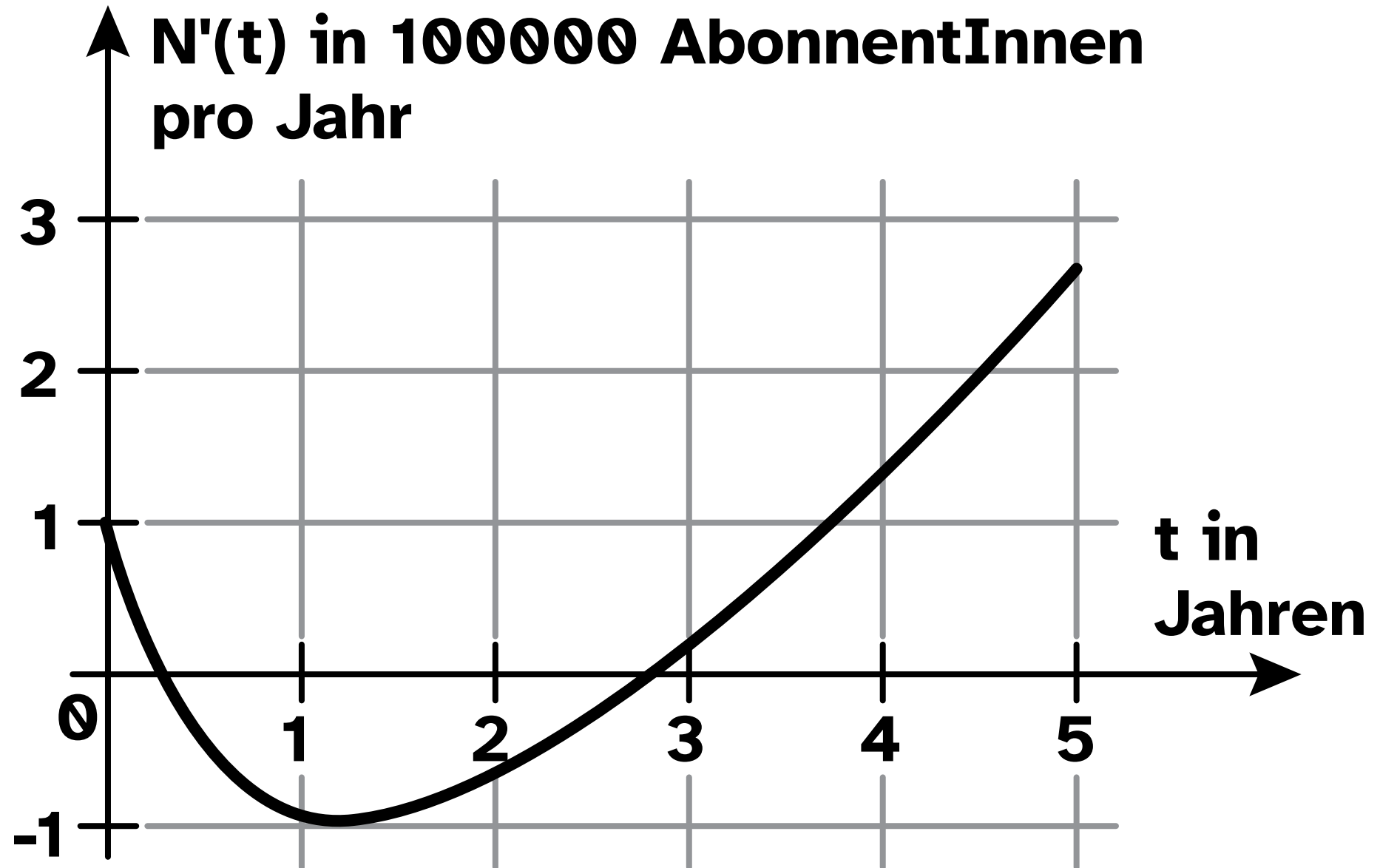
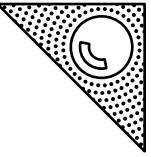
A.1  $\uparrow F_1(x), F_2(x)$



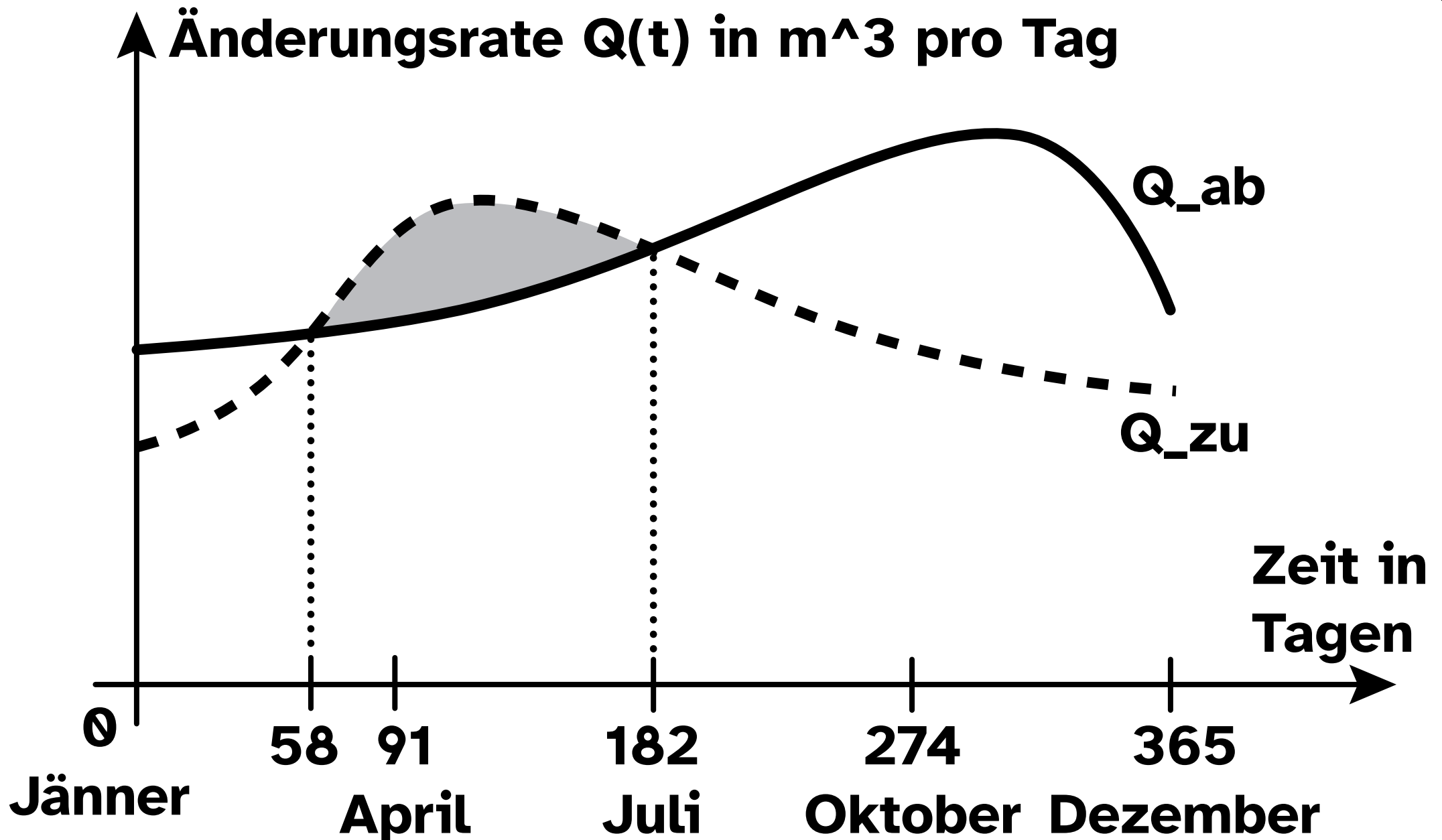
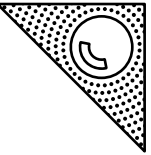
A.2  $\uparrow f(x)$

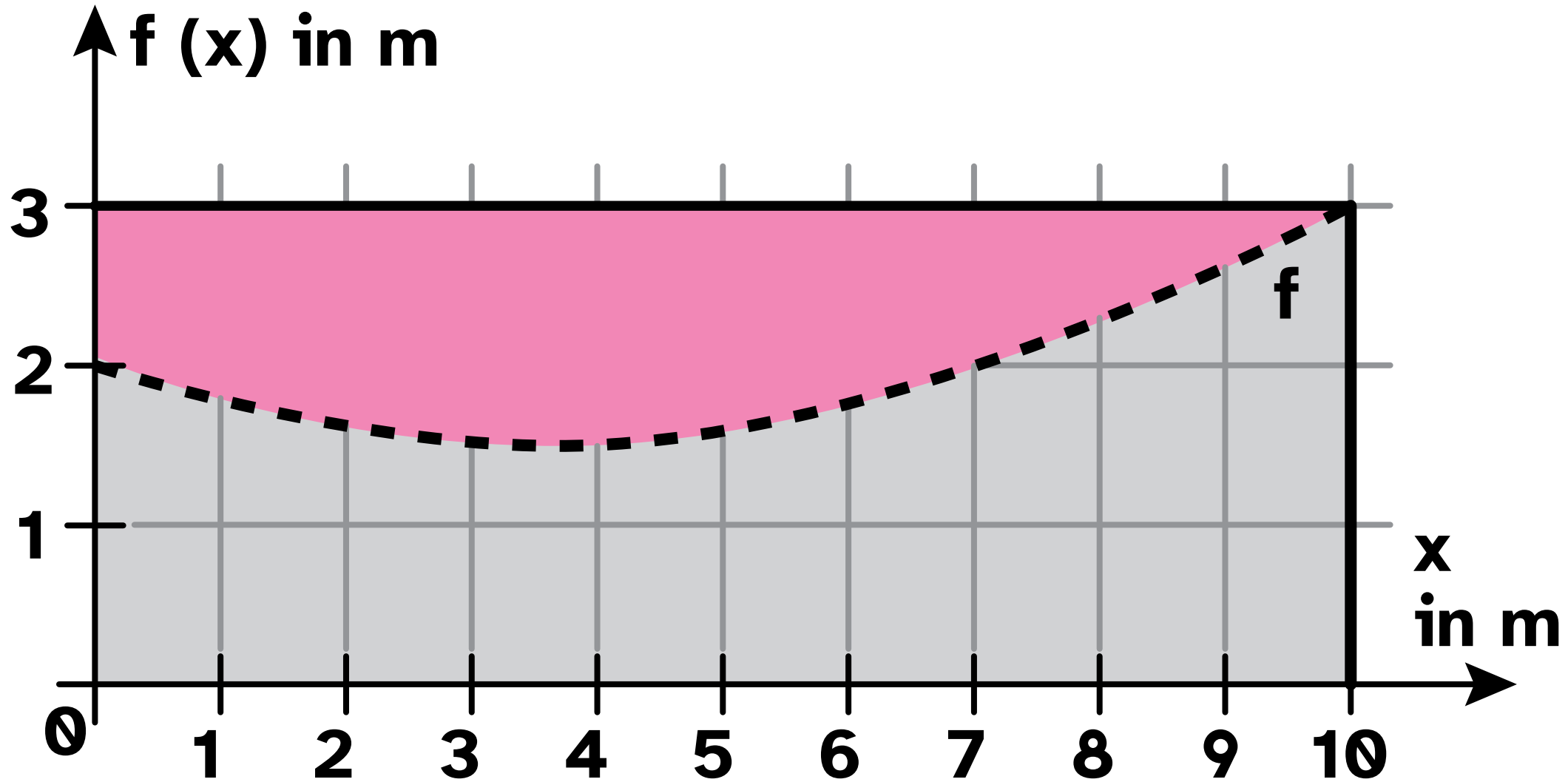
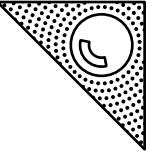


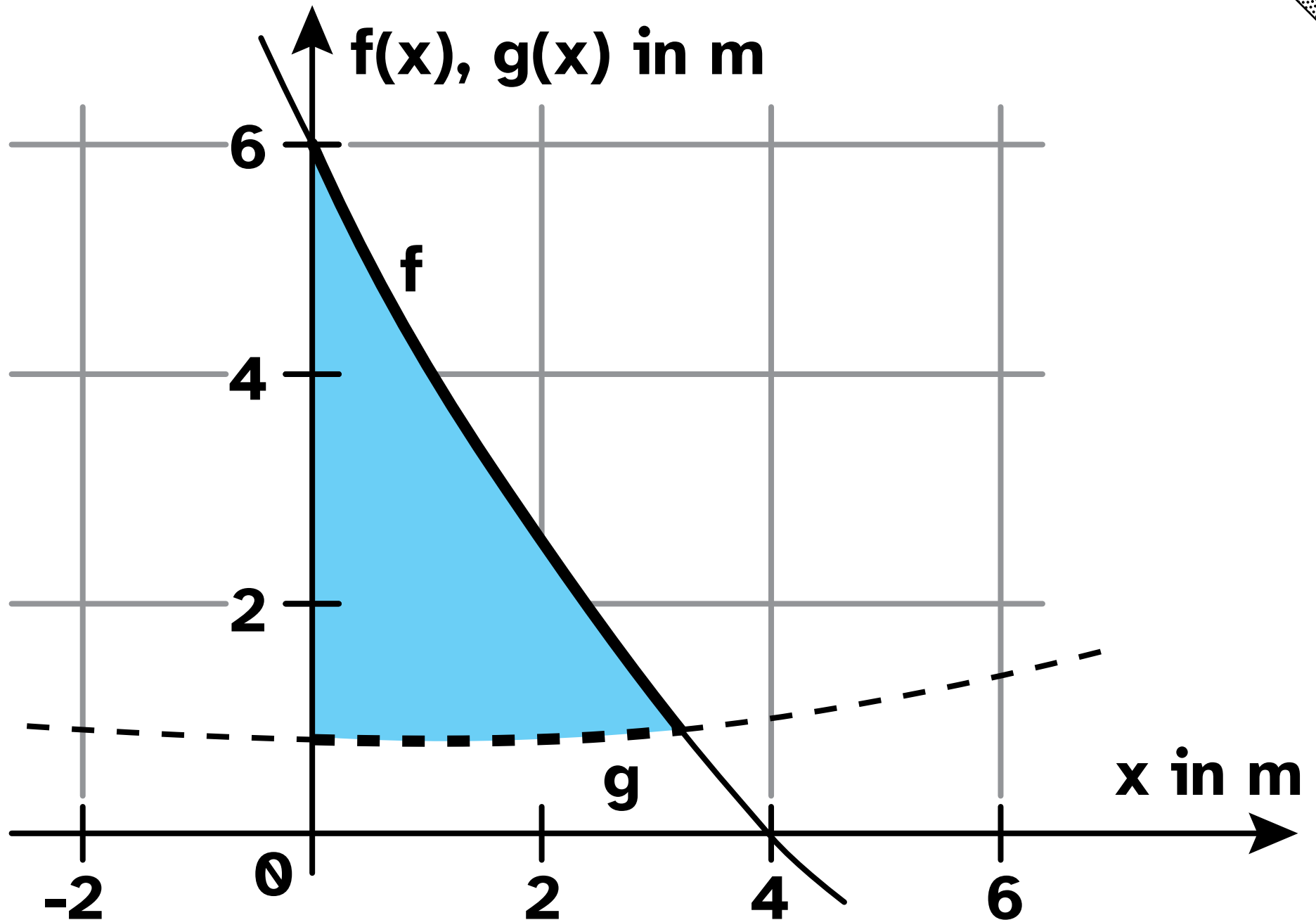
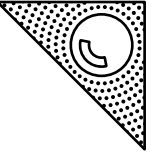


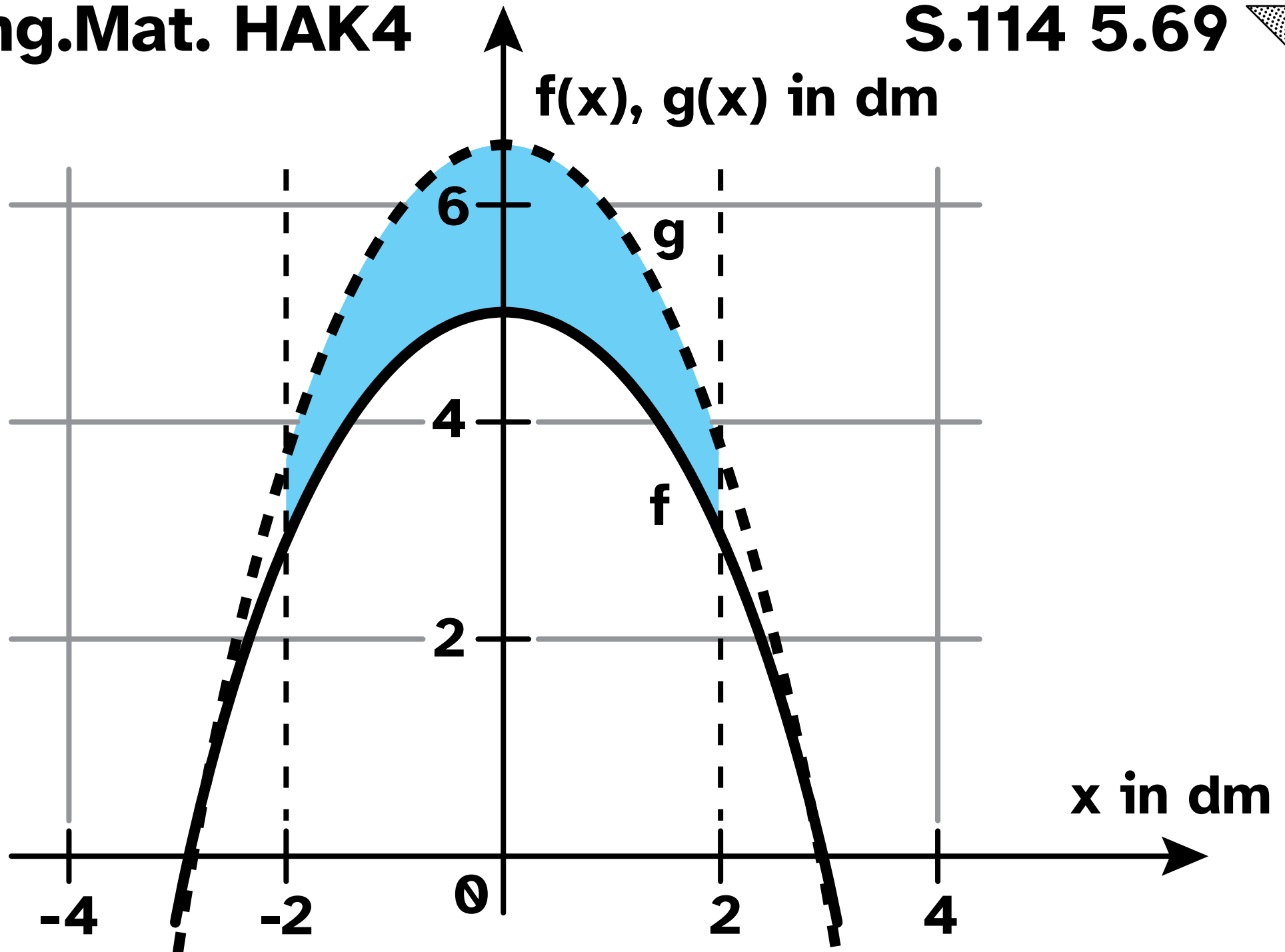
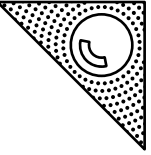


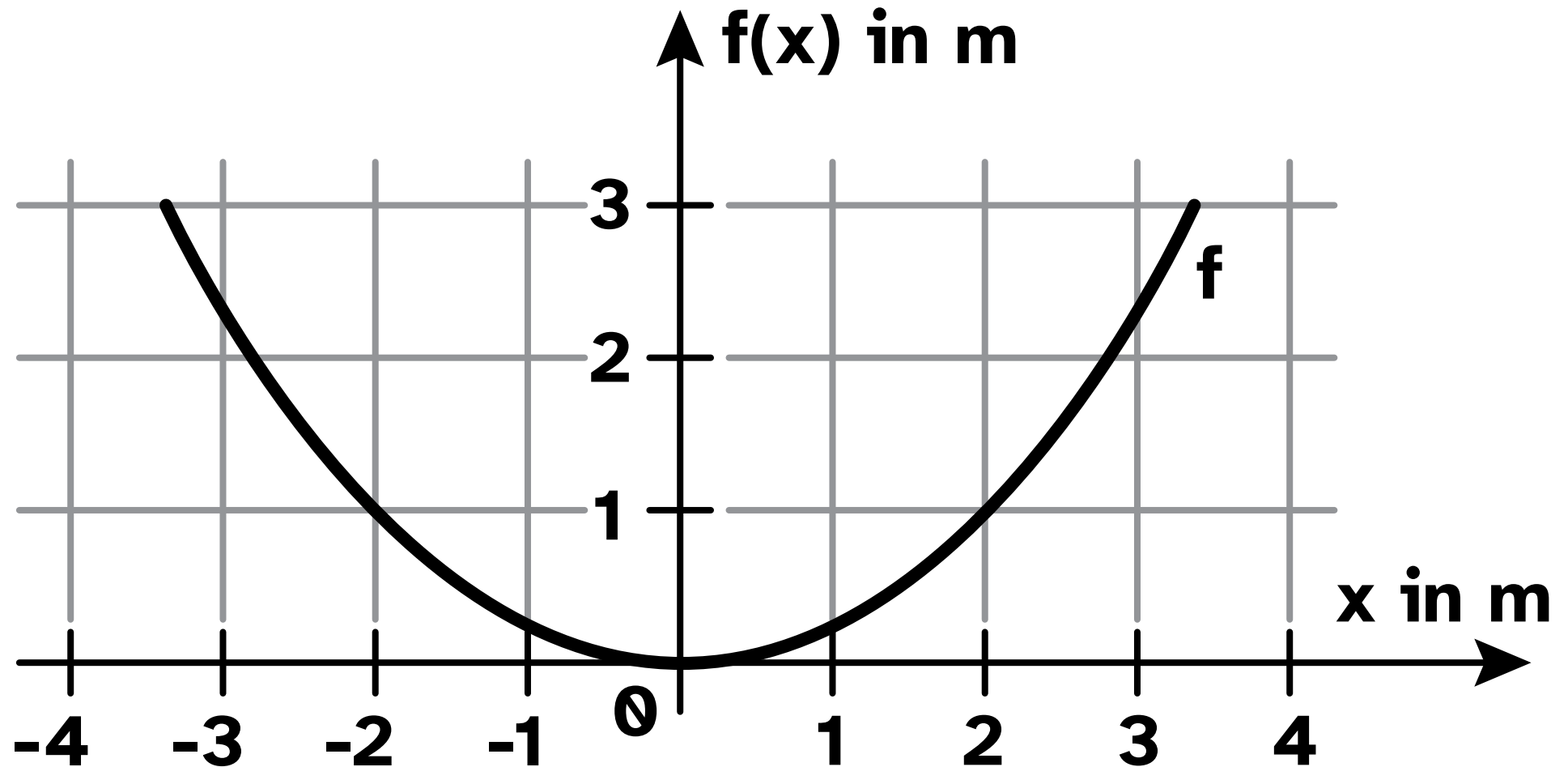
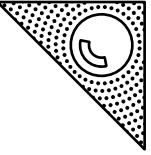


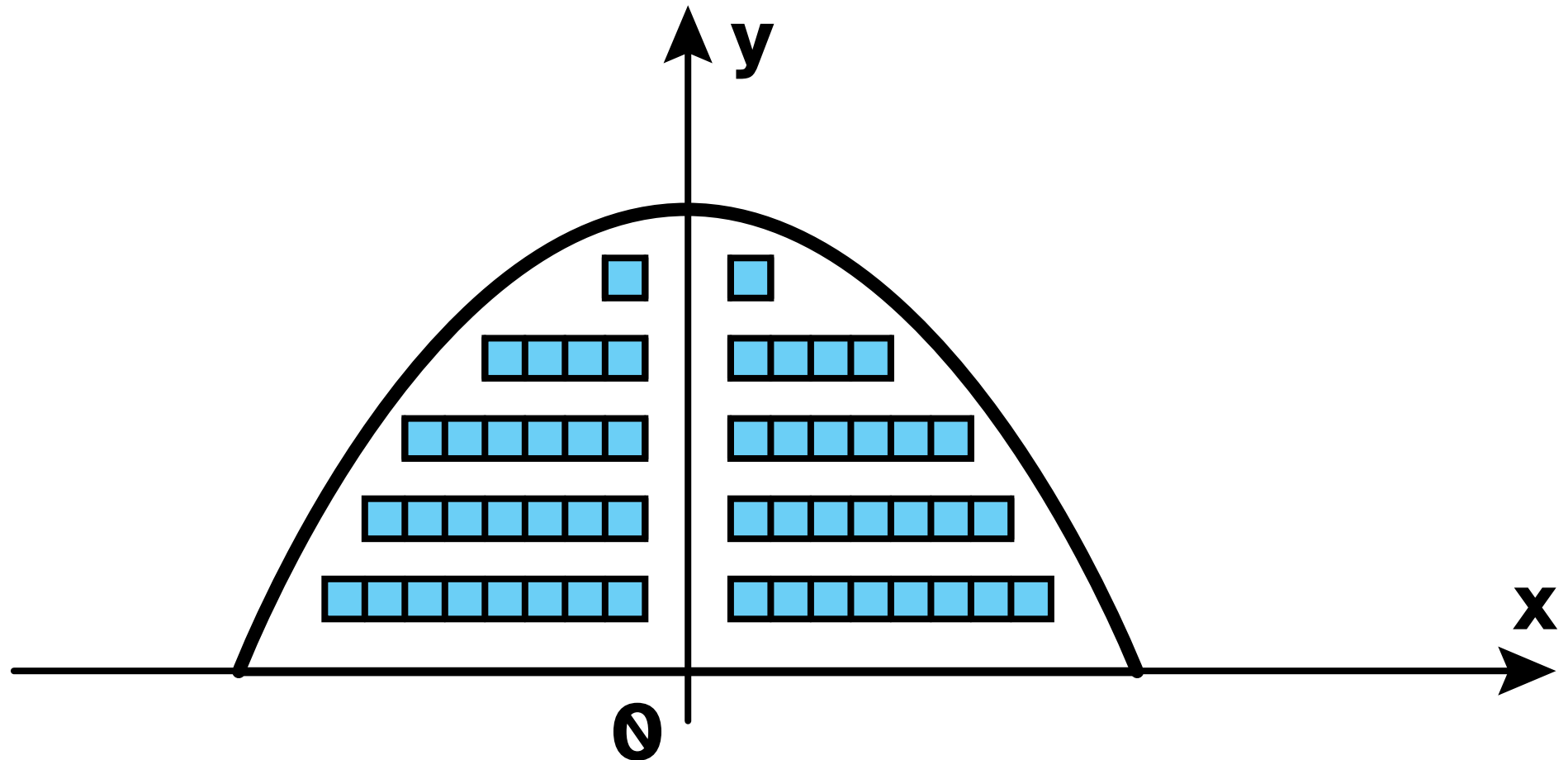
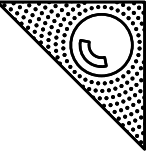


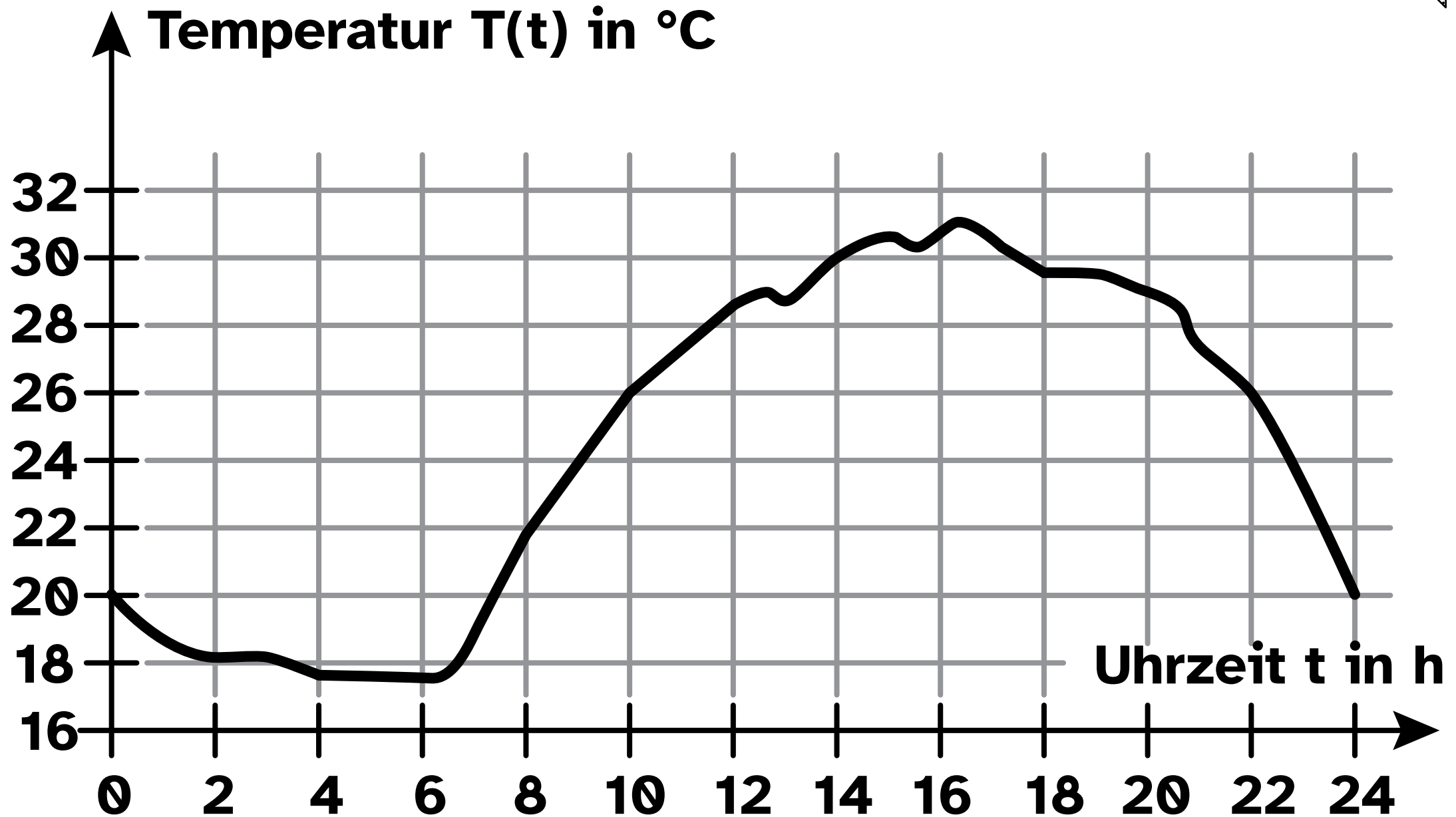
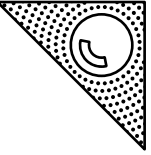




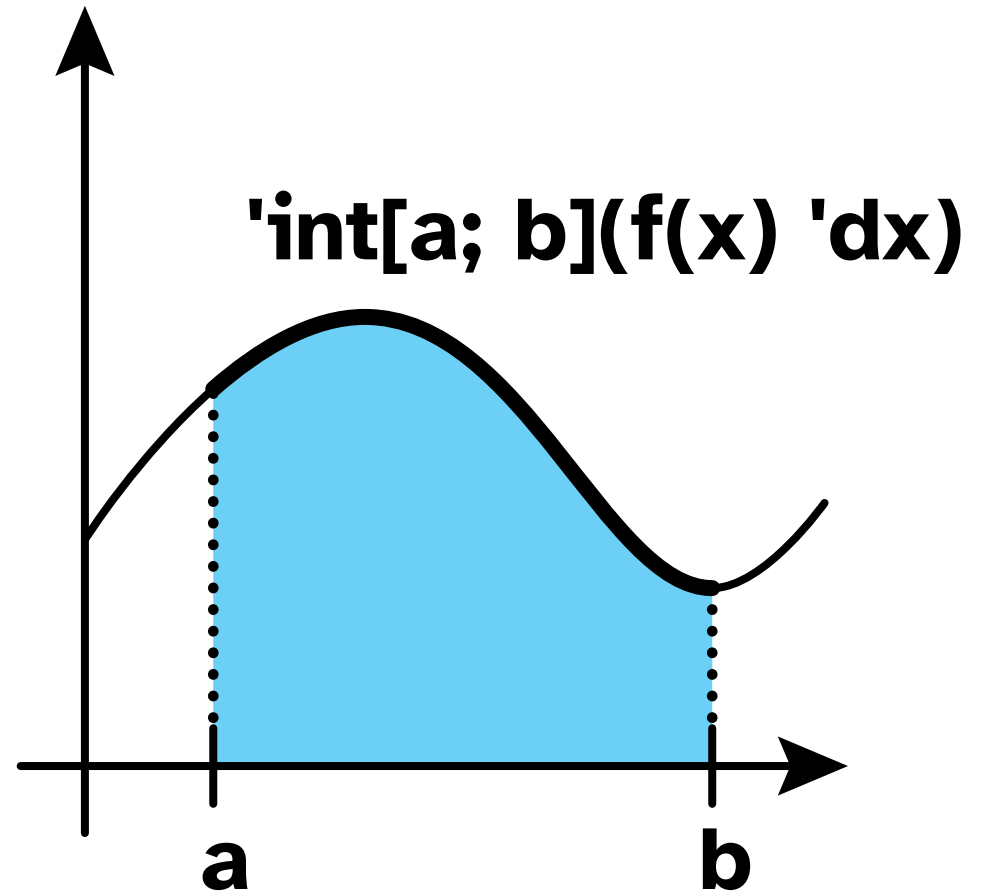
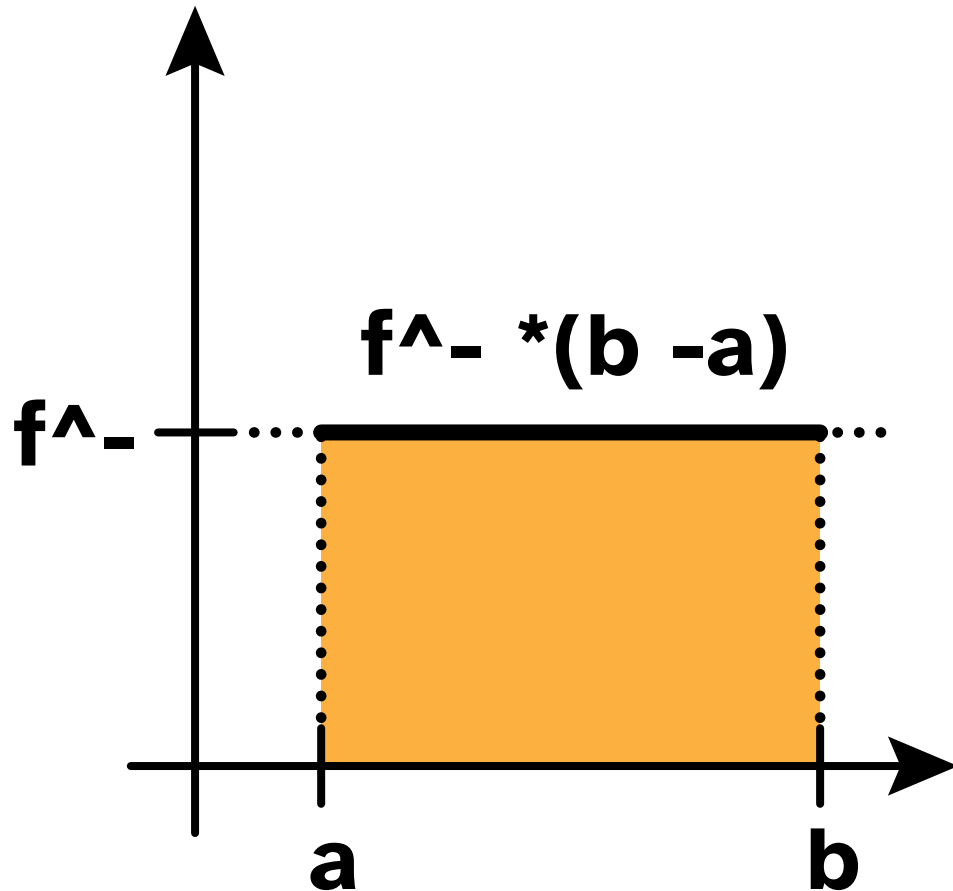




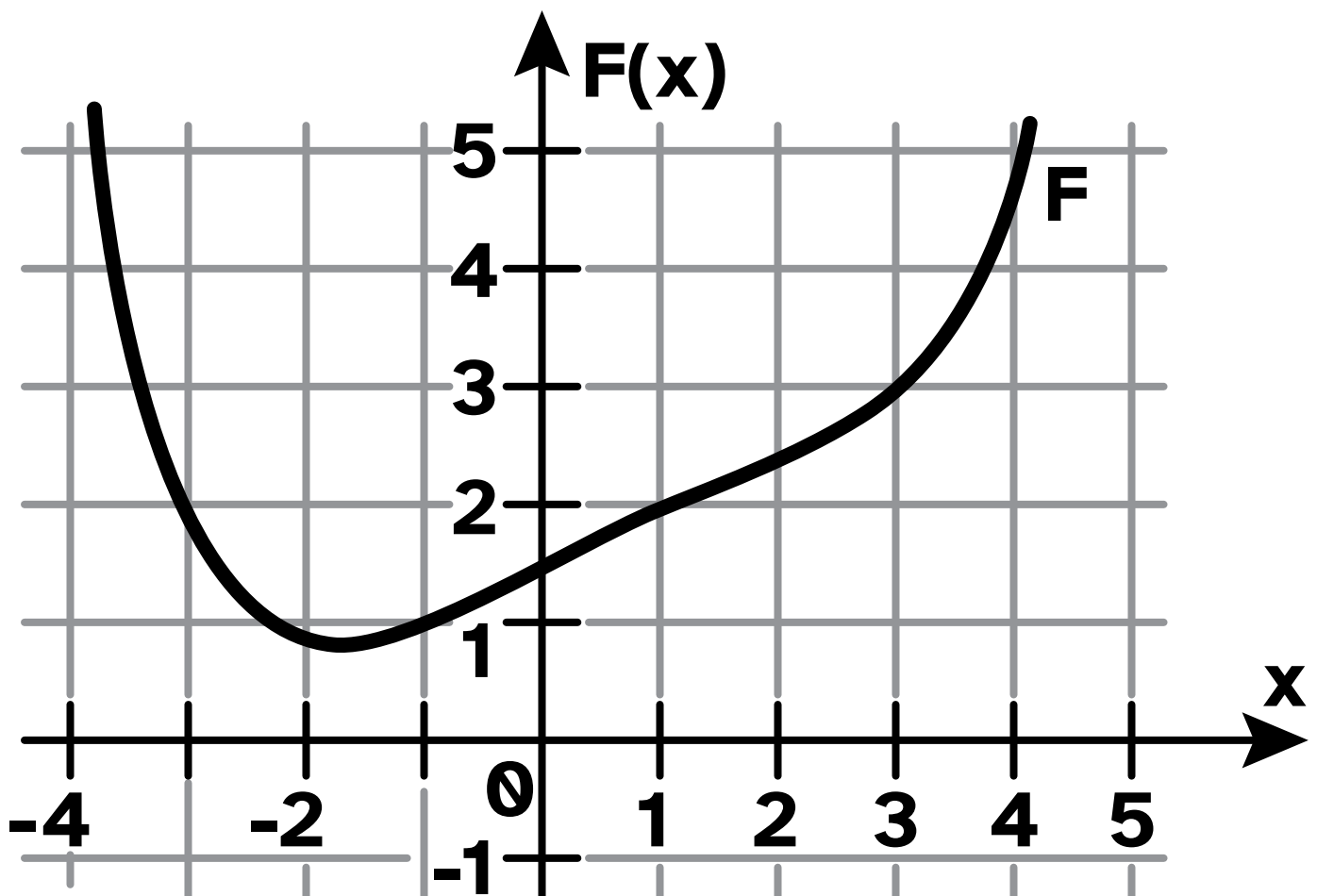
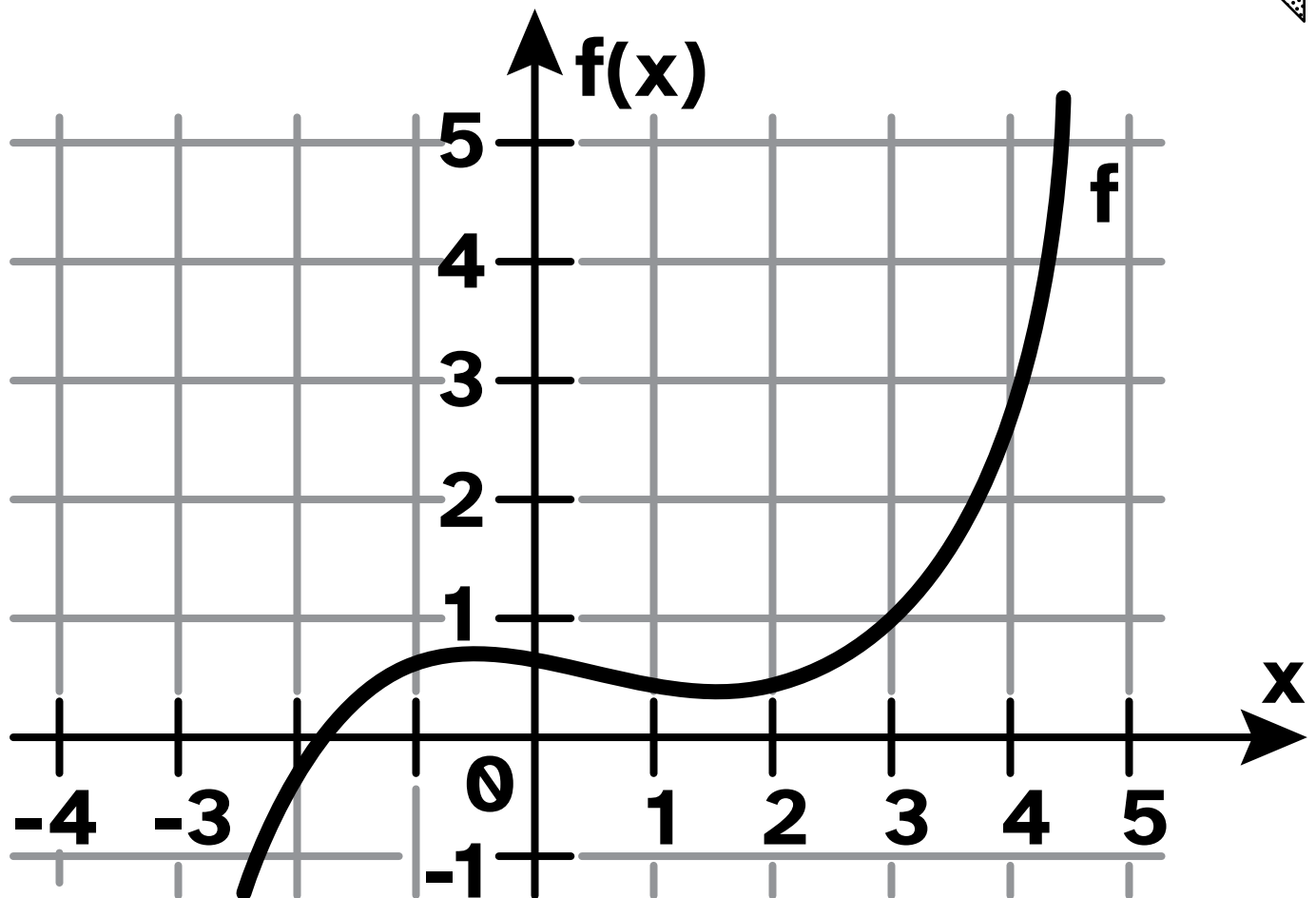
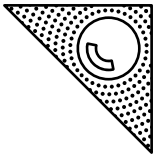


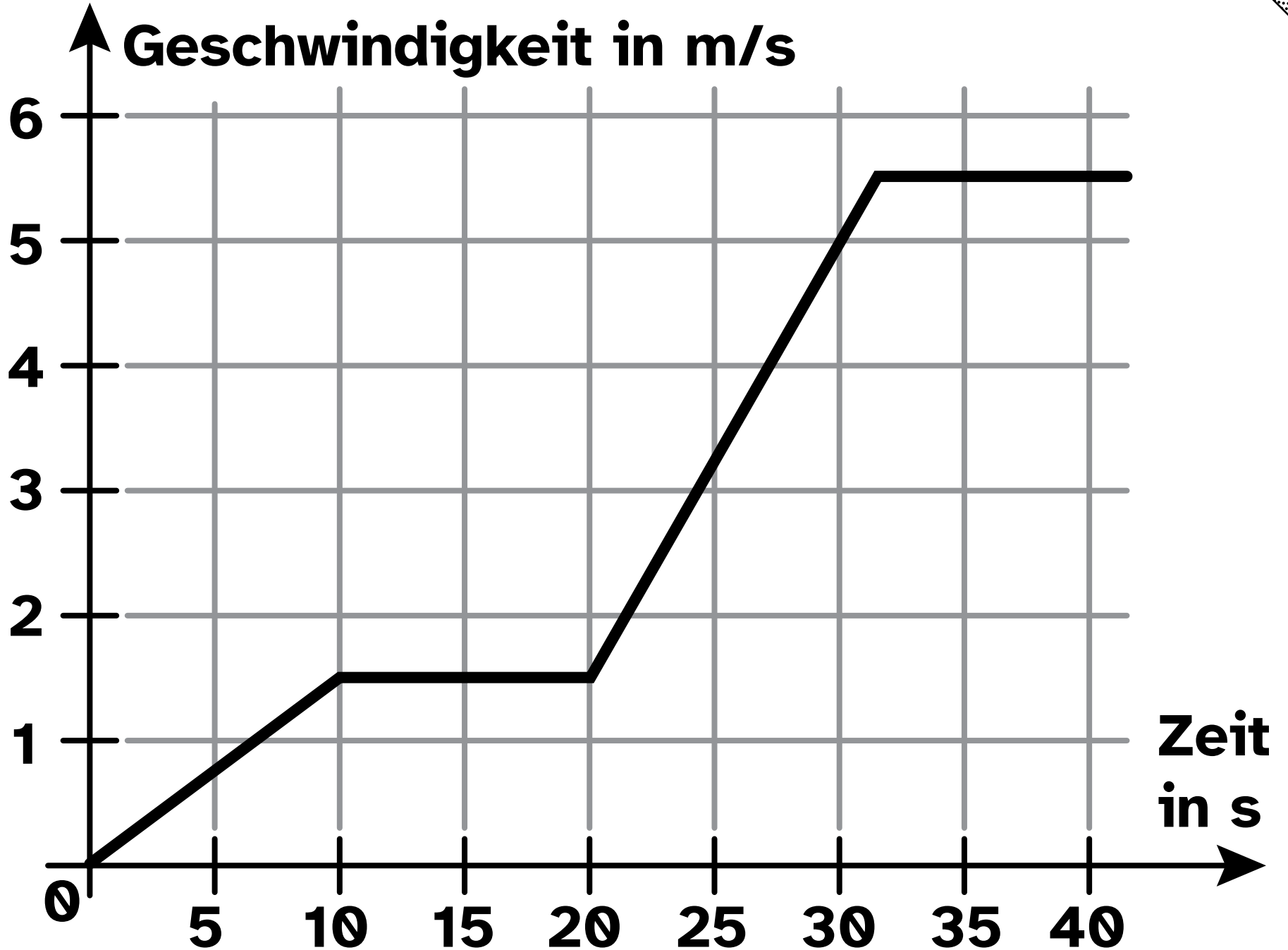
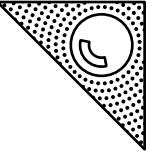


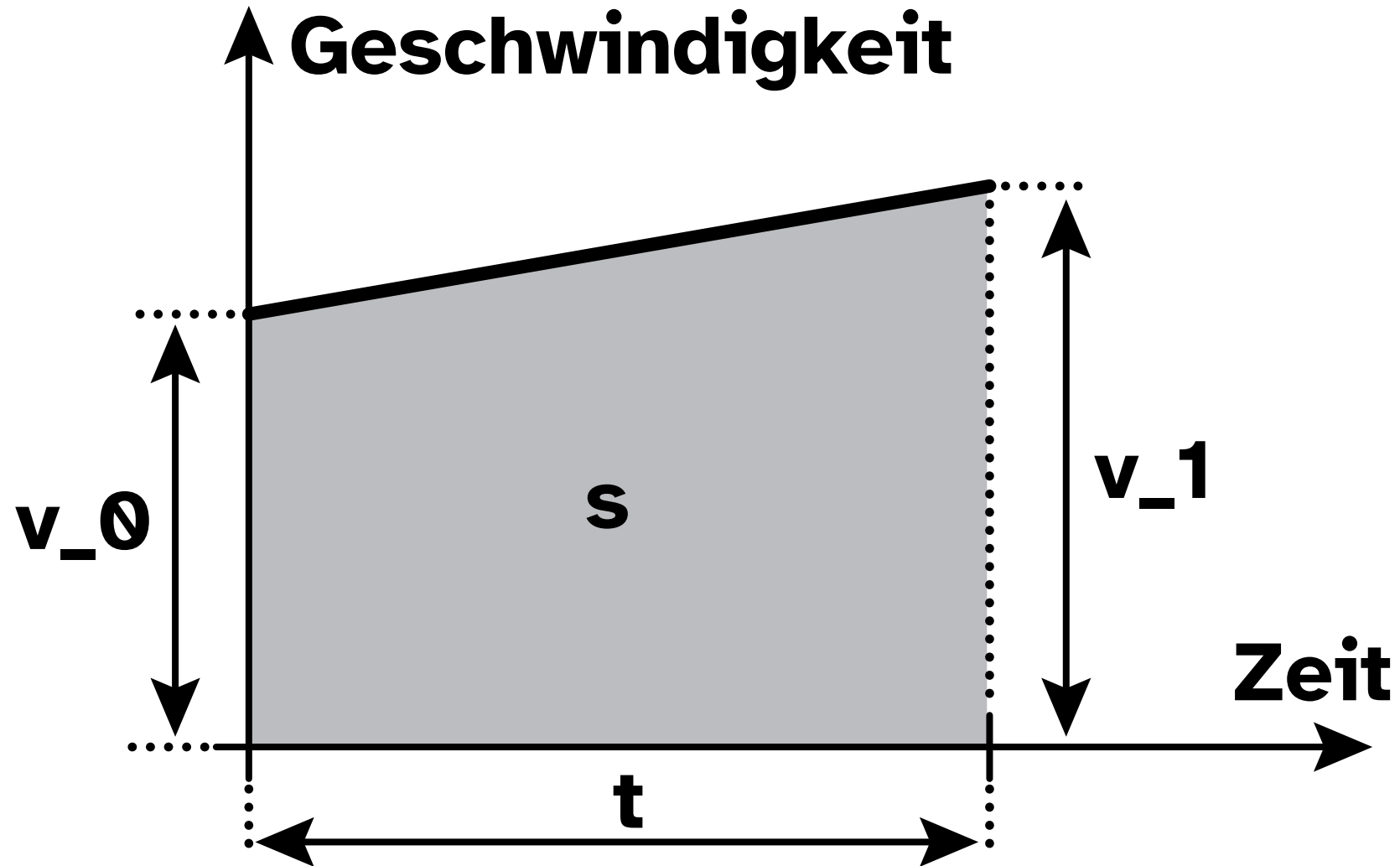
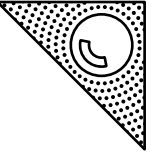
$$f^{\wedge}_{-} \cdot (b - a) = \int_a^b f(x) \, dx$$

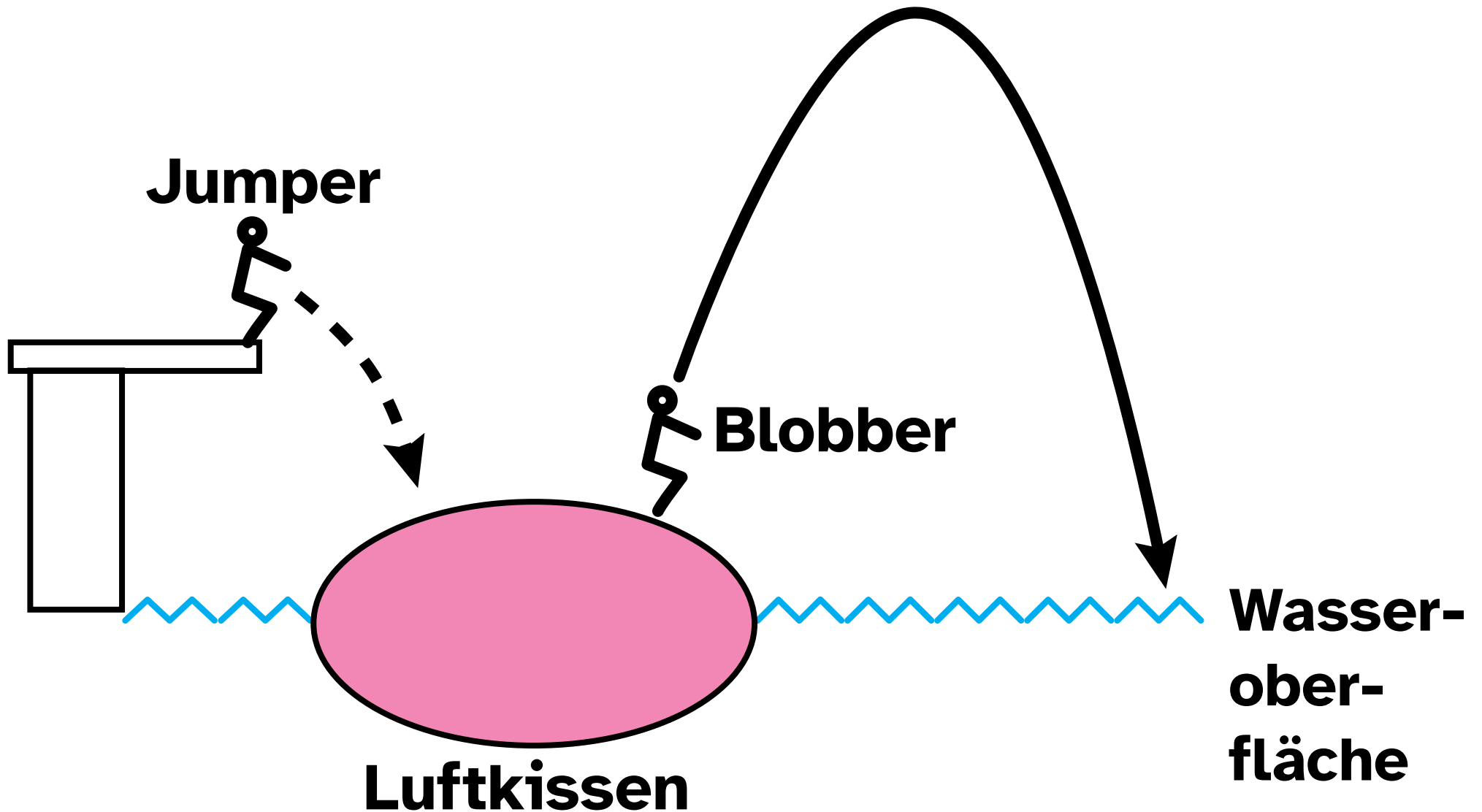
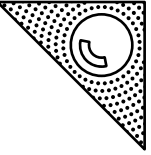


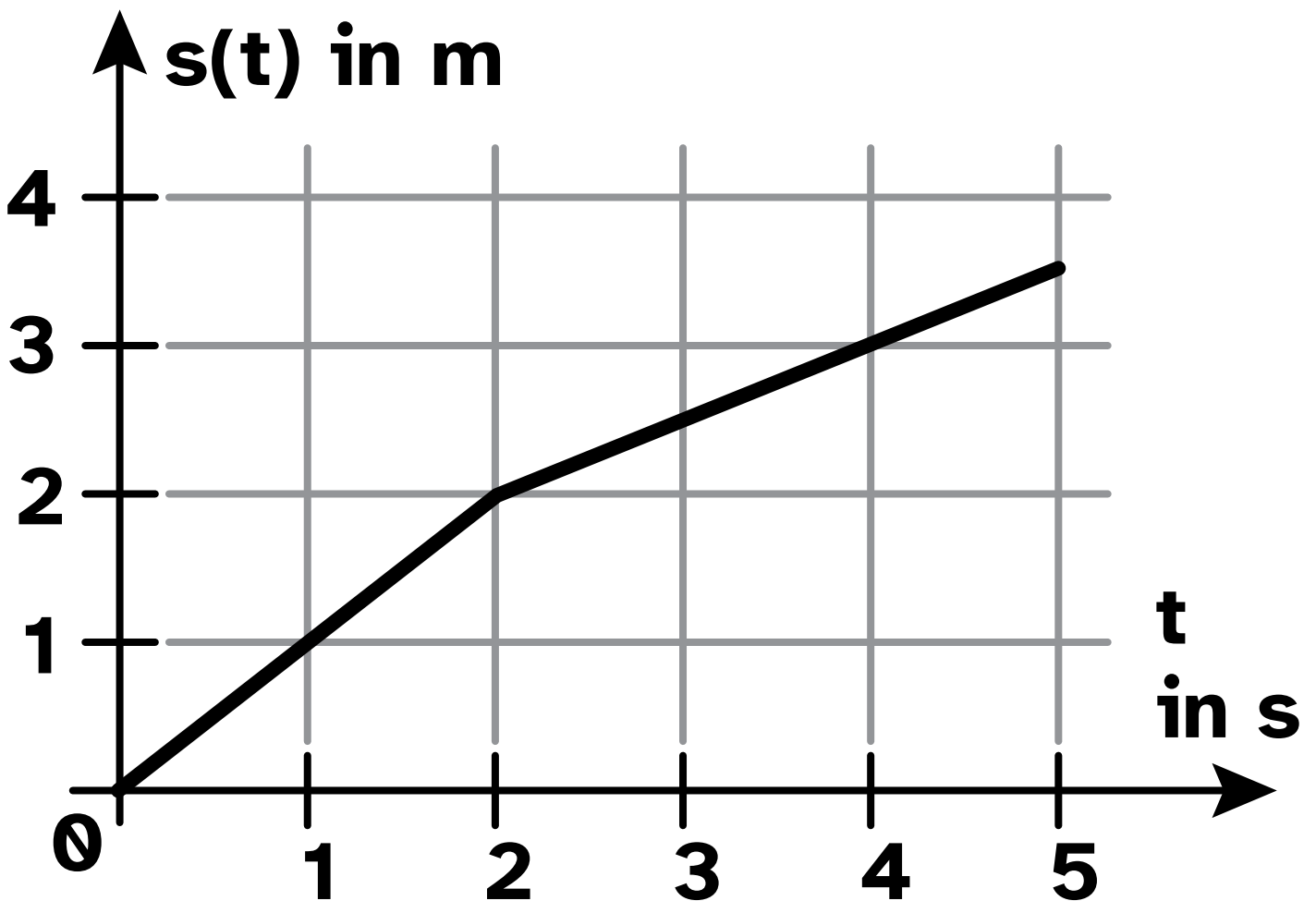
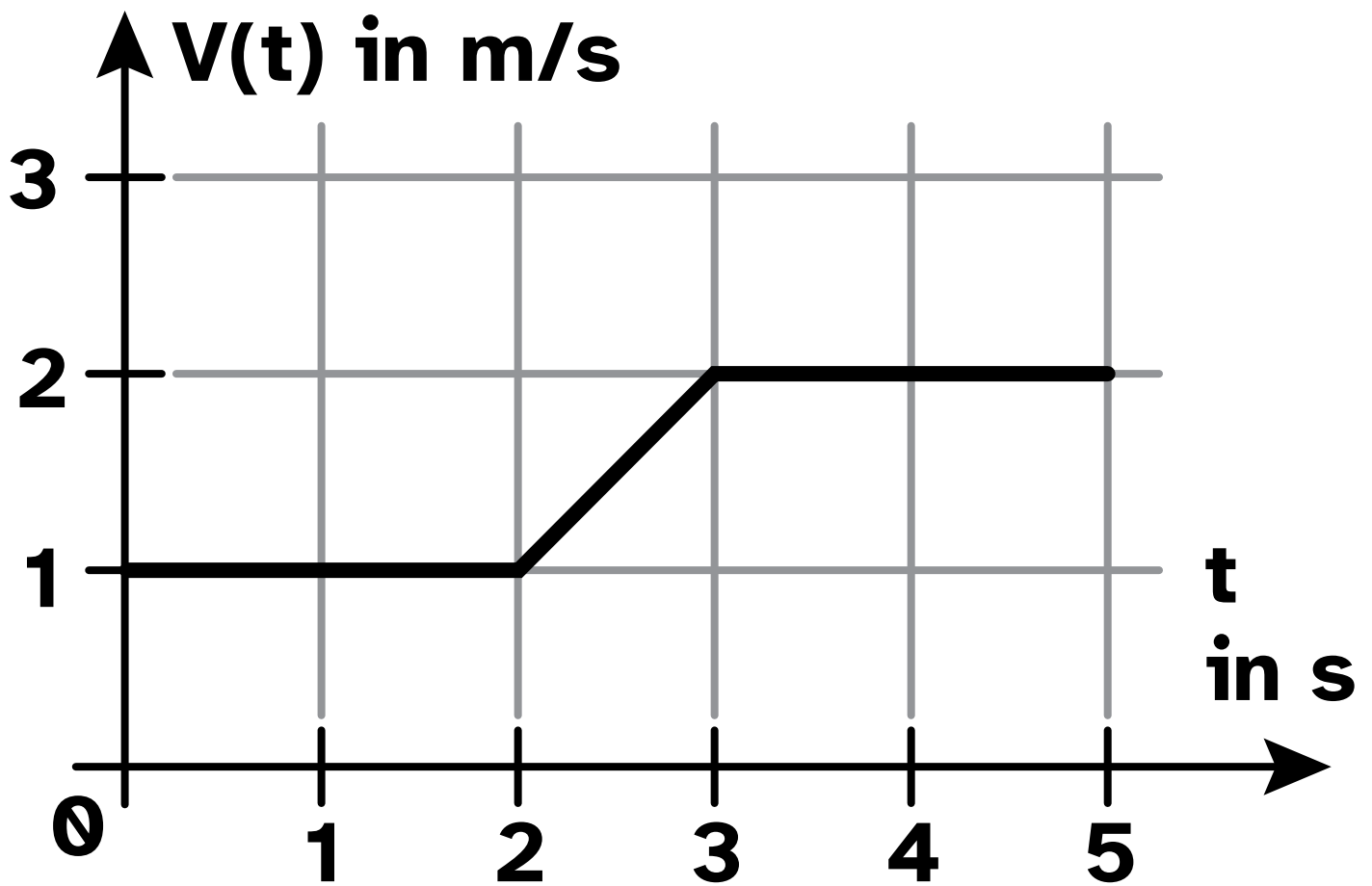
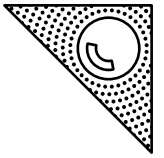


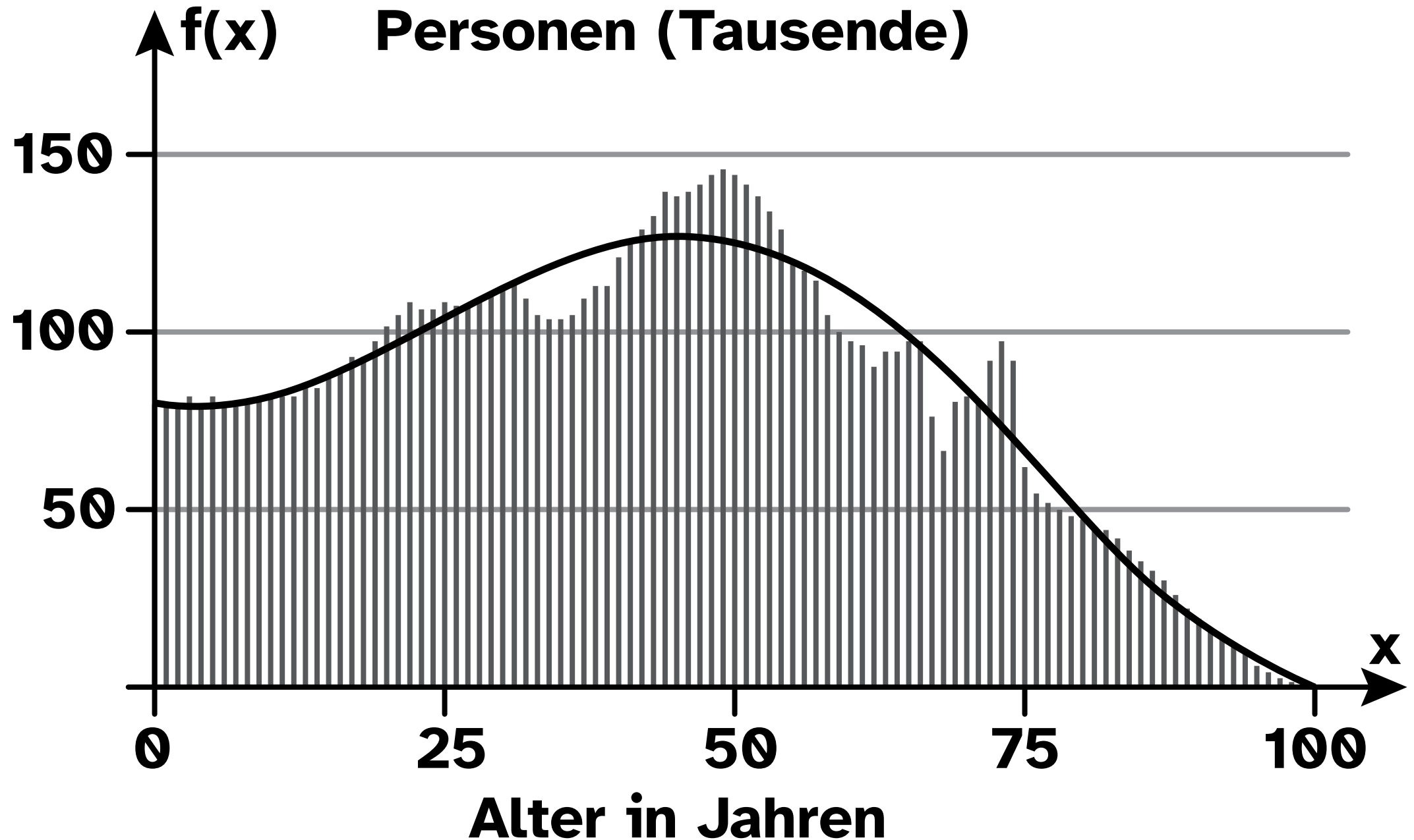
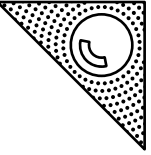


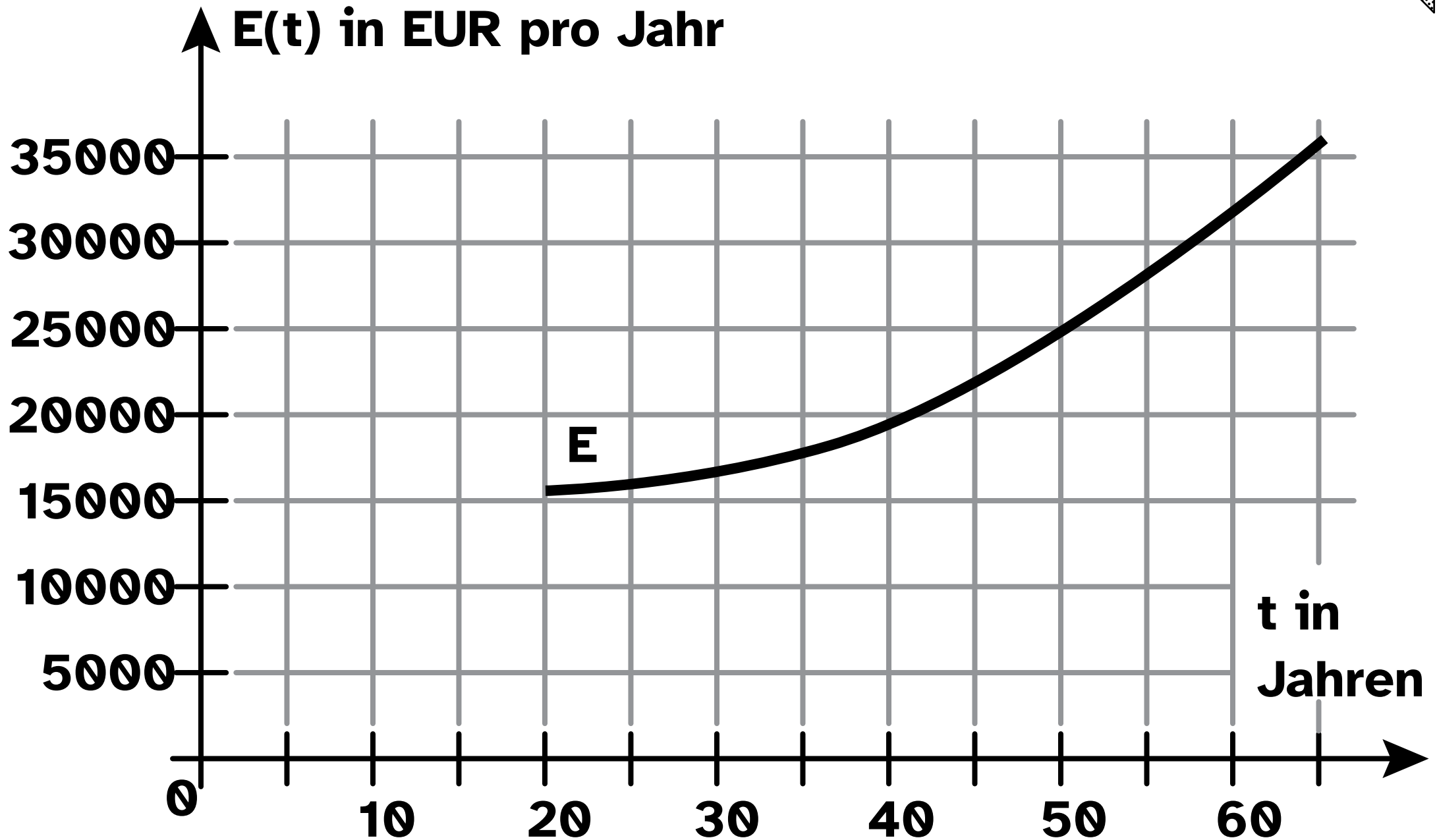
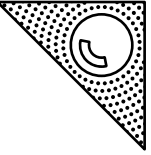


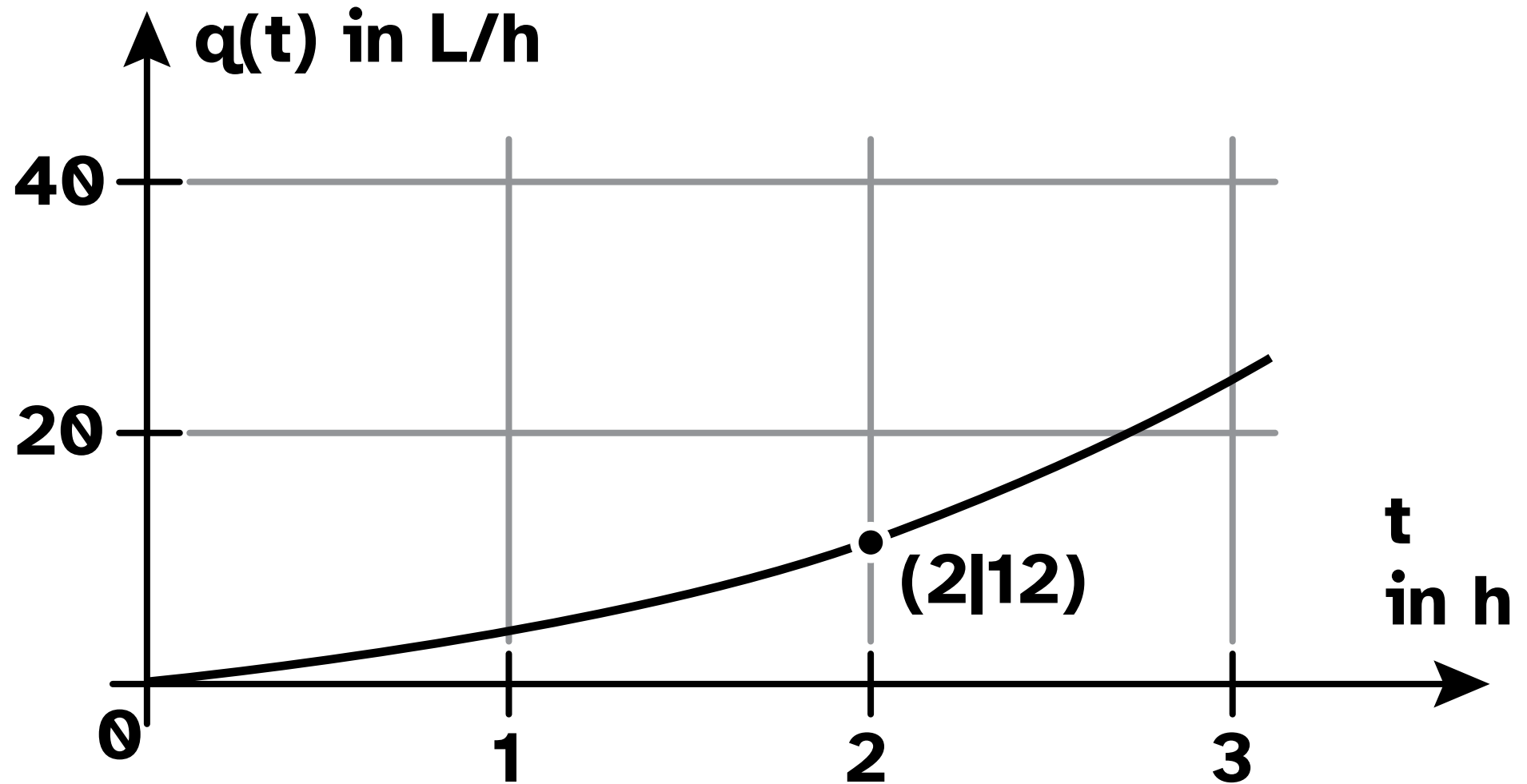
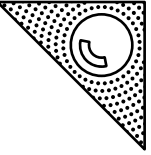




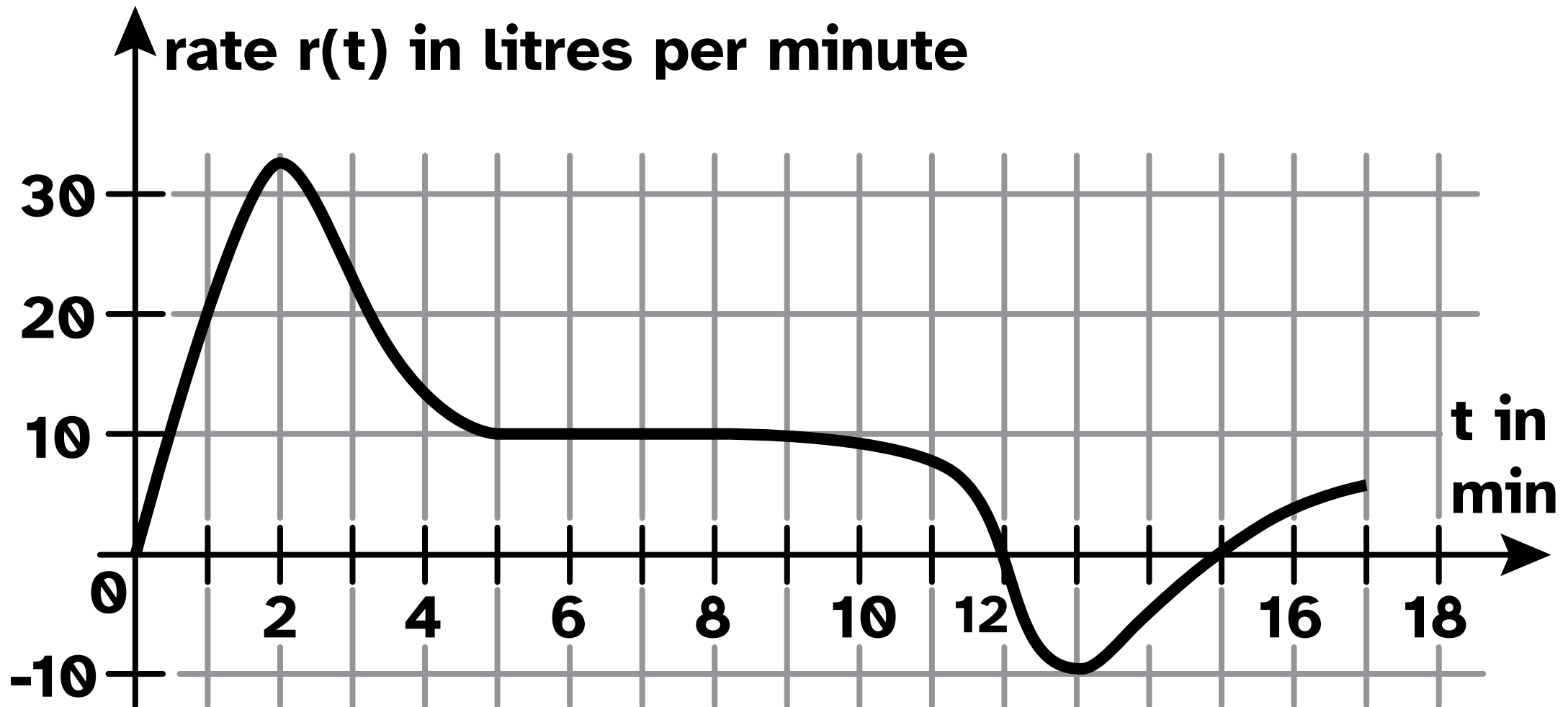
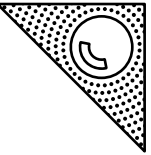


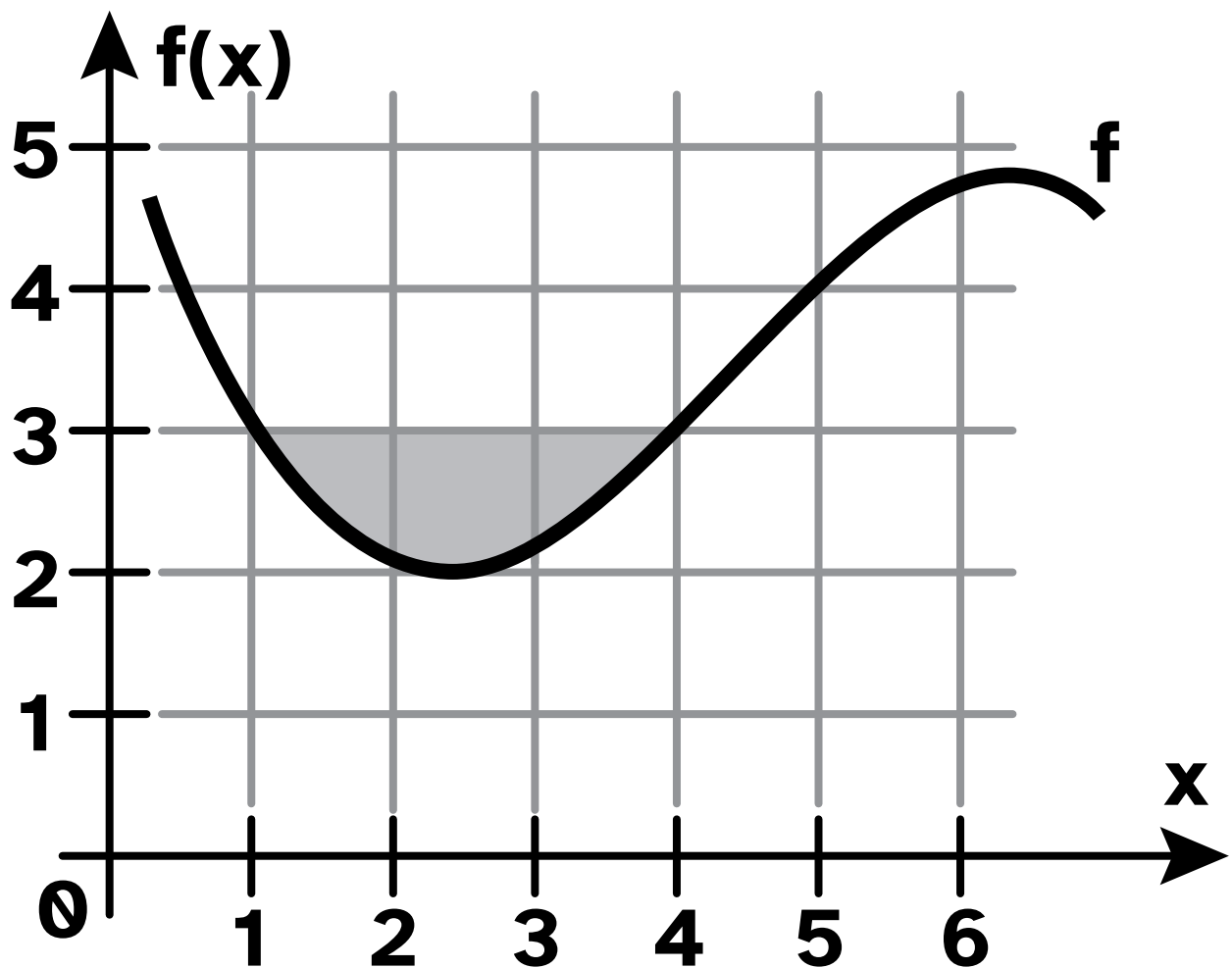
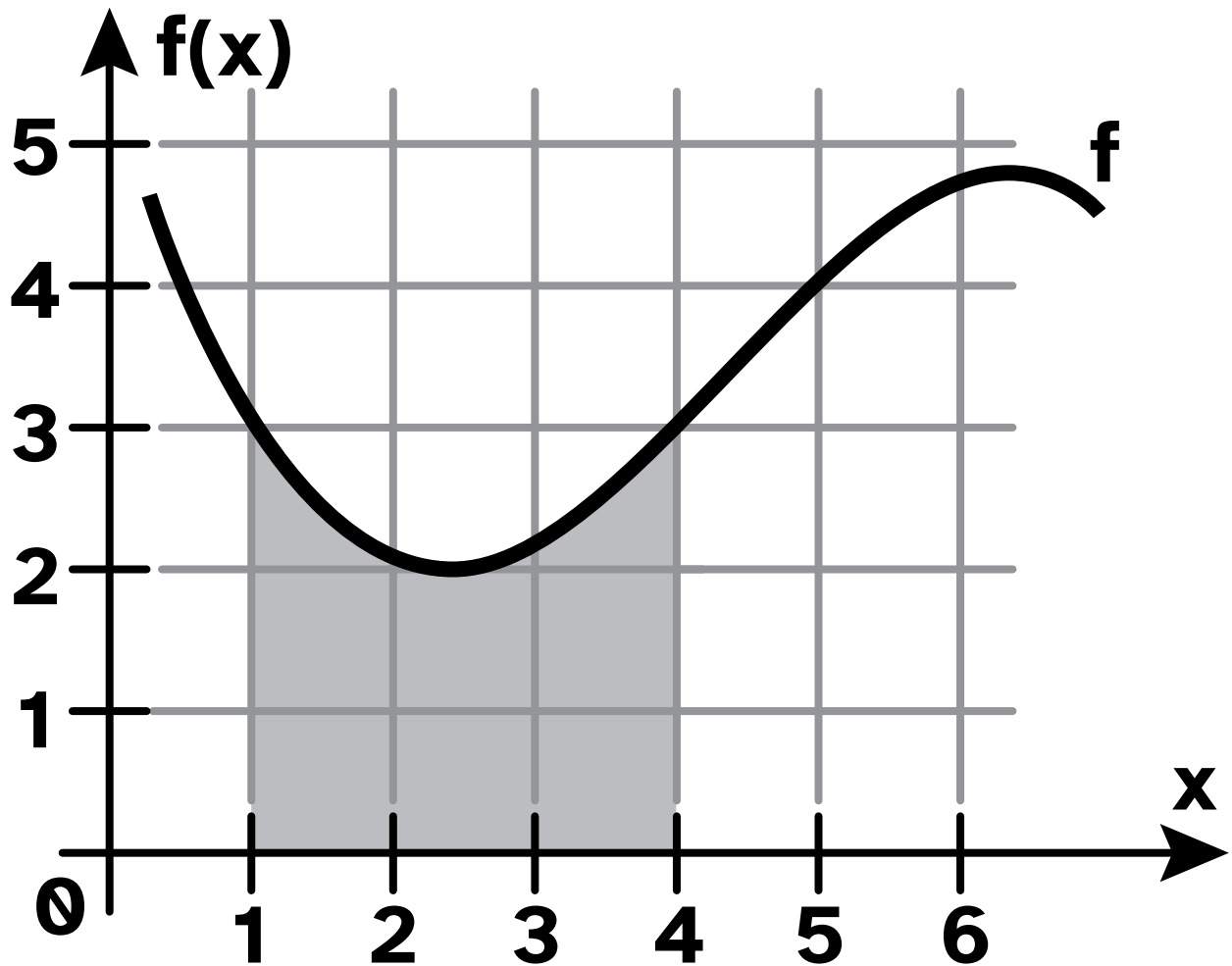
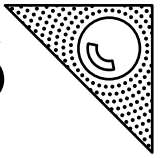


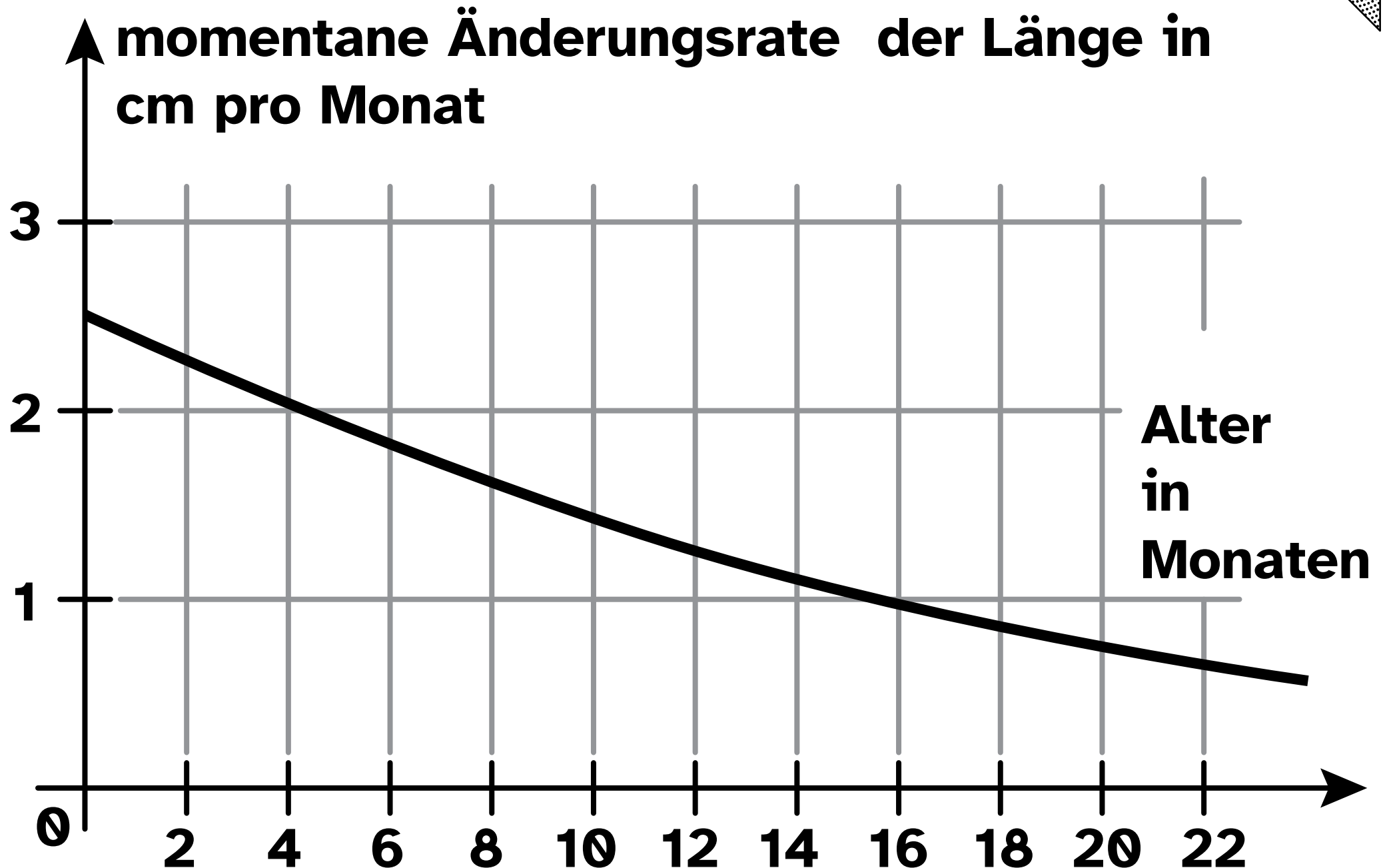
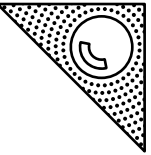


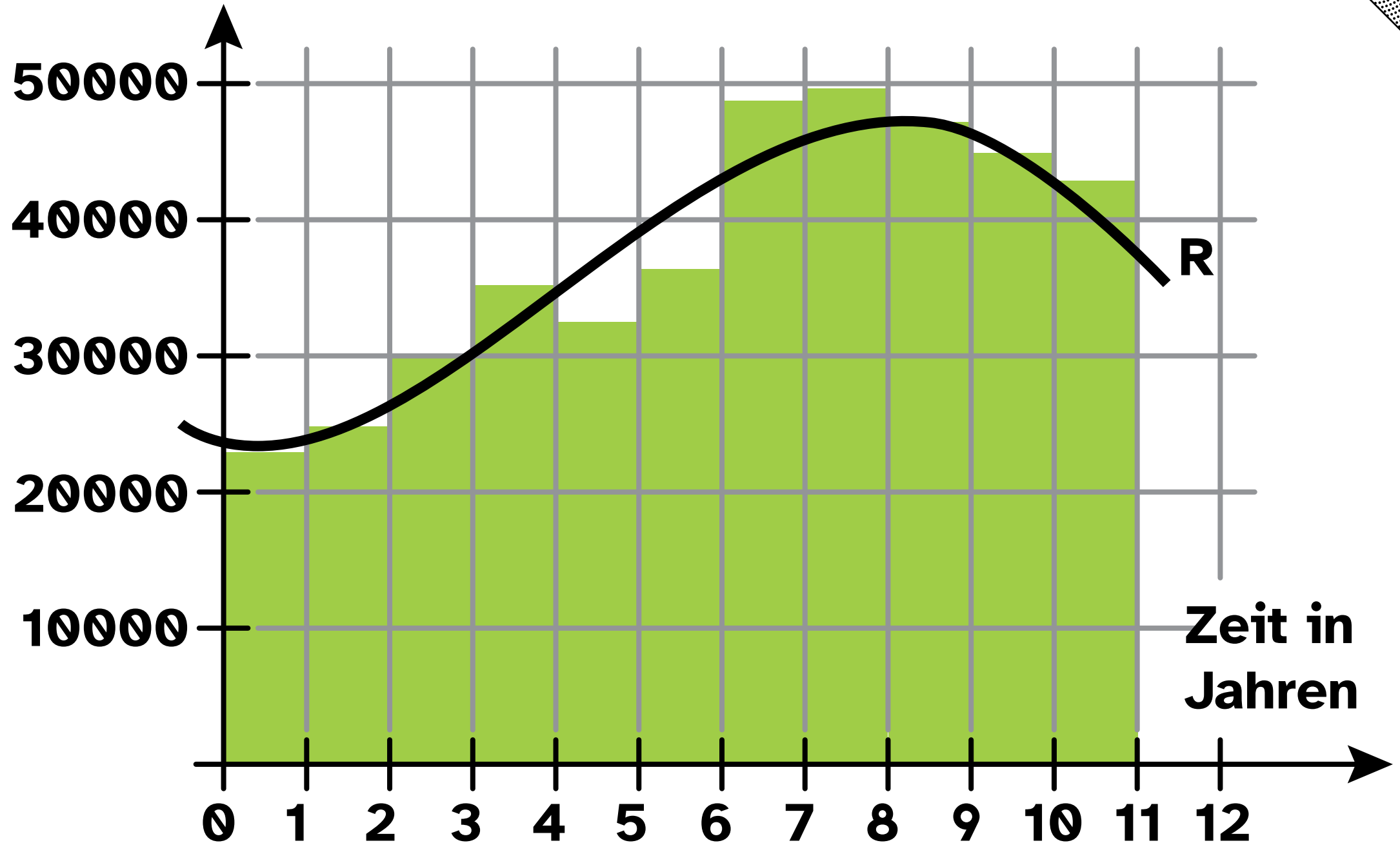
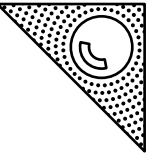


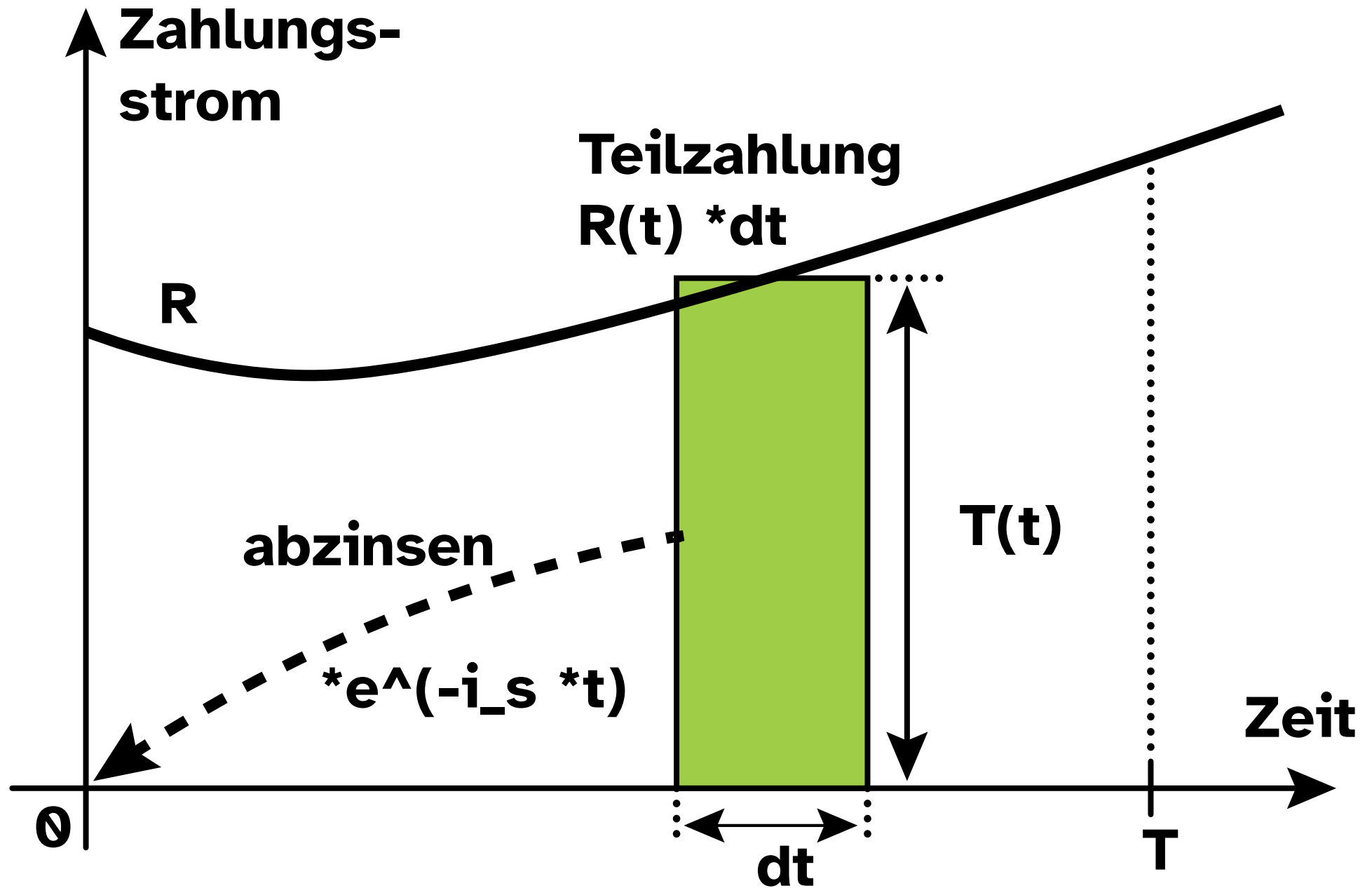
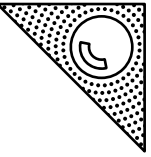


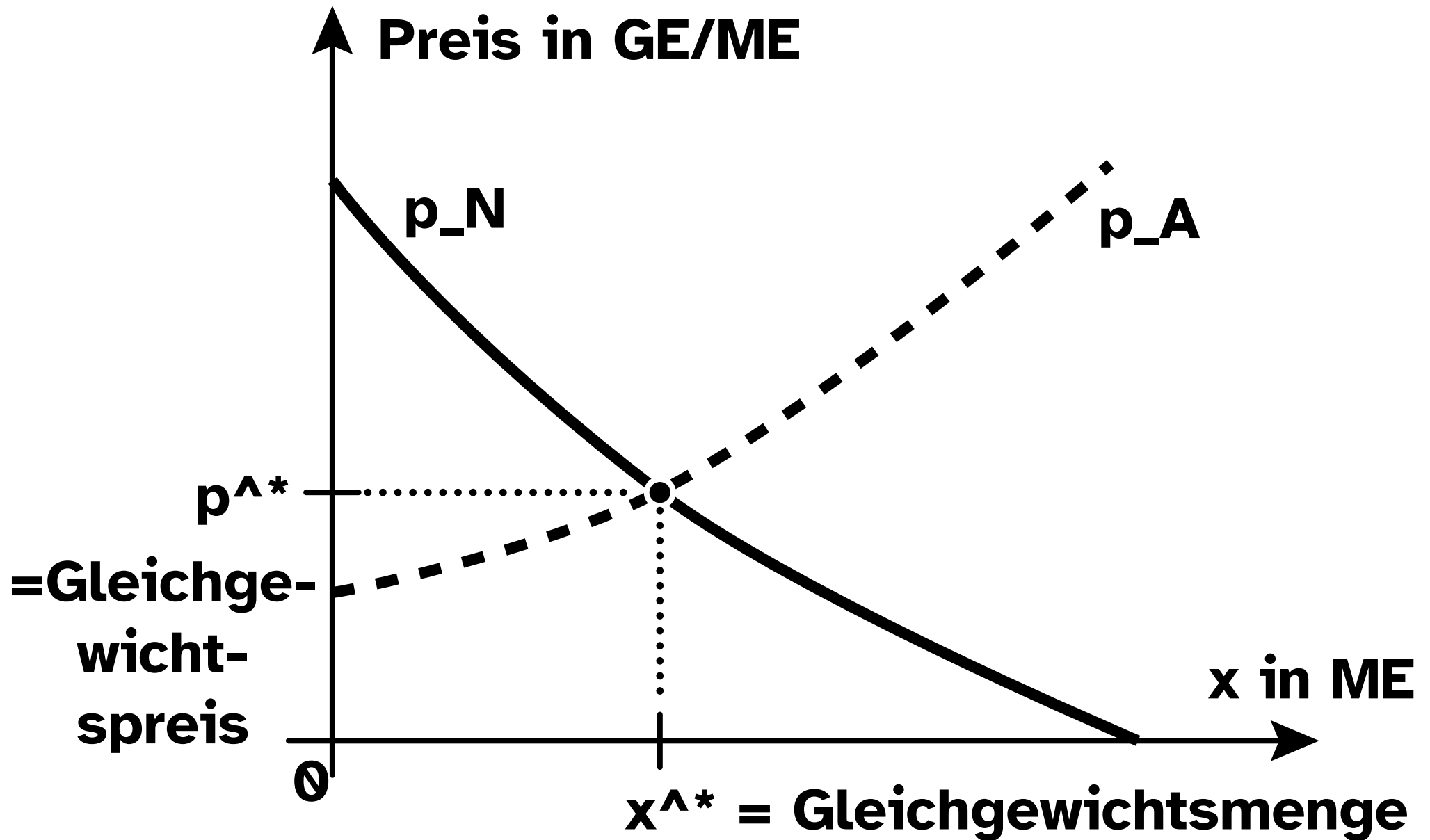
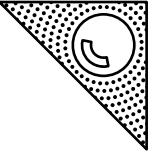




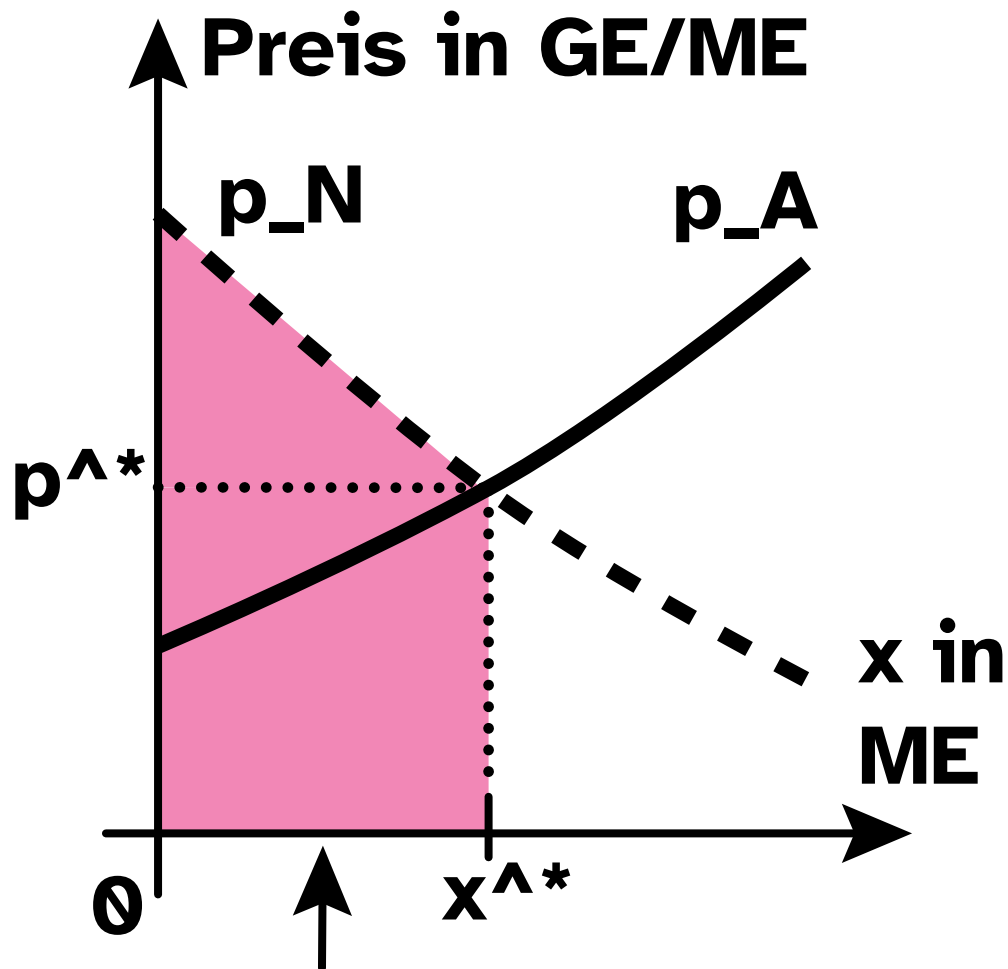
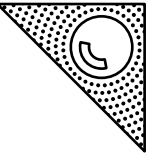




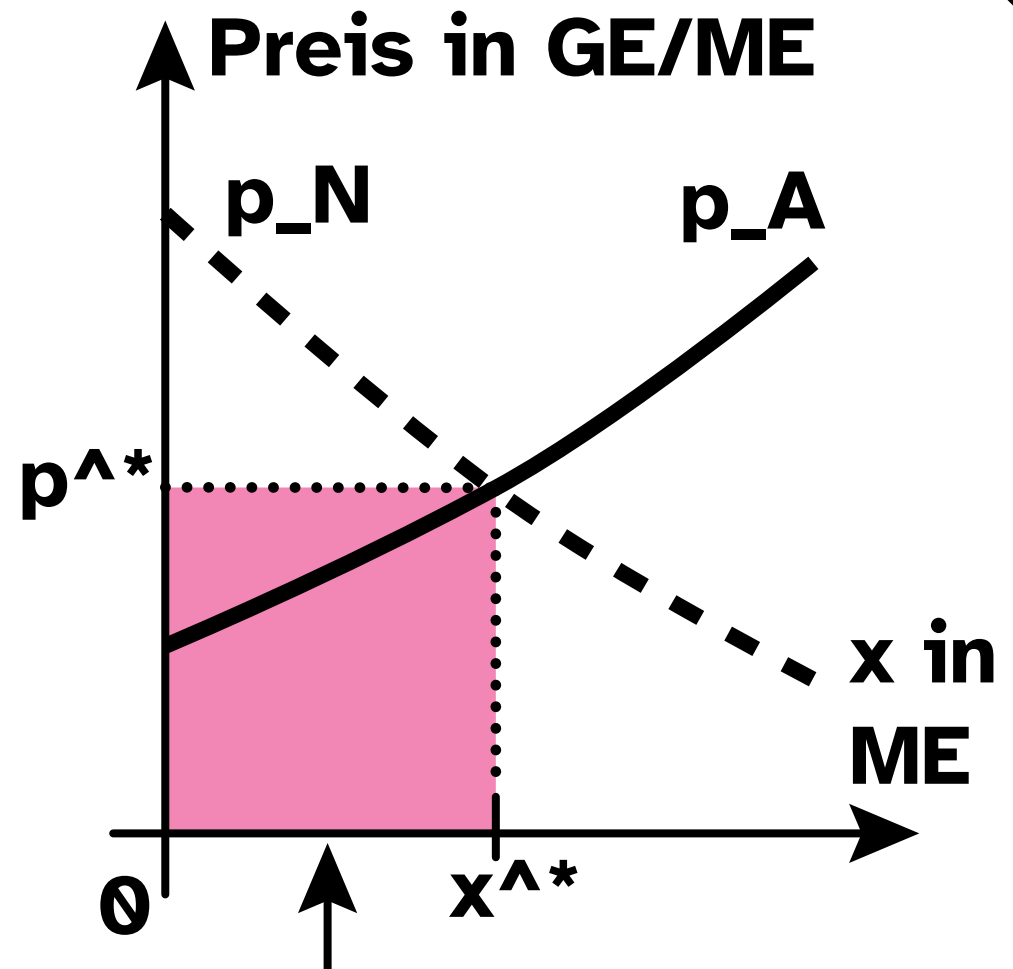




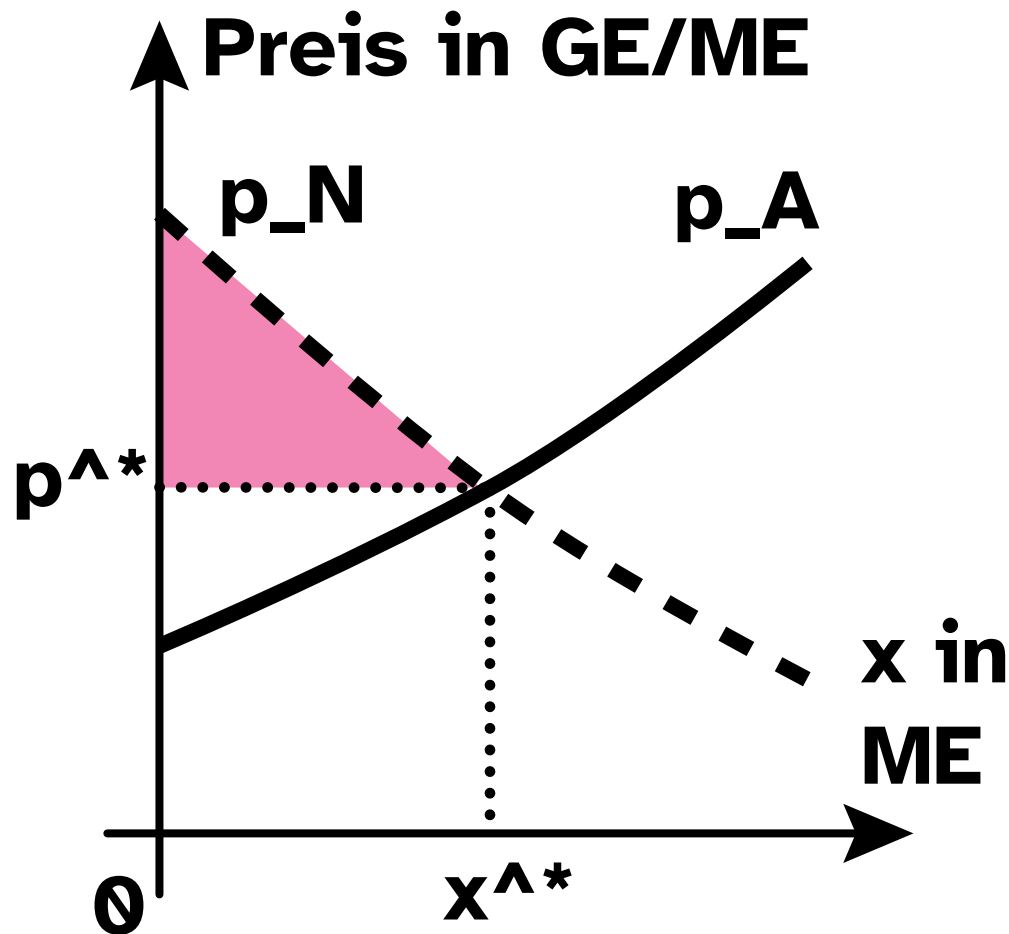
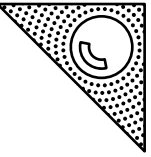
# Ang.Mat. HAK4 S.121 Konsumentenrente 1/2



Konsumenten sind bereit  
' $\int_0^{x^*} p_N(x)$ ' zu  
zahlen

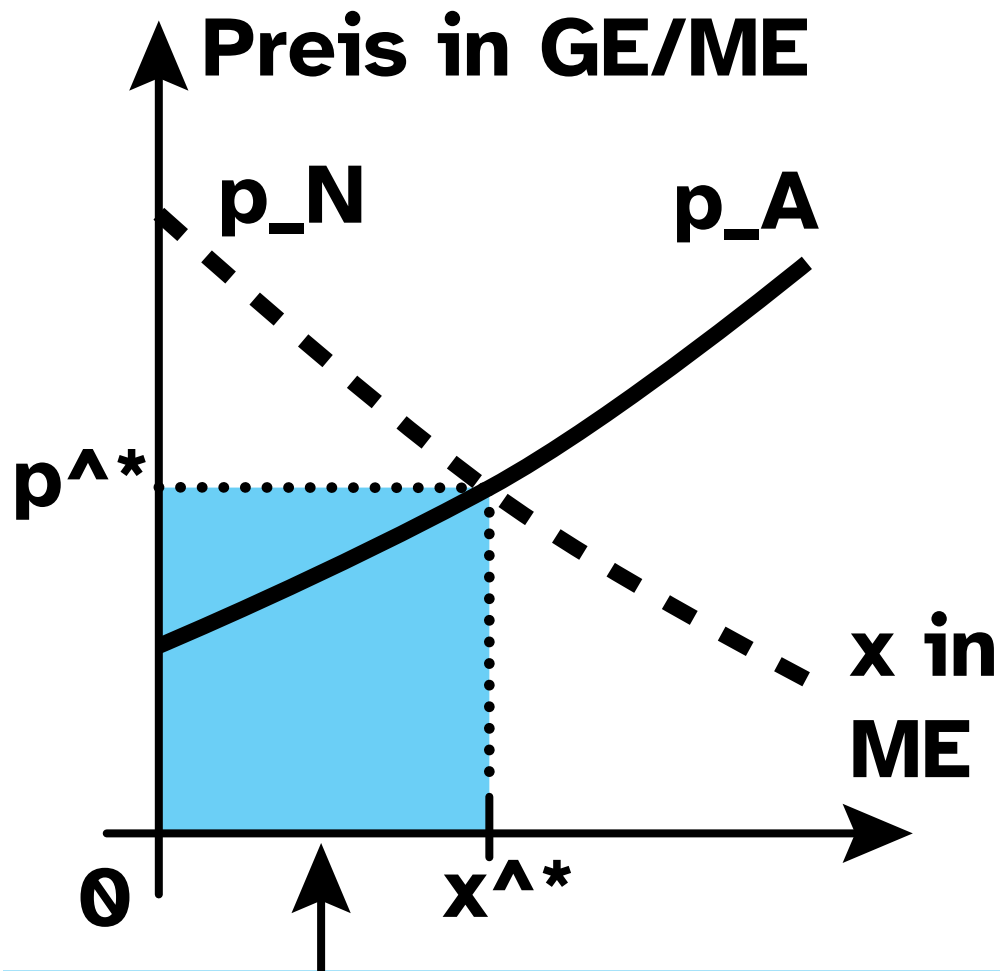
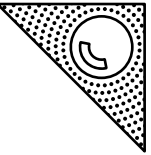


Tatsächlich zahlen  
sie aber nur  
 $p^* \cdot x^*$

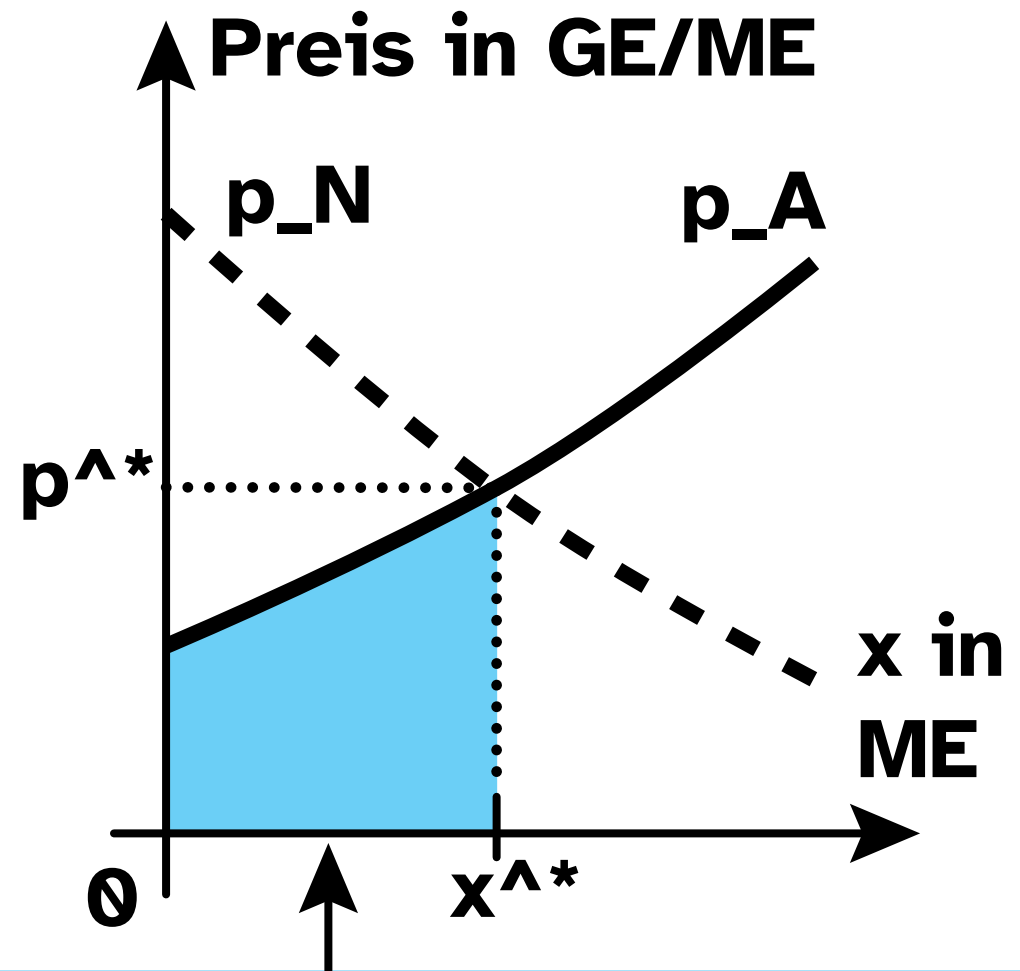


Konsumenten sparen also:  $\int_0^{x^*} (p_N(x) - p^* ) dx$ . Das nennt man Konsumentenrente

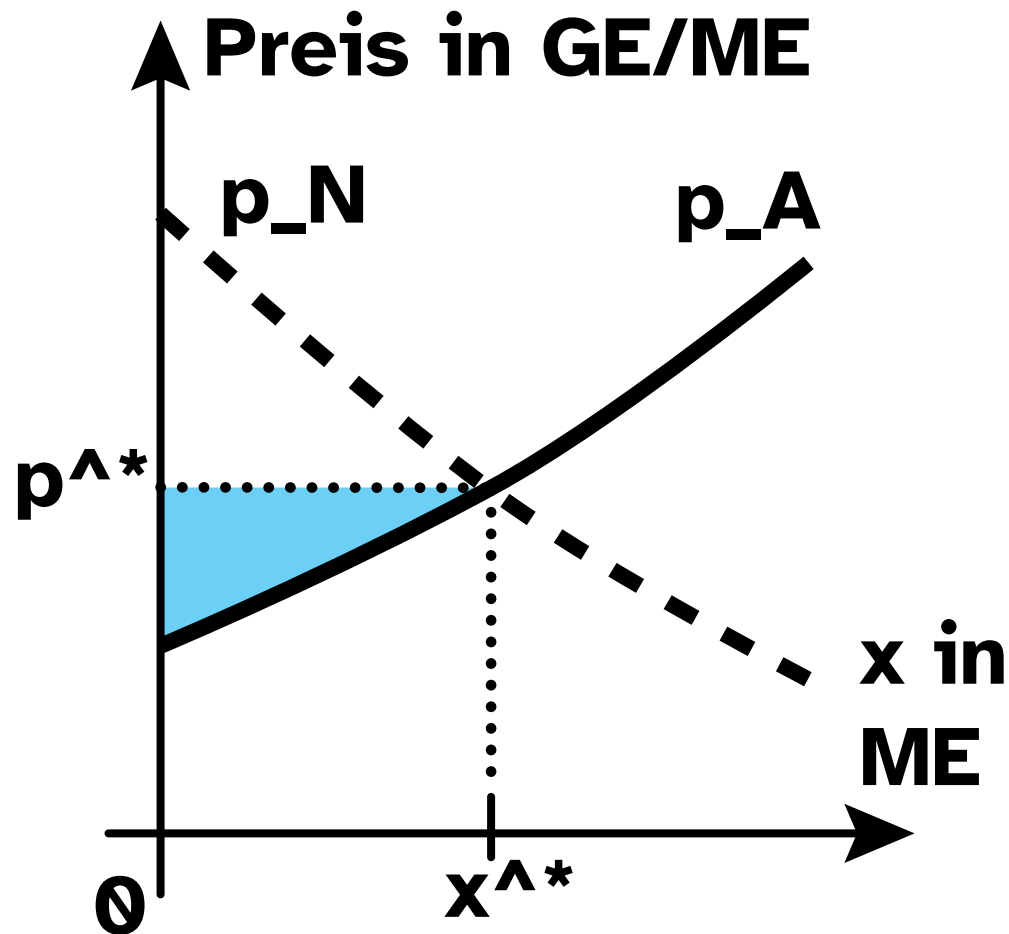
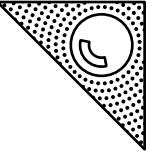




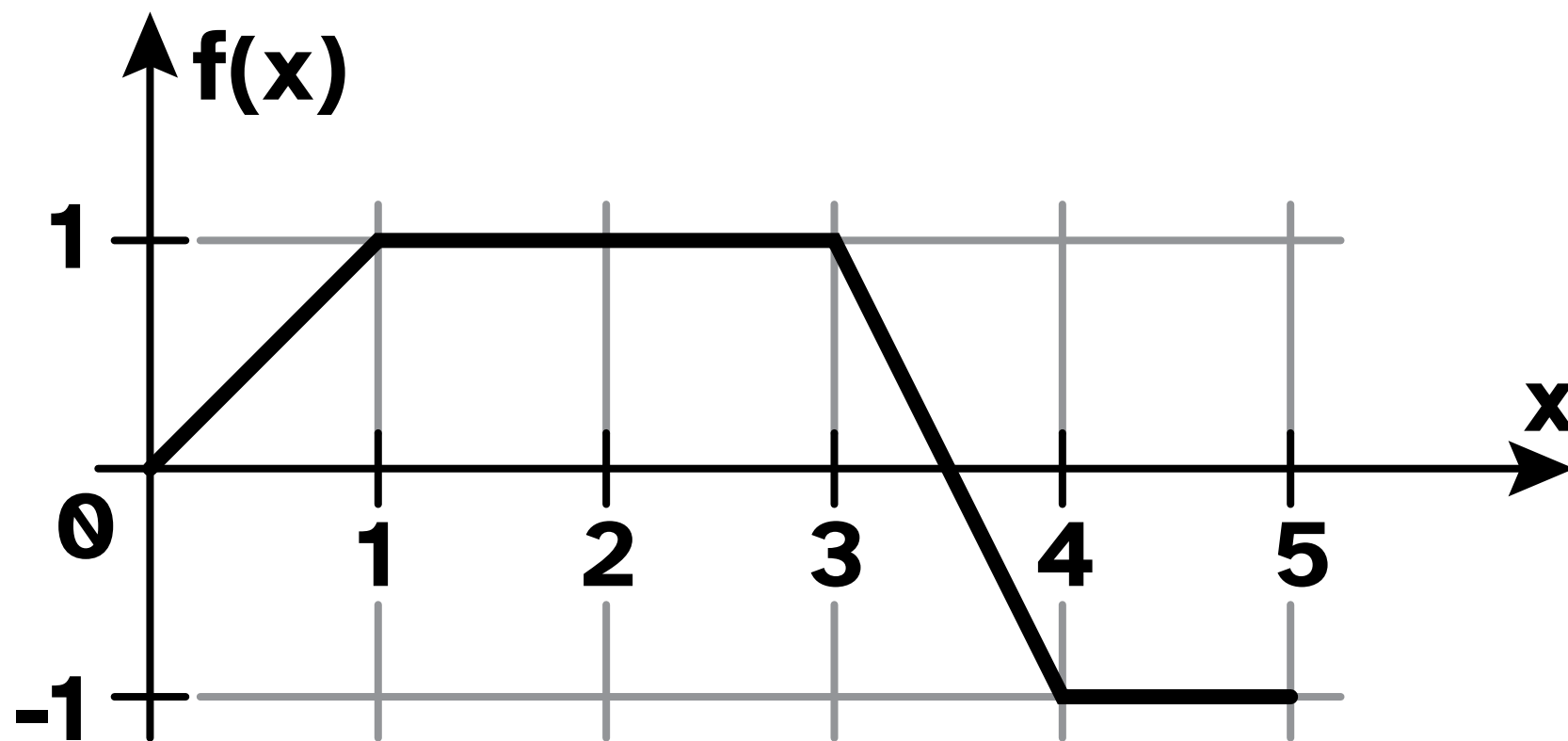
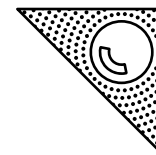
Produzenten erzielen  
einen Erlös von  
 $p^* \cdot x^*$

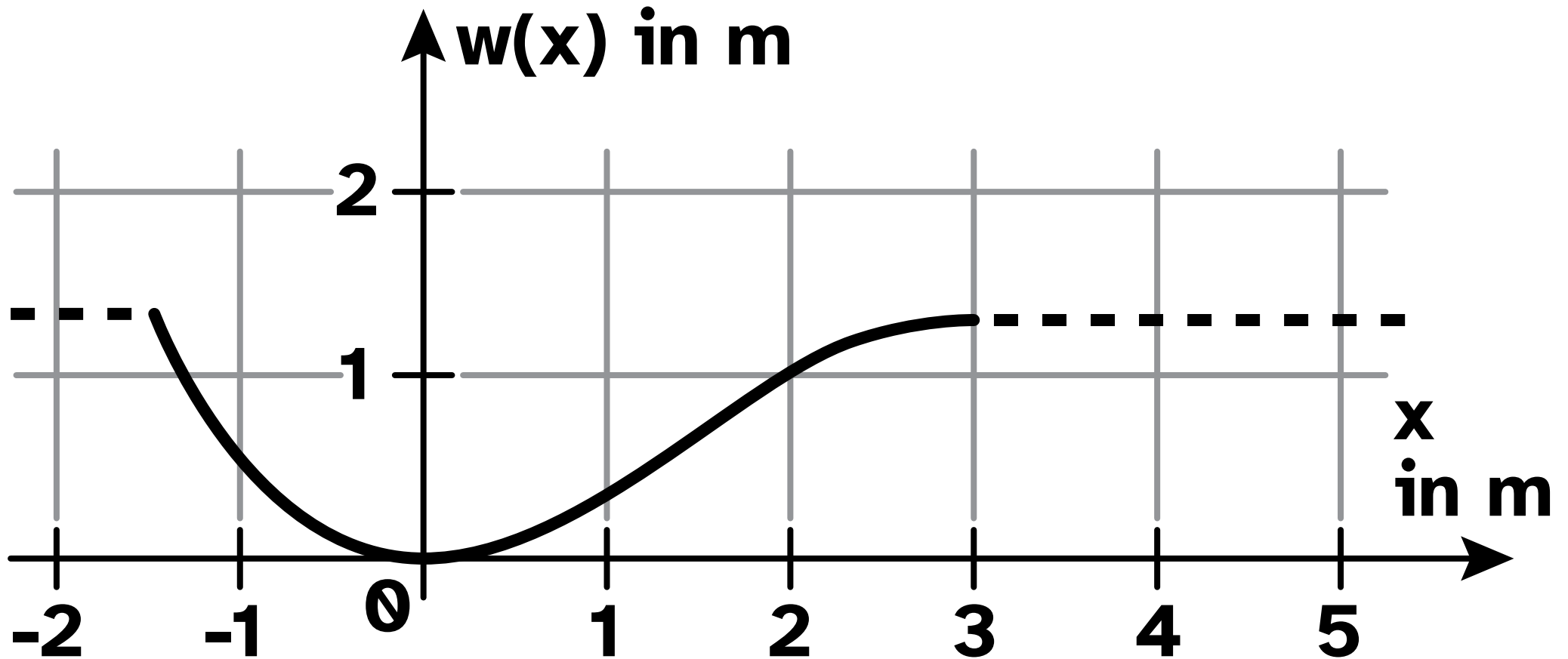
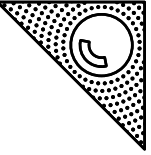


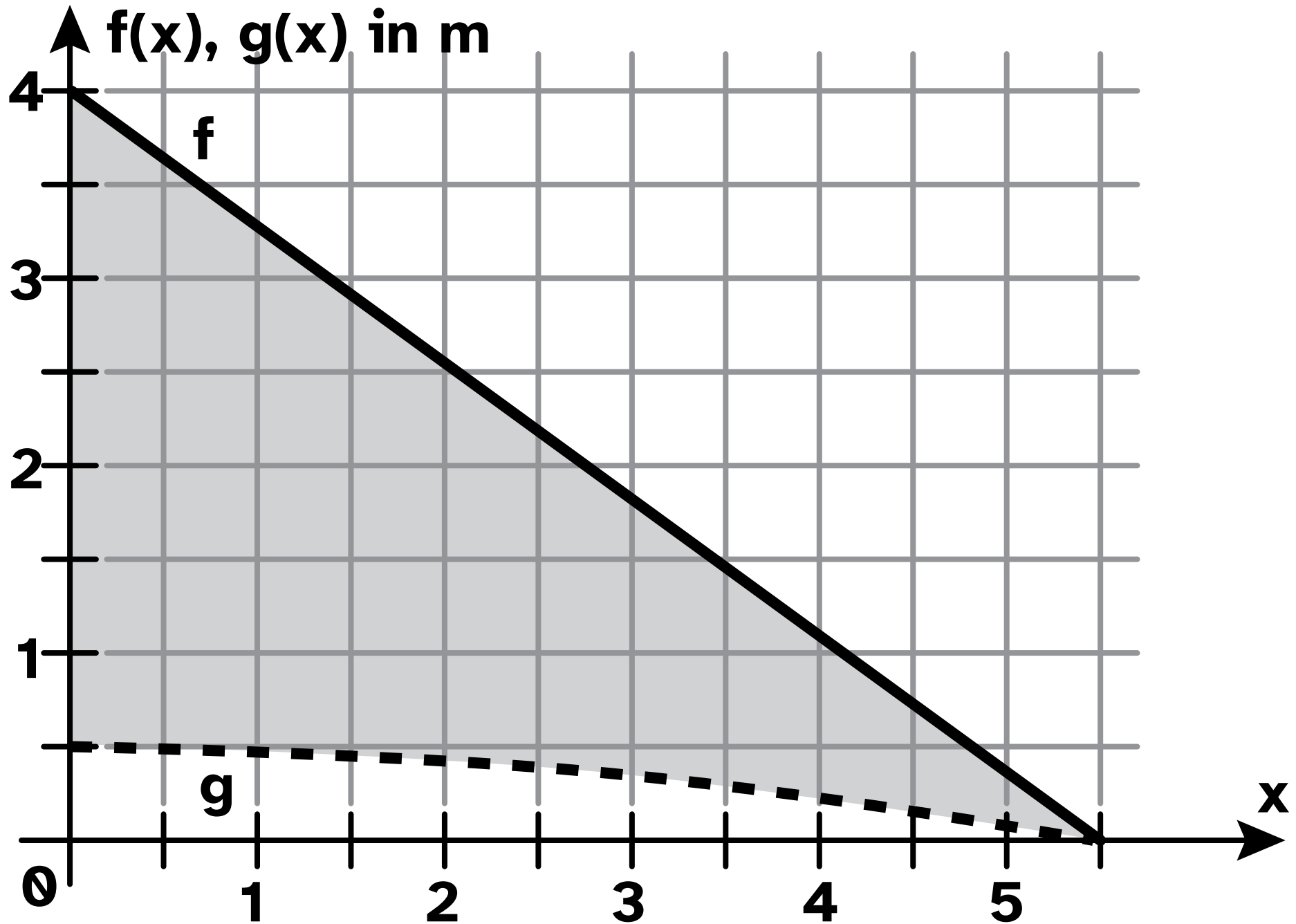
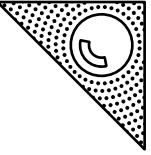
Würden aber Erlös von  
' $\int_0^{x^*} (p_N(x) \cdot dx)$ '  
akzeptieren



**Produzenten haben somit**  
 **$p^* \cdot x^* - \int_0^{x^*} p_N(x) \, dx$  gespart.**  
**Das nennt man Produzentenrente.**

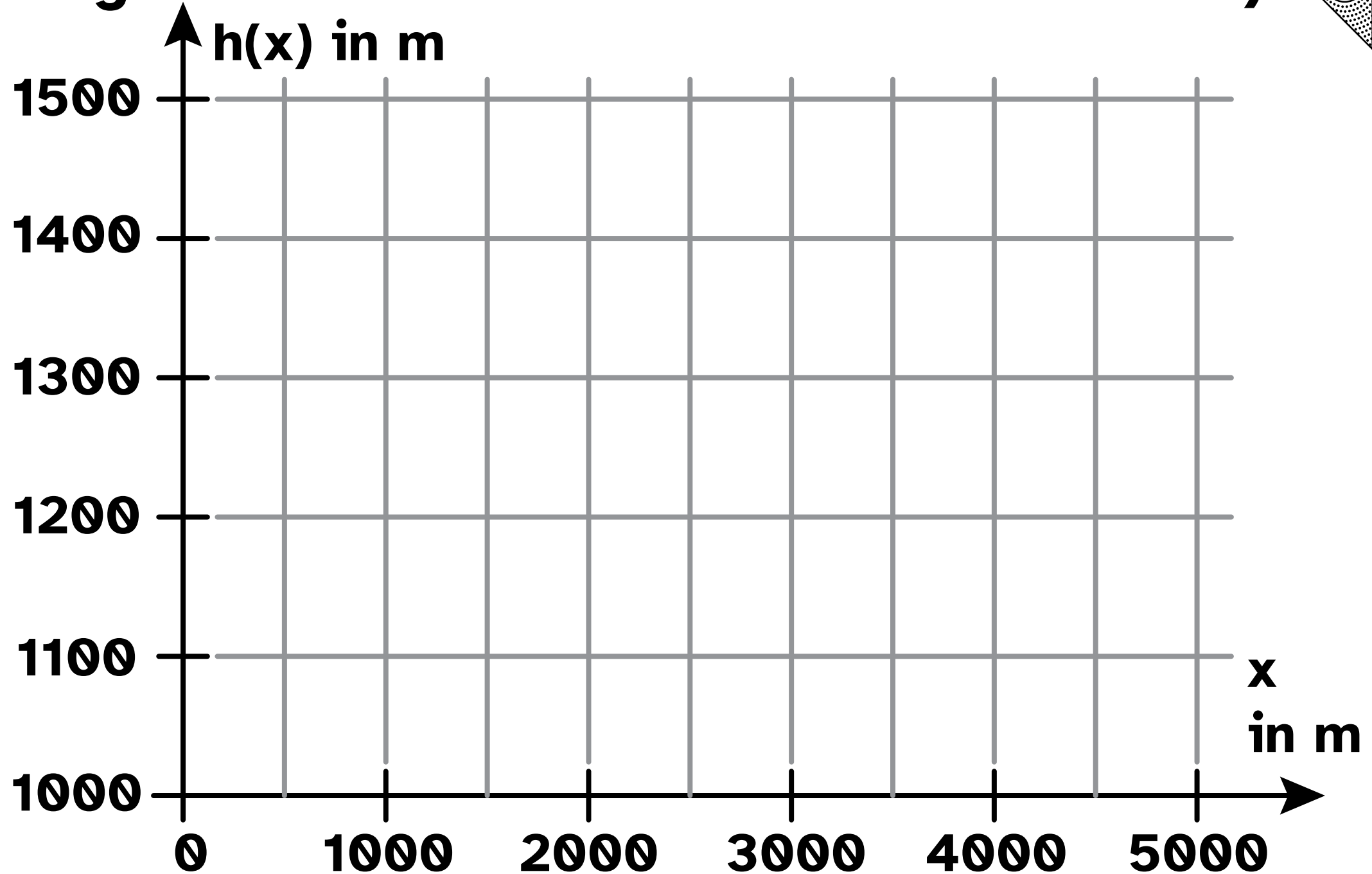
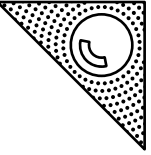


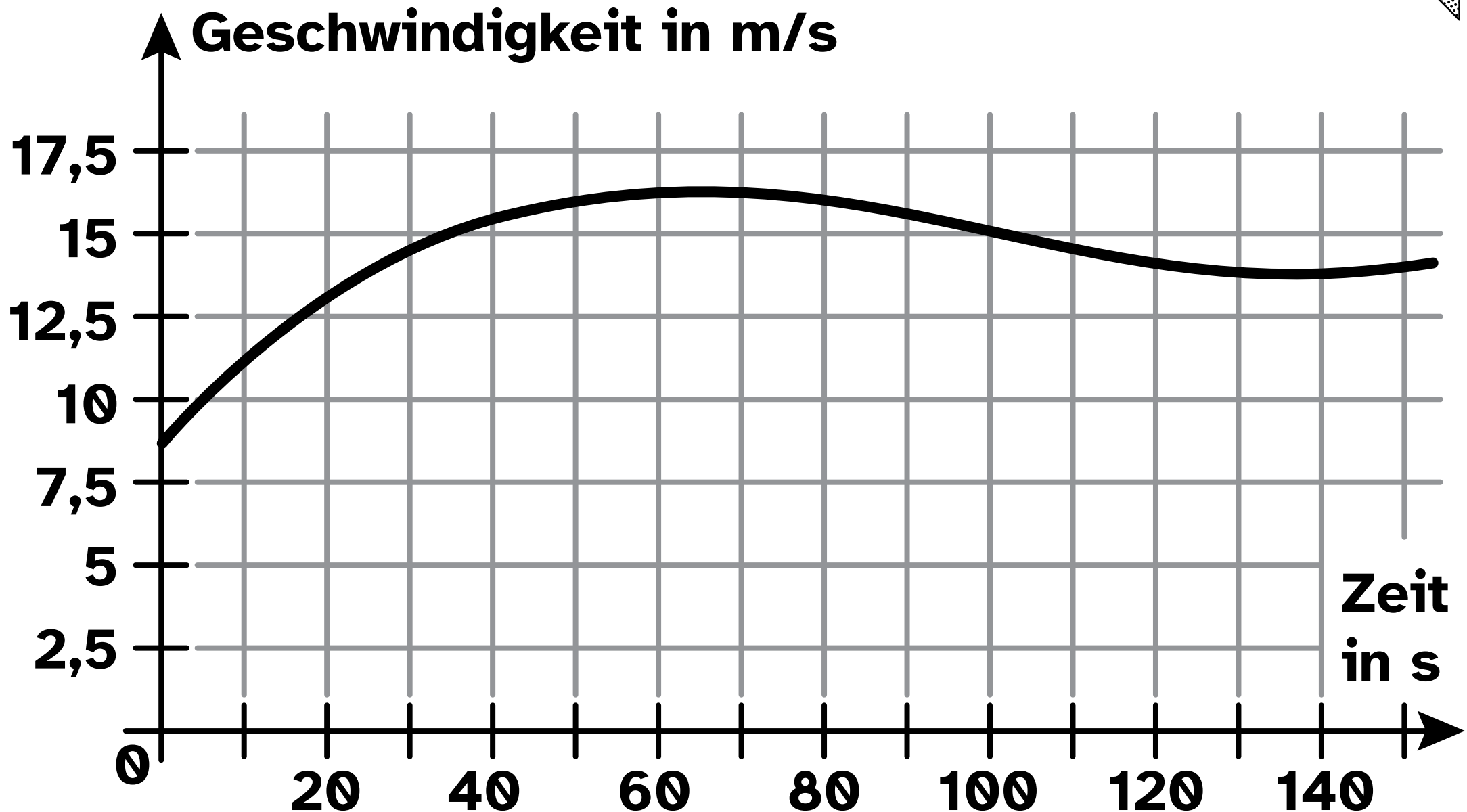
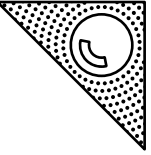


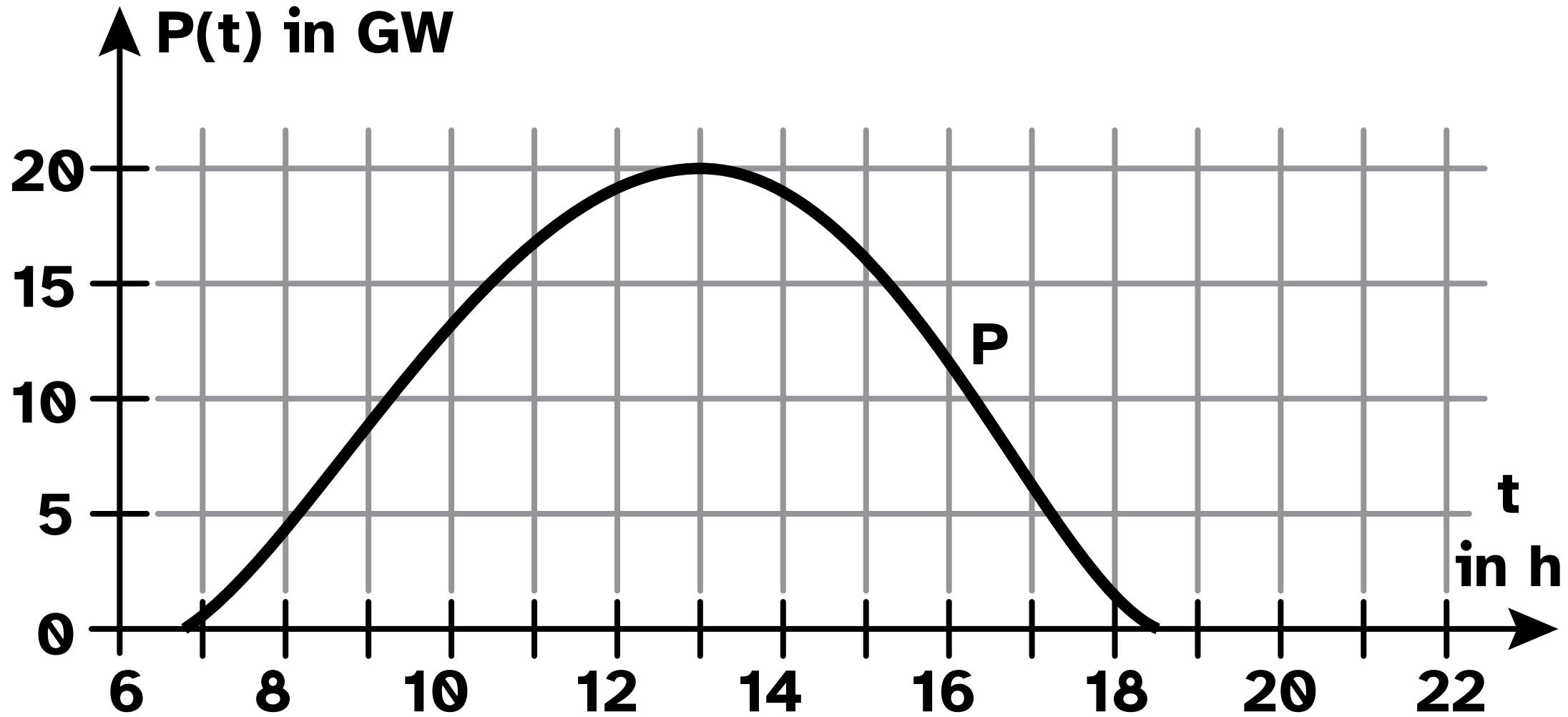
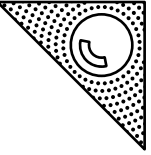


**Ang.Mat. HAK4**

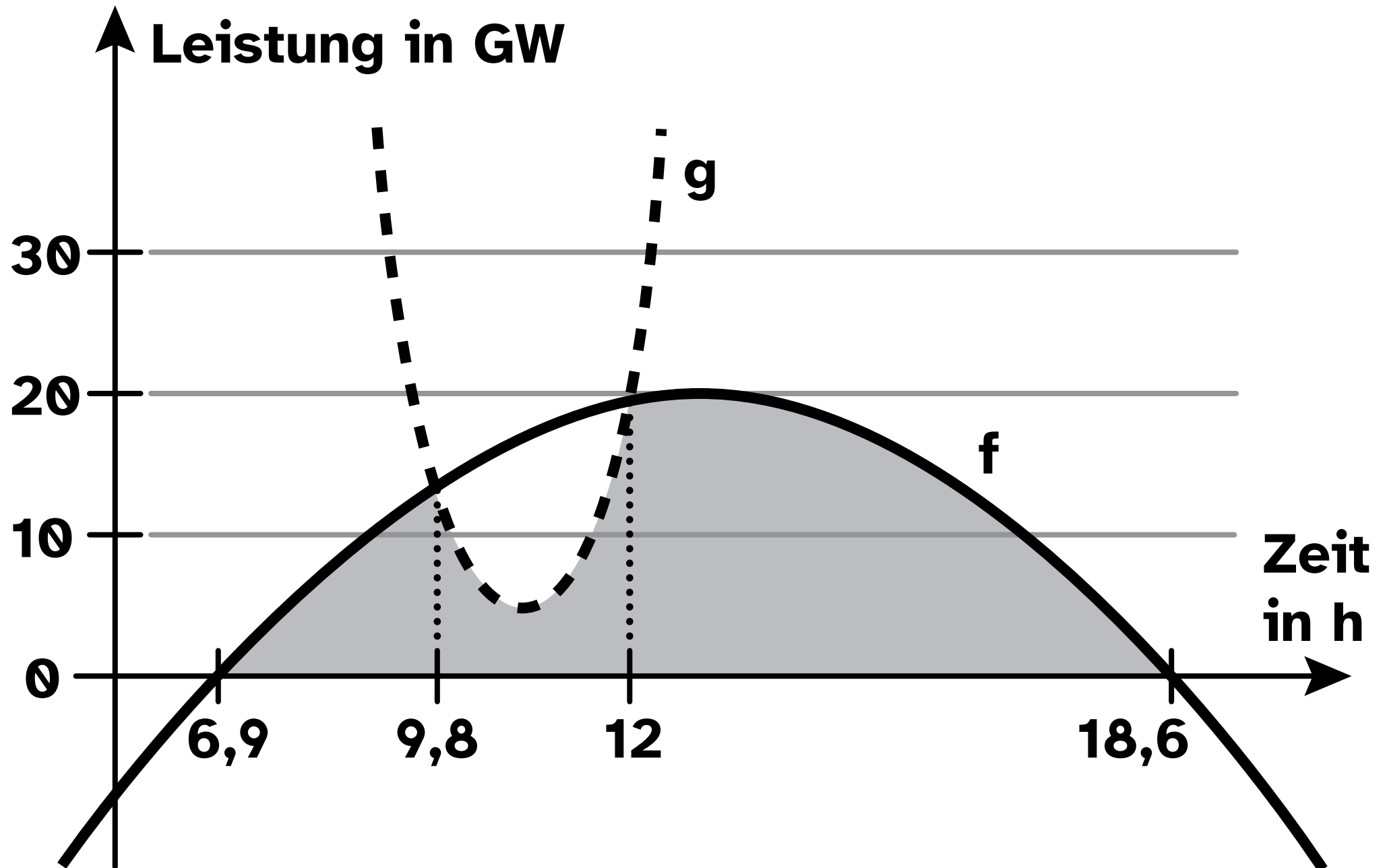
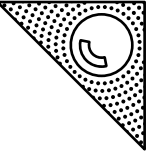
**S.123 5.92 b)**

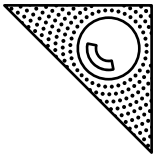










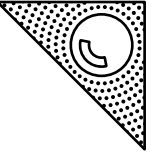


# **Angewandte Mathematik**

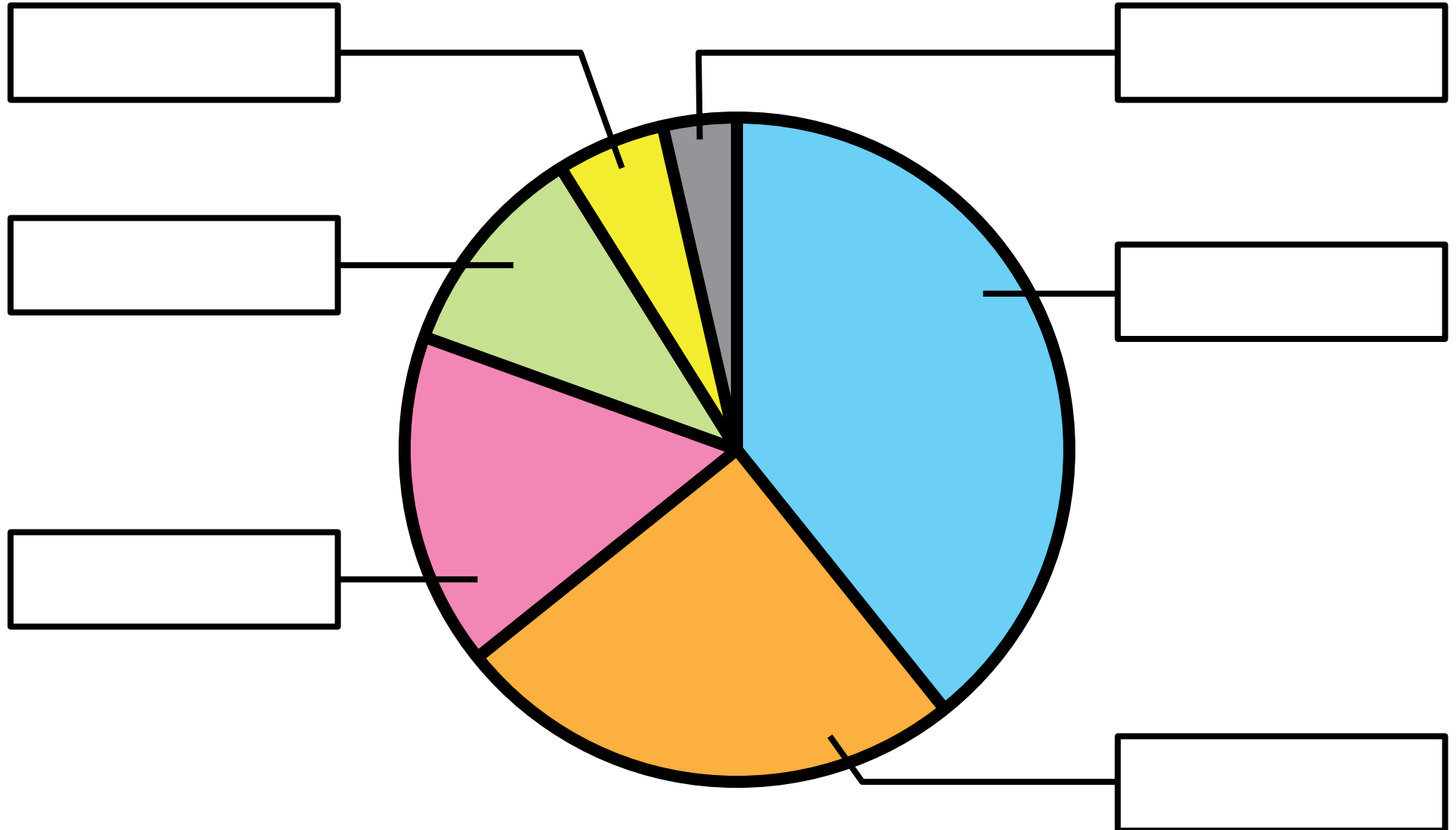
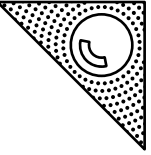
## **HAK Band 4**

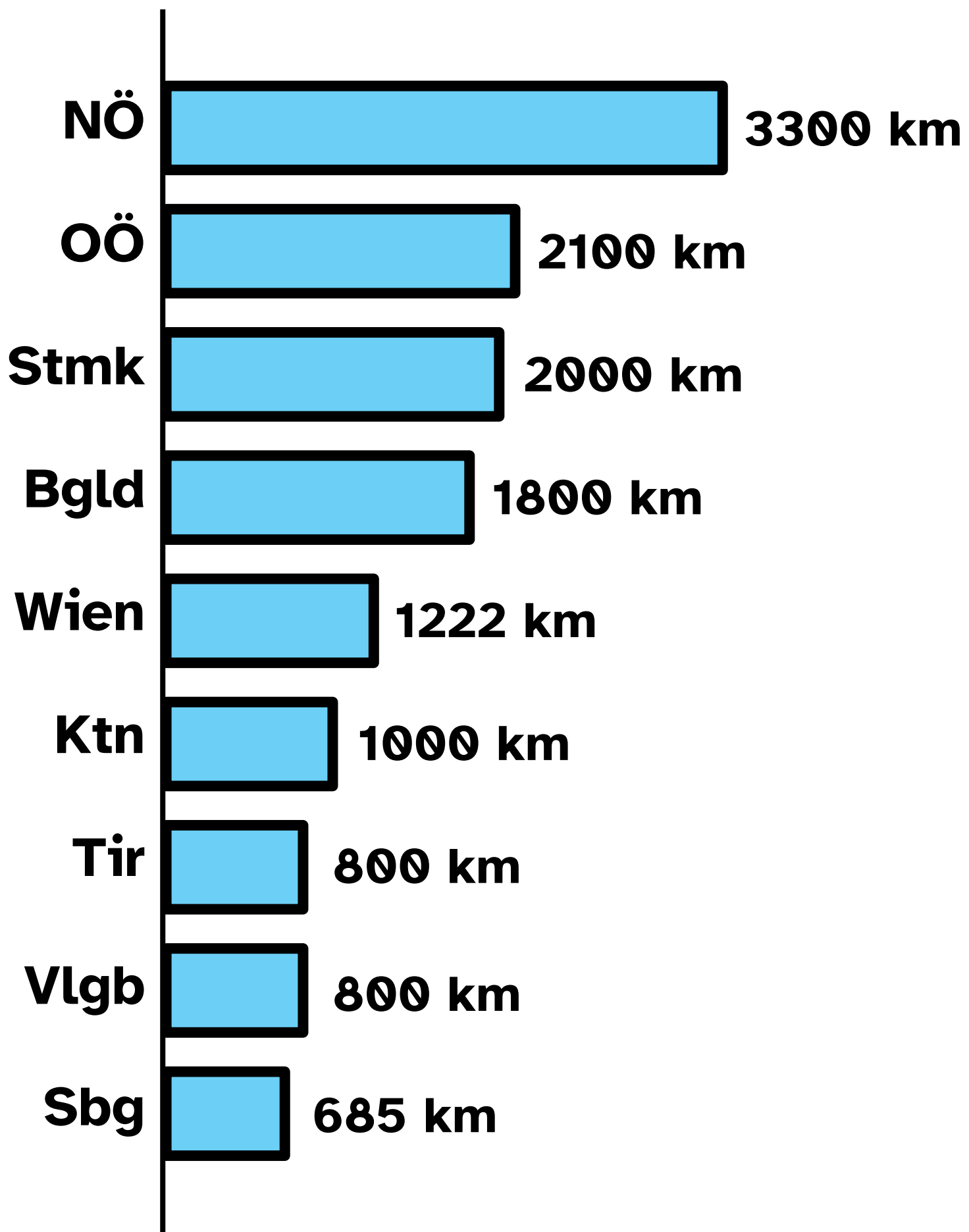
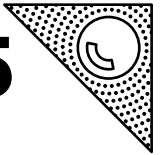
# **6.**

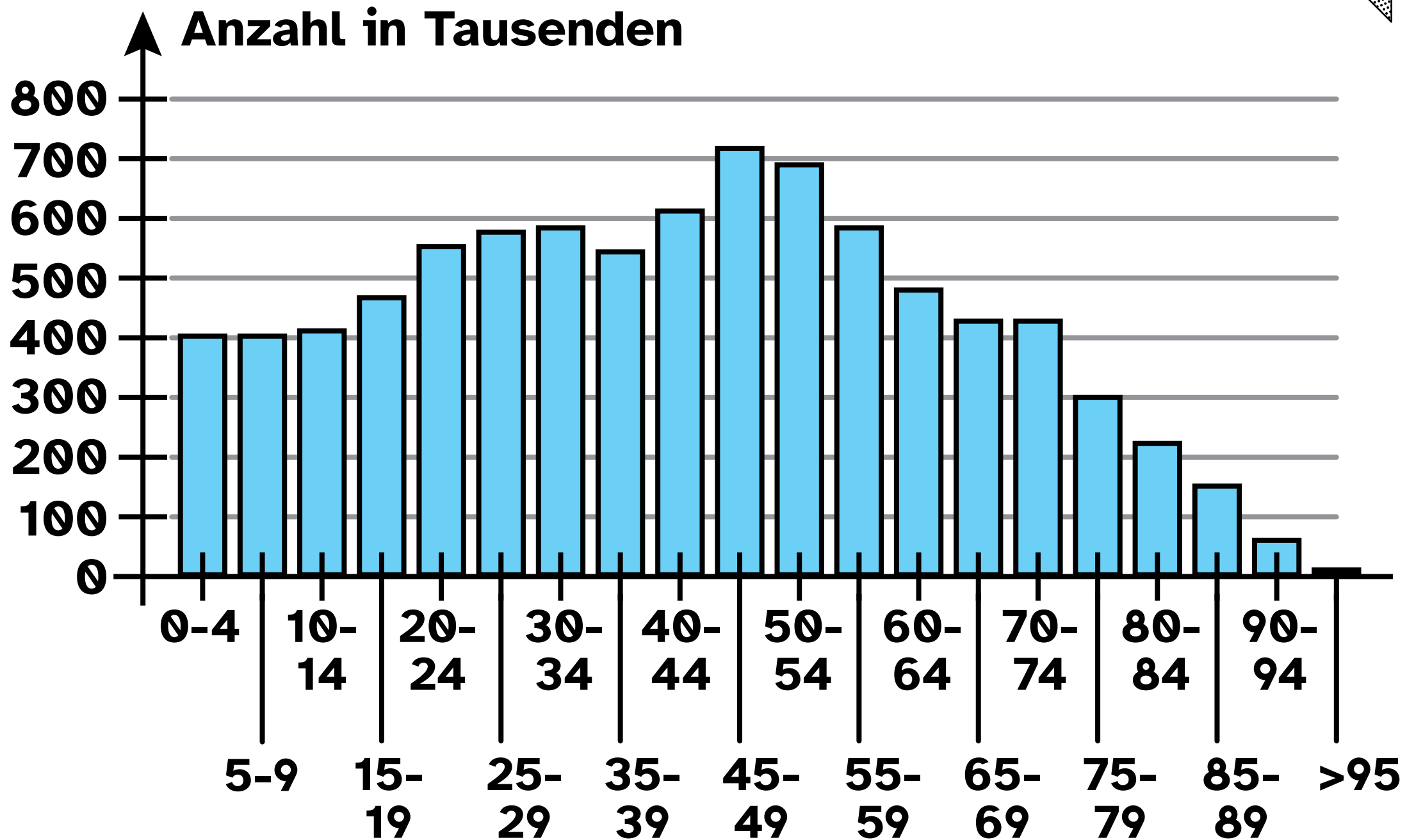
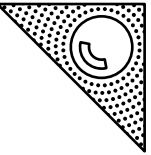
## **Beschreibende Statistik**

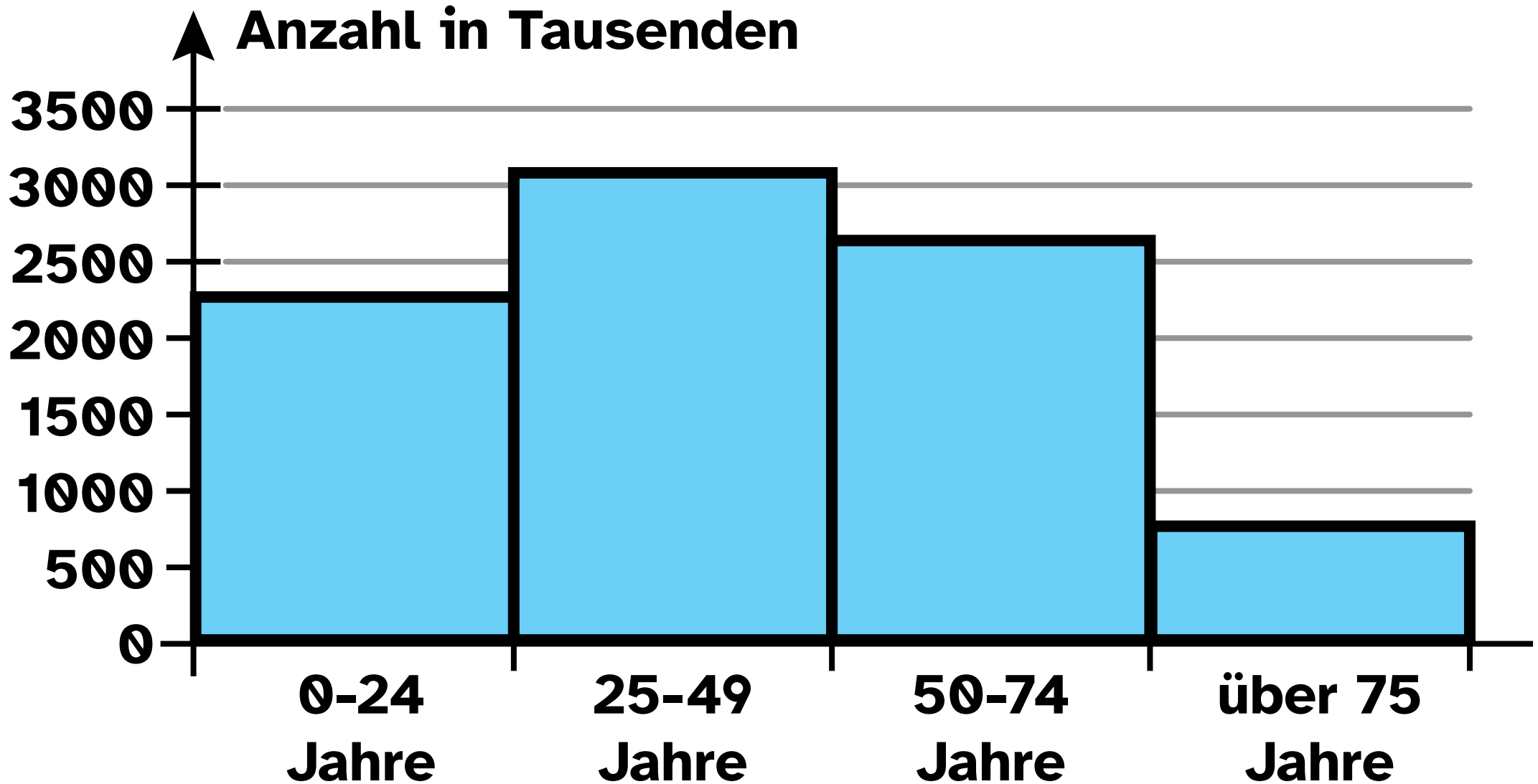
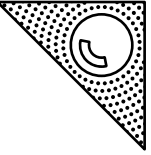


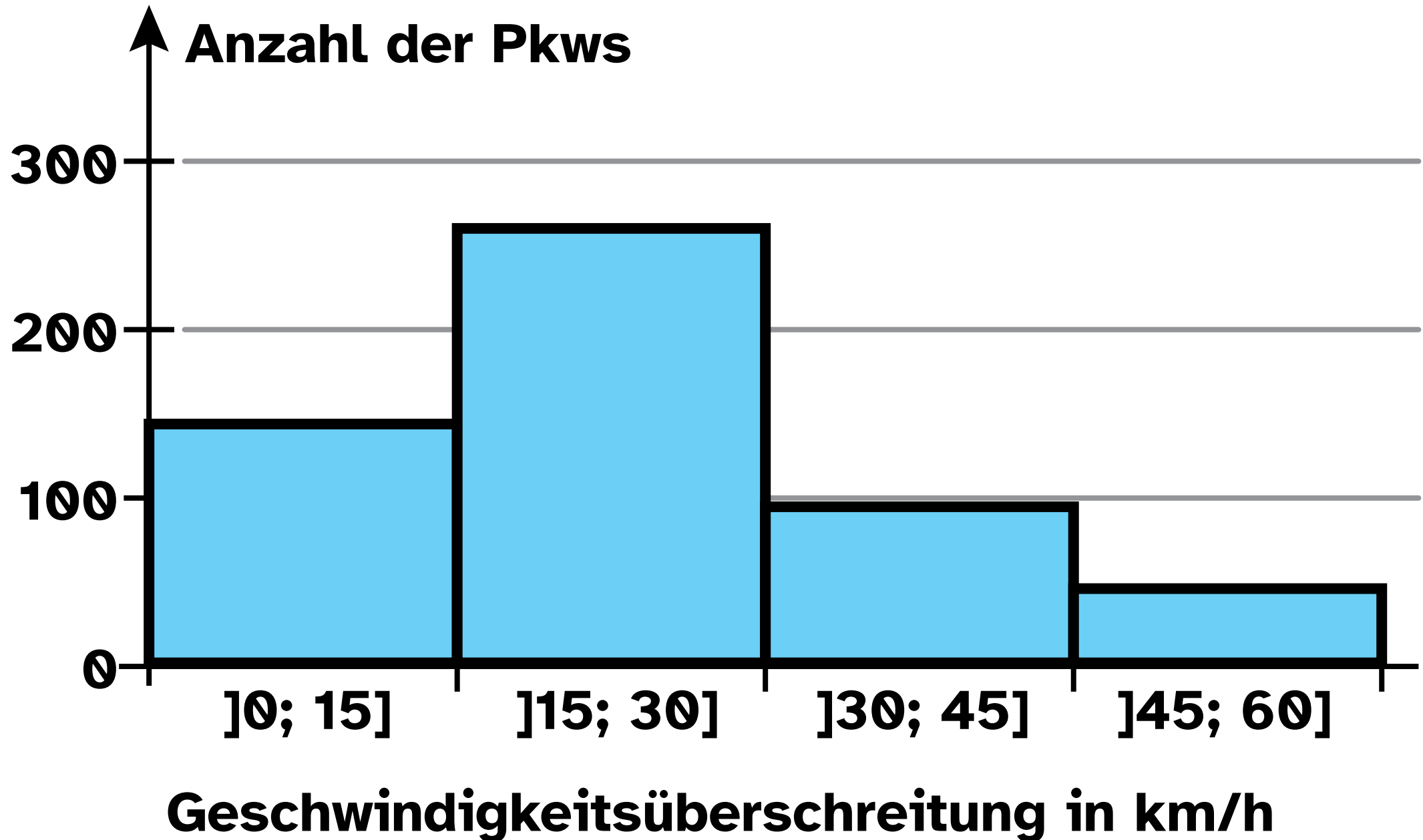
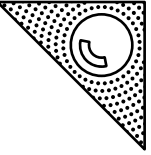
| <b>Getränk</b>      | <b>Anzahl</b> |
|---------------------|---------------|
| <b>Kakao</b>        | <b>22</b>     |
| <b>Milch</b>        | <b>14</b>     |
| <b>Vanillemilch</b> | <b>3</b>      |
| <b>Saft</b>         | <b>9</b>      |
| <b>Tee</b>          | <b>6</b>      |
| <b>Sonstiges</b>    | <b>2</b>      |



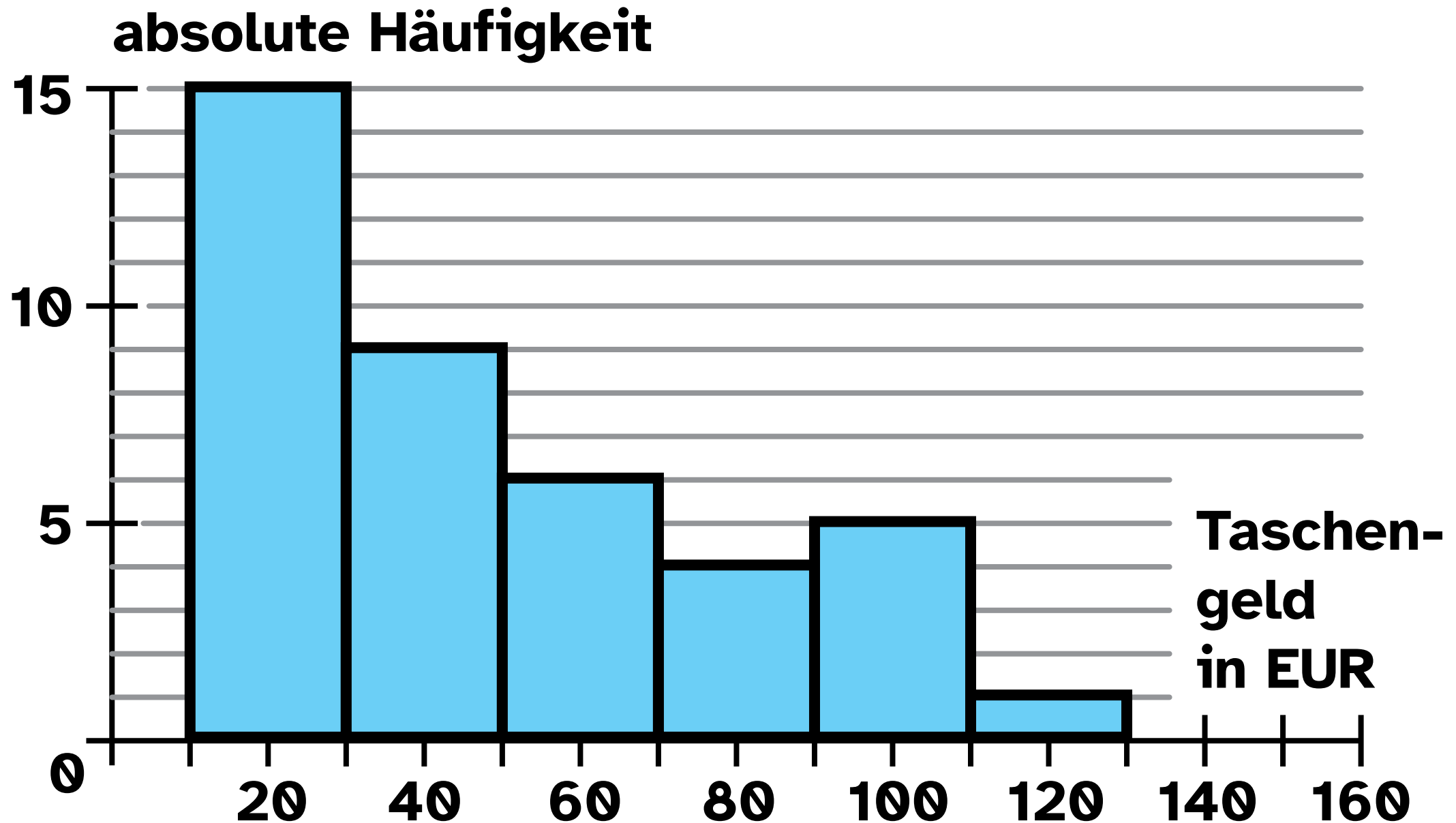
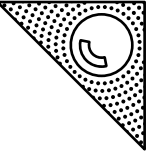


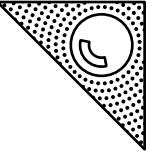






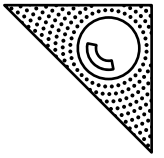




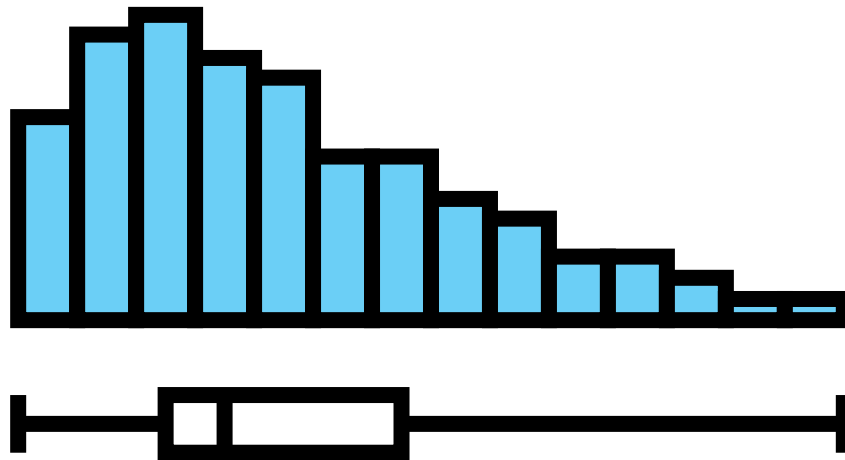


**3,7 von 5 Sternen**

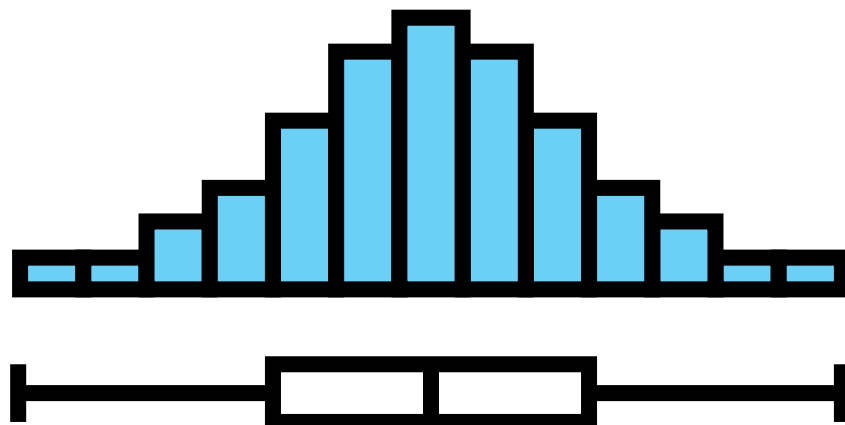




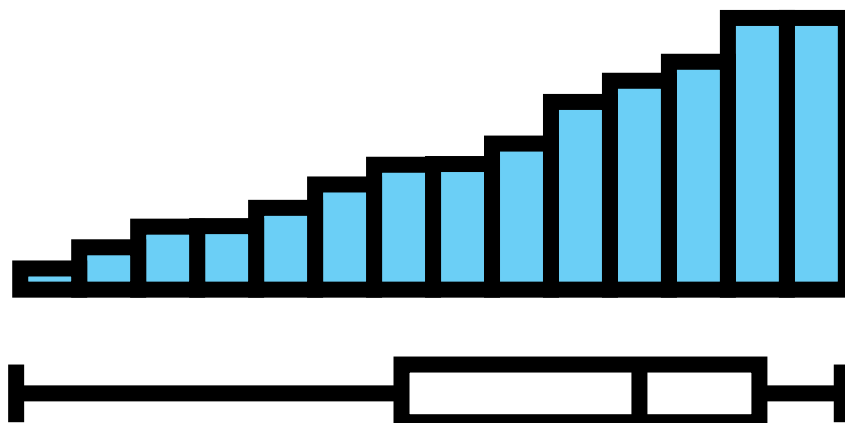
## rechtsschiefe Verteilung

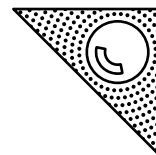


## symmetrische Verteilung

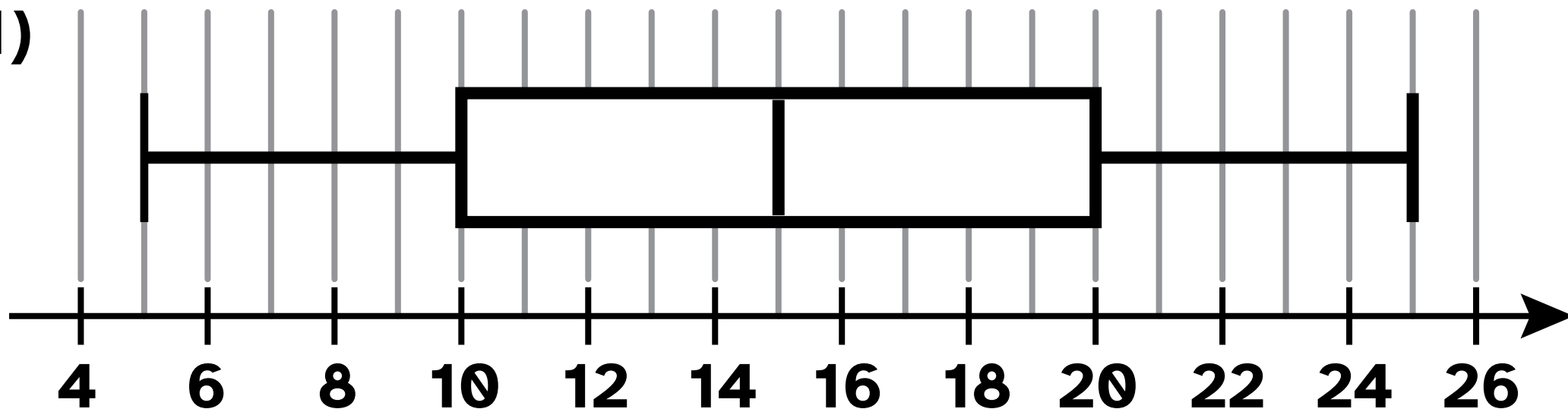


## linksschiefe Verteilung

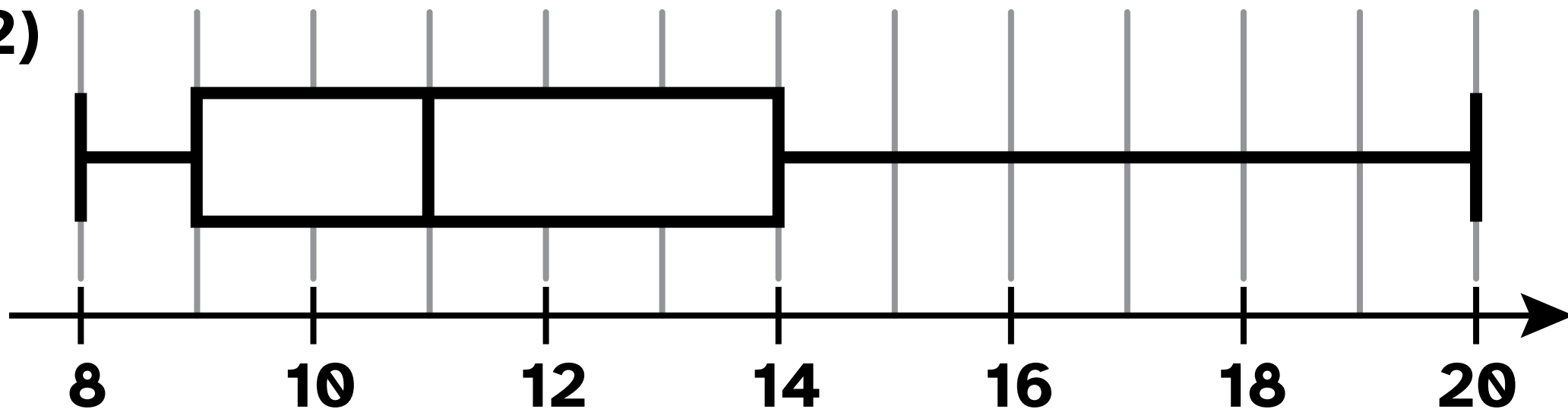


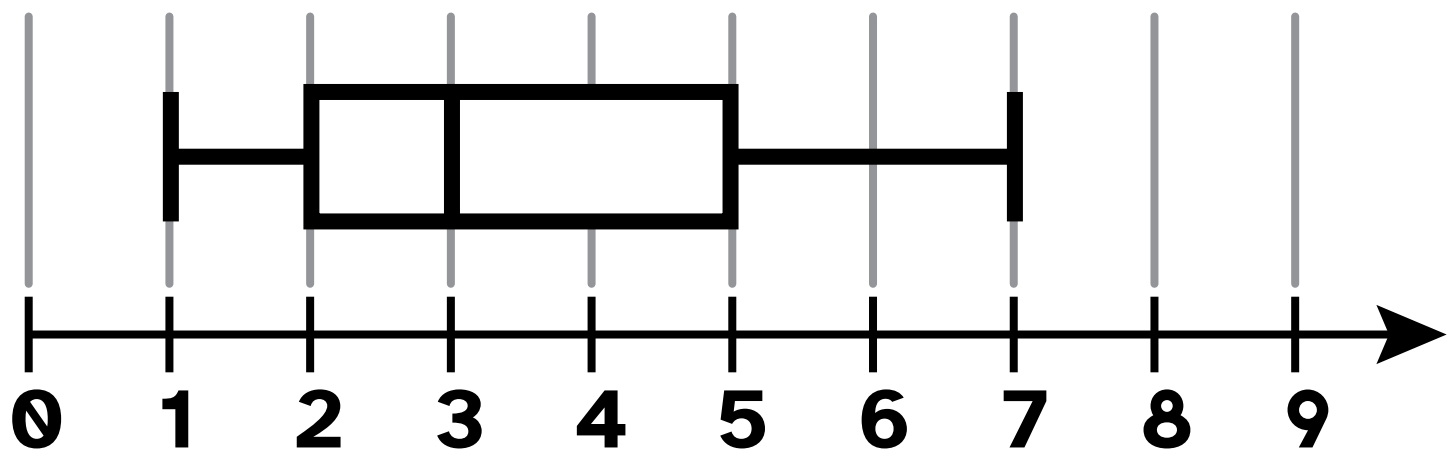
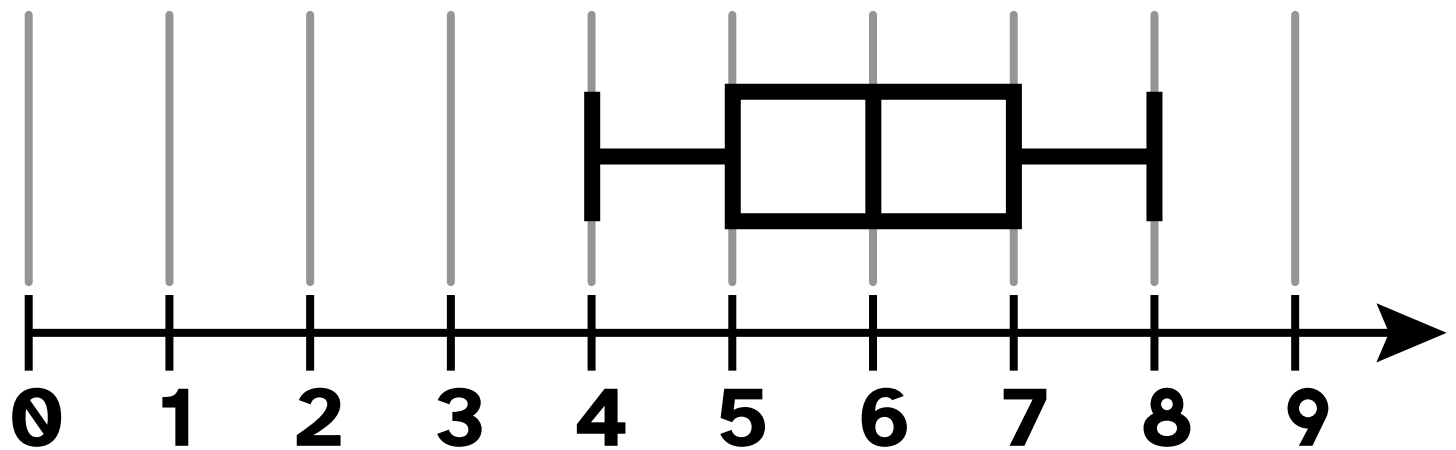
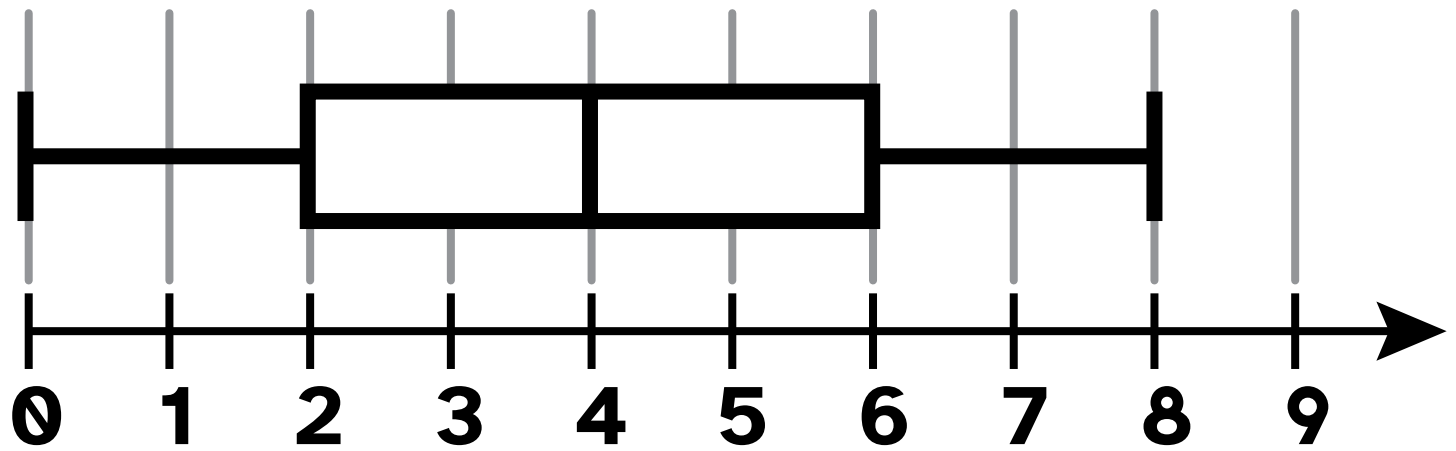
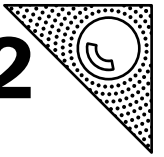


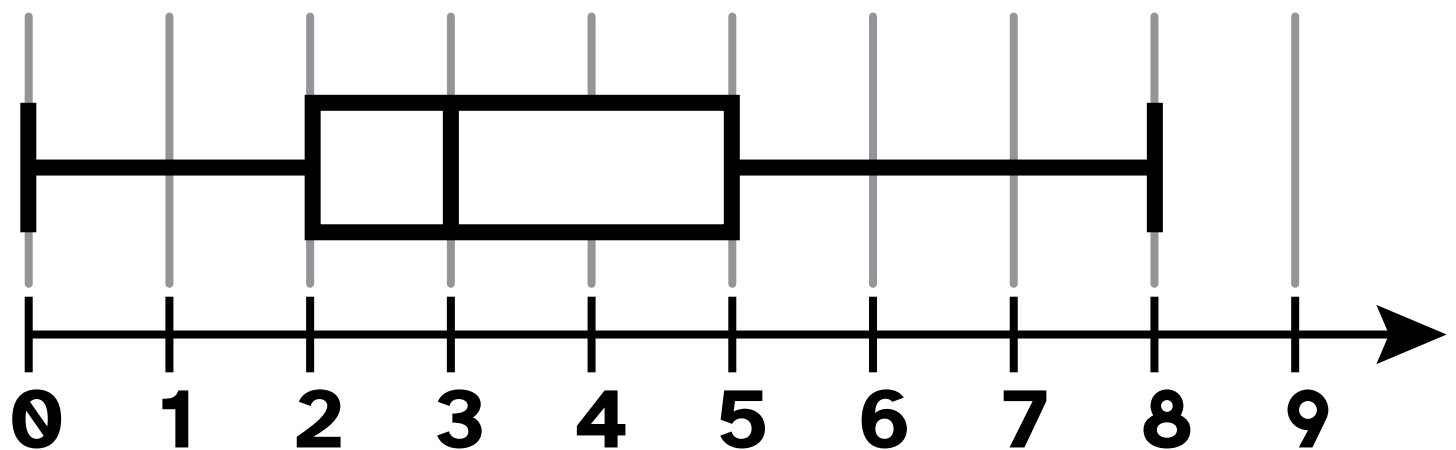
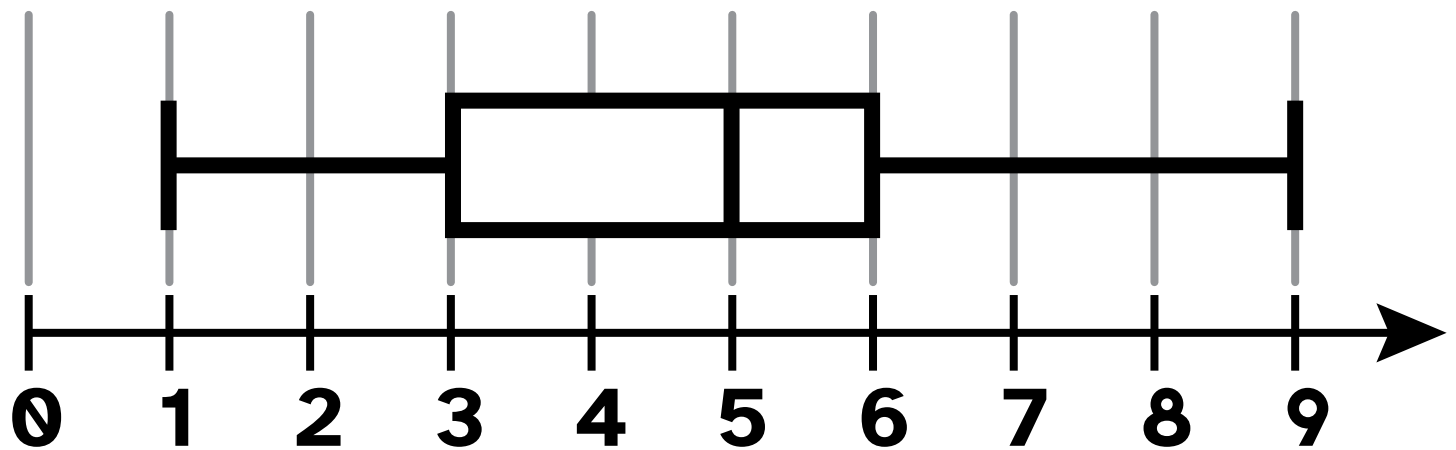
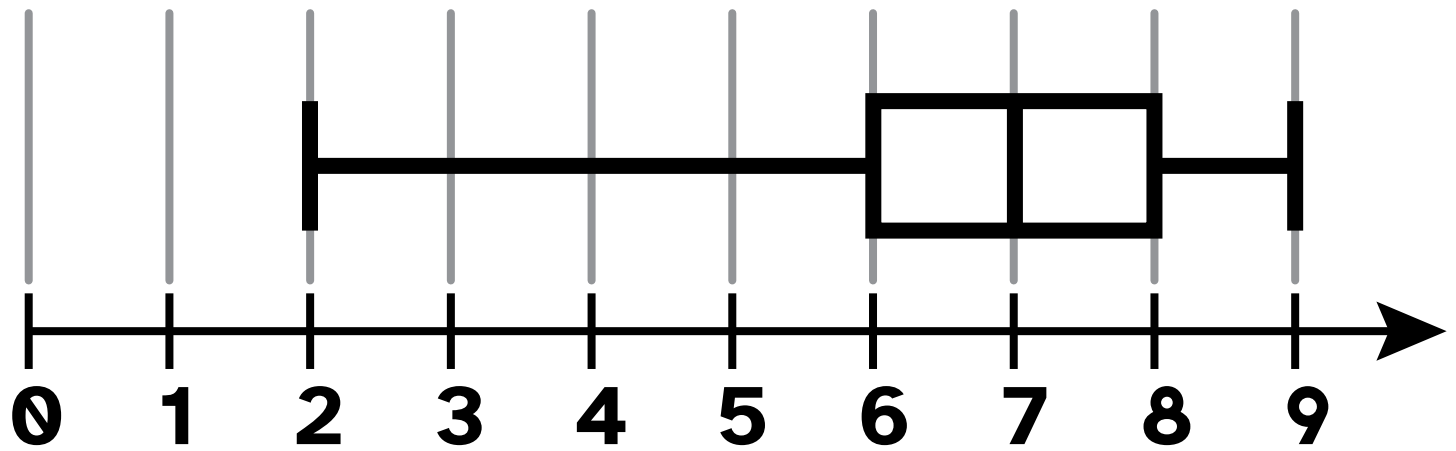
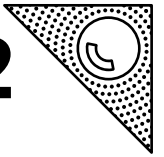
(1)

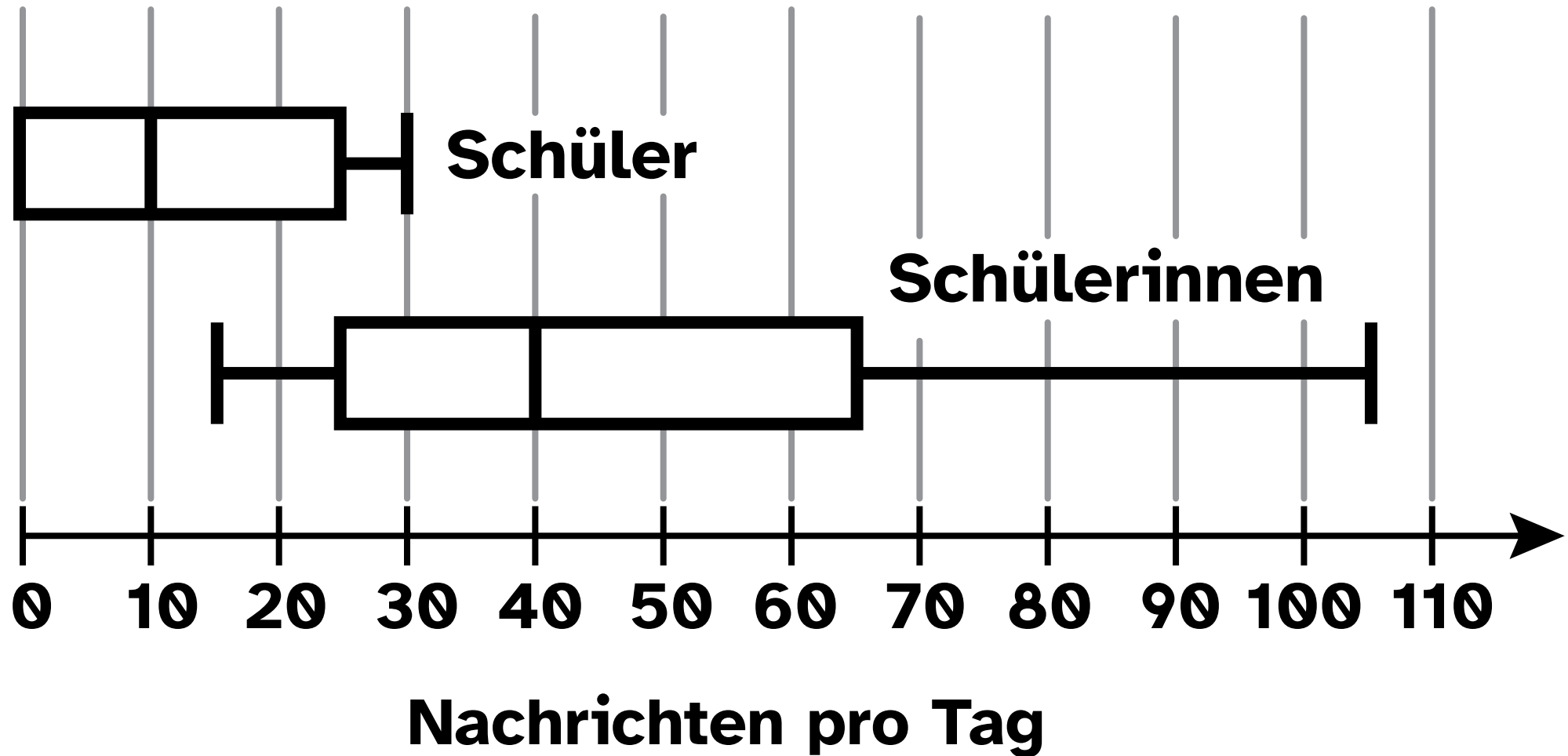
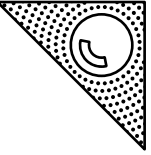


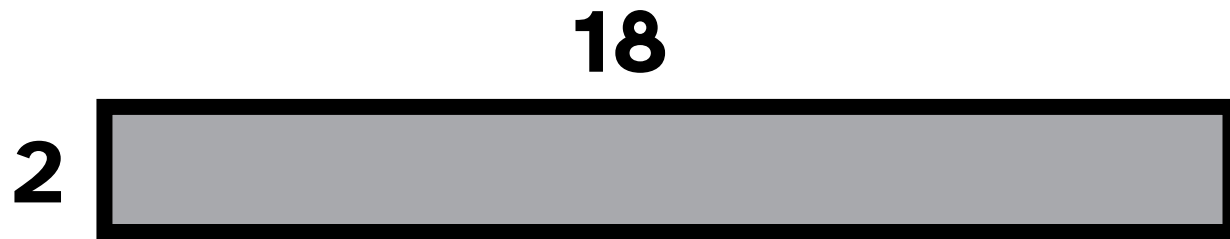
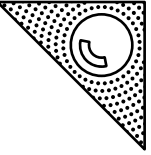
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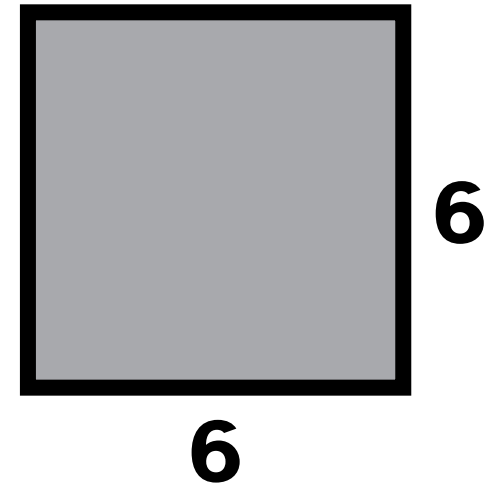




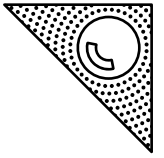




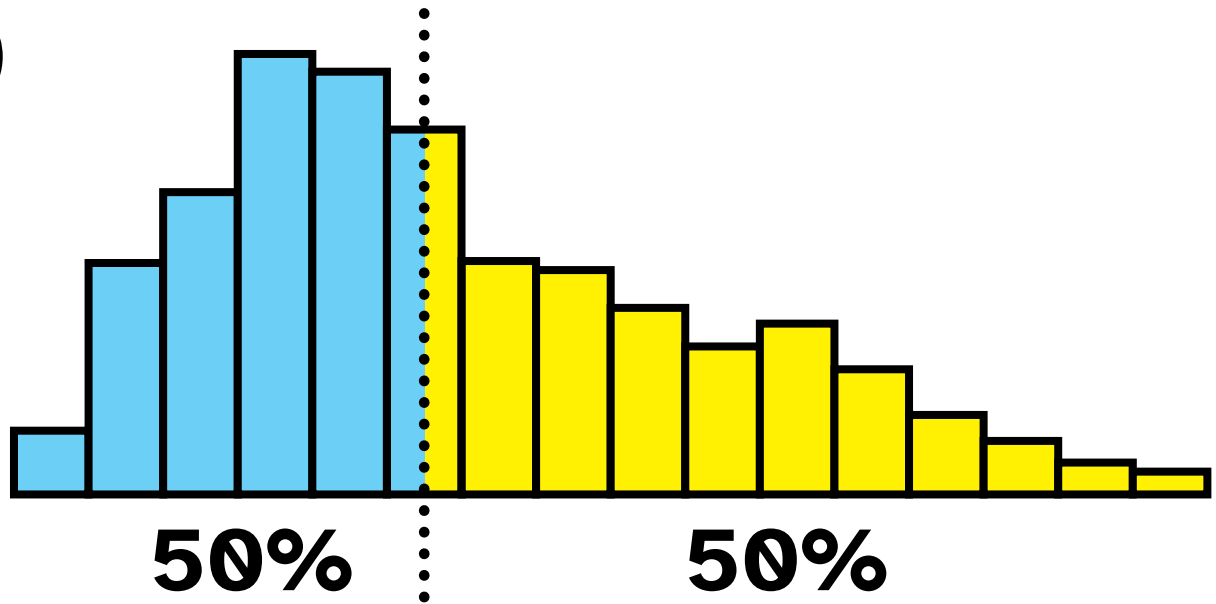
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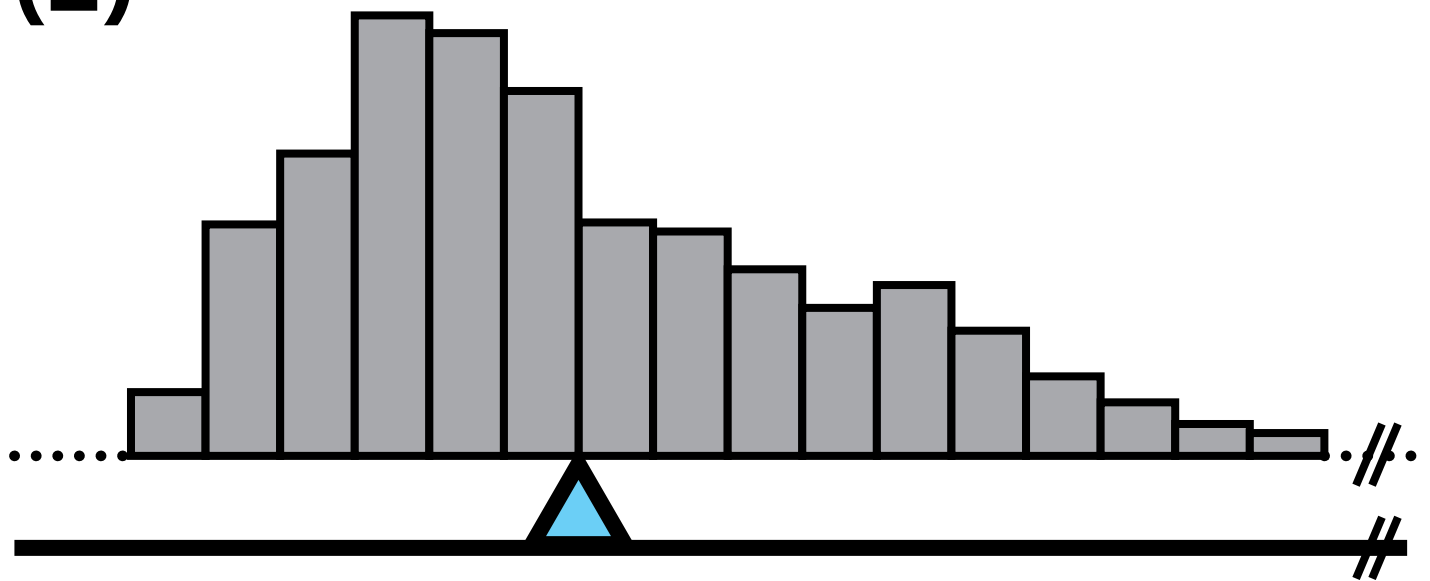




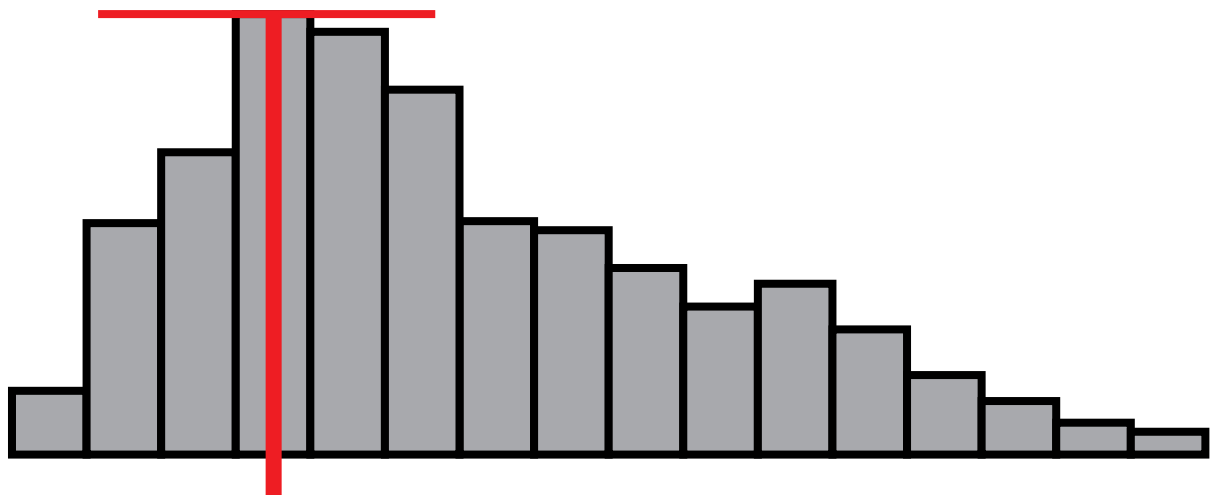
(1)



(2)



(3)



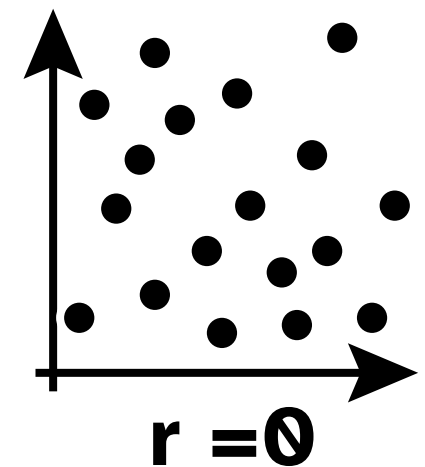
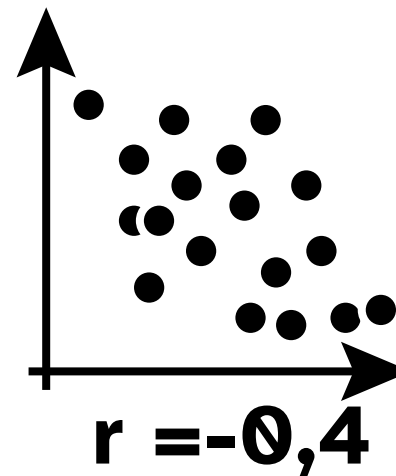
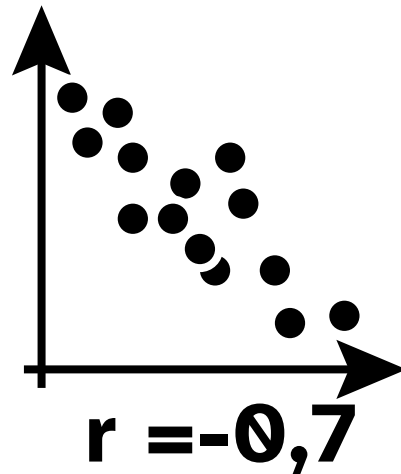
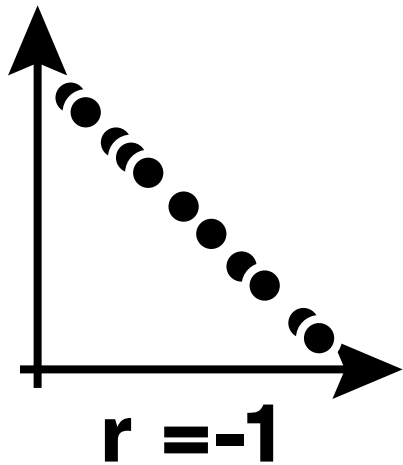
**negative  
Korrelation**

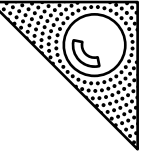
**keine  
Korrelation**

**starke**

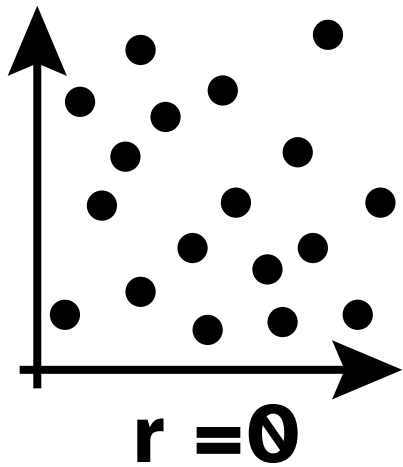


**schwache**





**keine  
Korrelation**

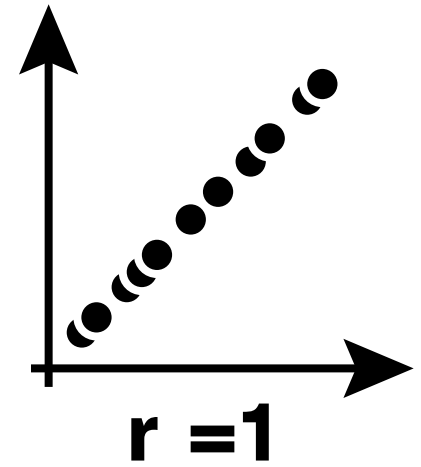
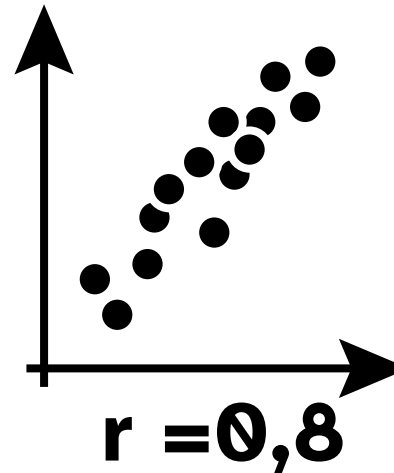
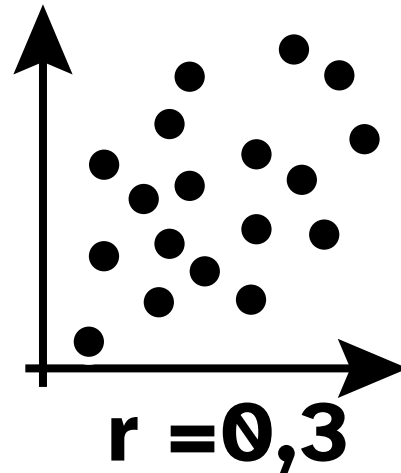


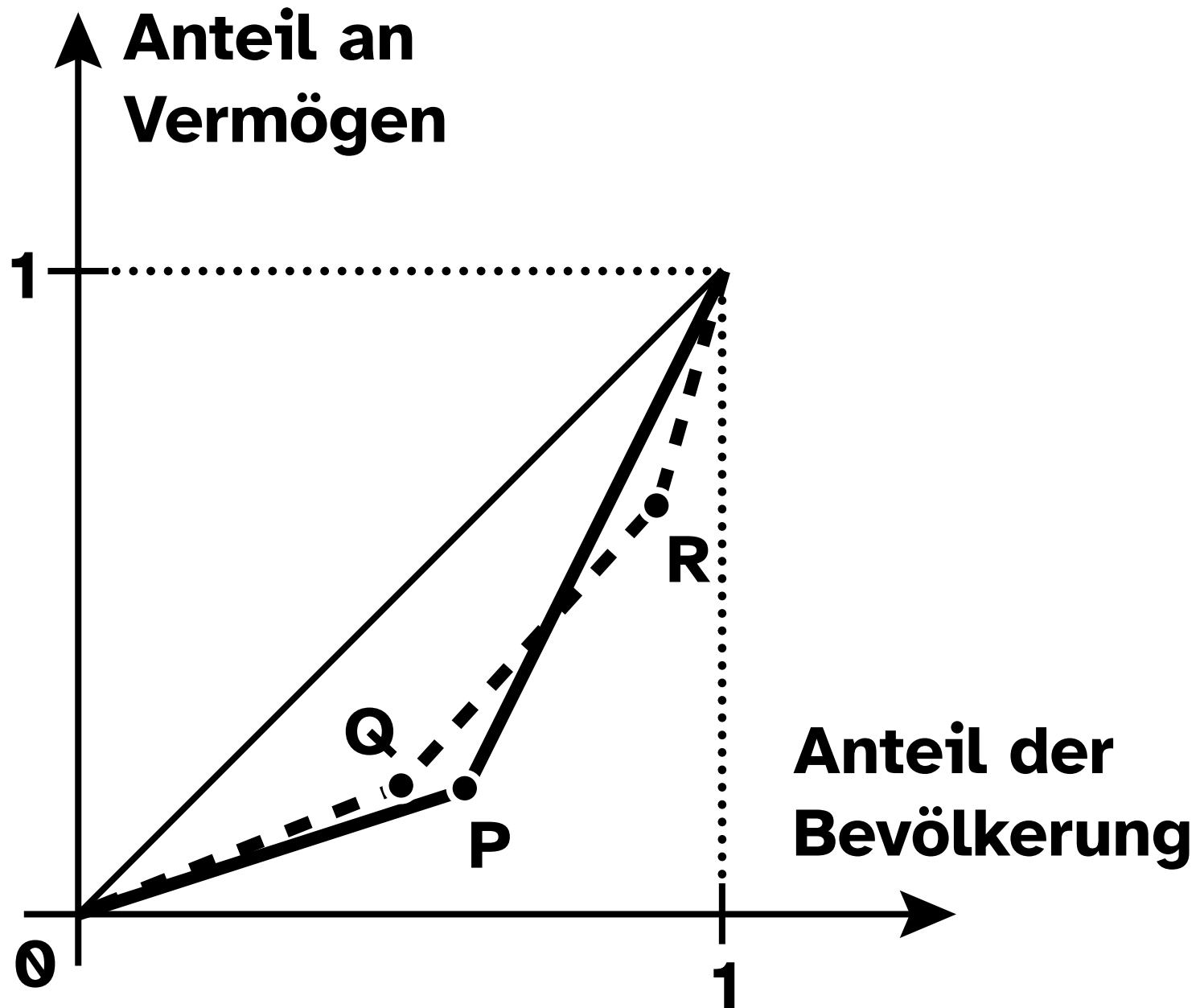
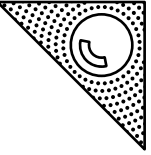
**positive  
Korrelation**

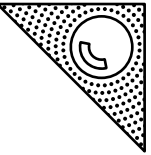
**schwache**



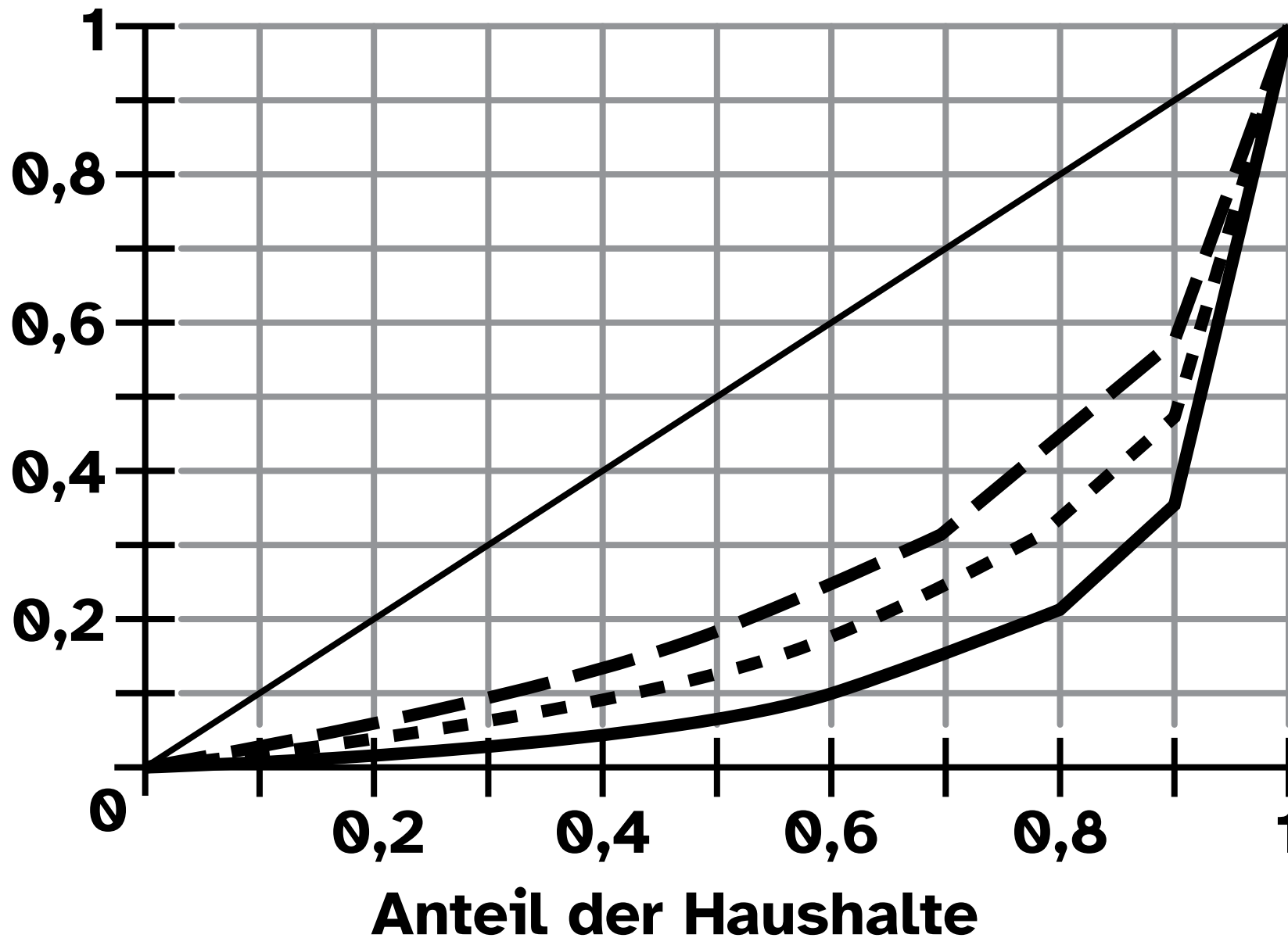
**starke**







### Anteil am Gesamteinkommen



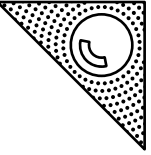
Legende:

— GV

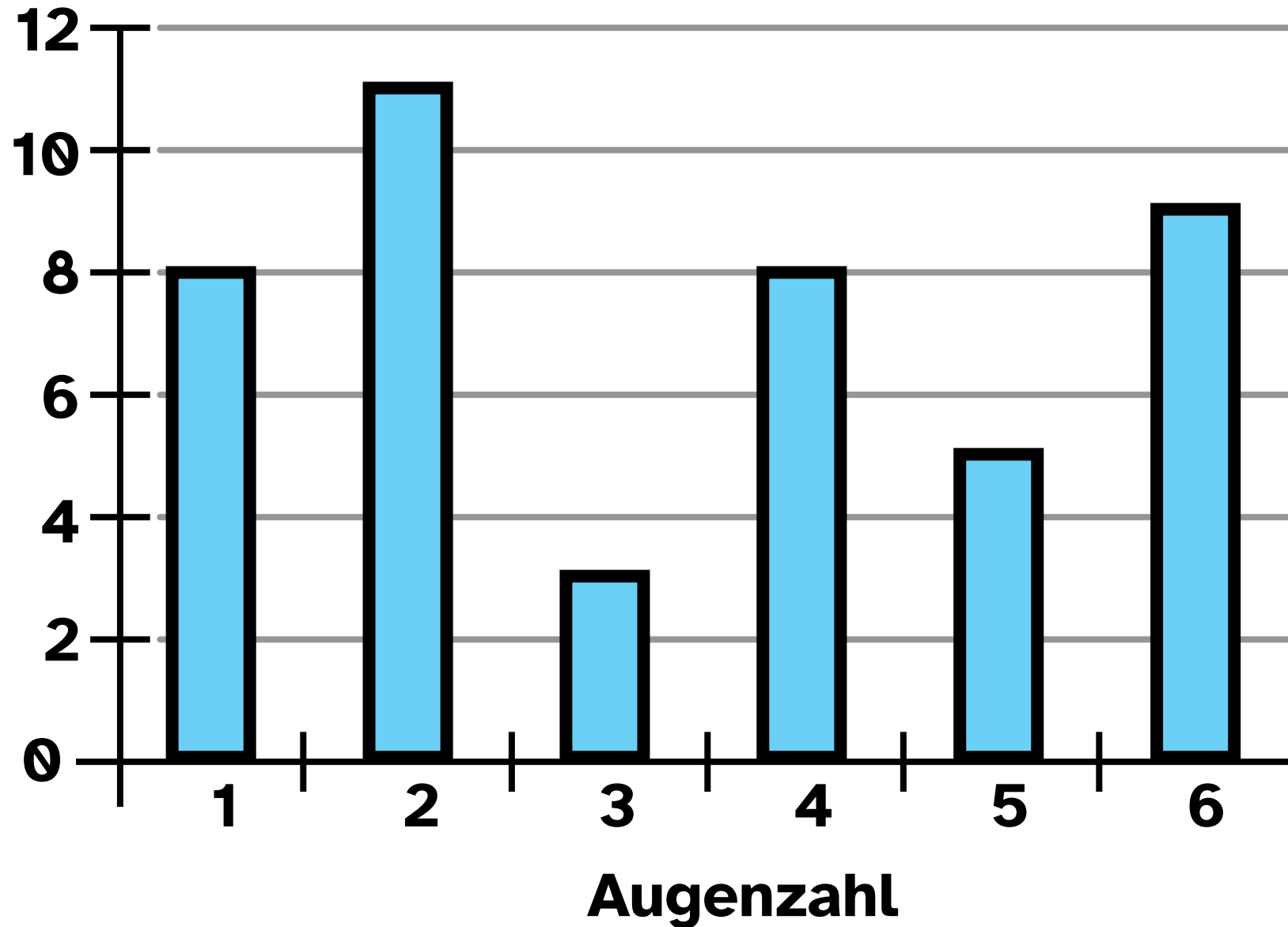
— 1993-94

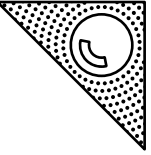
- - - 2003-04

- · - 2009-10

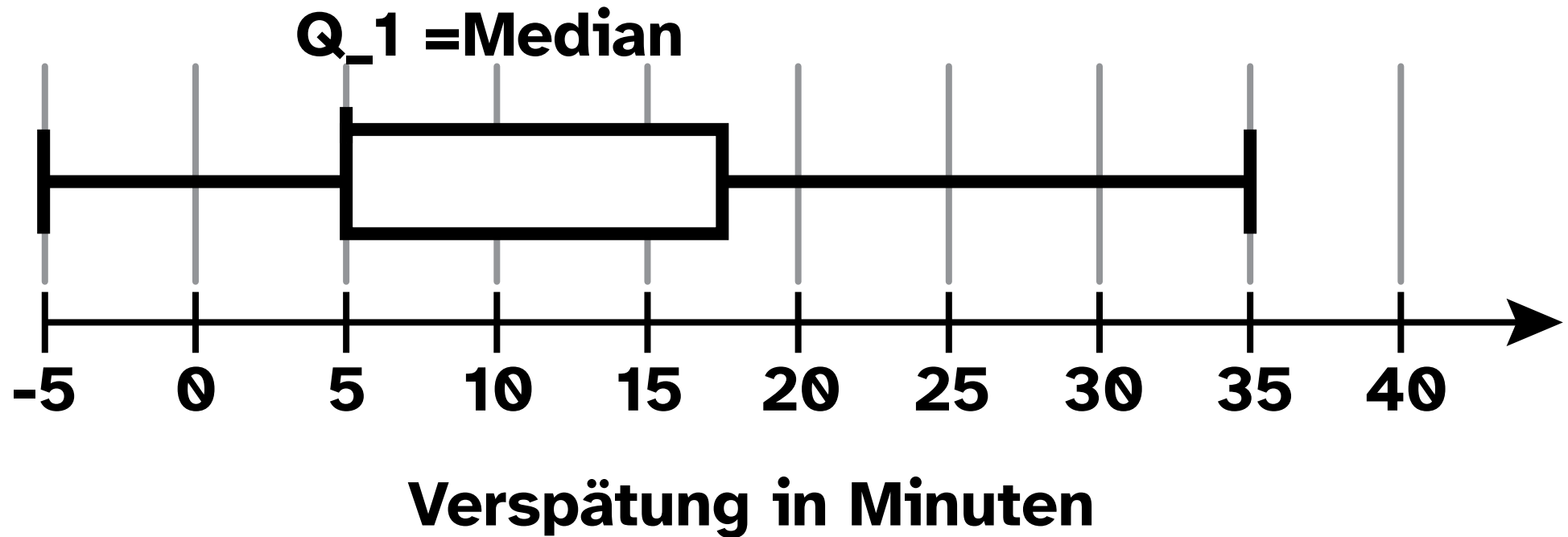


## Absolute Häufigkeit



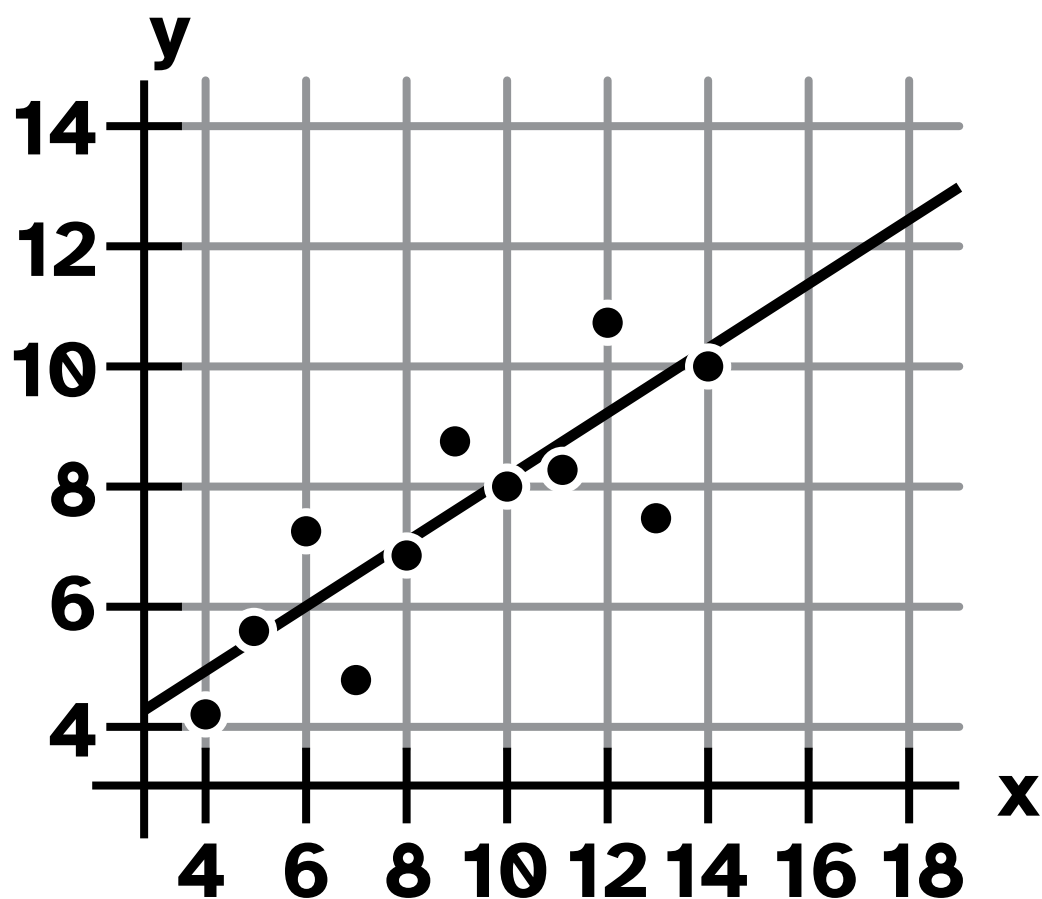


| Versp. in min  | -5 | 0 | 5  | 10 | 15 | 20 | 25 | 30 | 35 |
|----------------|----|---|----|----|----|----|----|----|----|
| Anzahl d. Züge | 1  | 3 | 10 | 3  | 2  | 2  | 0  | 1  | 3  |

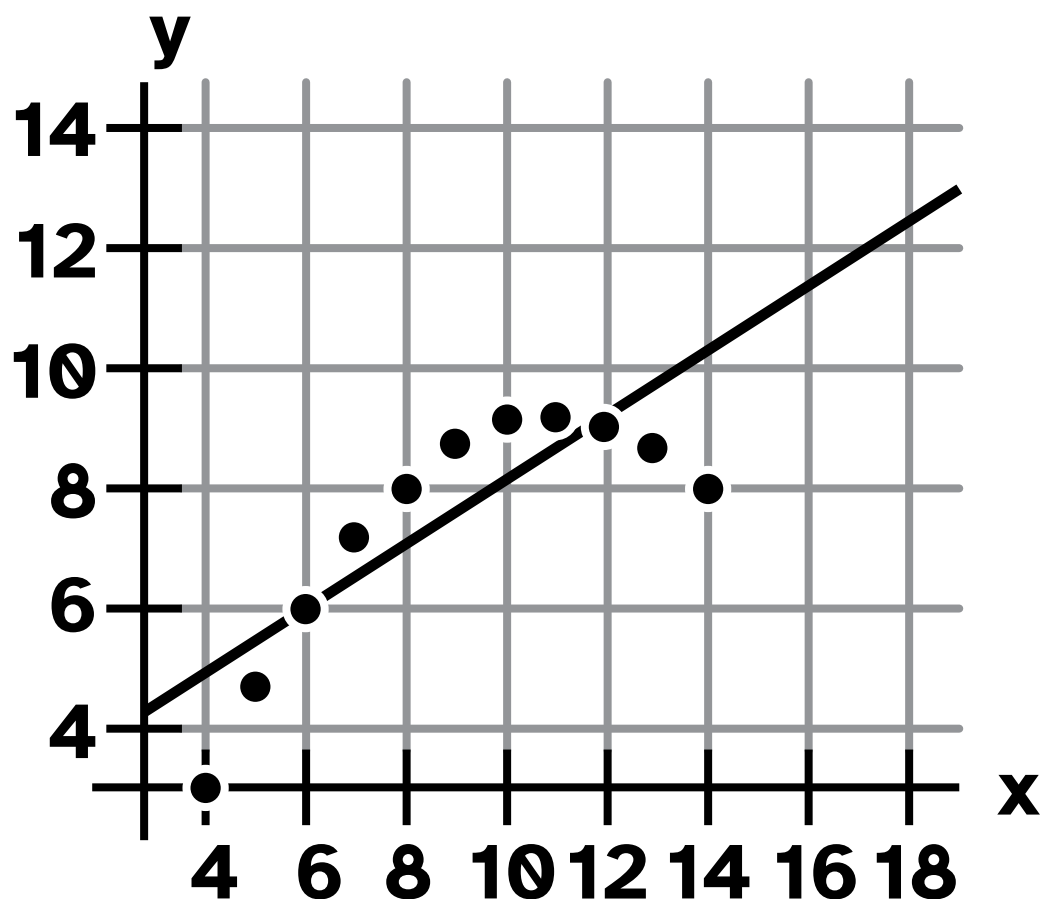




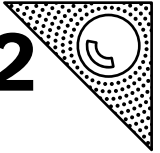
(1)



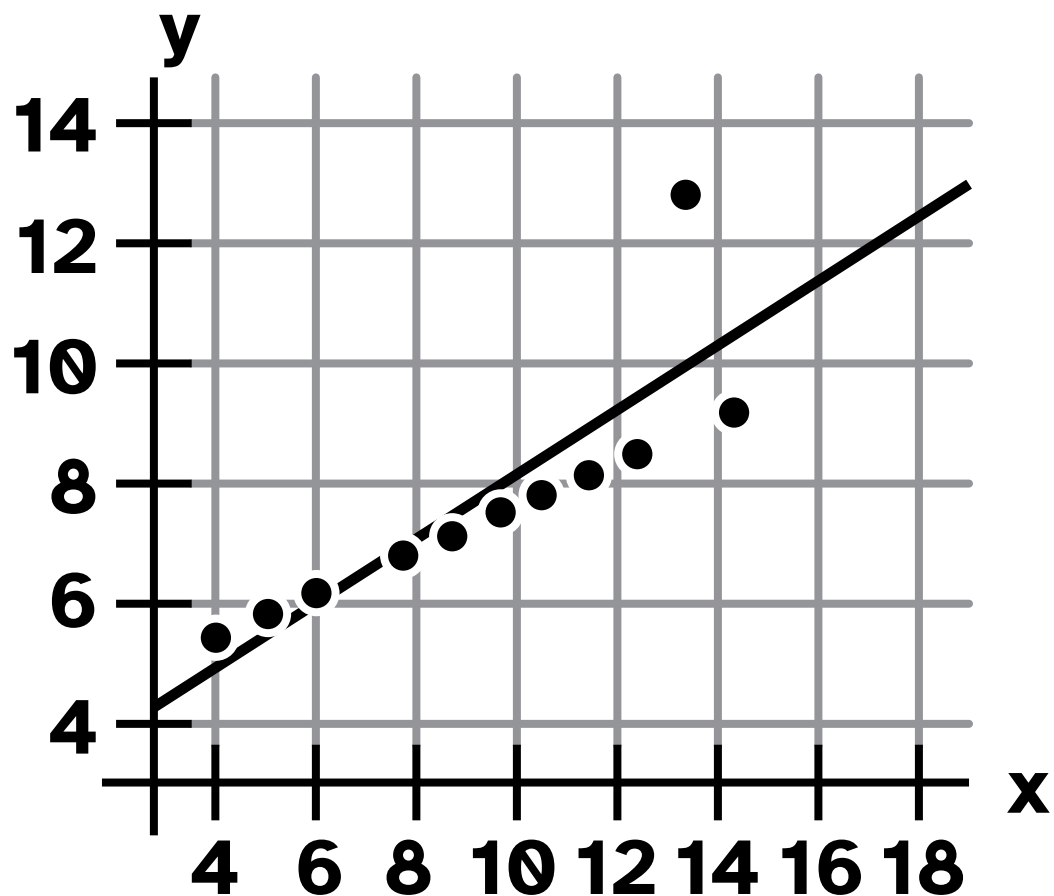
(2)



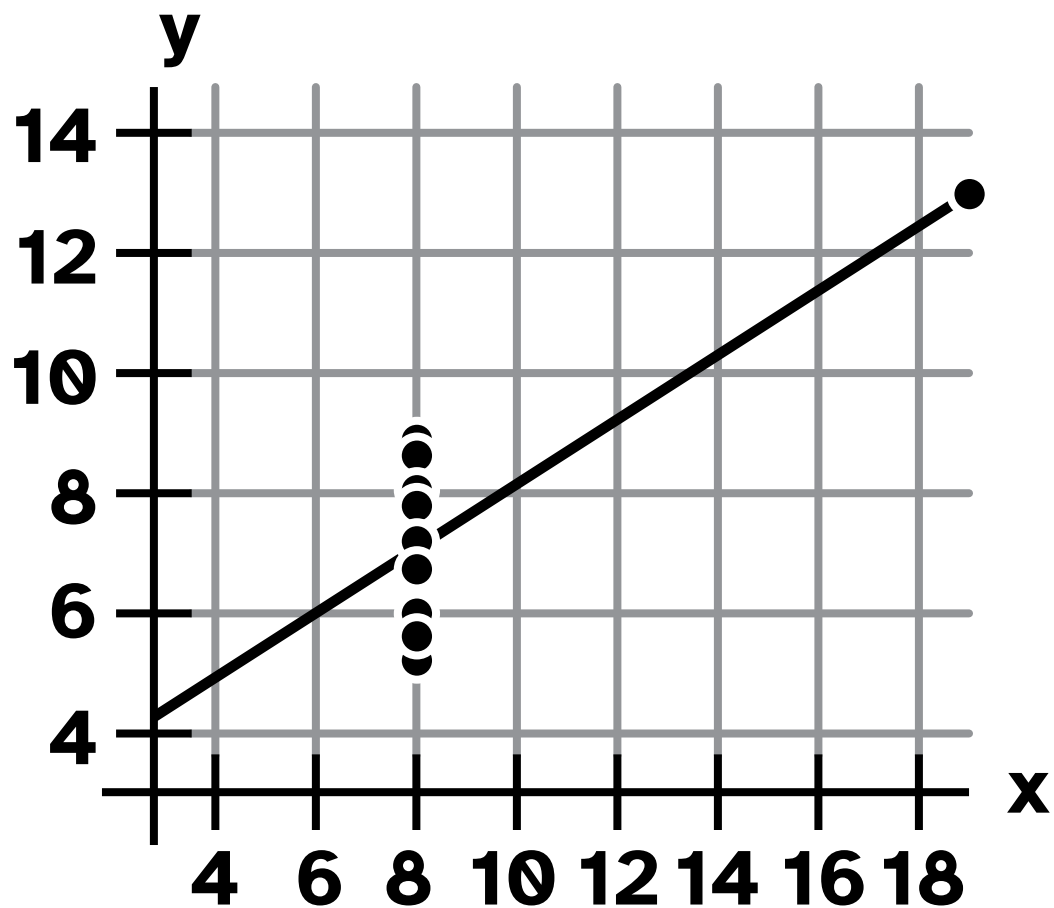




(3)



(4)



Ang.Mat. HAK4

S.149 6.78



--- P97

--- P90

--- P75

--- P50

--- P25

--- P10

--- P3

Masse (kg)

90

80

70

60

50

40

30

20

10

0

0

2

4

6

8

10

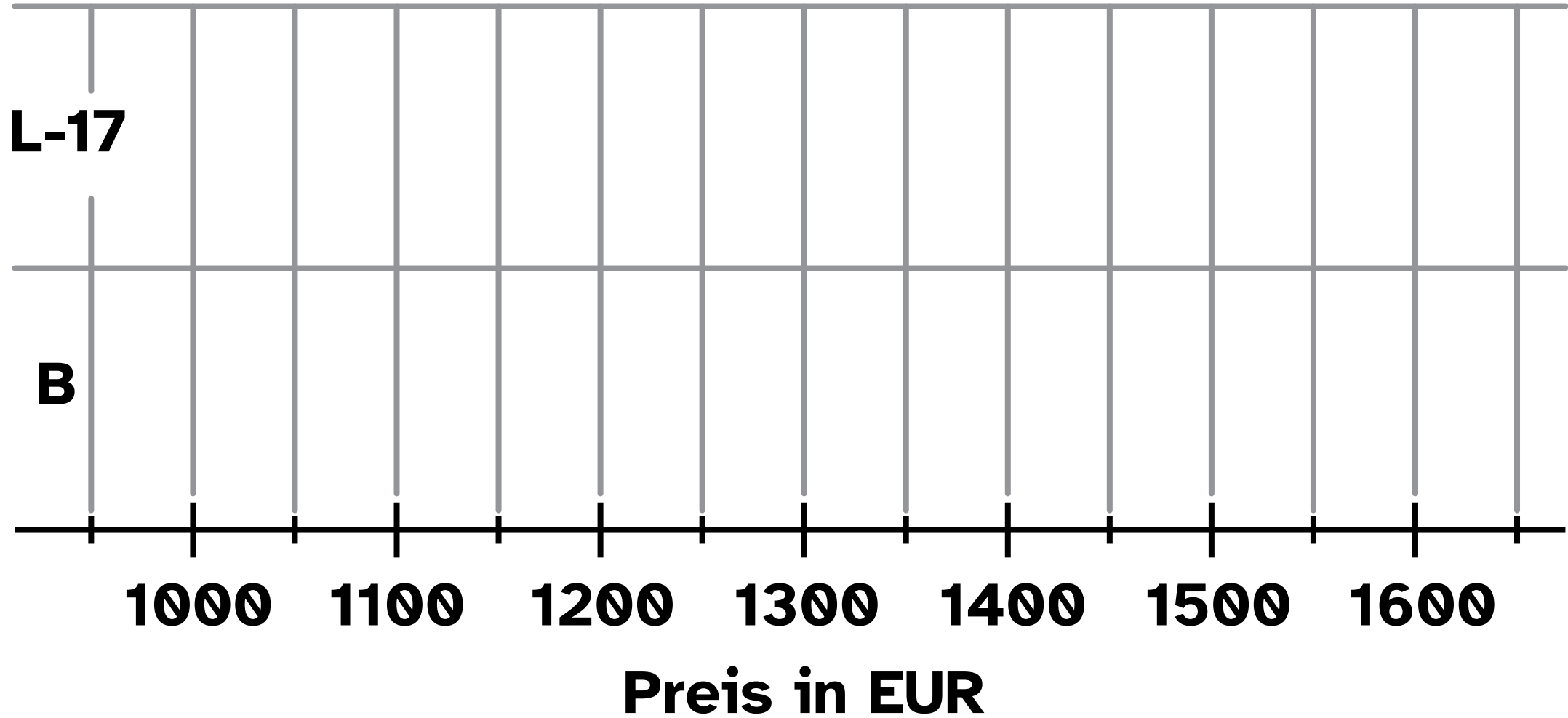
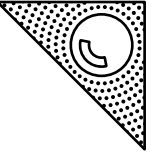
12

14

16

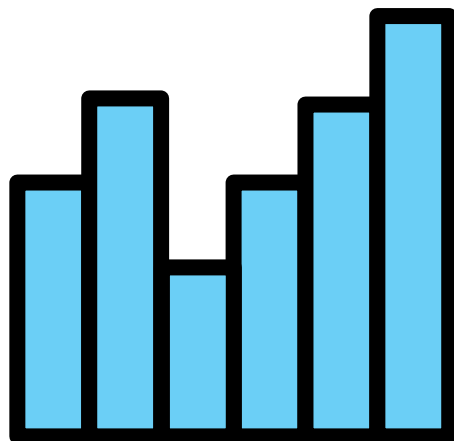
18

Alter (Jahre)

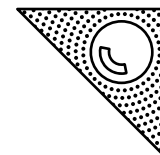


**Ang.Mat. HAK4**

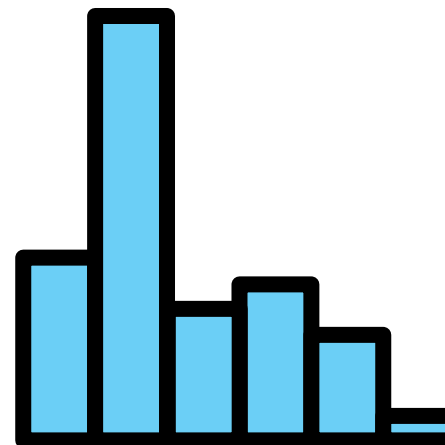
**1**



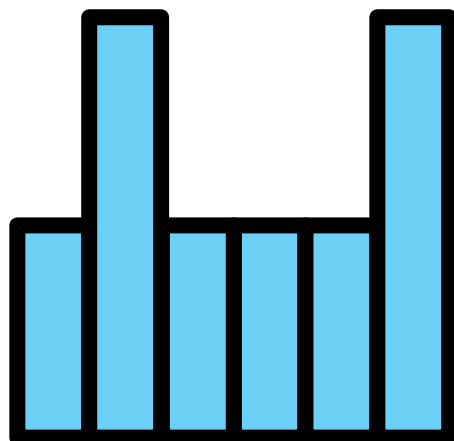
**S.150 6.80 b)**



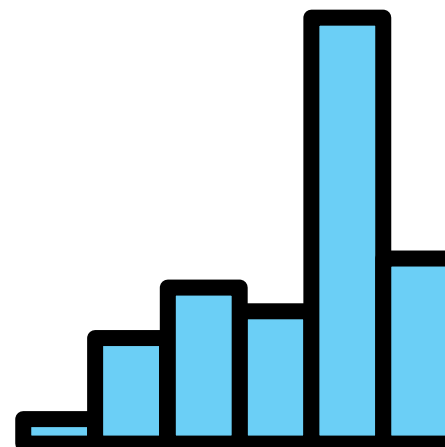
**2**

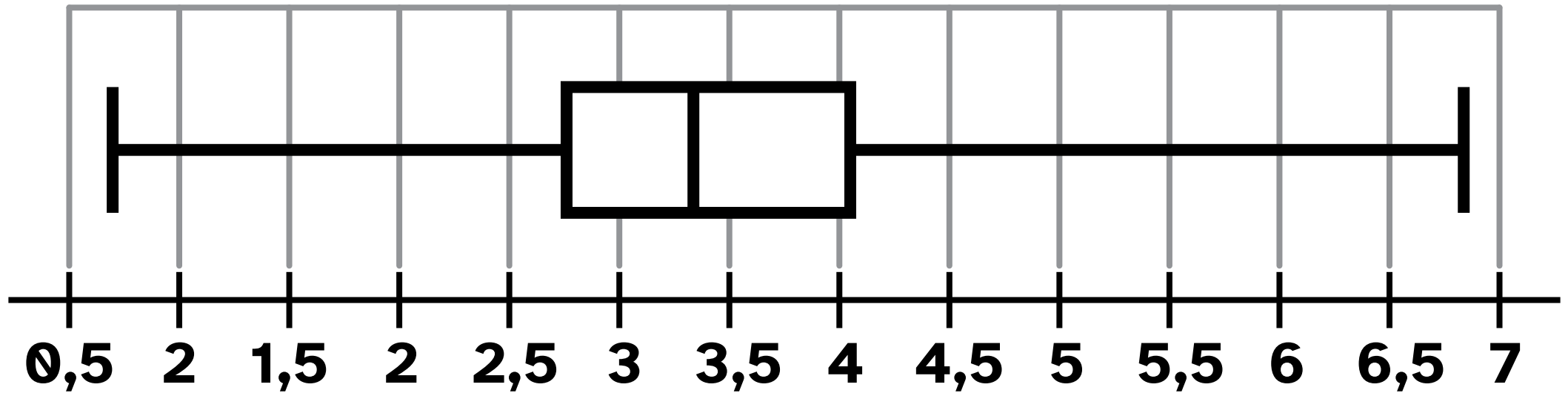
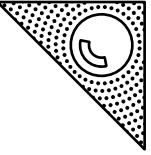


**3**

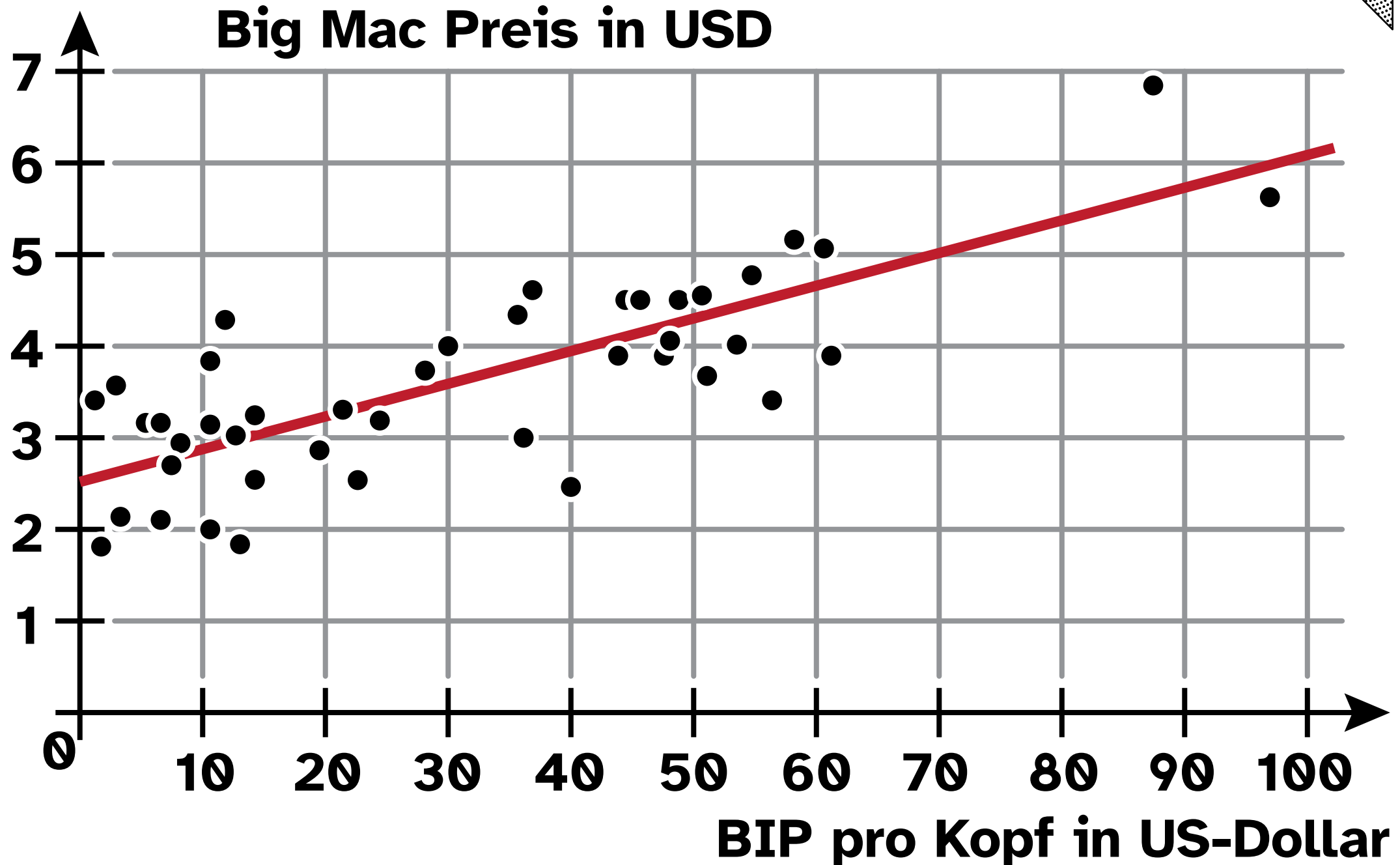
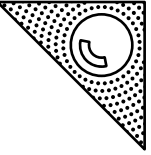


**4**





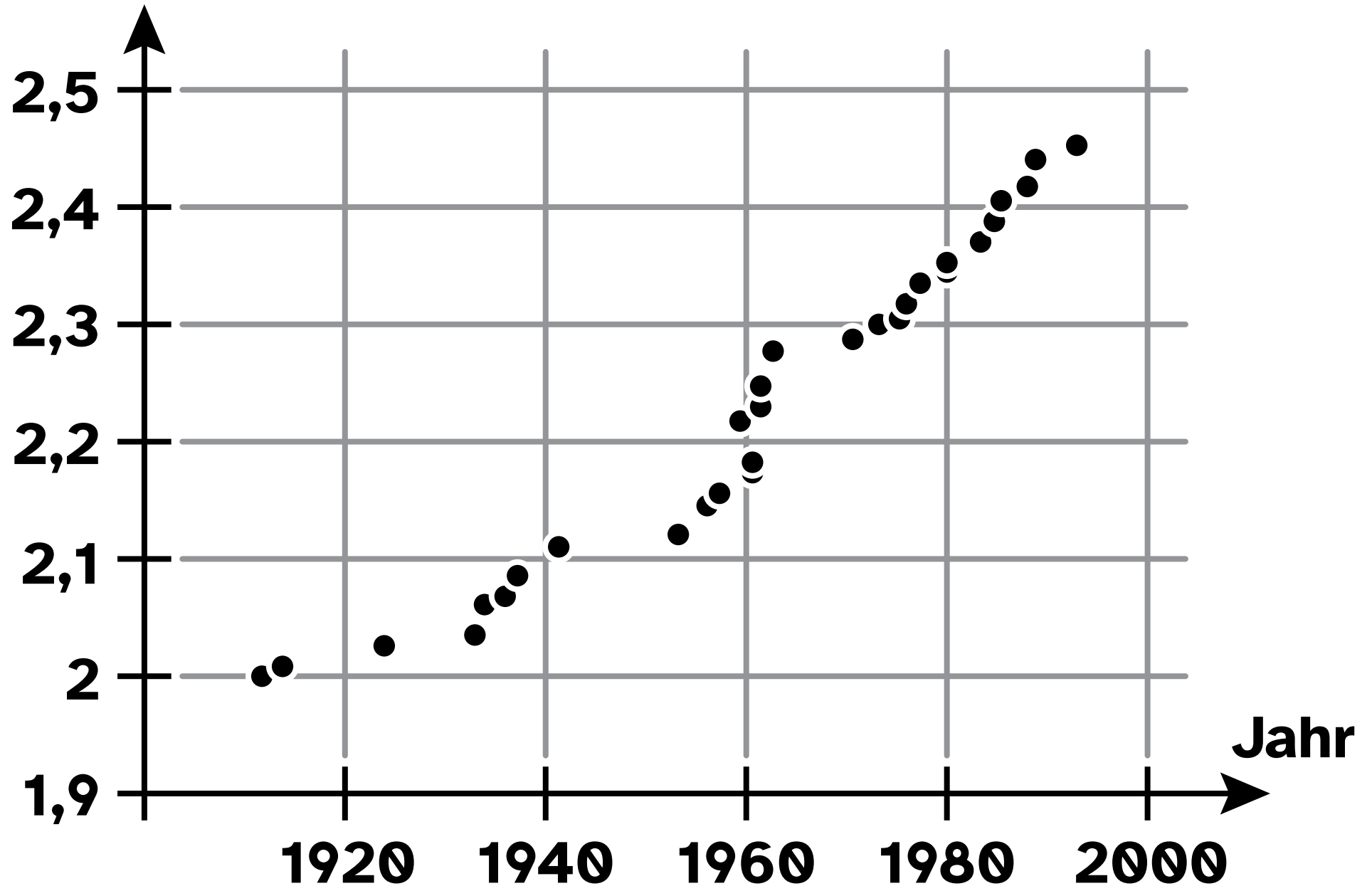
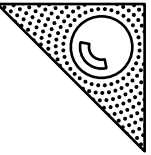
Preis eines Big Mac in US-Dollar



# Ang.Mat. HAK4

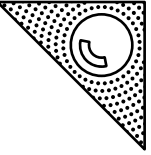
## Höhe in m

S.152 6.83

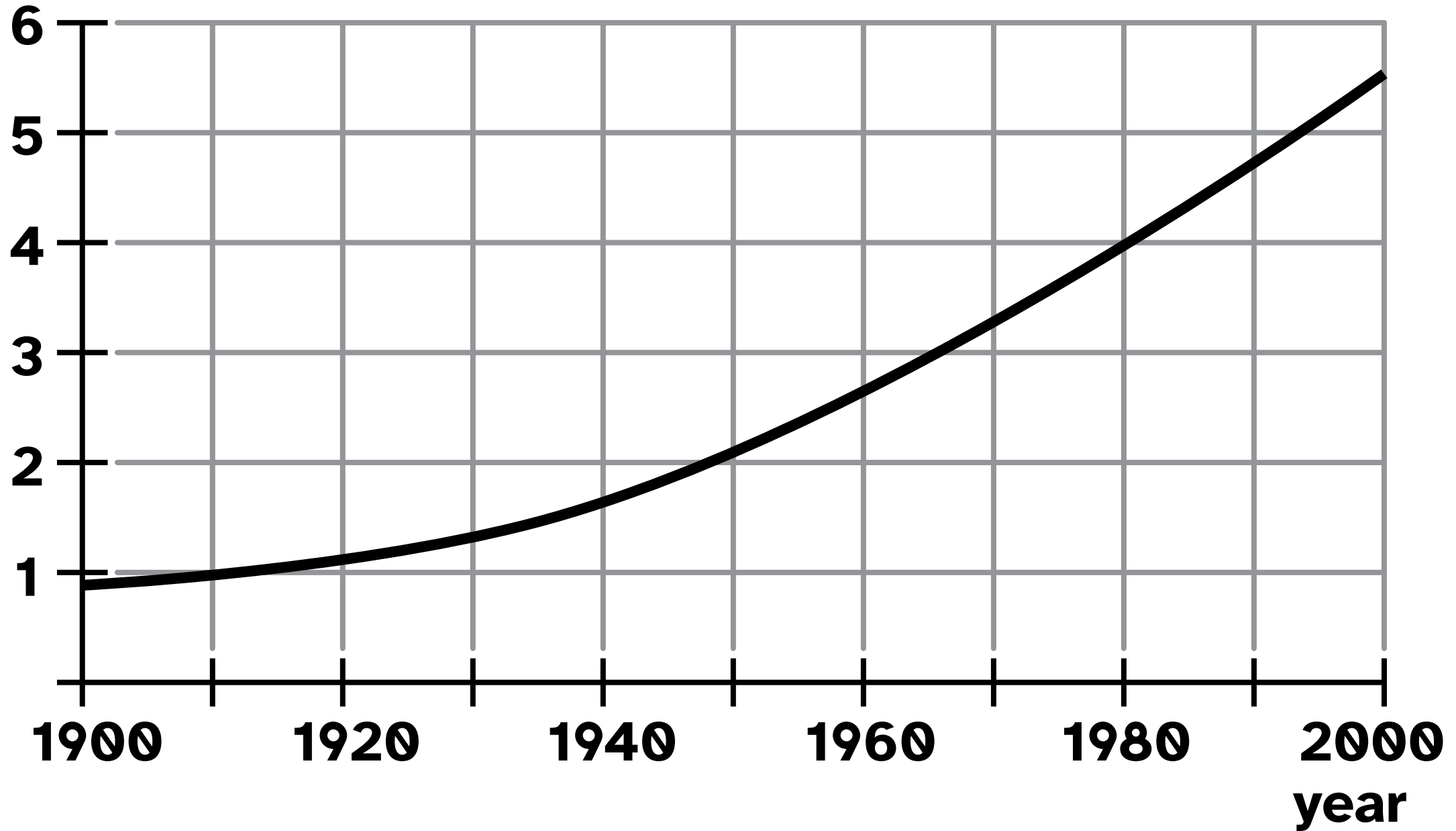


**Ang.Mat. HAK4**

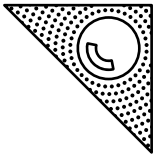
**S.152 6.84**



**population in thousands**







a)



h)

Abbildung 1

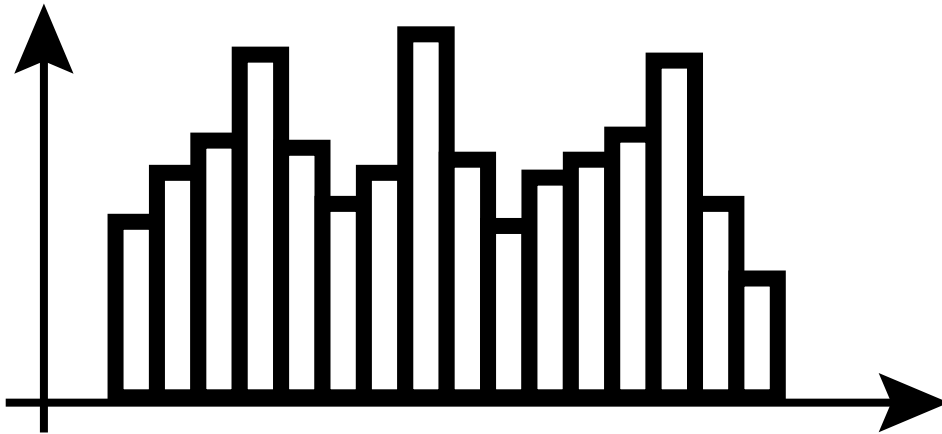
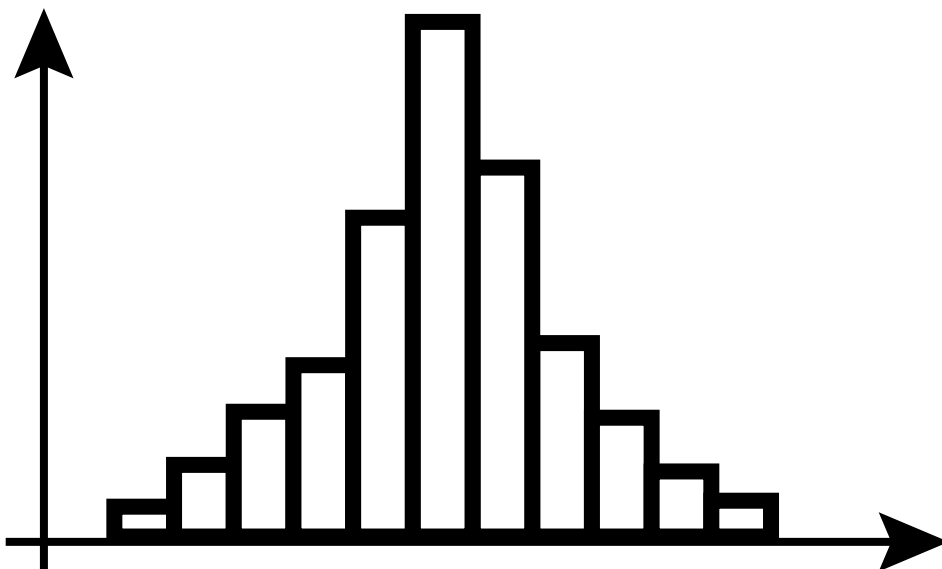
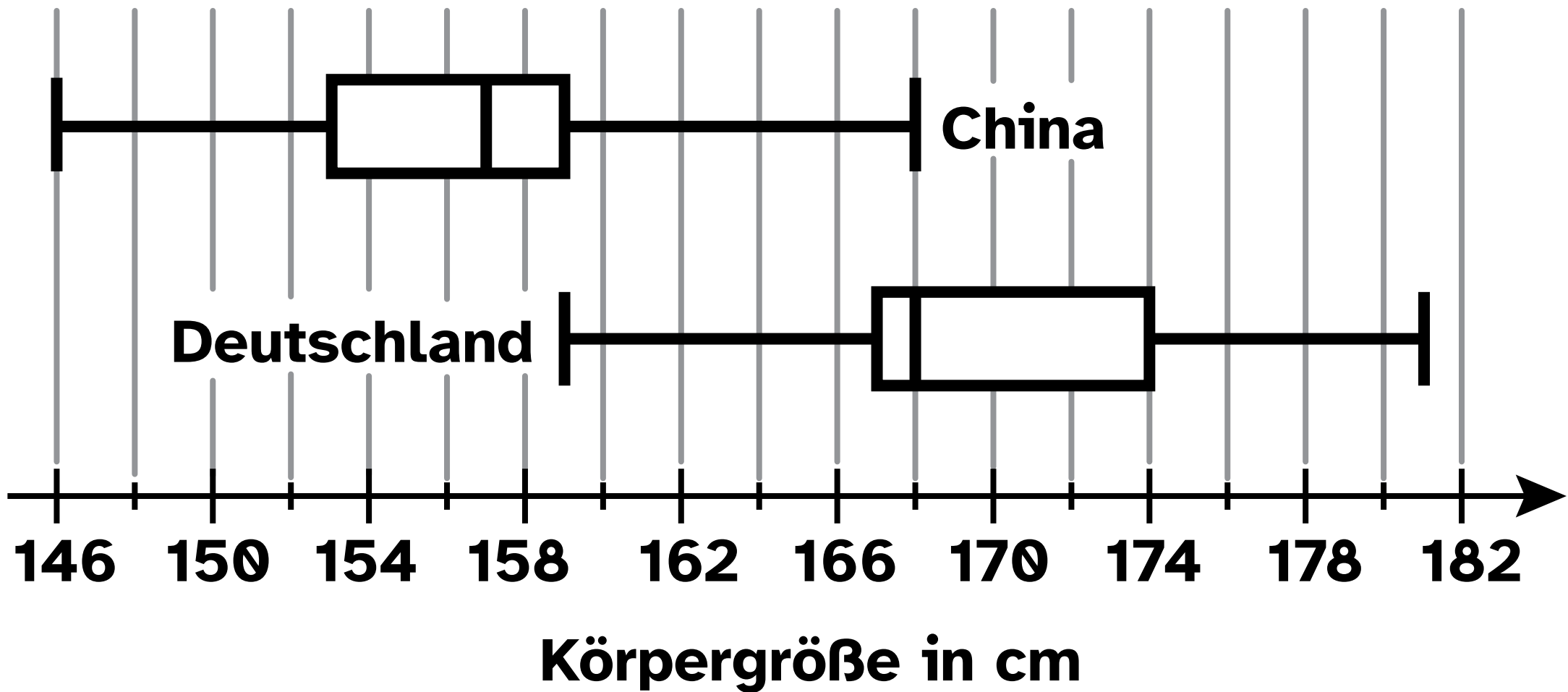
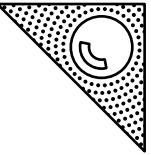
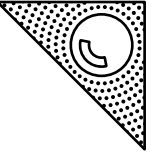


Abbildung 2







**Gewinn i 1000 EUR**

