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LeMa
Lehr- und Lernmaterialien

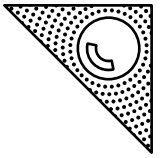
Vergrößerte und aufbereitete Versionen
ausgewählter Grafiken aus dem Schulbuch

Angewandte Mathematik HAK 4



Grafiken: Tomáš Batha

LEMA - Lehr- und Lernmaterialien
Hergestellt von der Abteilung für Inklusion und Lehrmittel am Bundes-Blindeninstitut Wien,
Wittelsbachstraße 5, 1020 Wien, Tel.: 01/728 08 66-405, 406, lmz@bbi.at, www.bbi.at

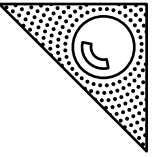


Angewandte Mathematik

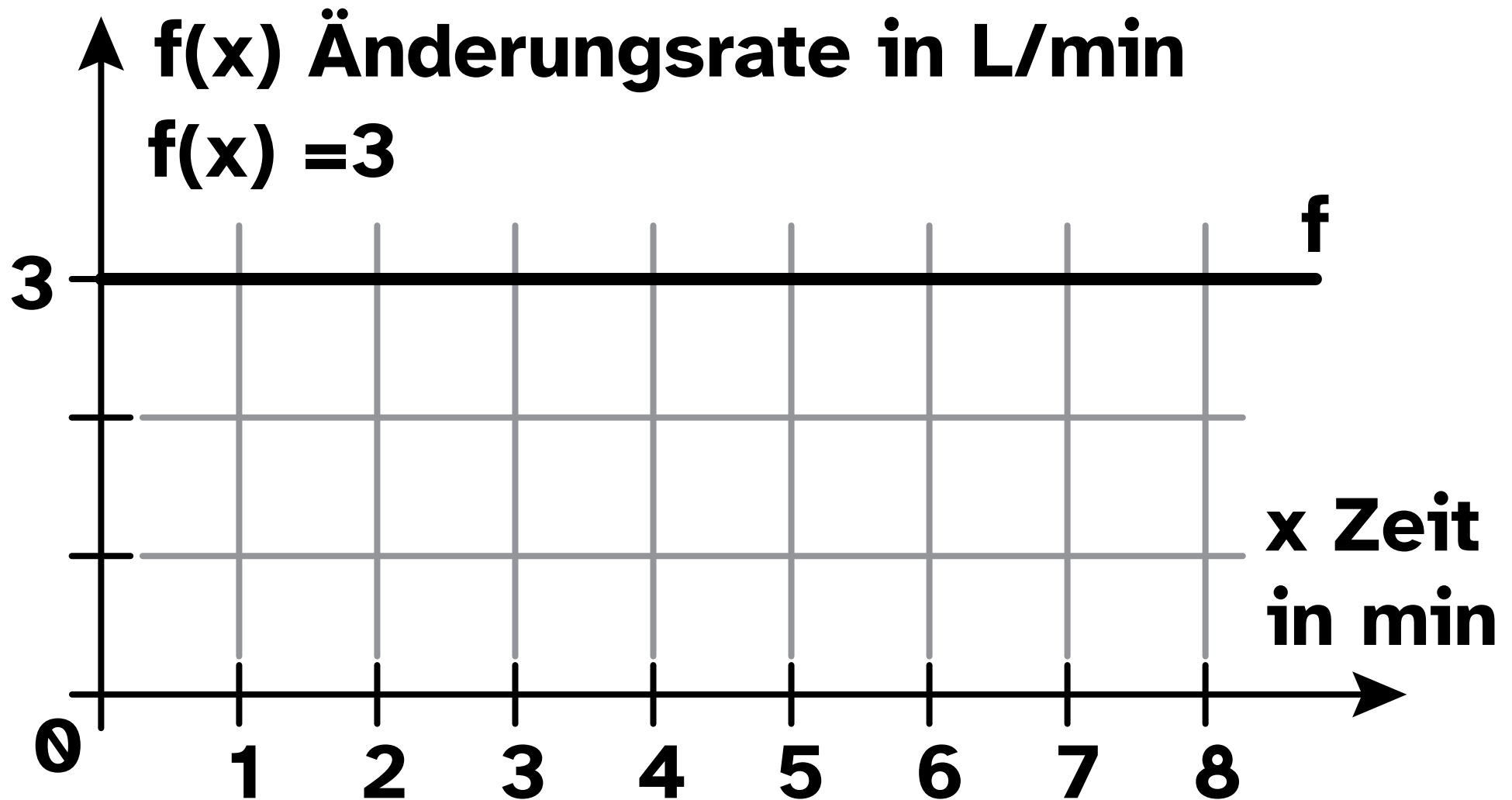
HAK Band 4

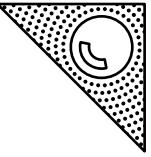
5.

Integralrechnung

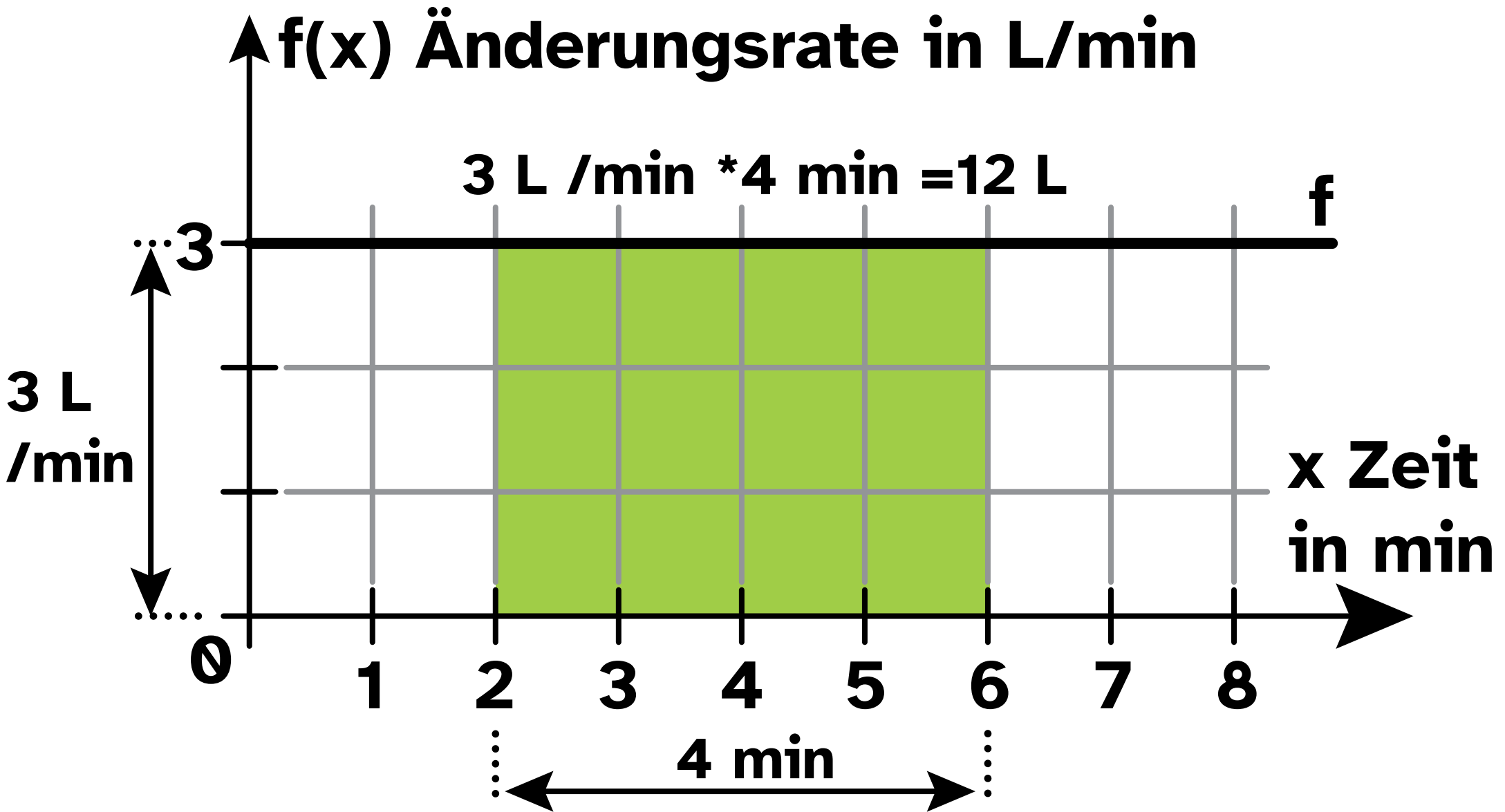


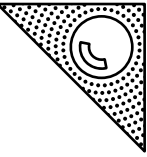
Beispiel mit konstanter Zuflussrate:



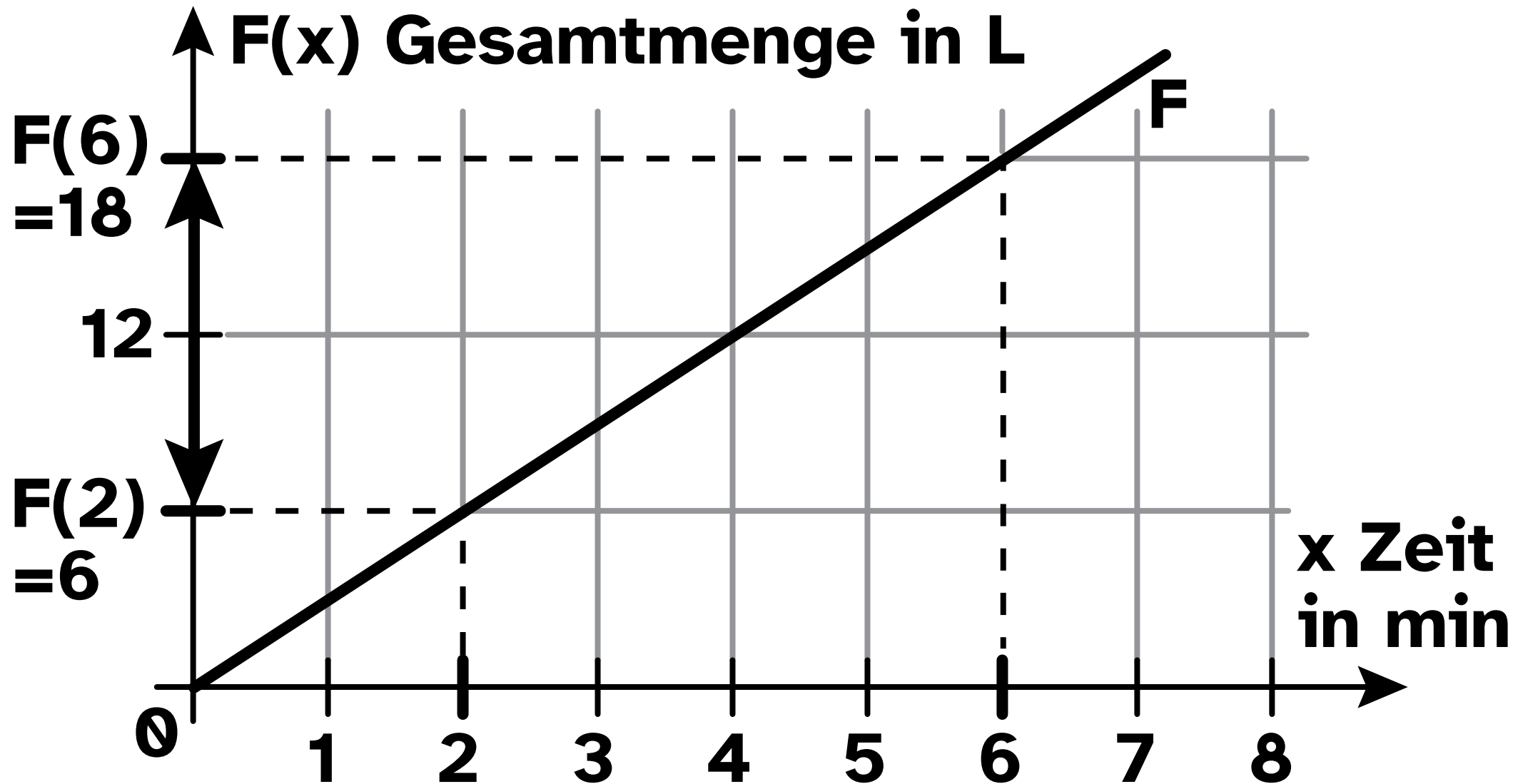


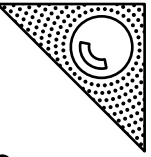
Zuflussmenge als Flächeninhalt:



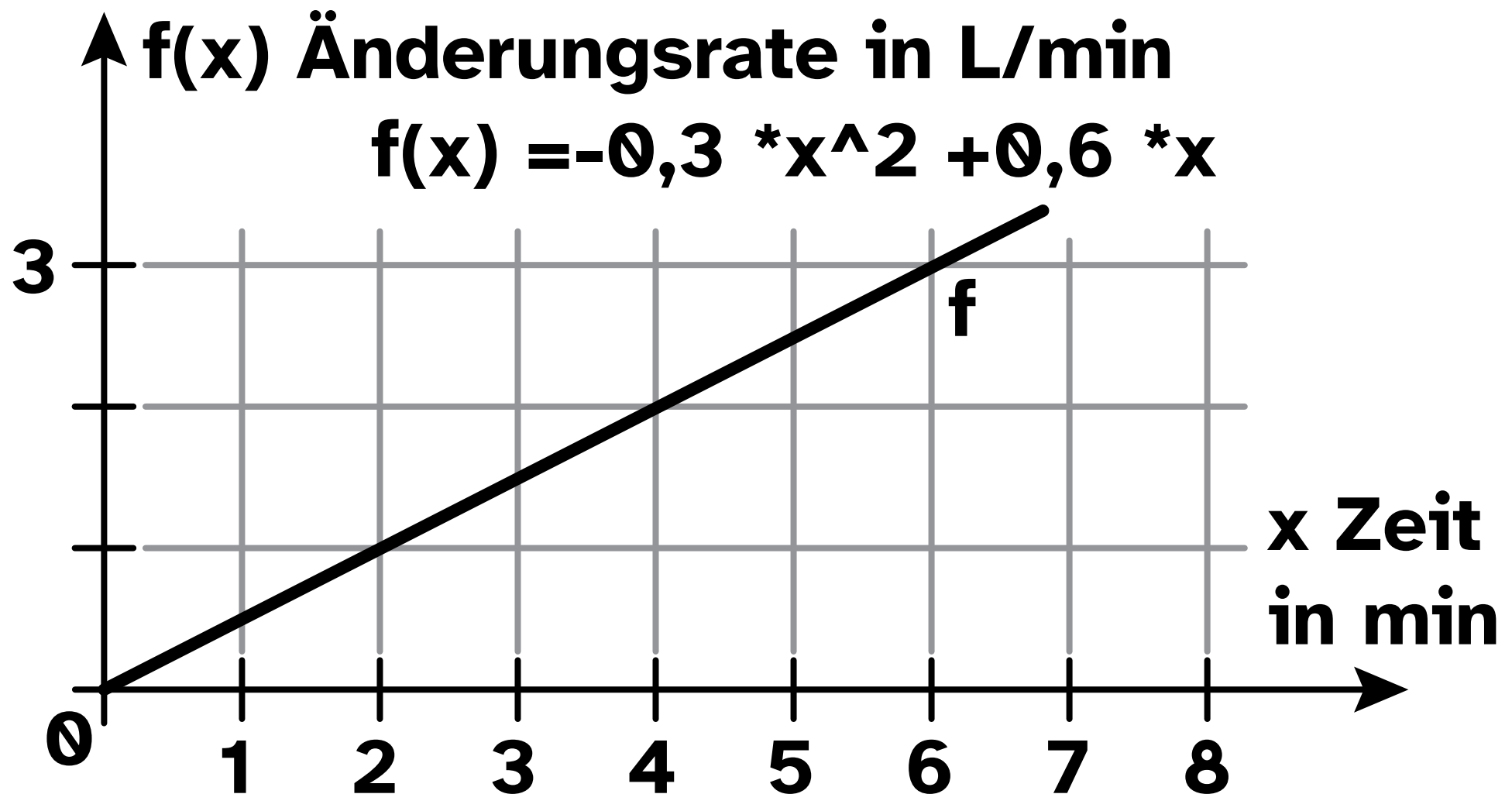


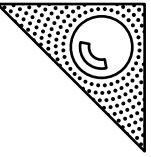
Zuflussmenge mithilfe der Bestandsfunktion



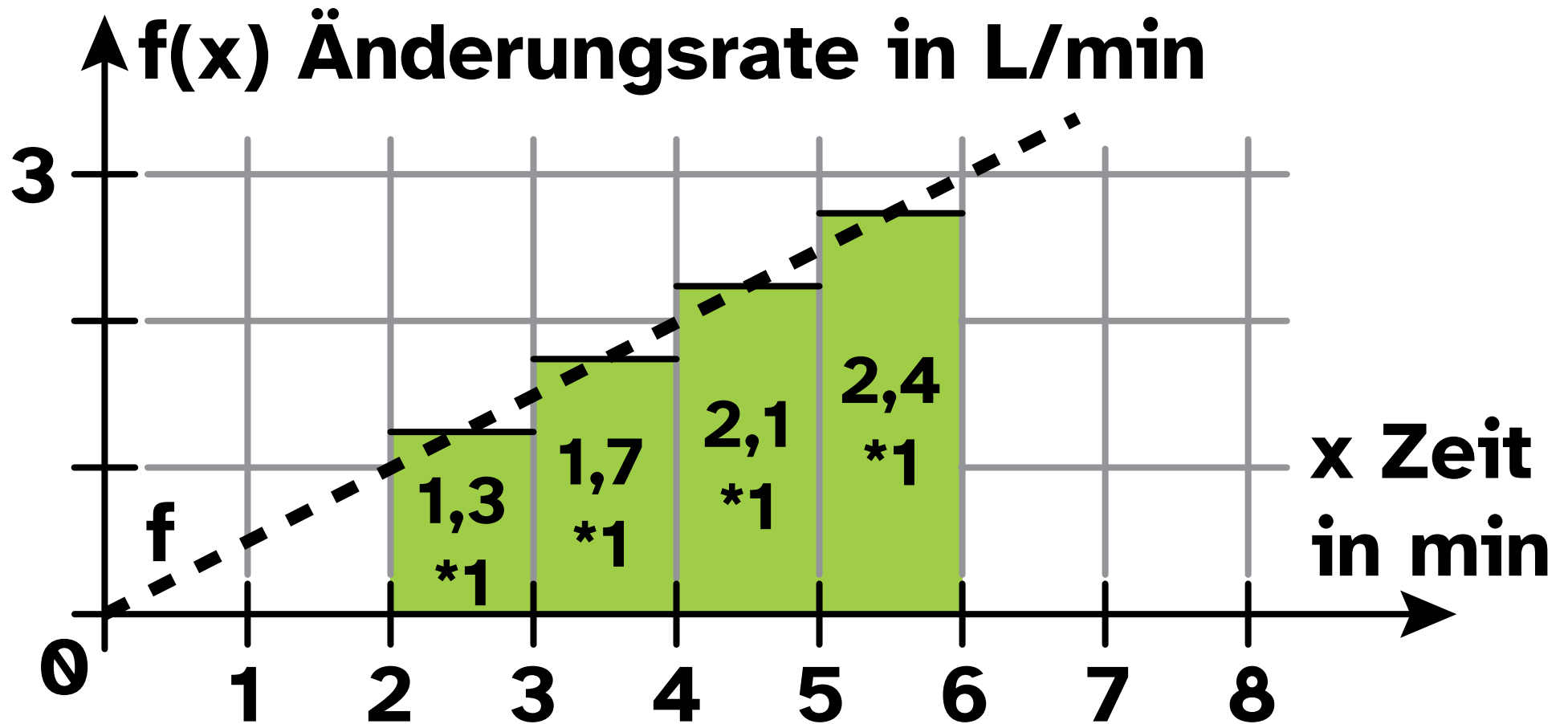


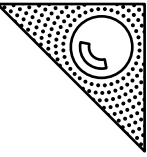
Beispiel mit nicht konstanter Zuflussrate:



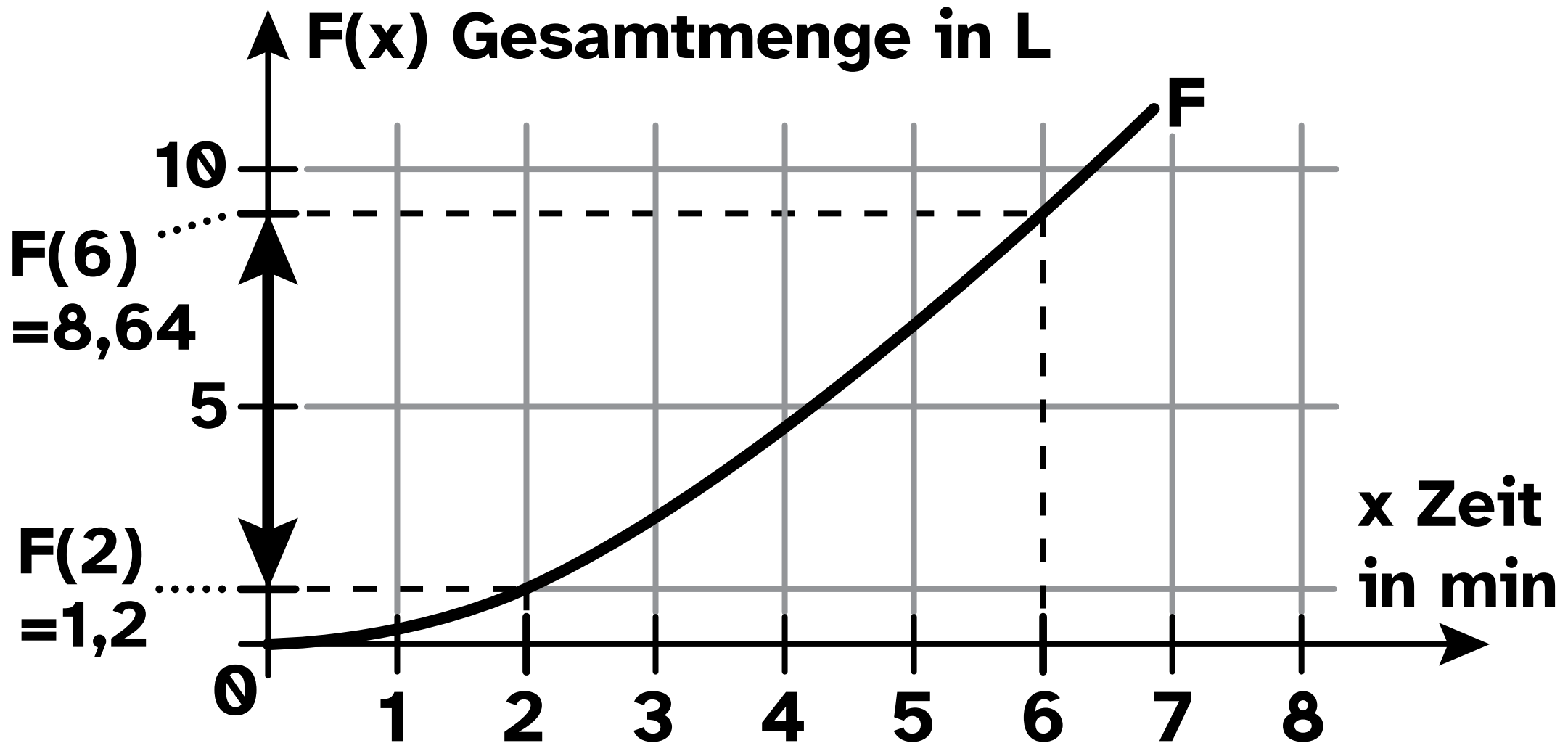


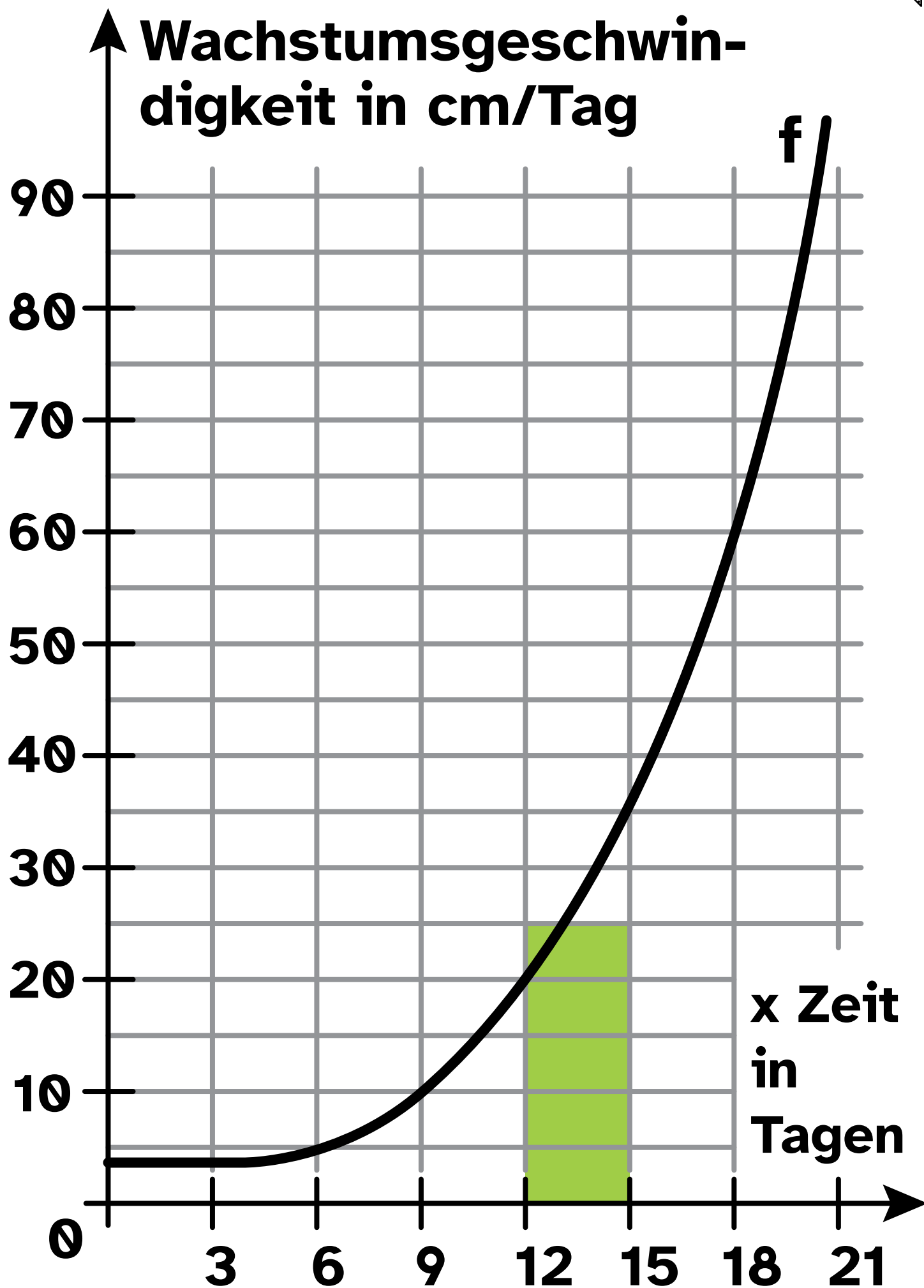
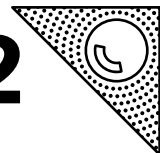
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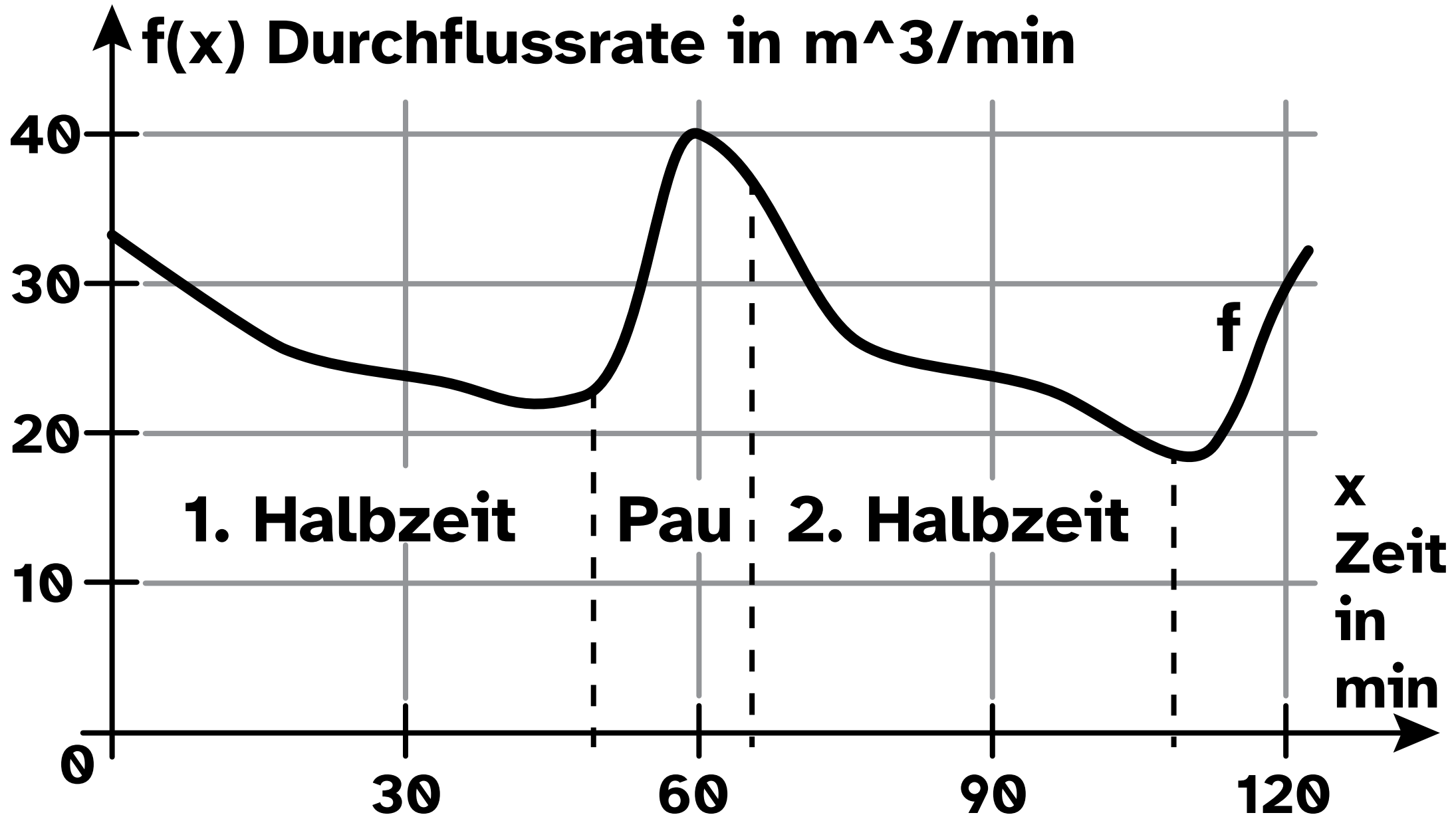
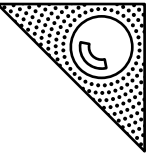


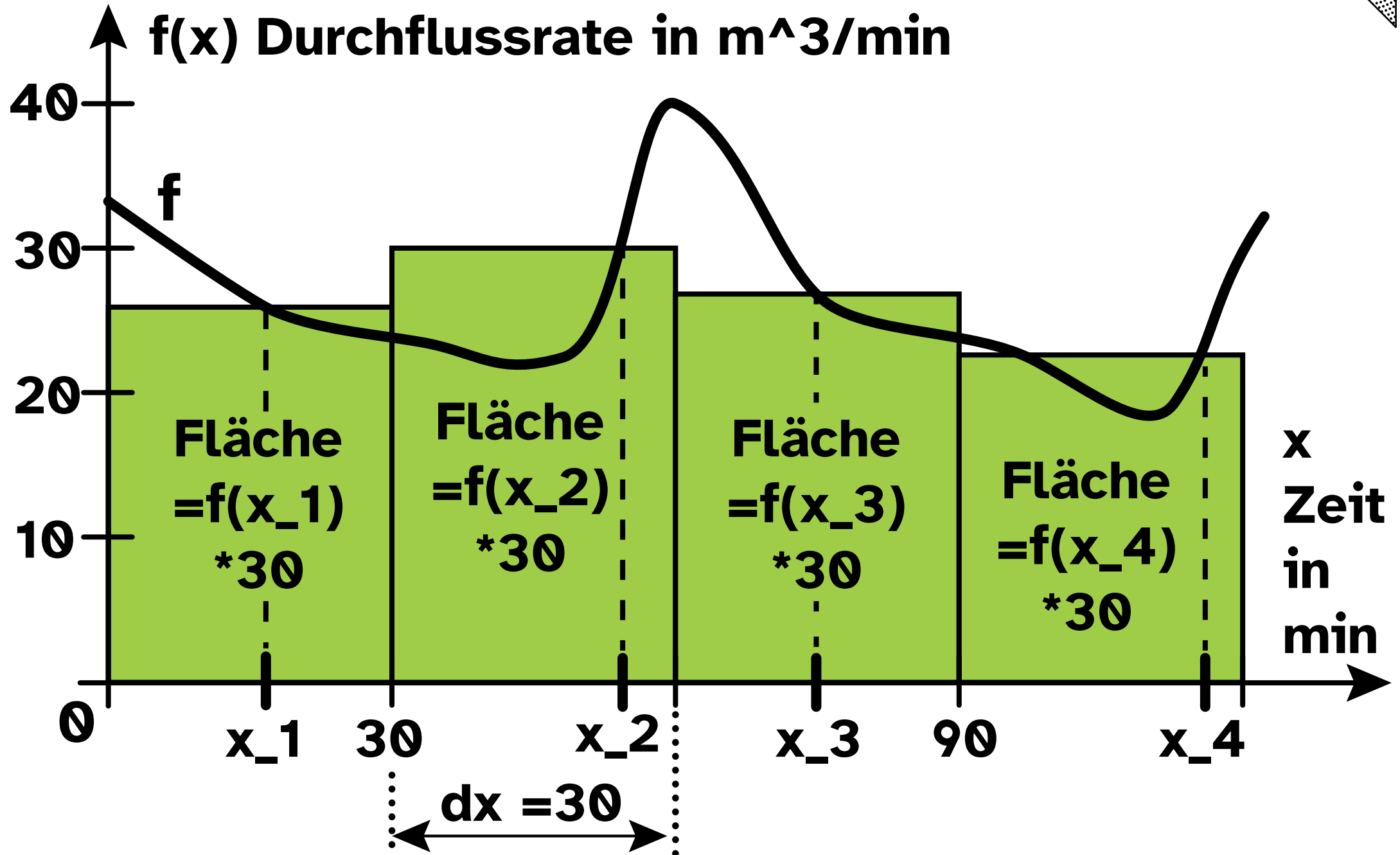
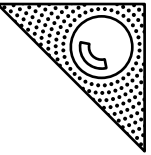


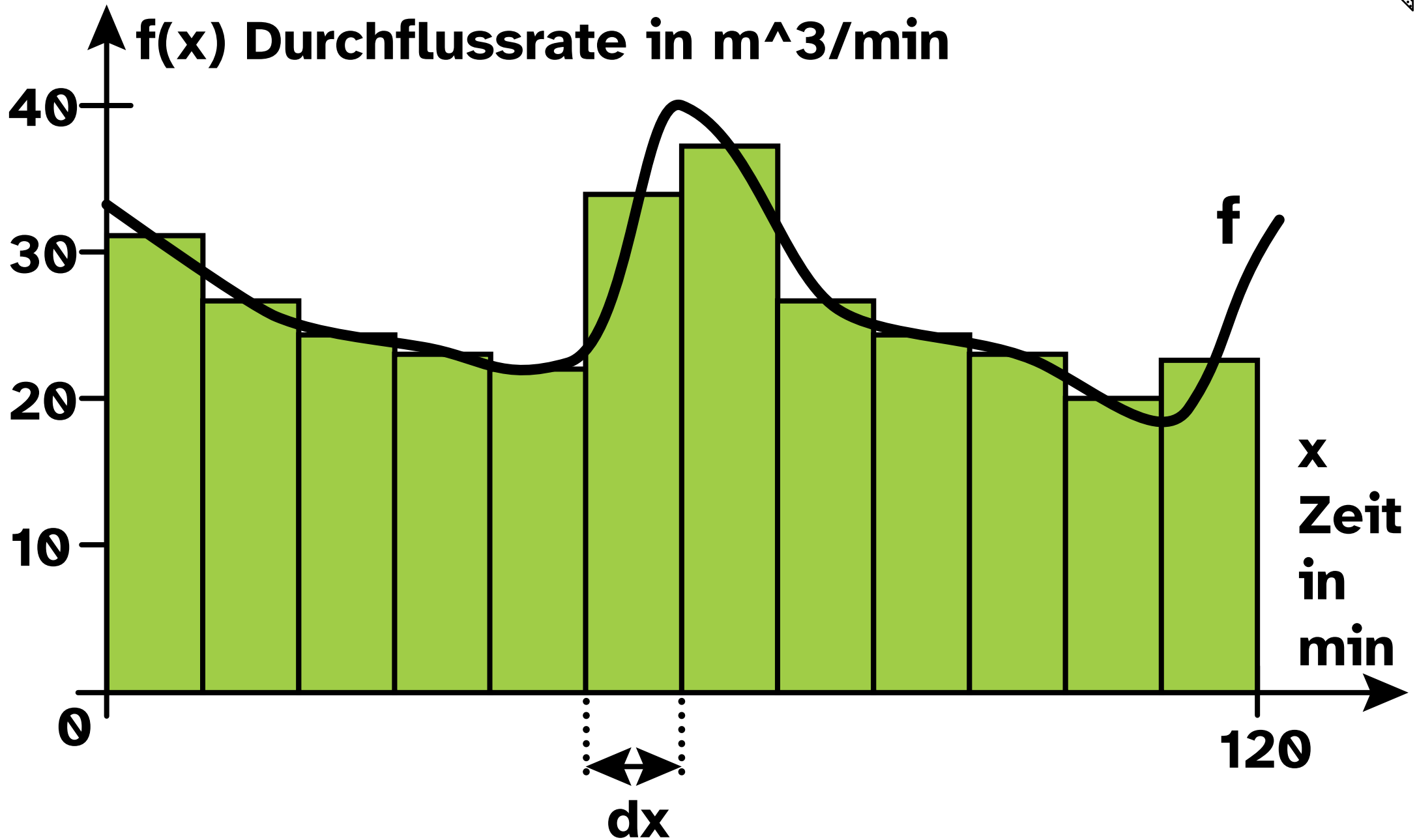
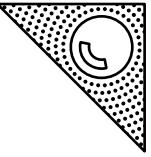
Zuflussmenge mithilfe der Bestandsfunktion

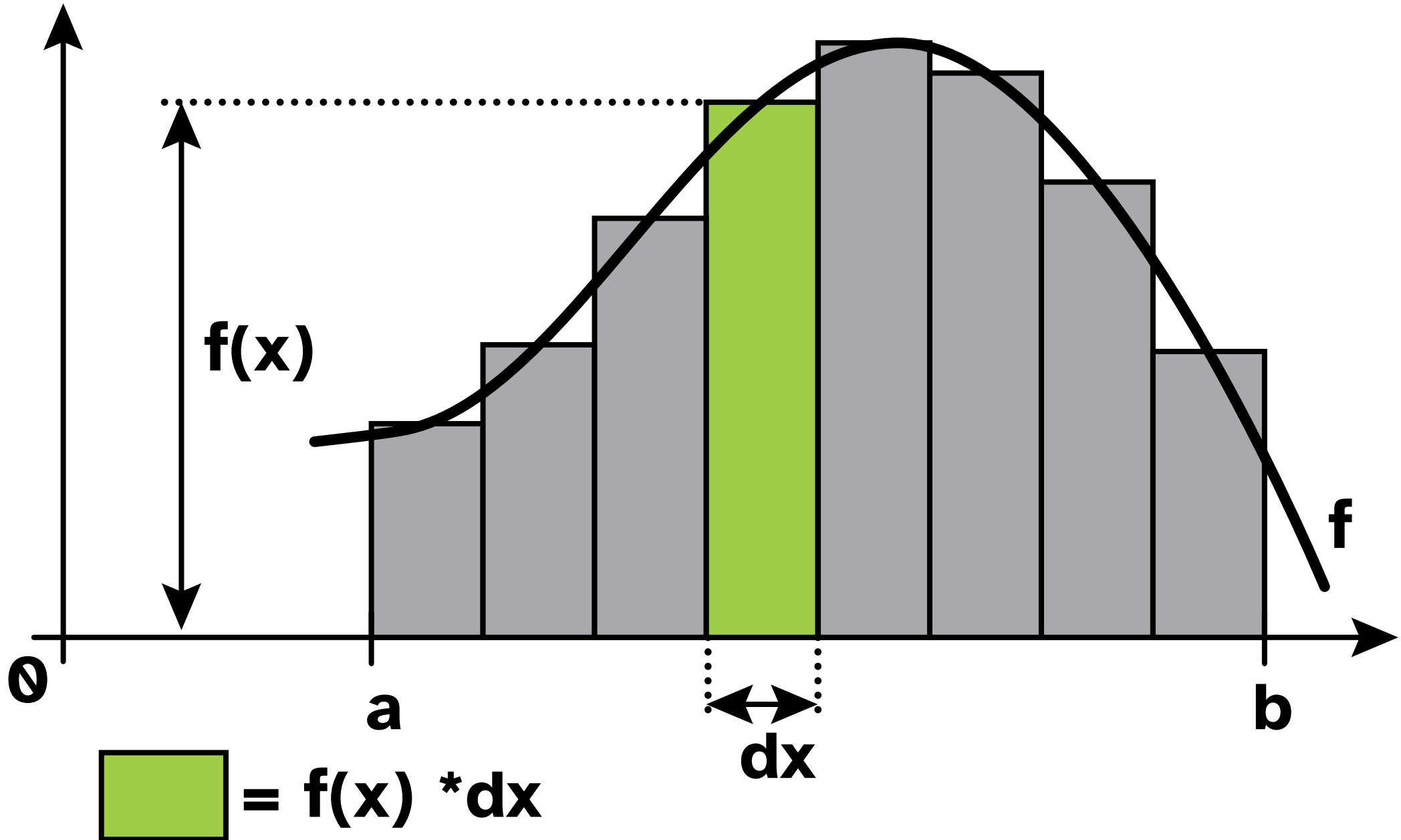
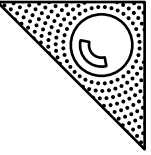


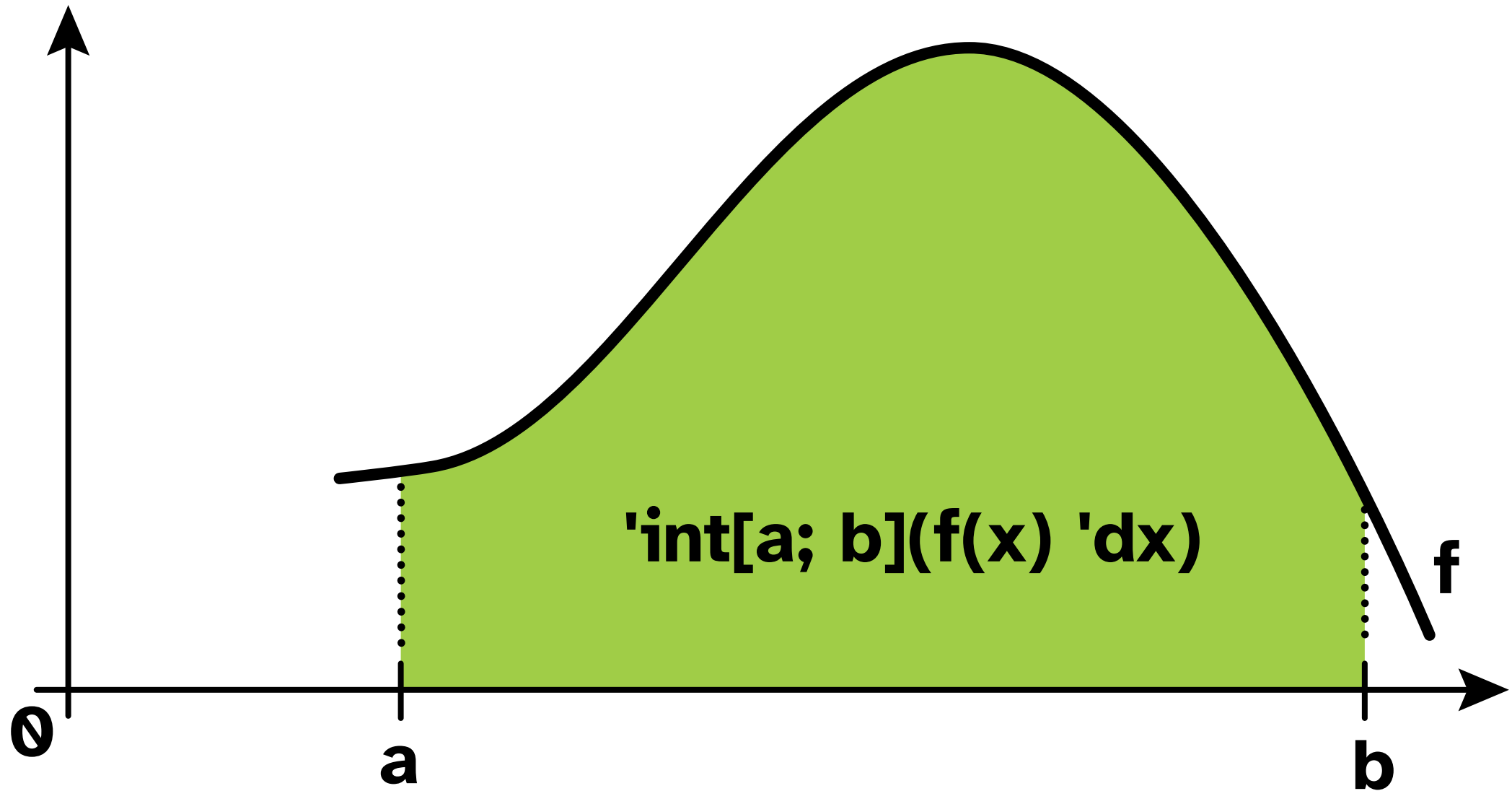
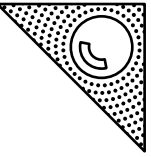


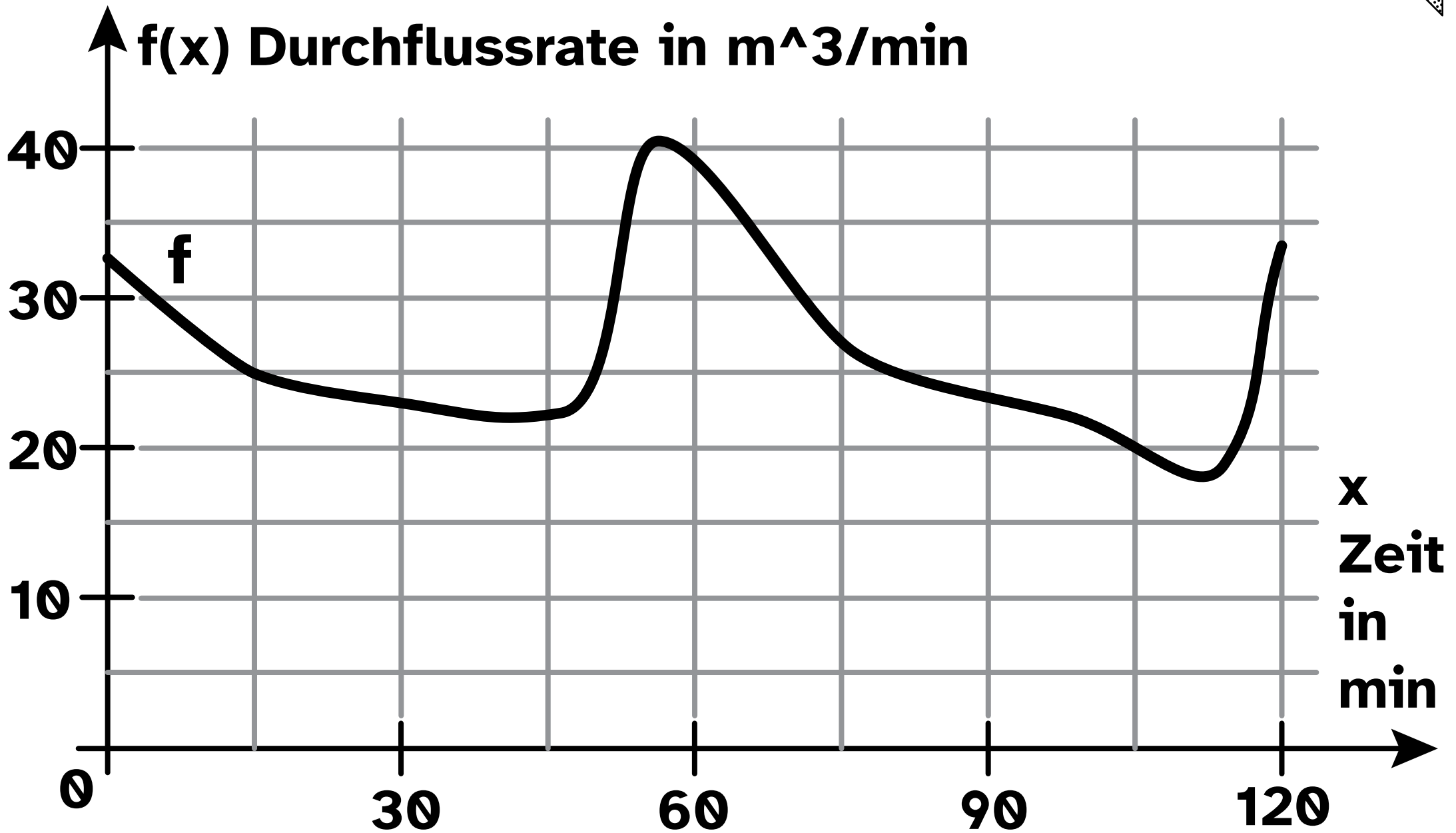
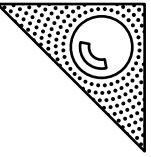


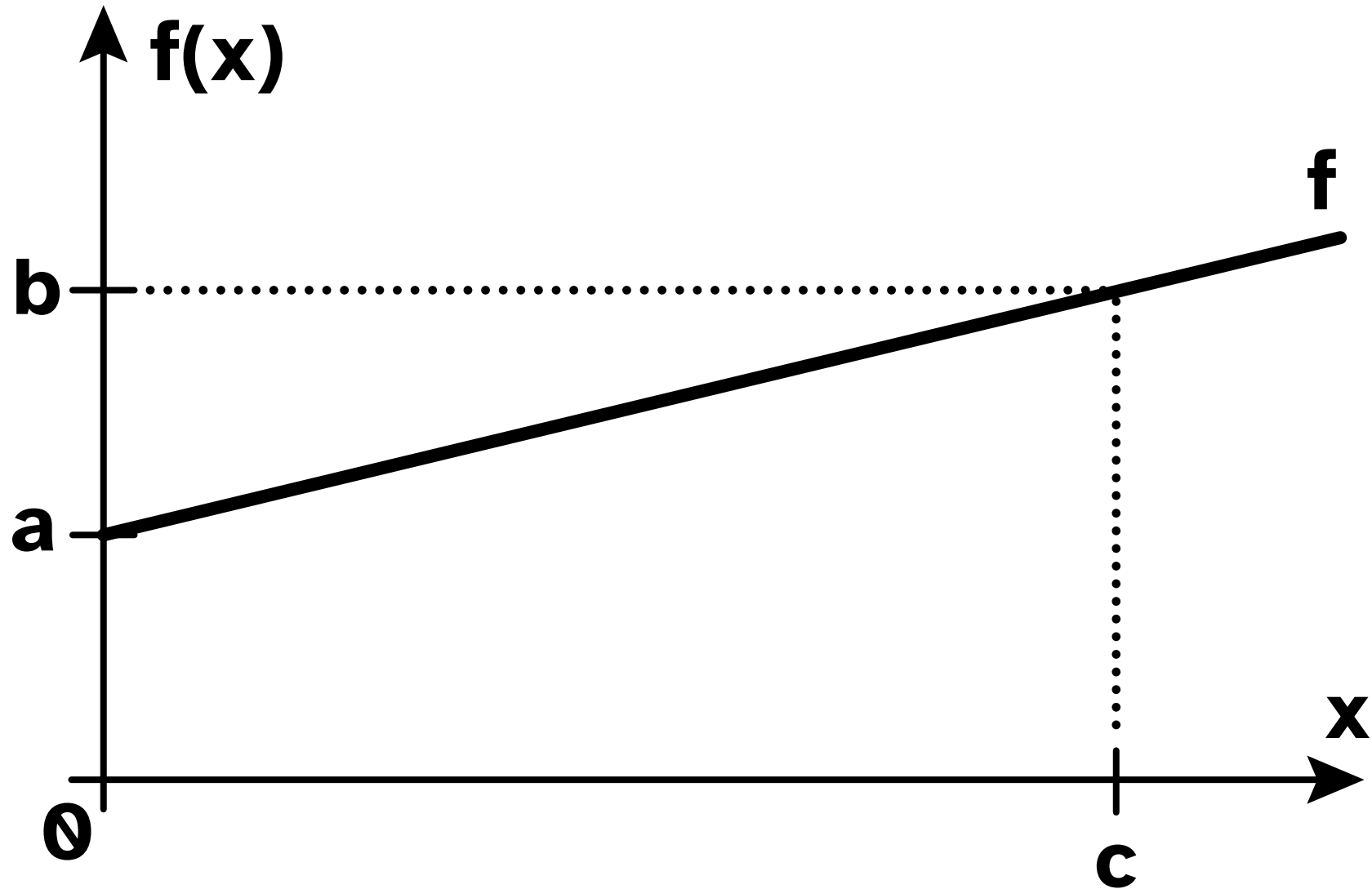
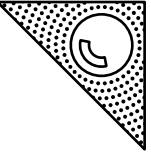


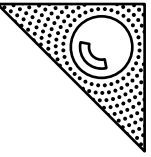




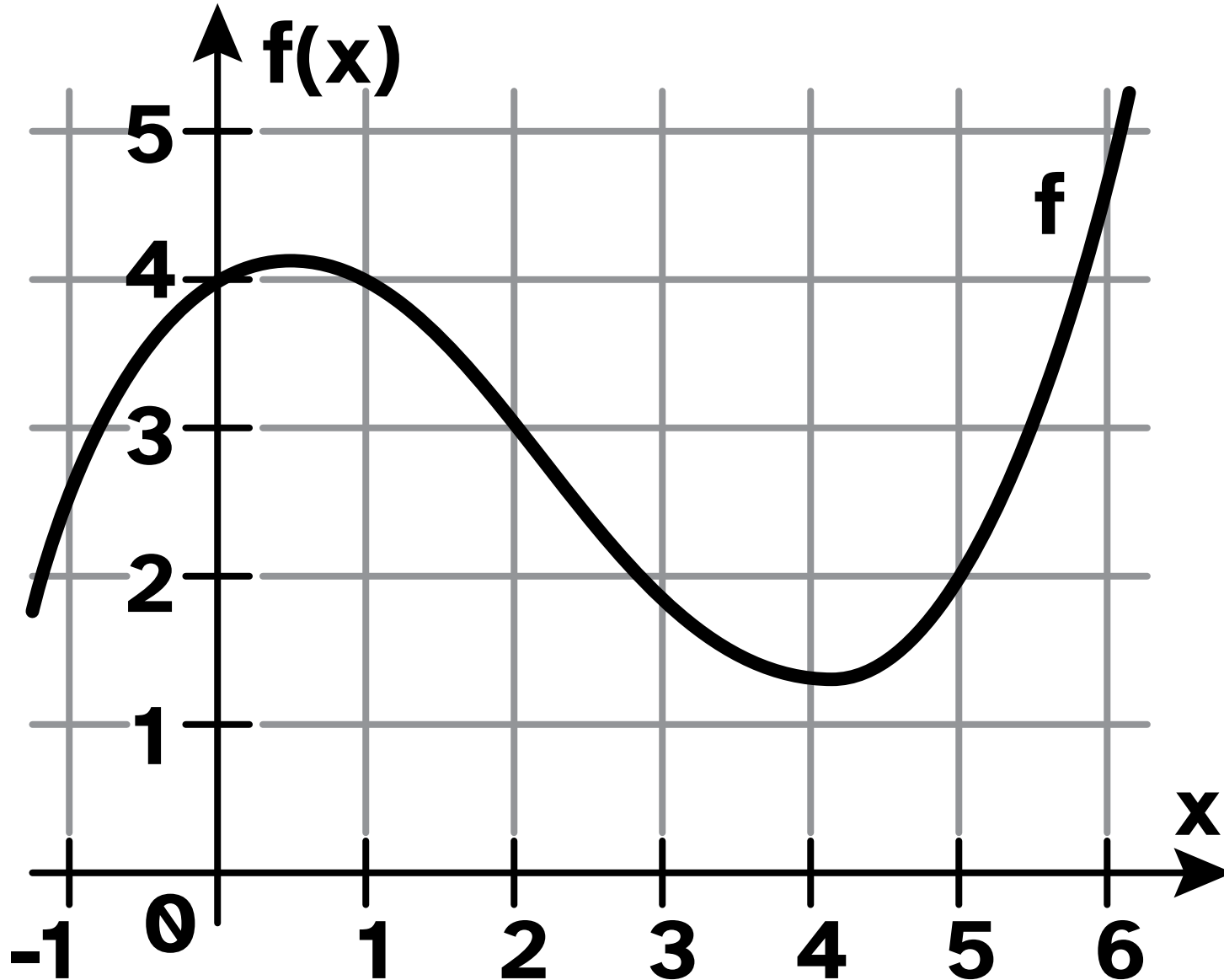


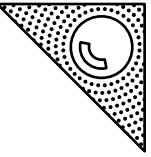




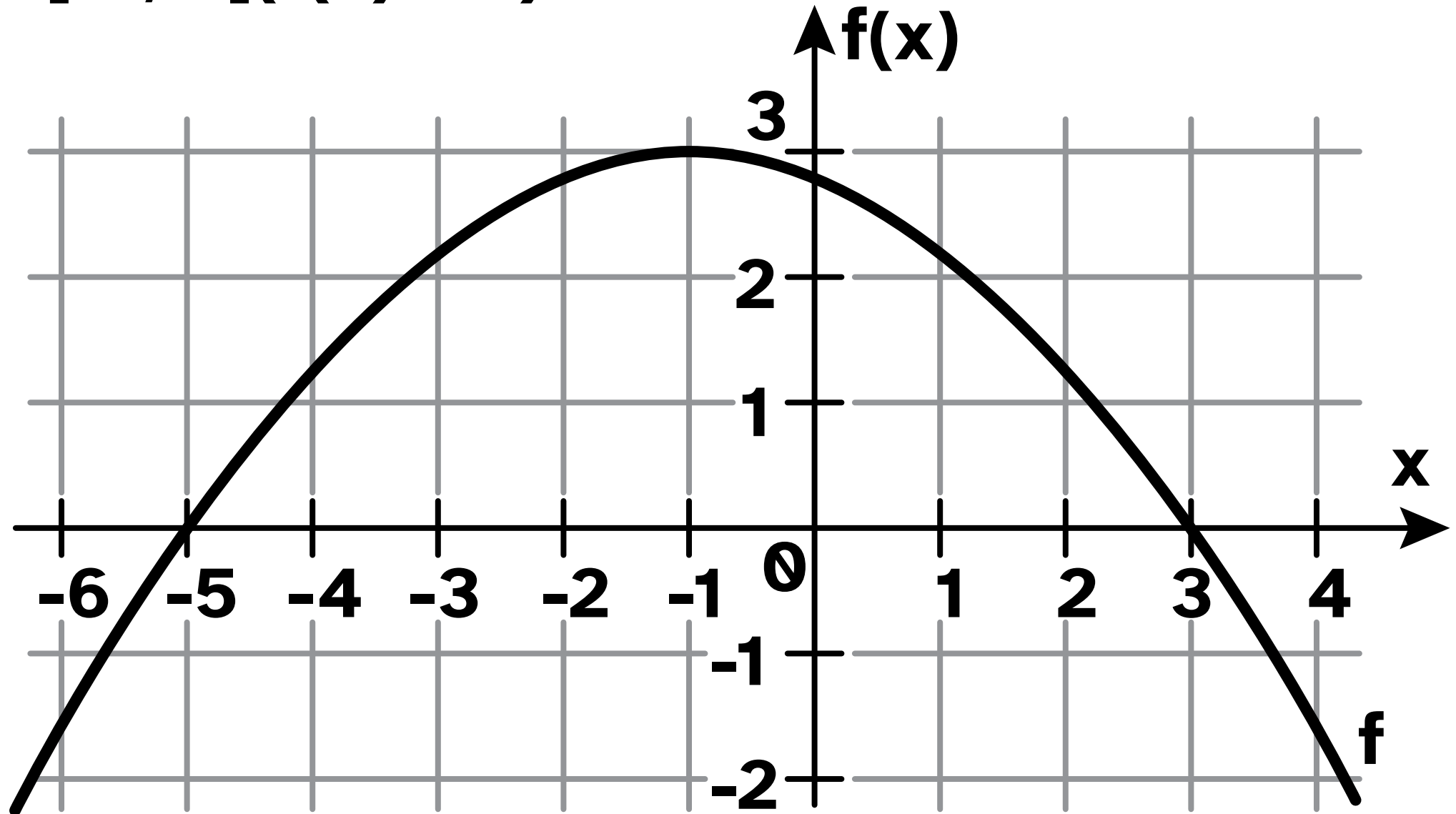


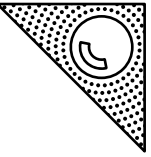
$\int_0^5 f(x) dx$



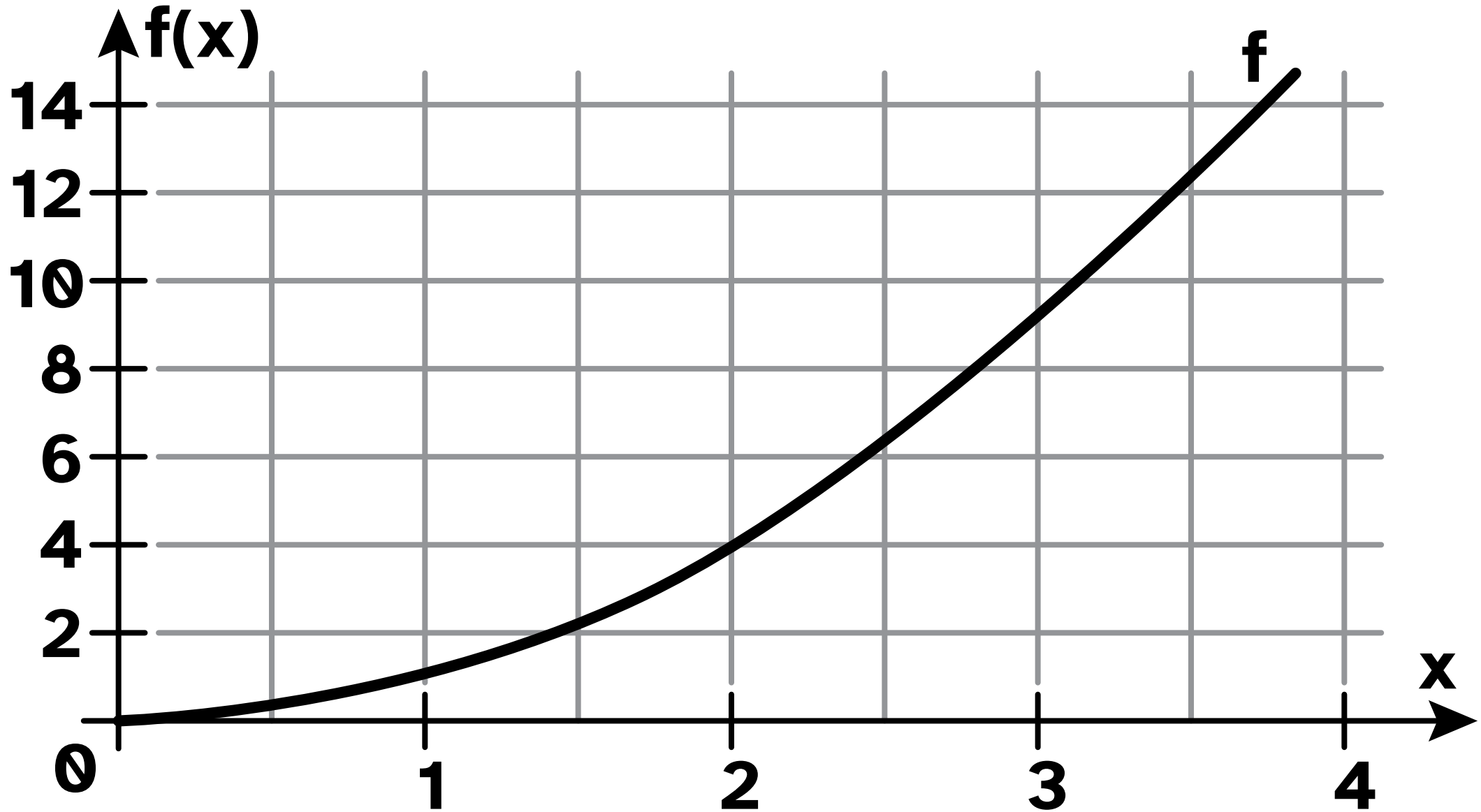


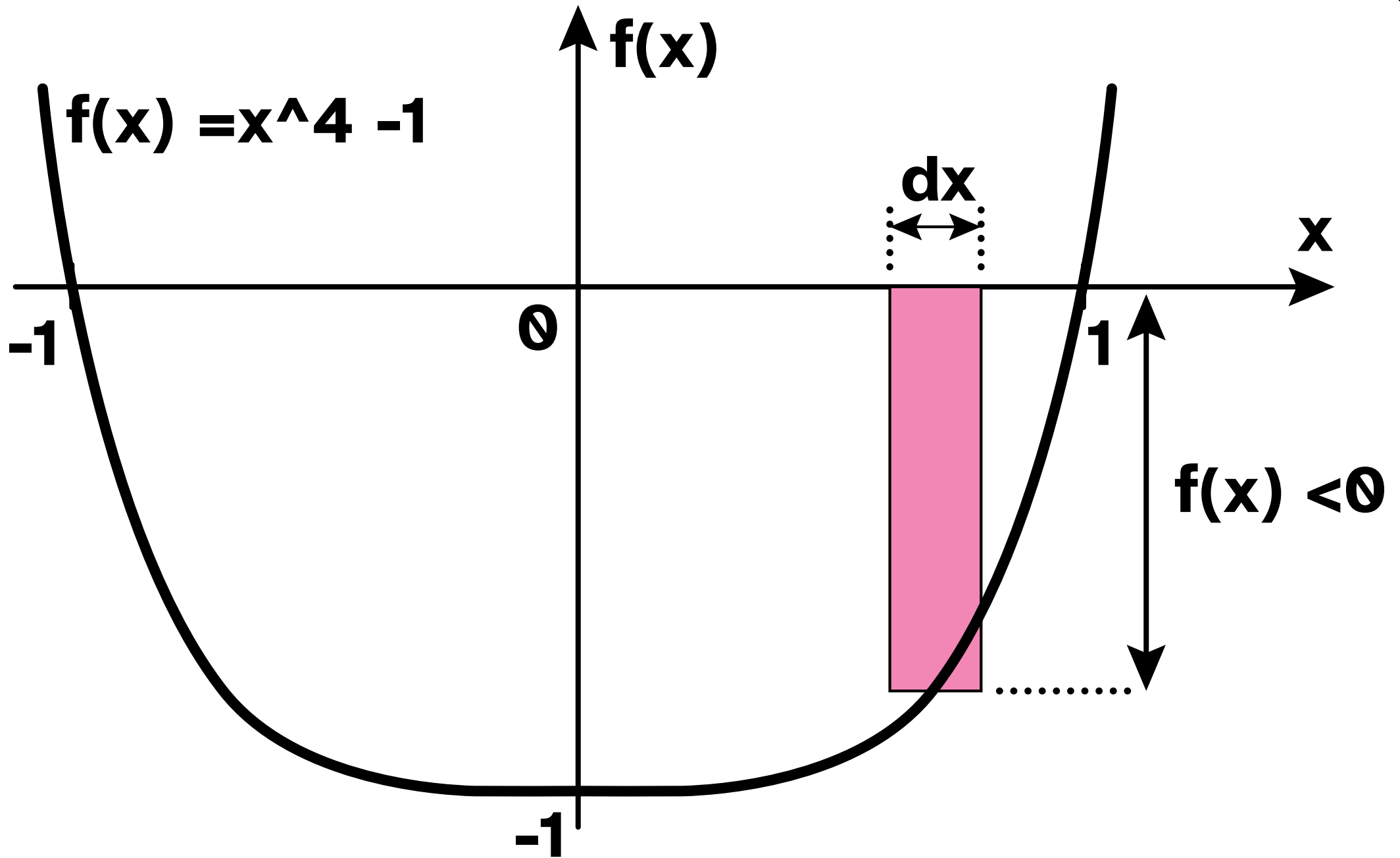
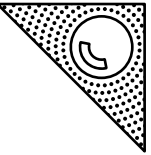
$\int_{-2}^3 (f(x))' dx$

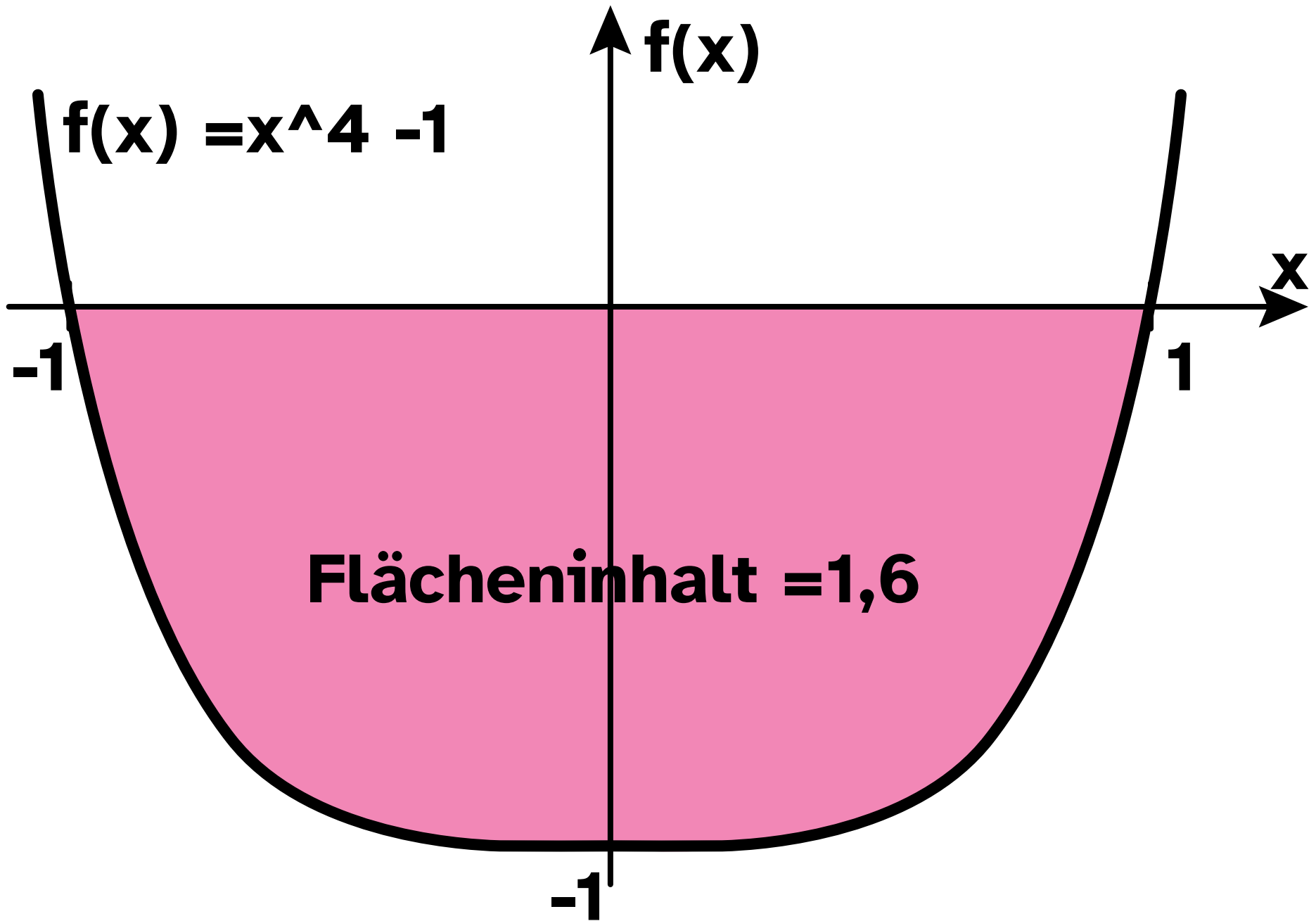
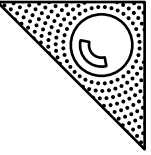


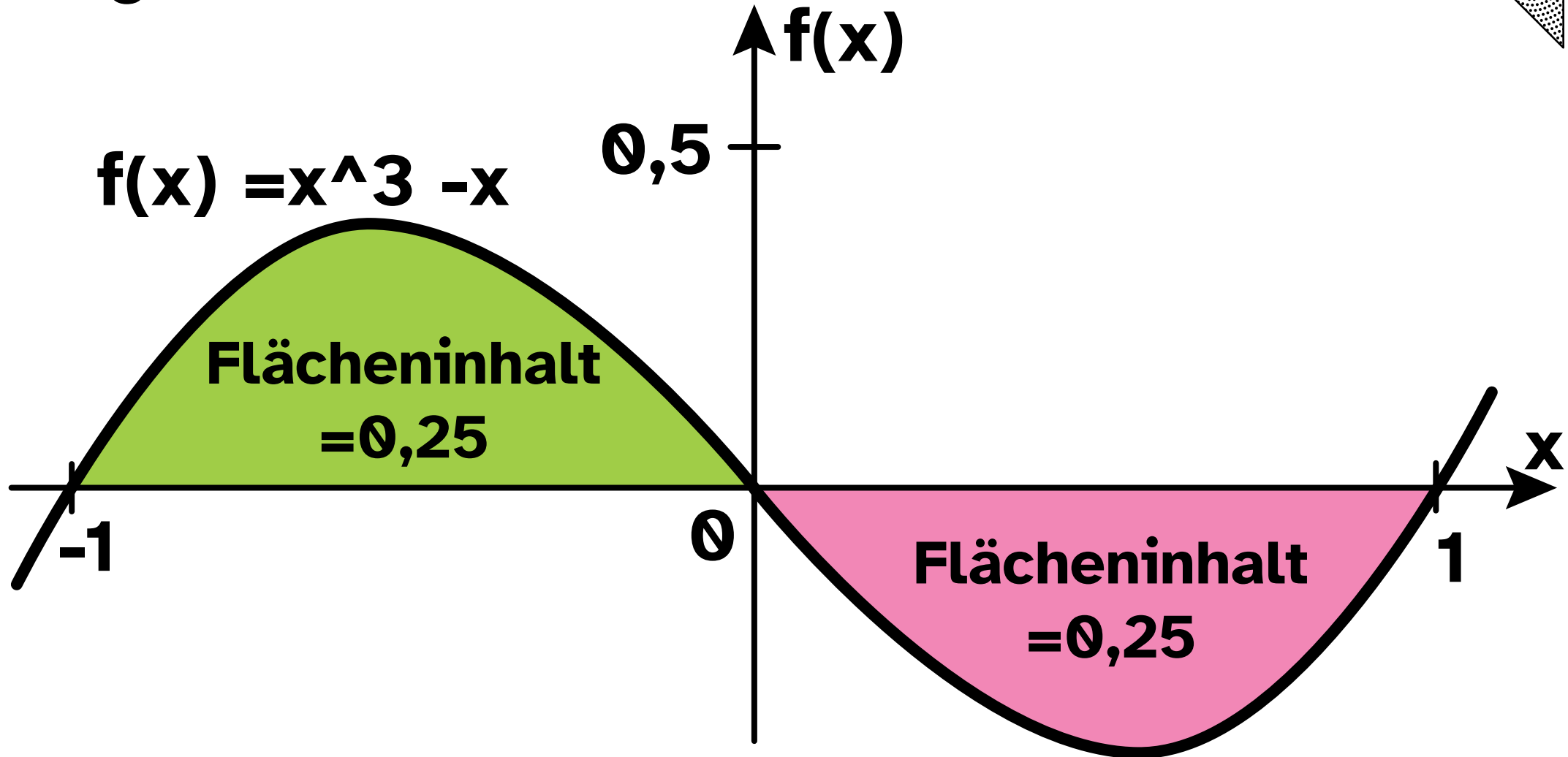
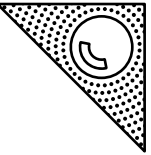


$\int_1^3 f(x) dx$

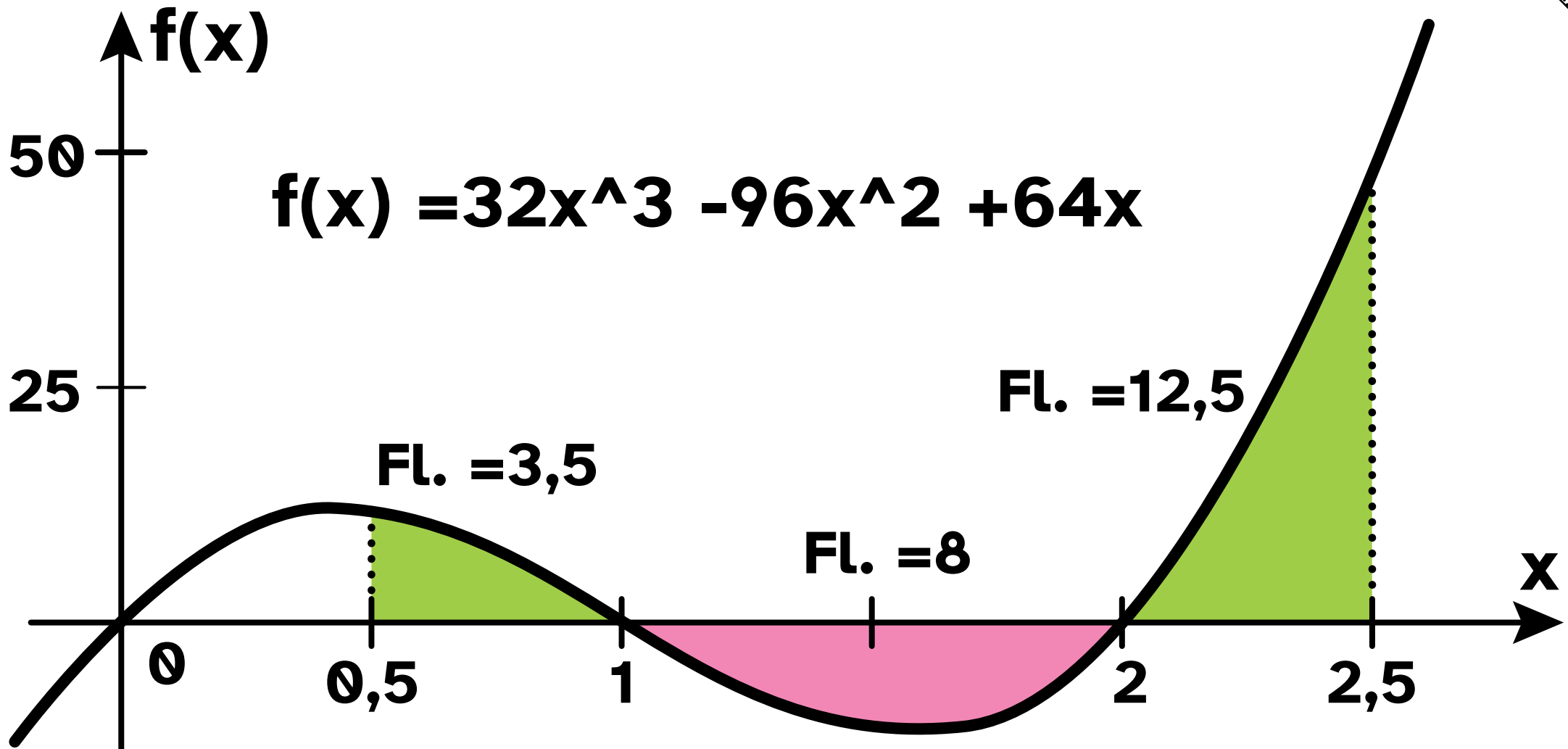
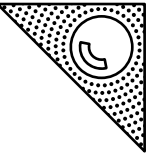




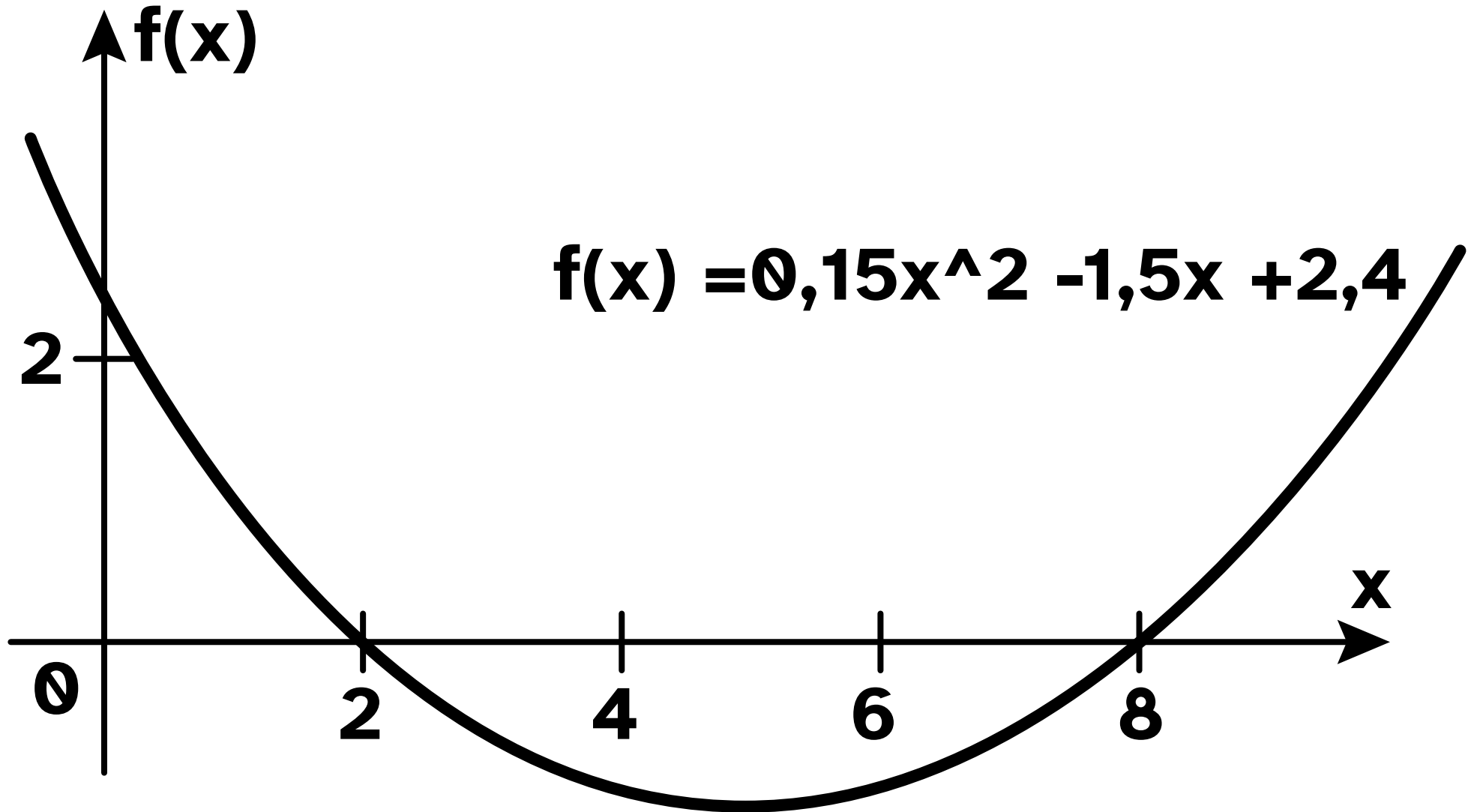
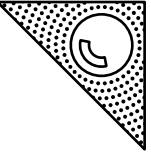


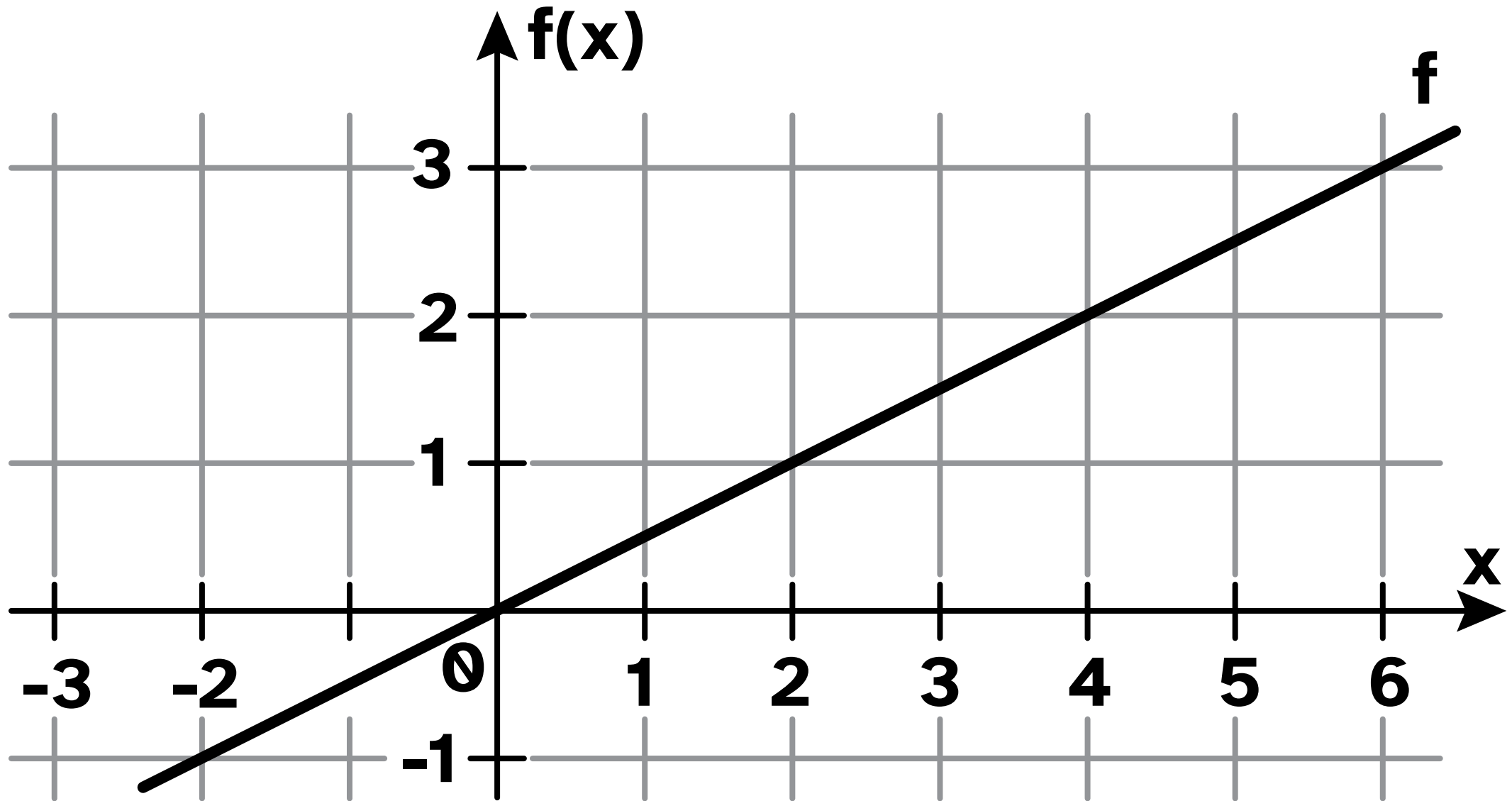
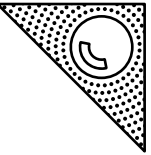


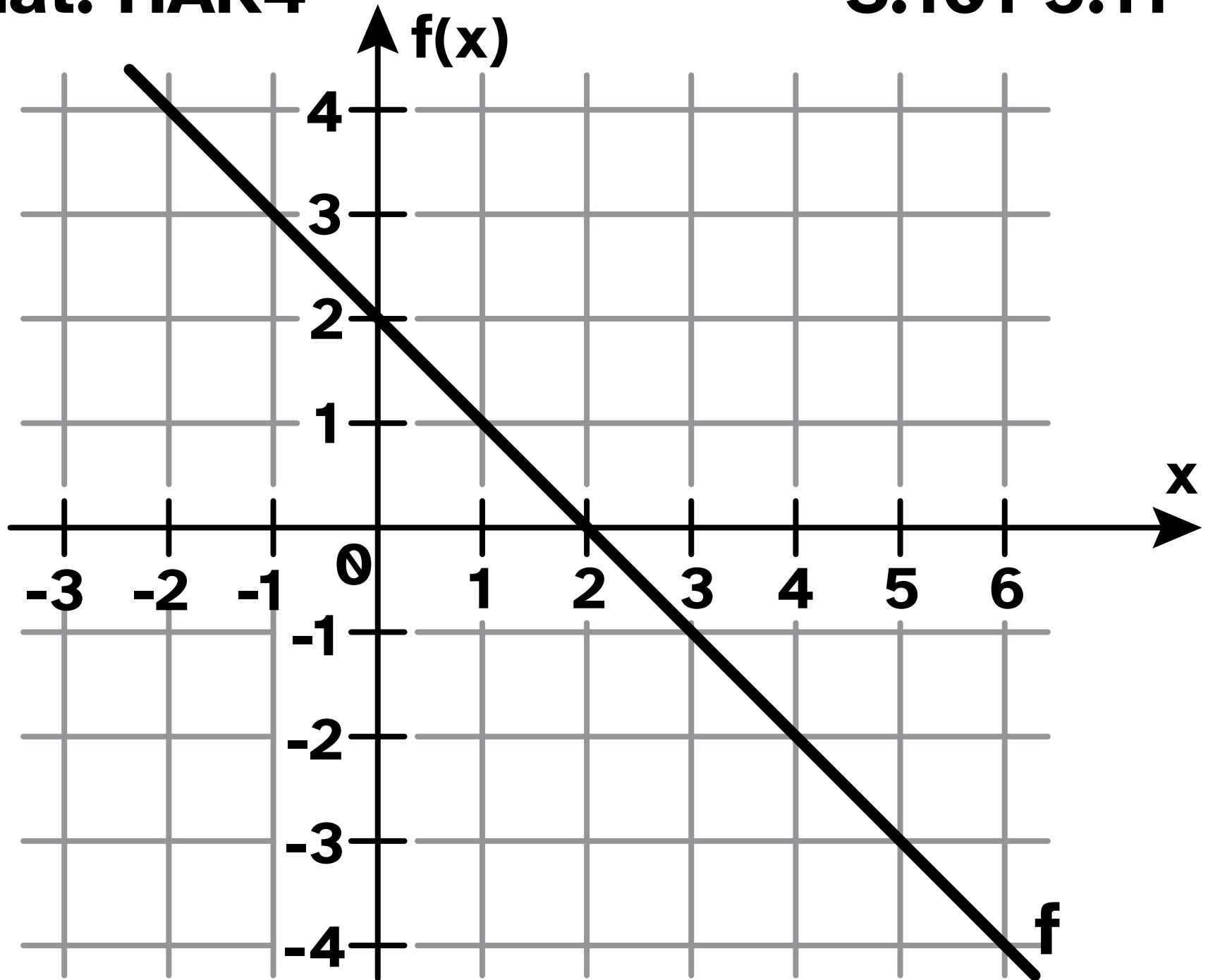
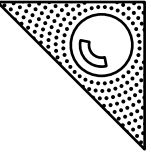
$$\int_{-1}^1 (f(x)) \, dx = 0,25 - 0,25 = 0$$

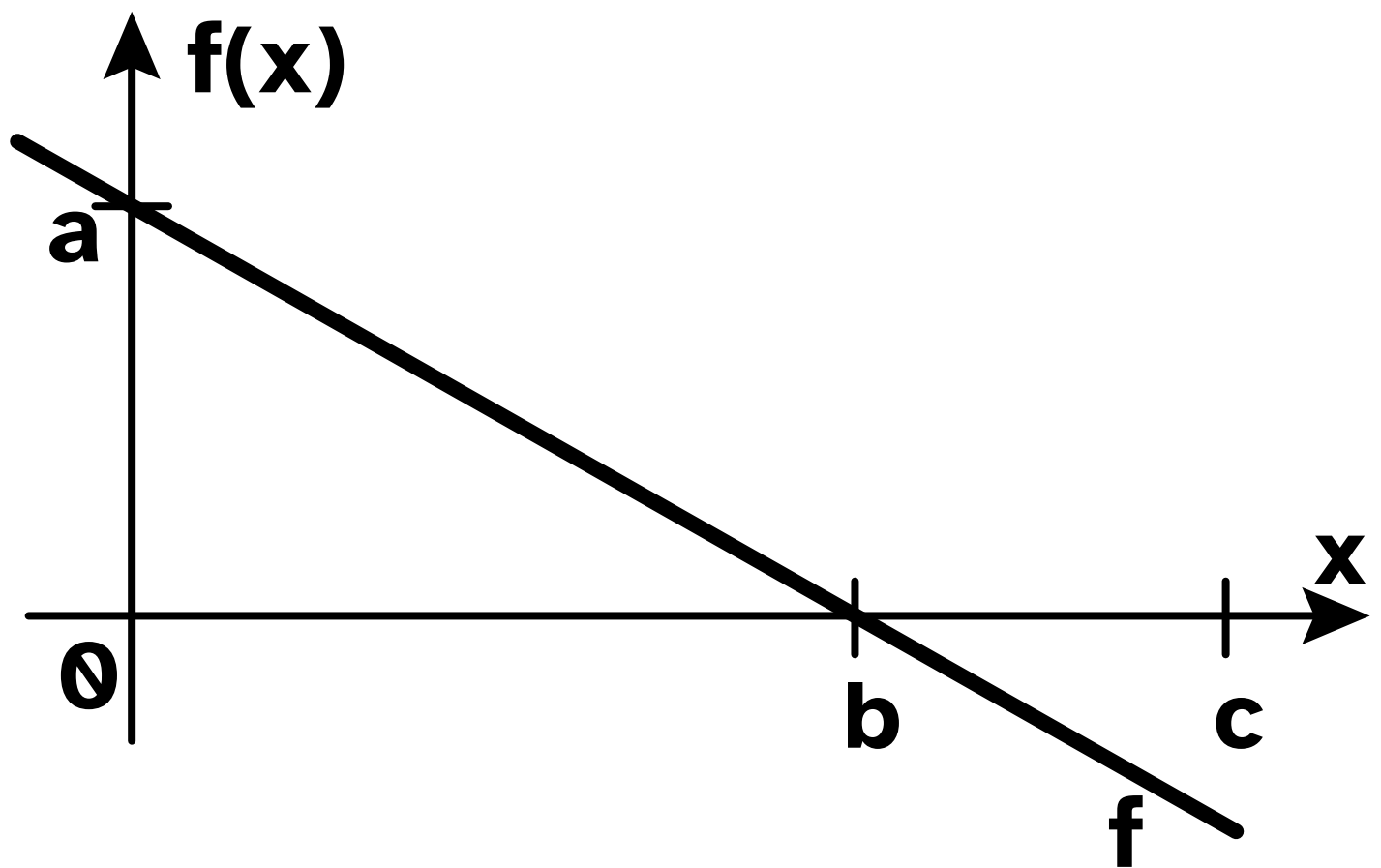
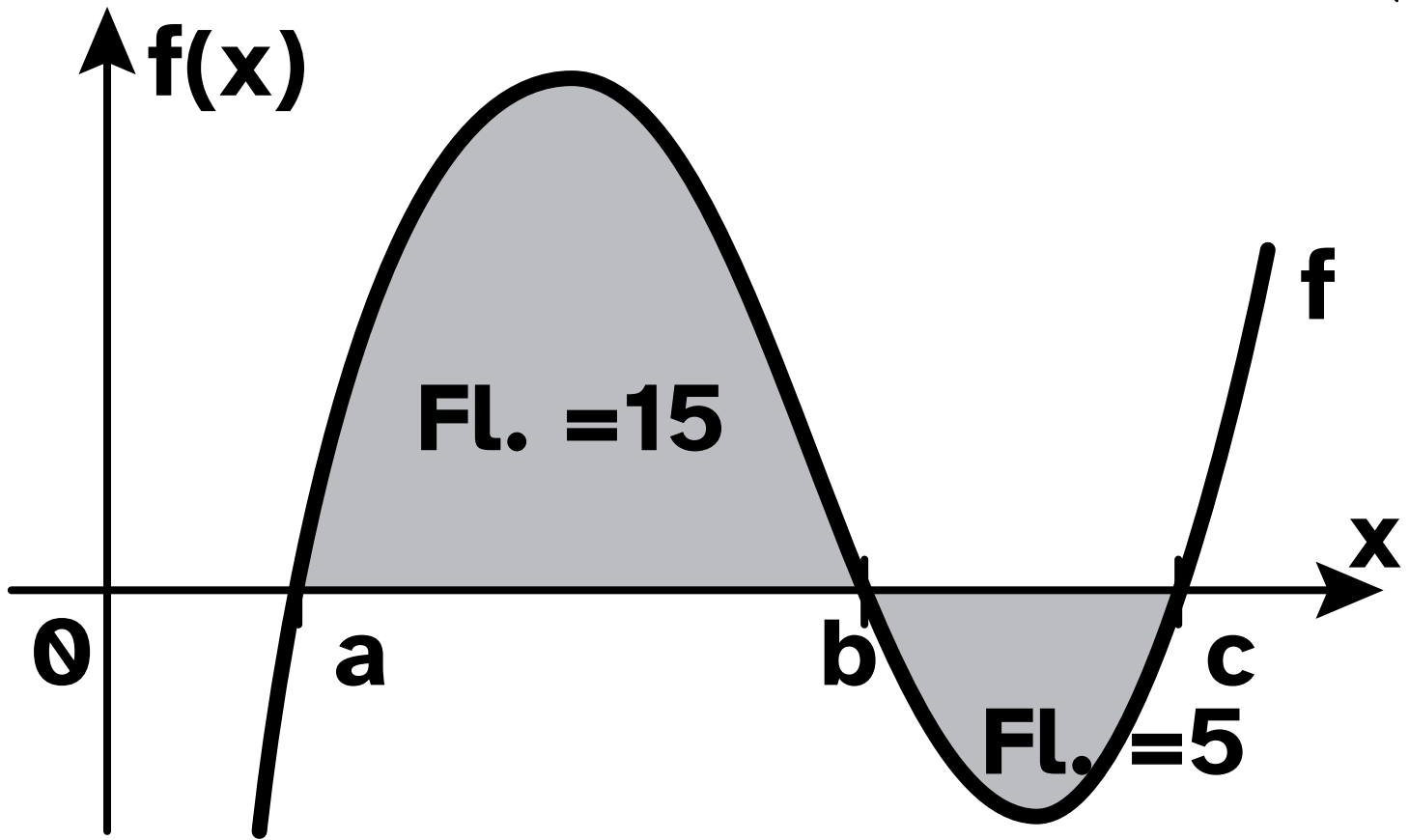
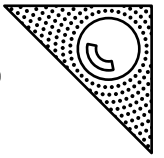


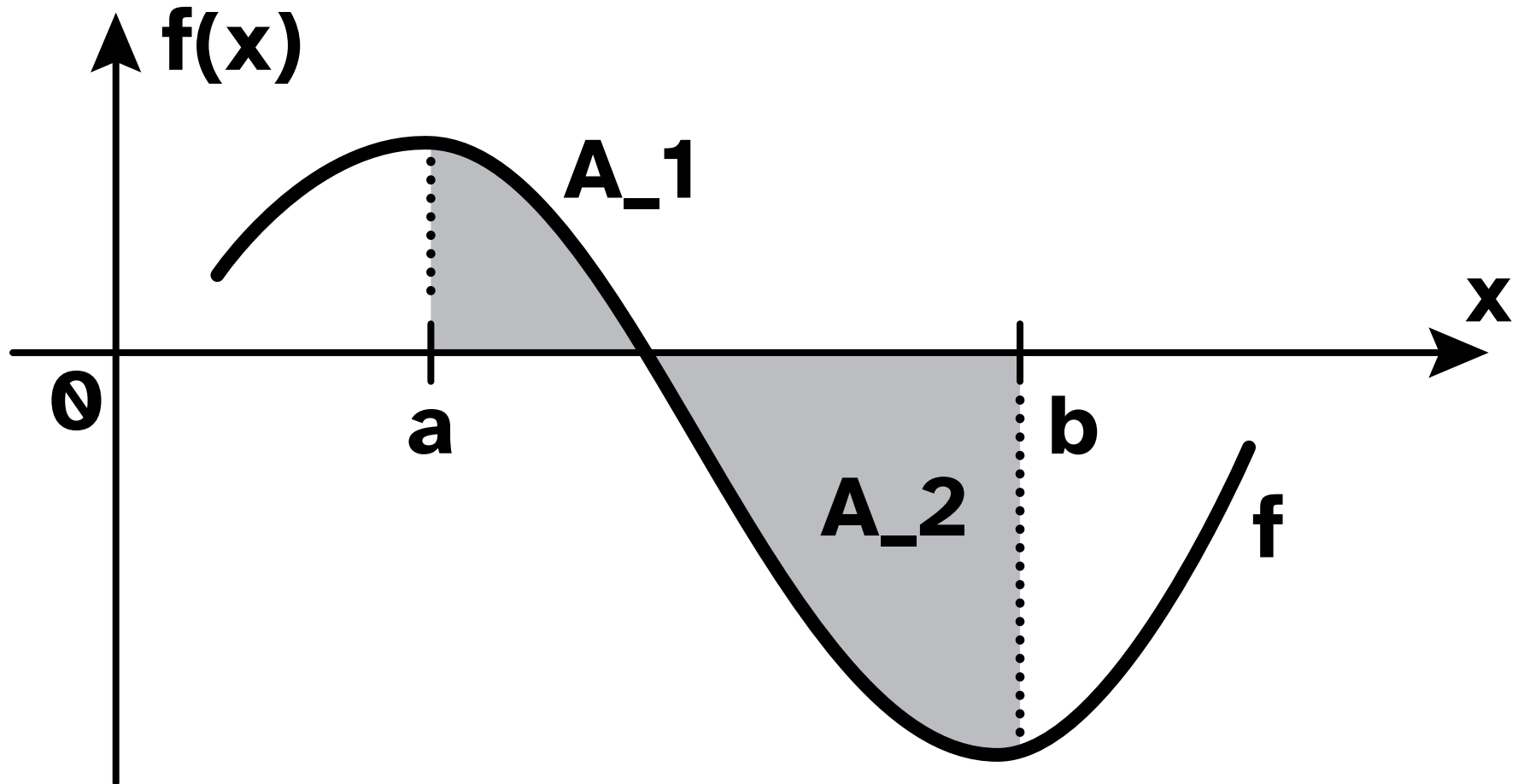
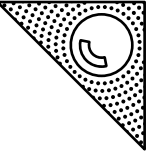
$$\int_{0,5}^{2,5} (f(x) \, dx) = 3,5 - 8 + 12,5 = 8$$

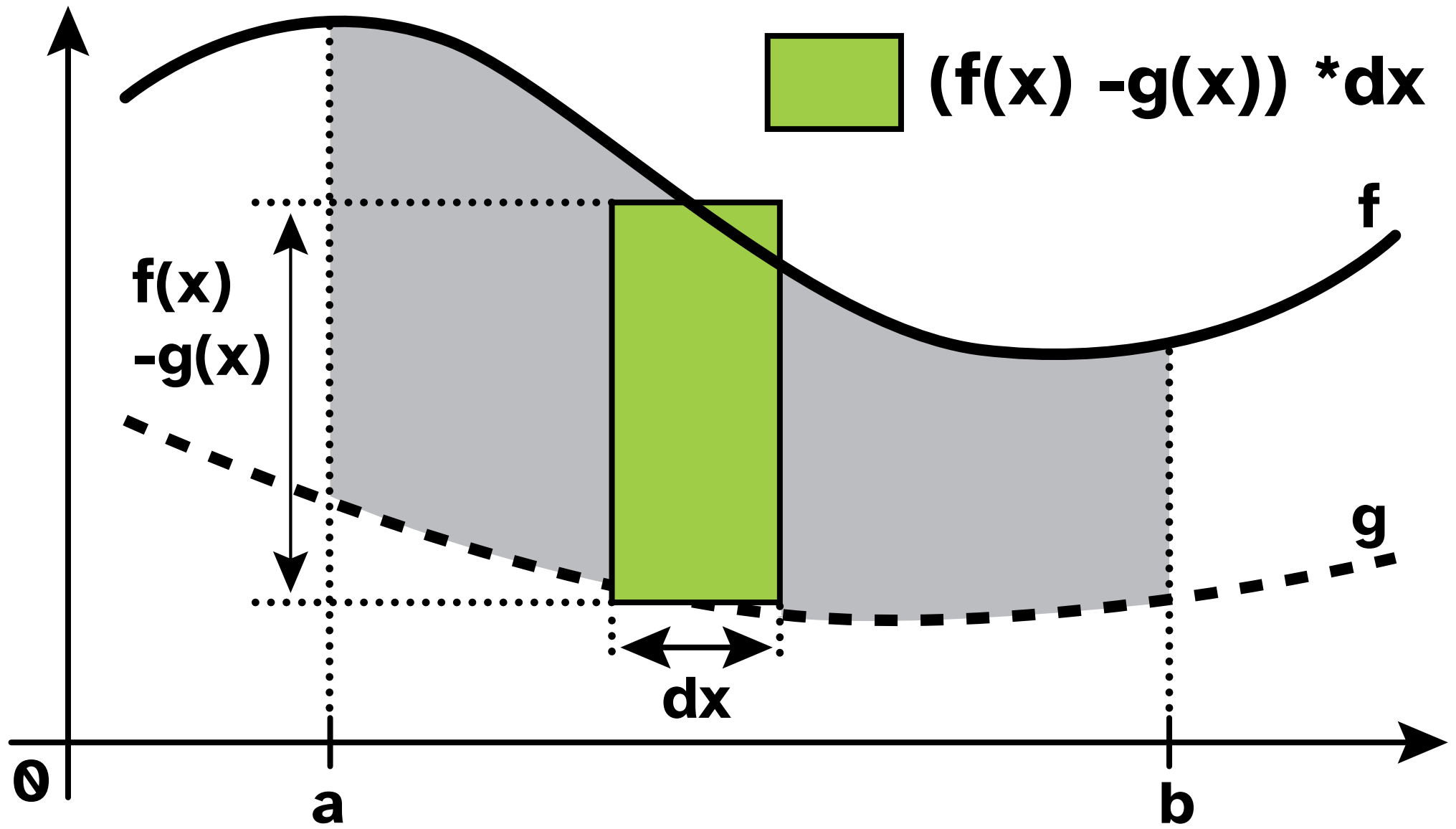
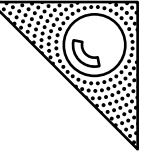




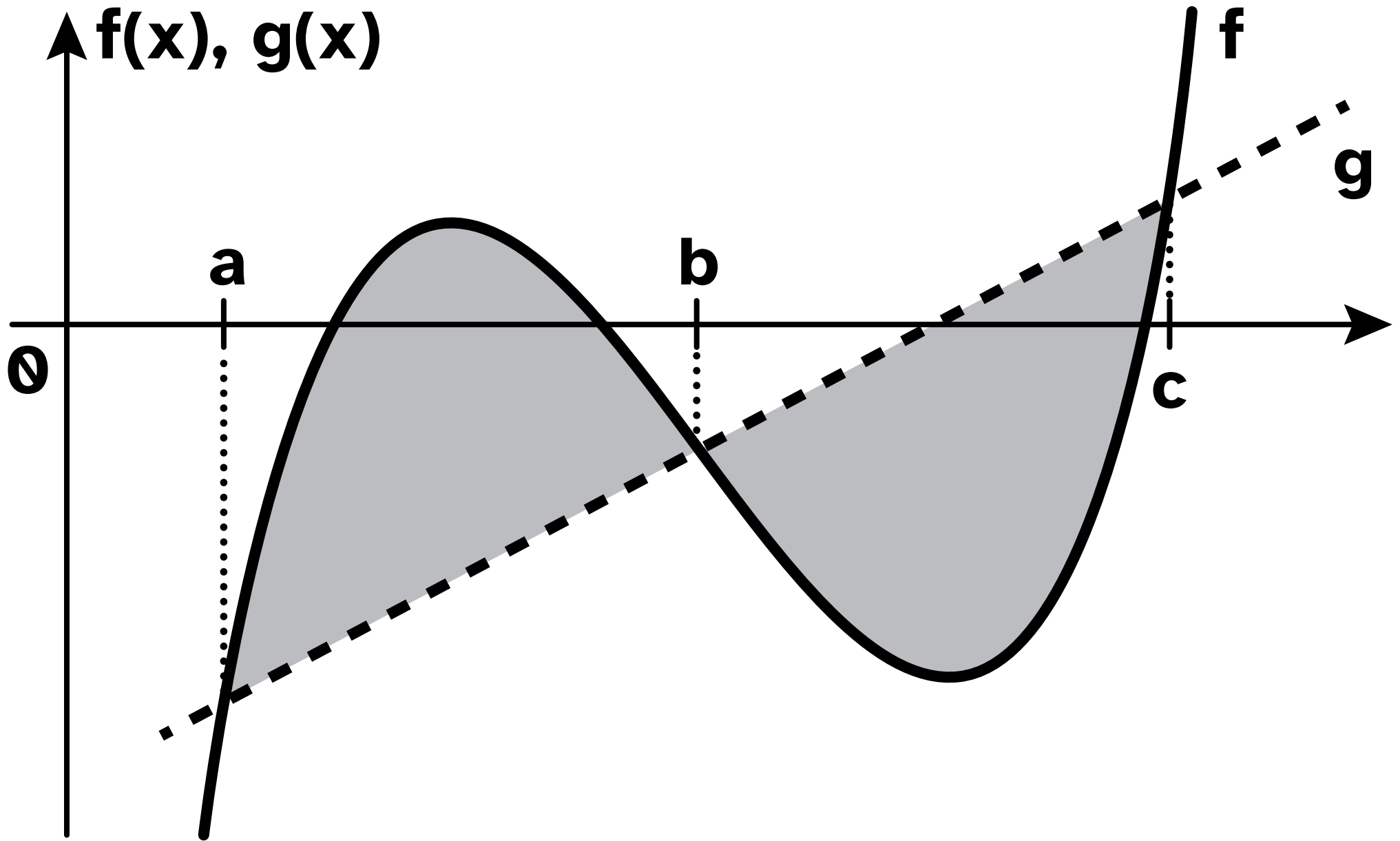
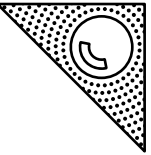


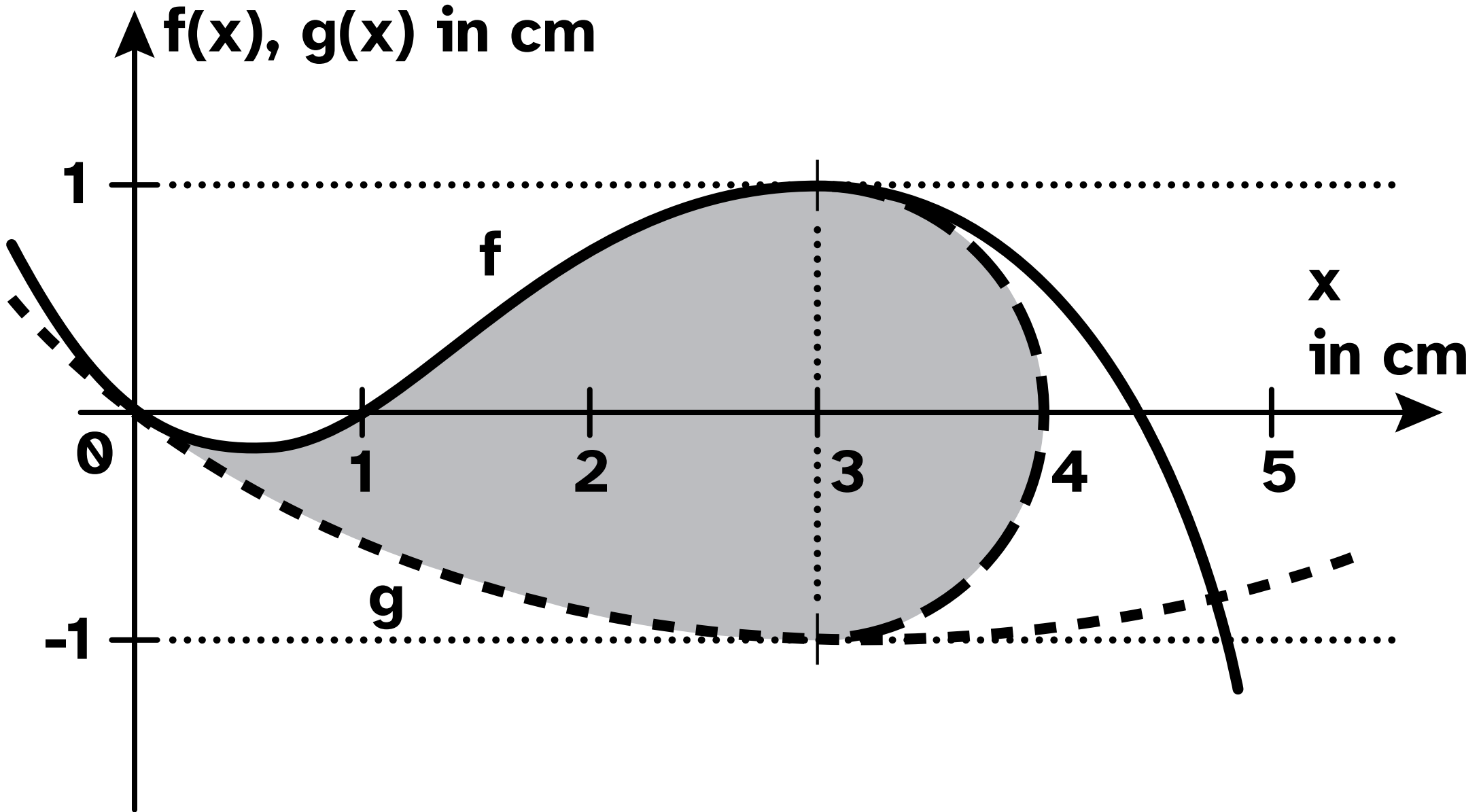
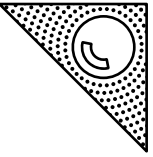


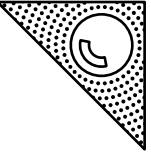




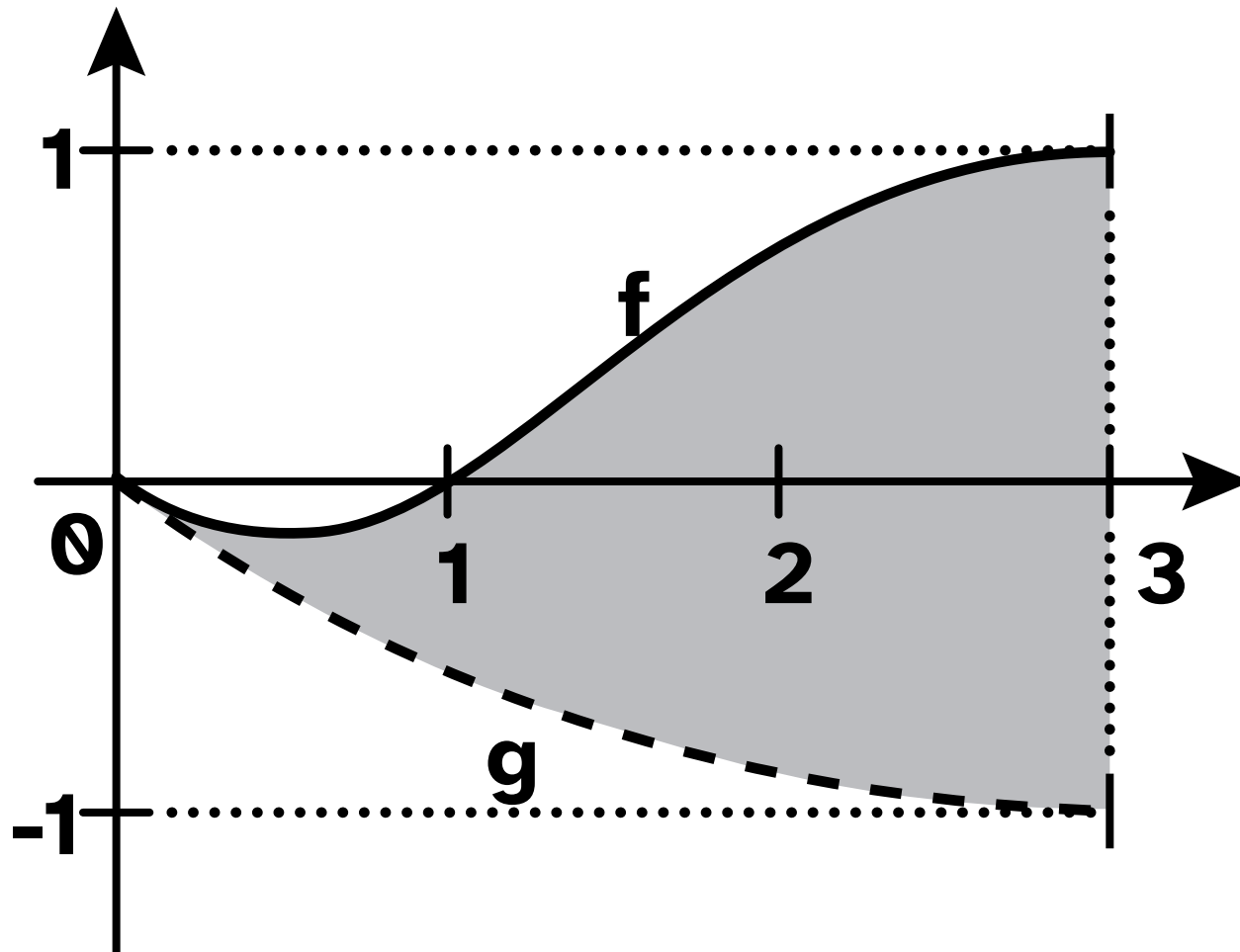
 $\int [a; b] (f(x) - g(x)) dx$



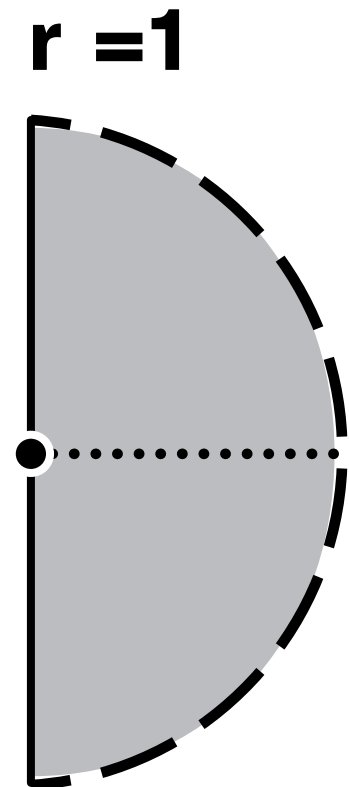


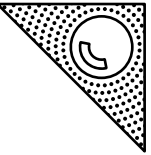


Teilfläche 1:



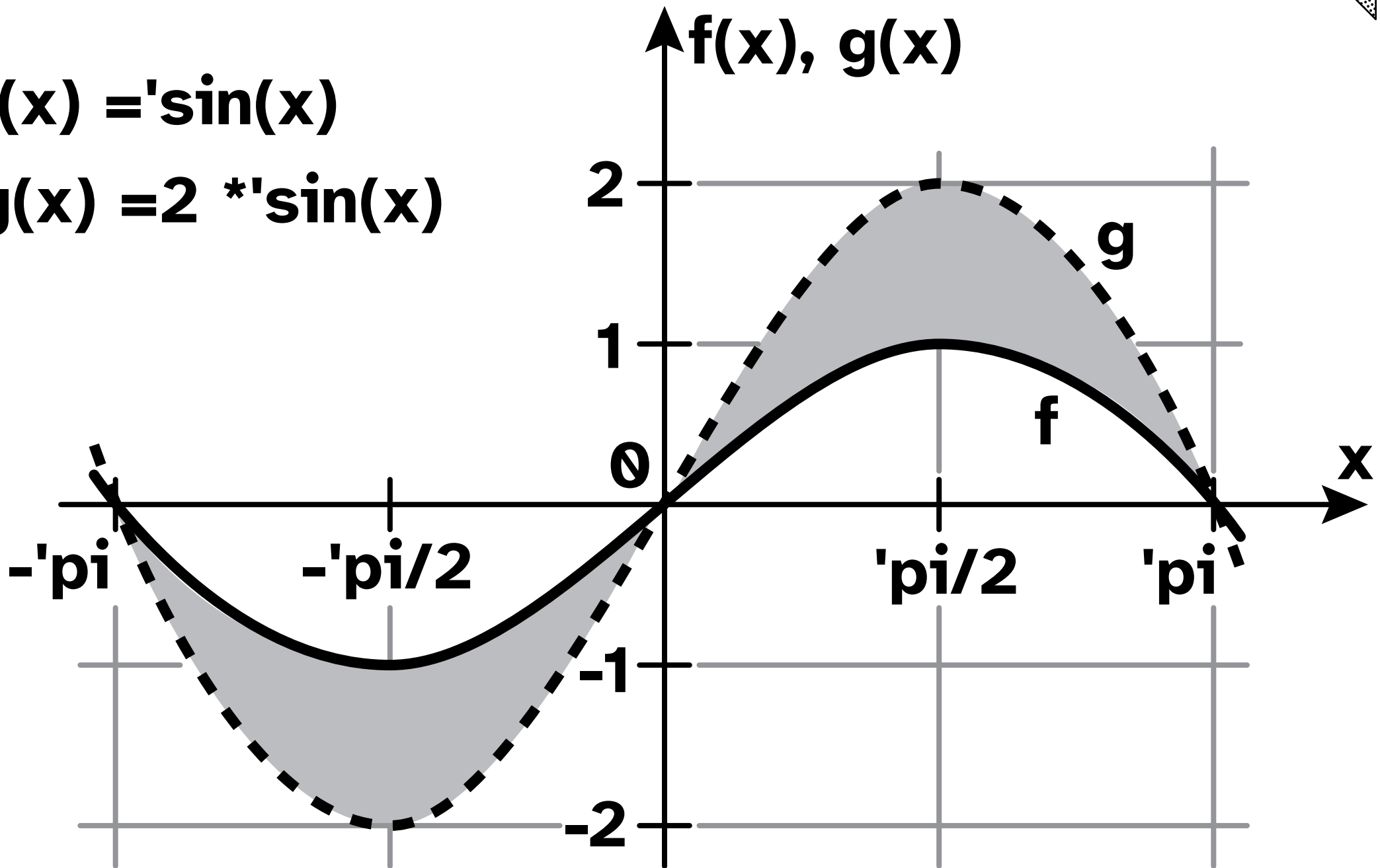
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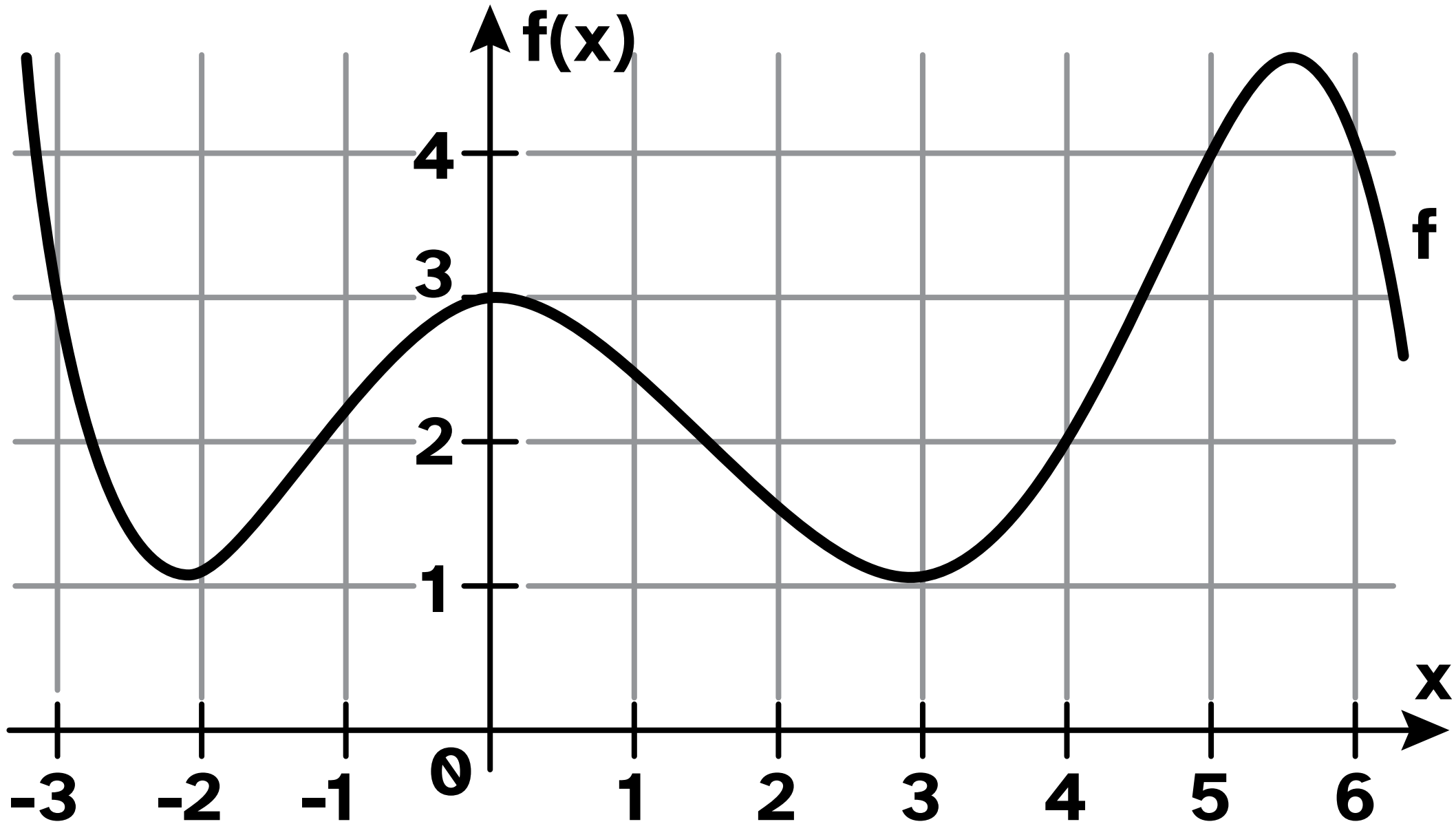
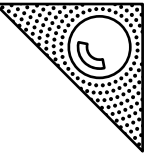


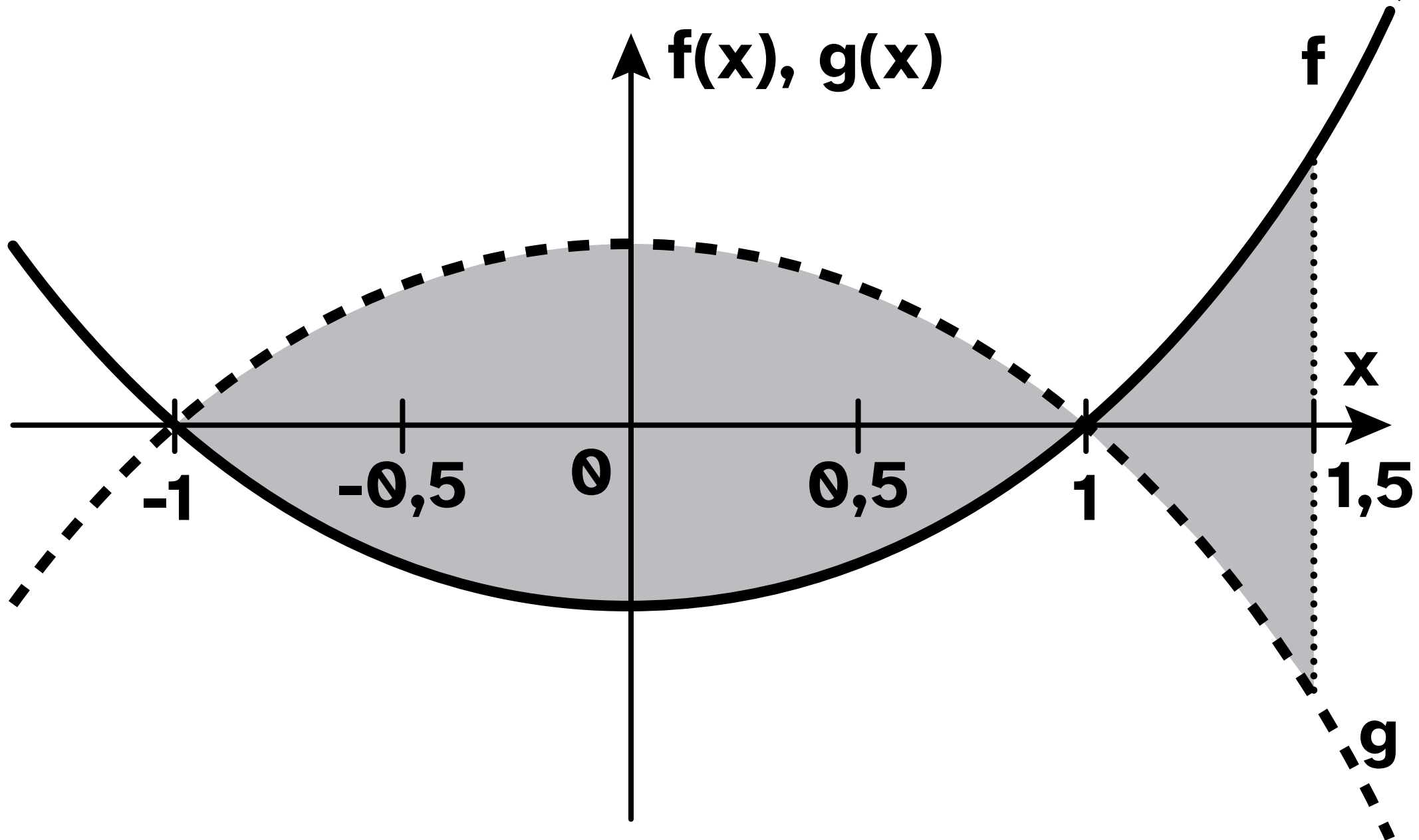
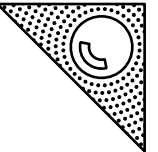


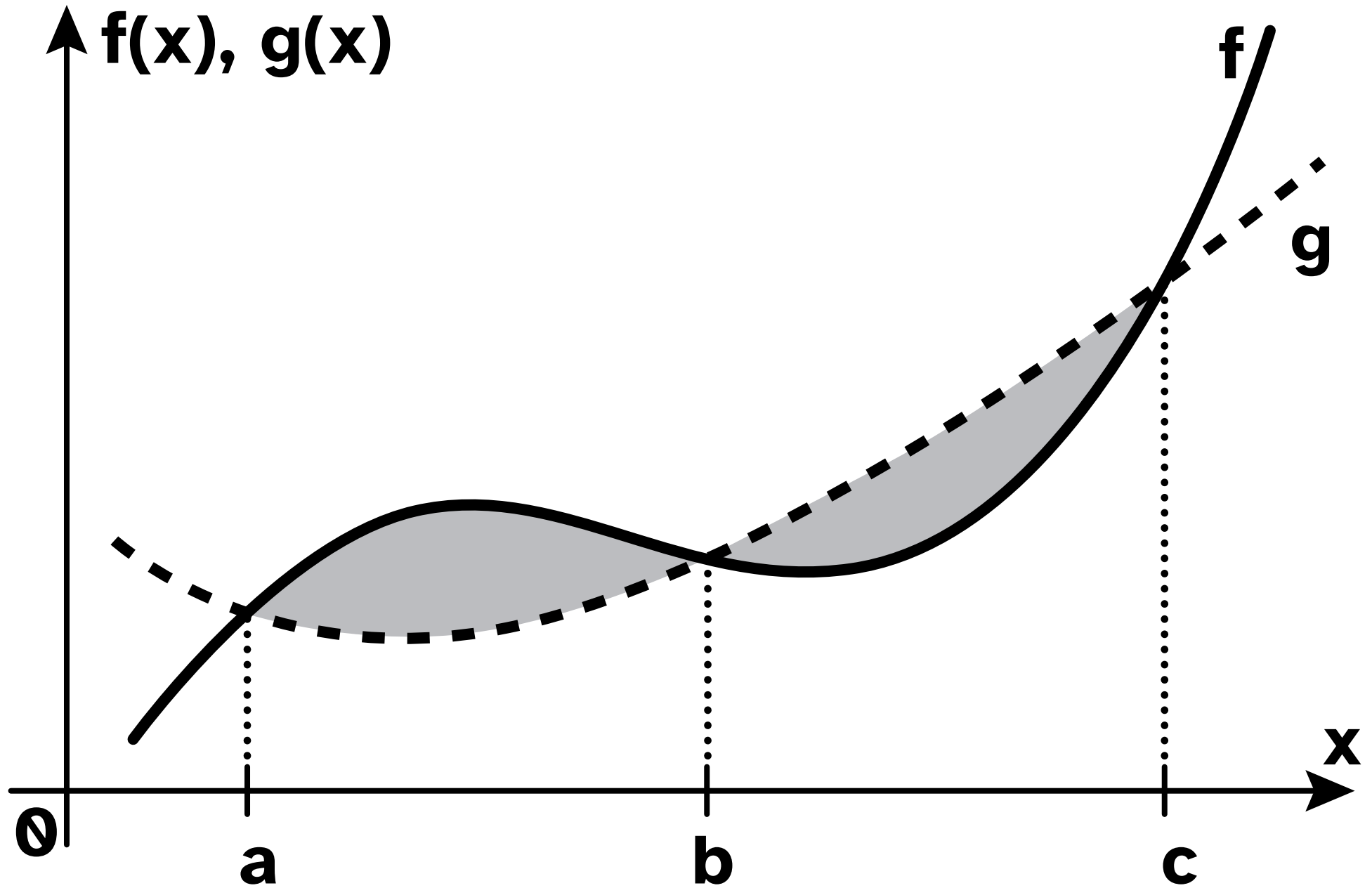
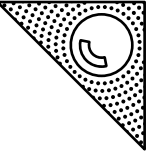
$$f(x) = \sin(x)$$

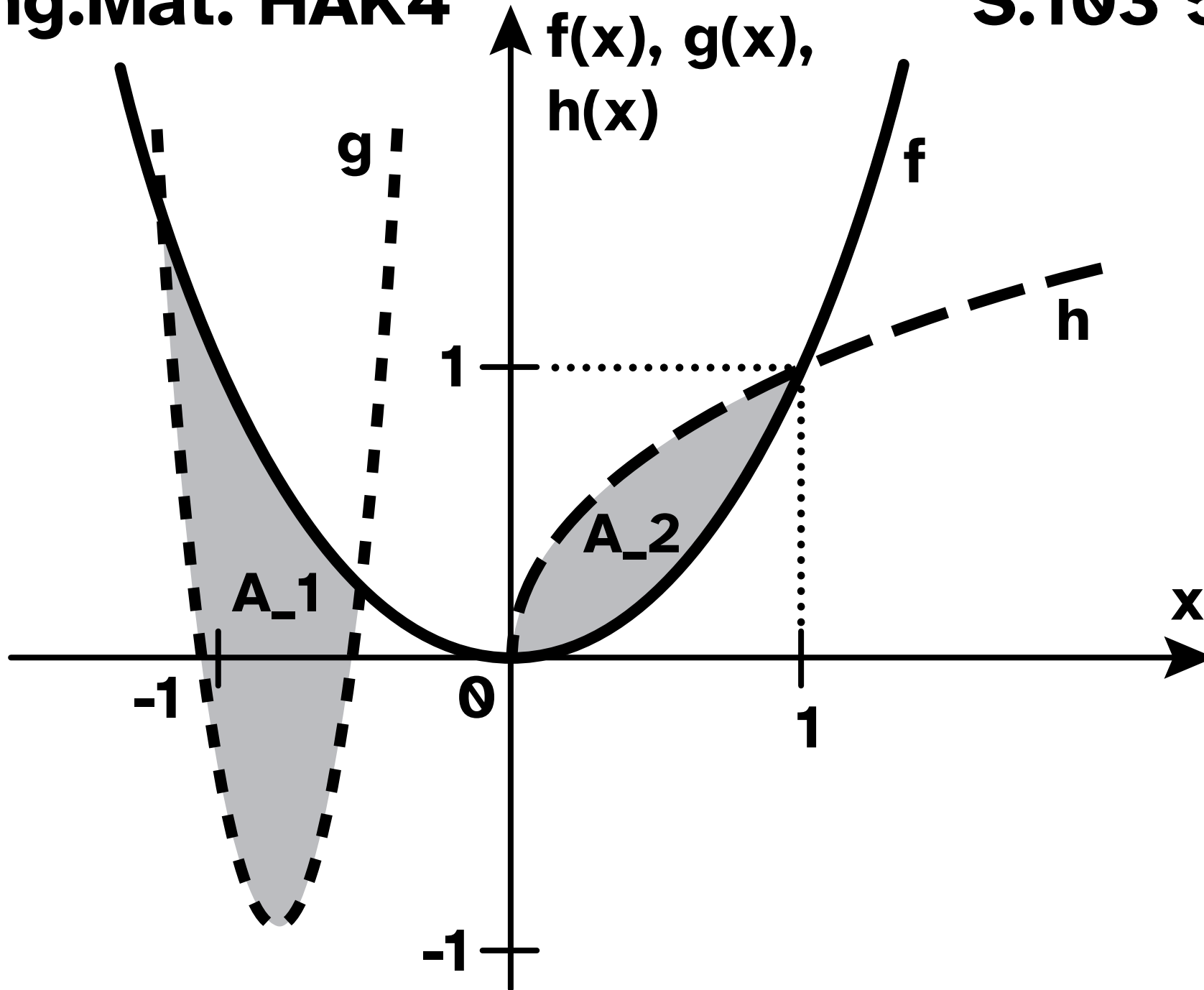
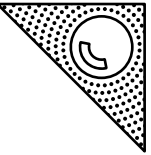
$$g(x) = 2 \cdot \sin(x)$$

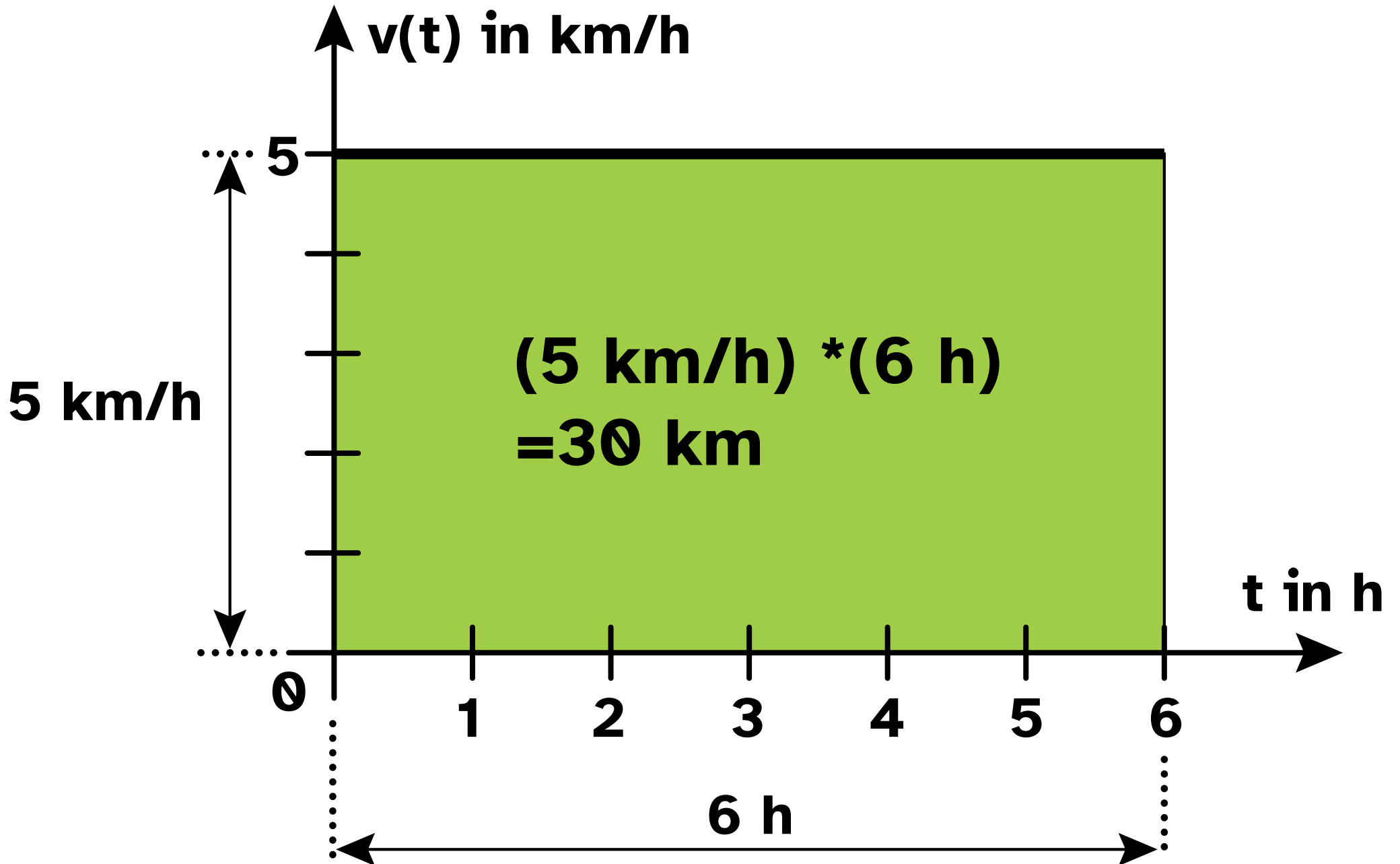
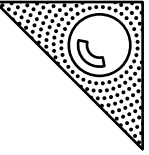


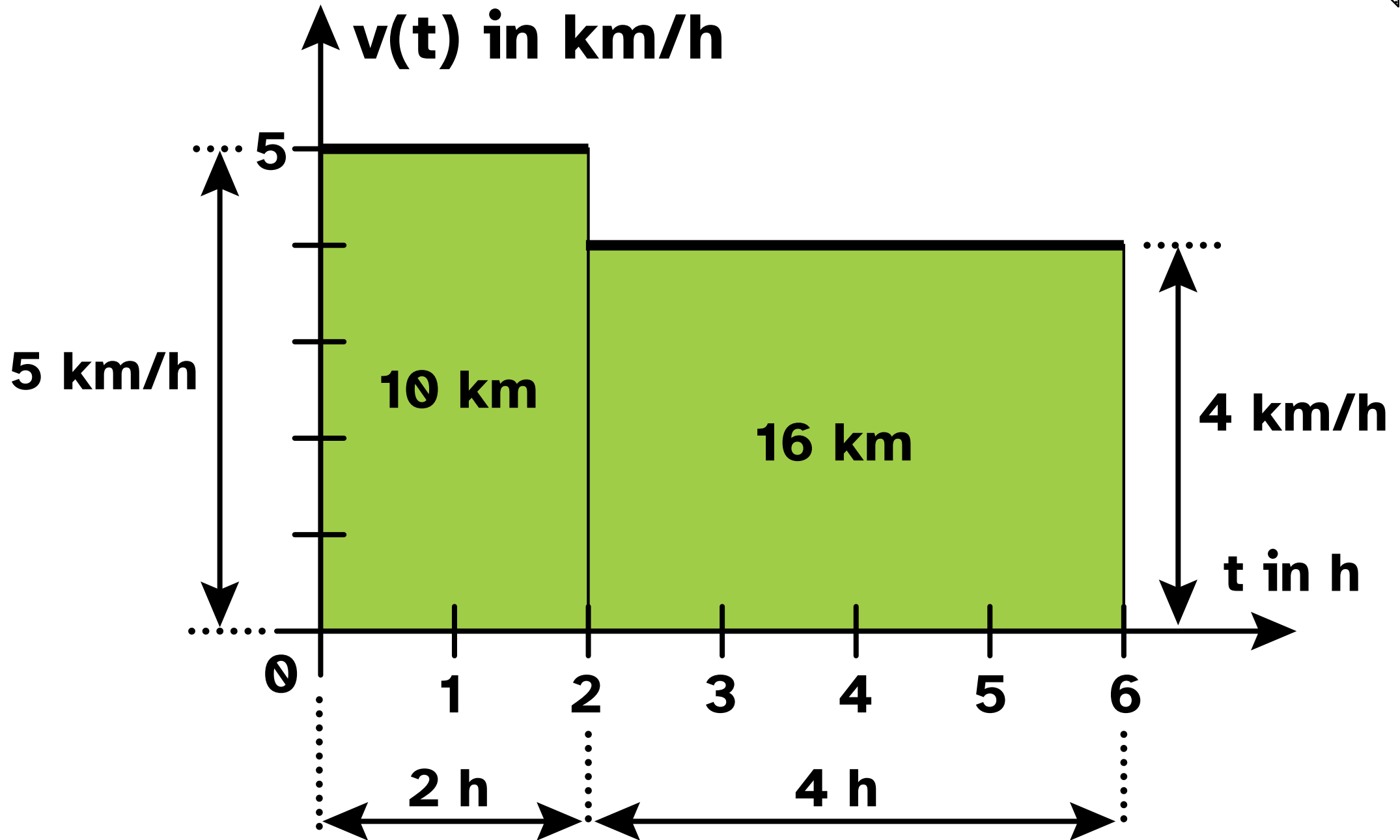
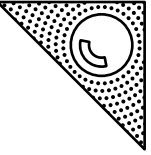


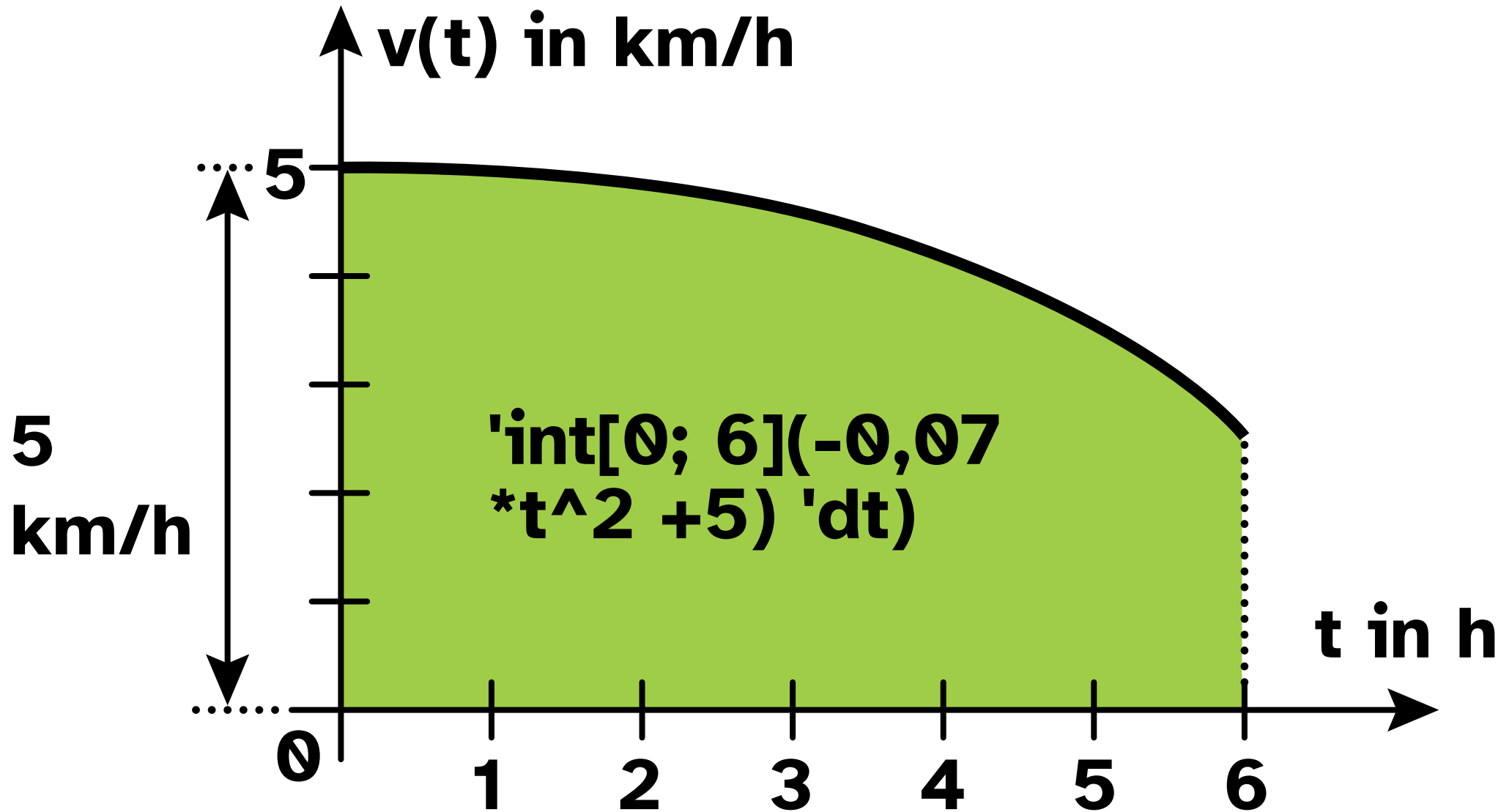
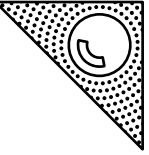


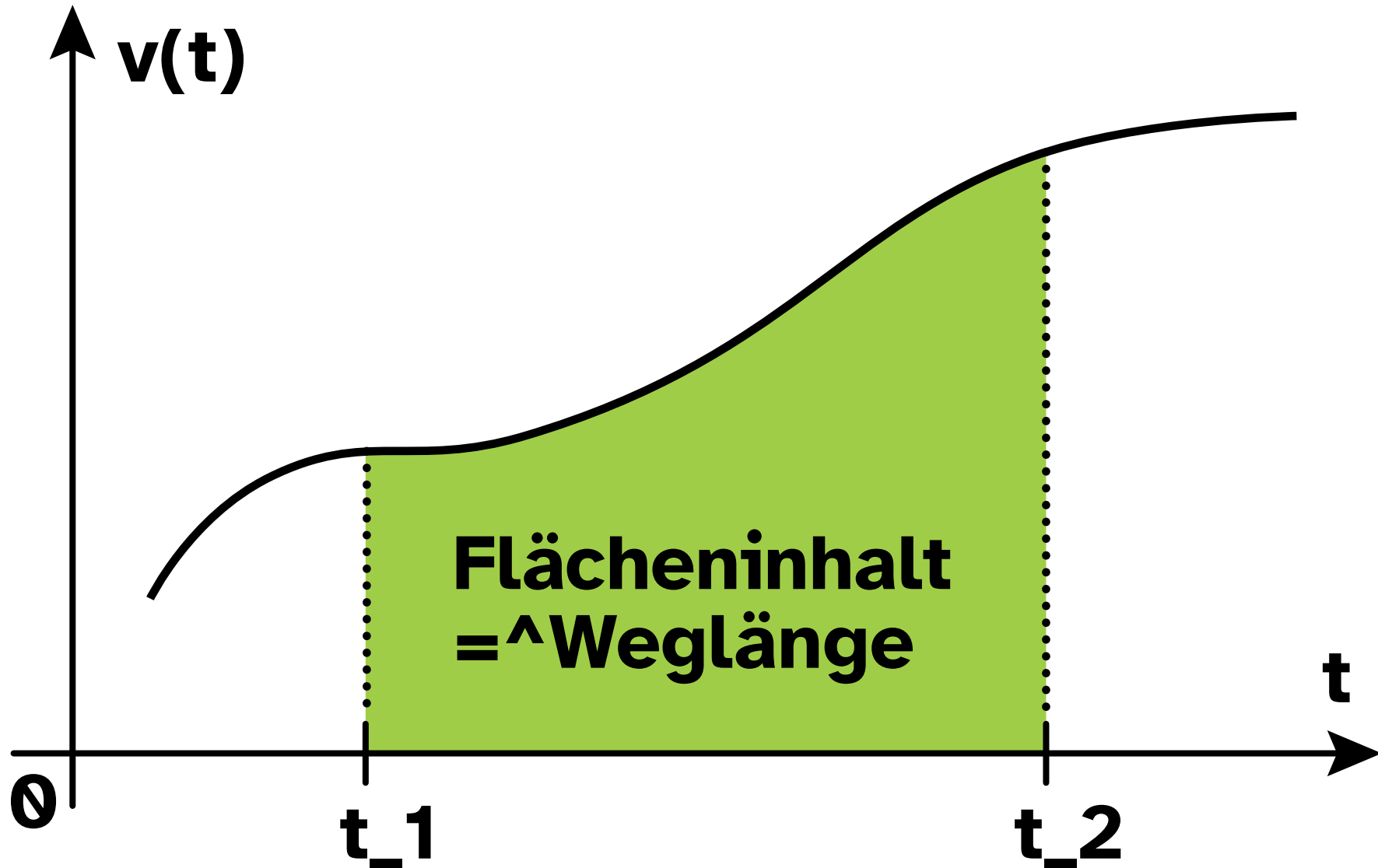
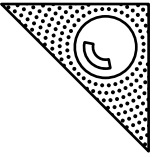


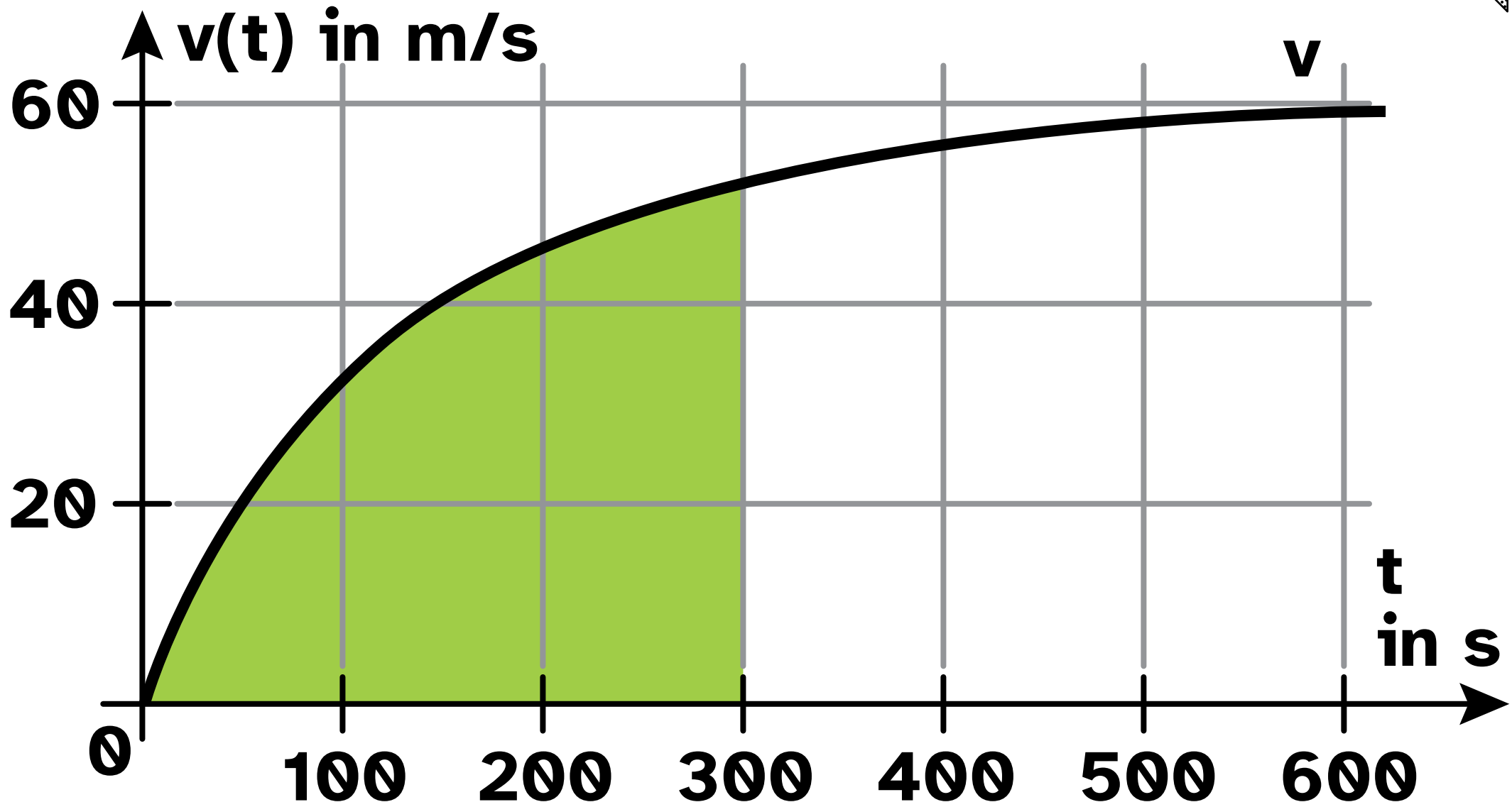
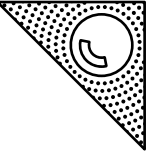




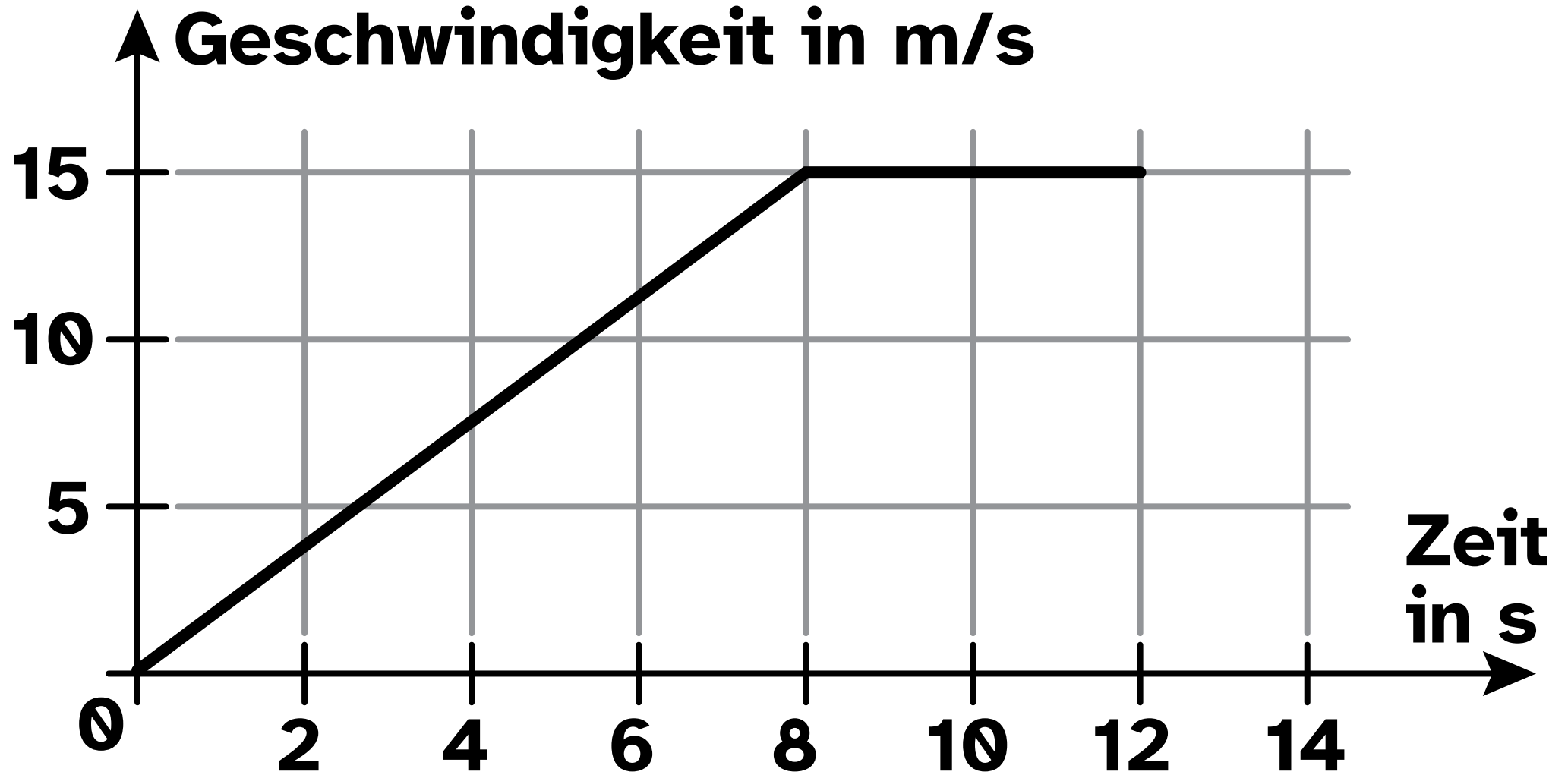
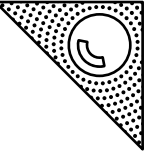


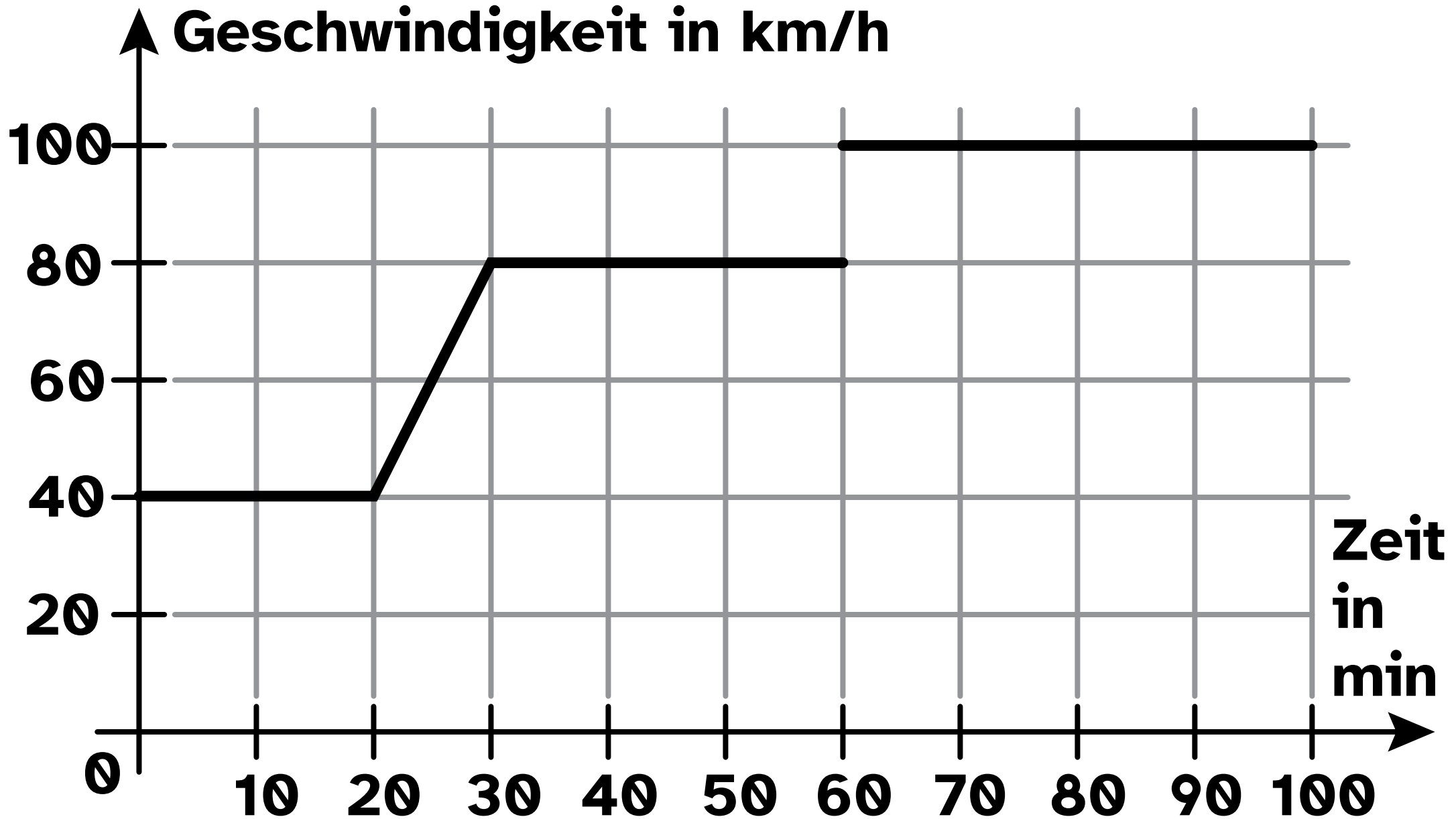
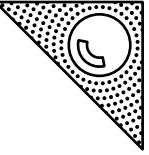


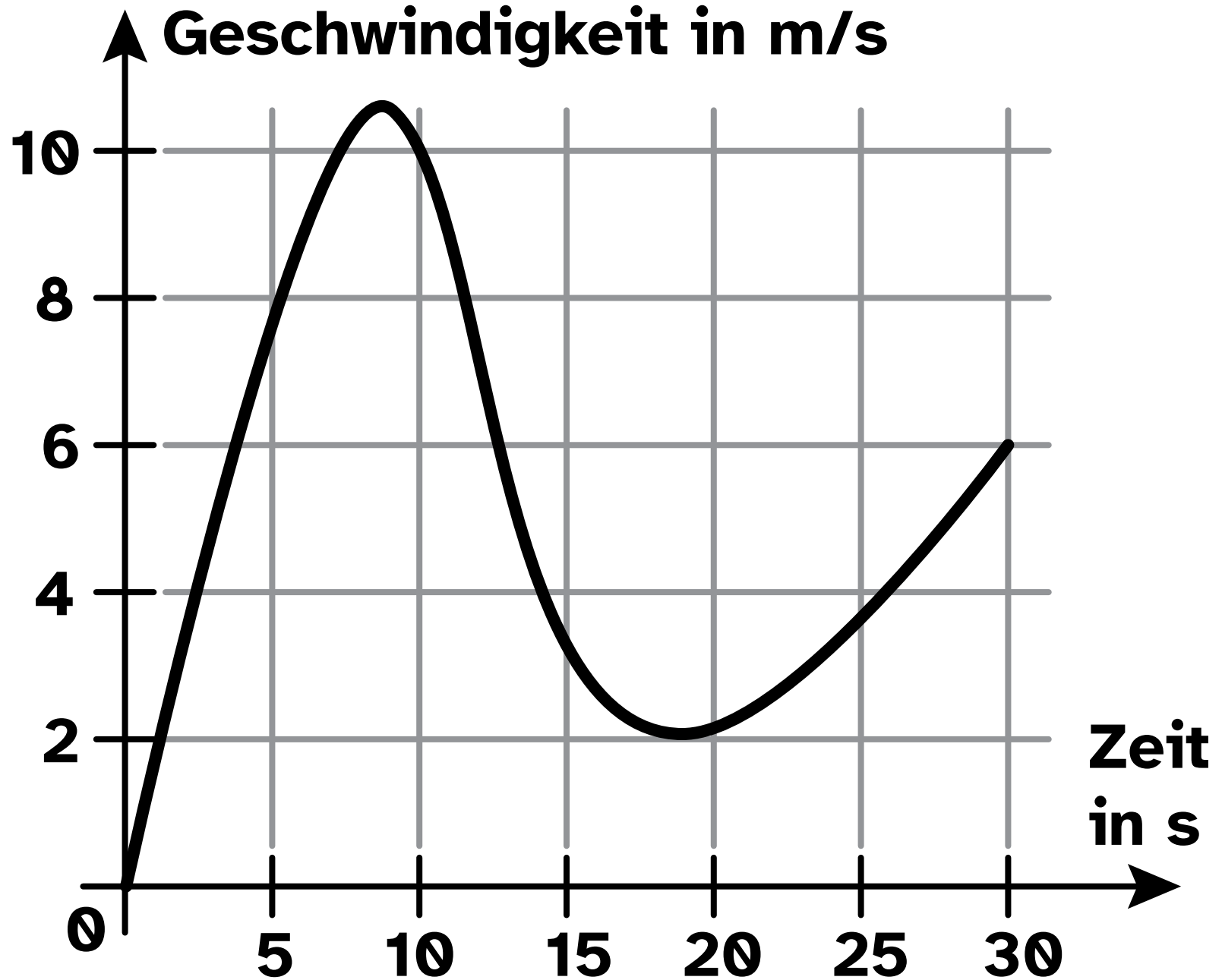
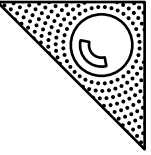


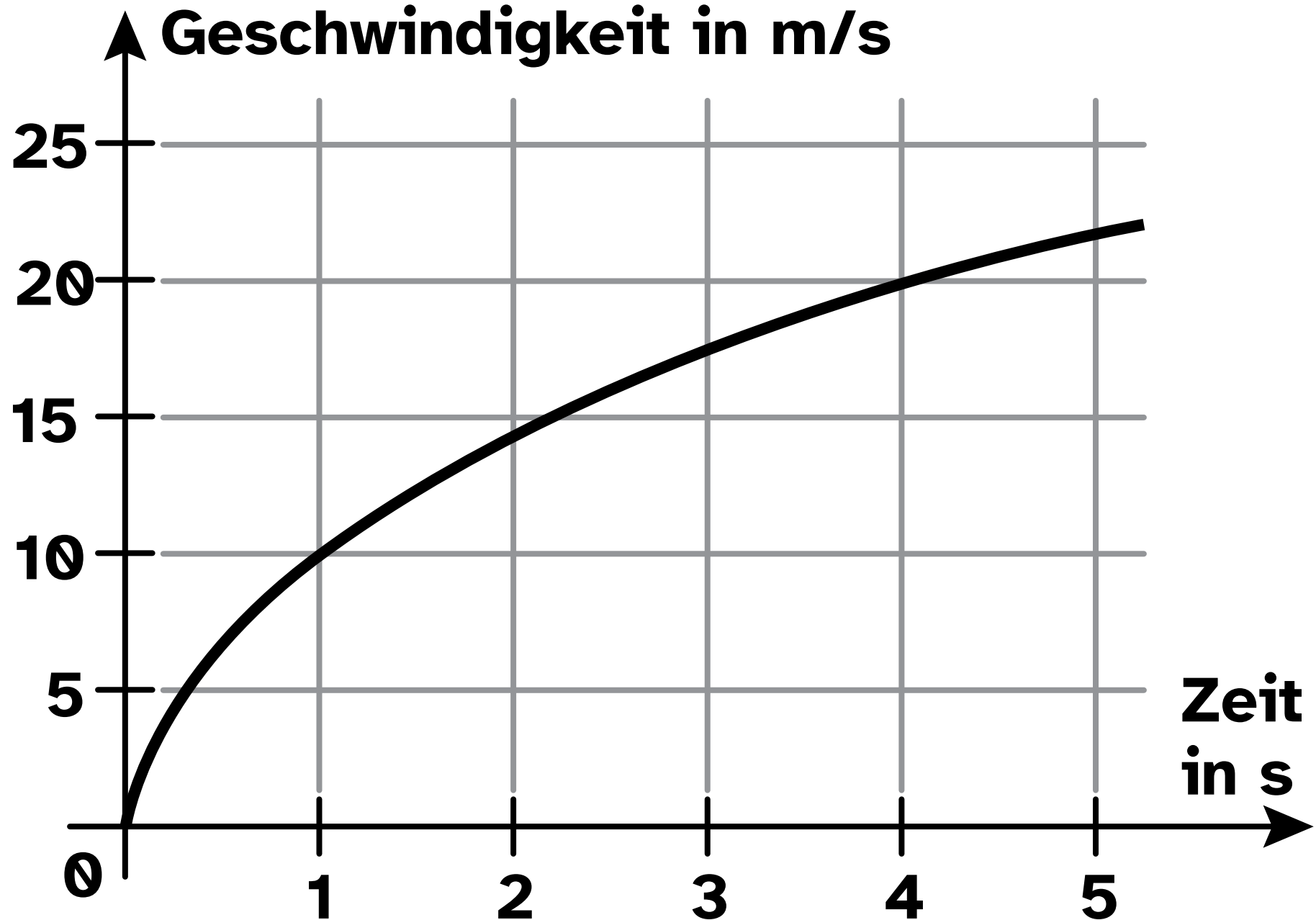
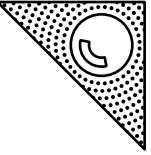


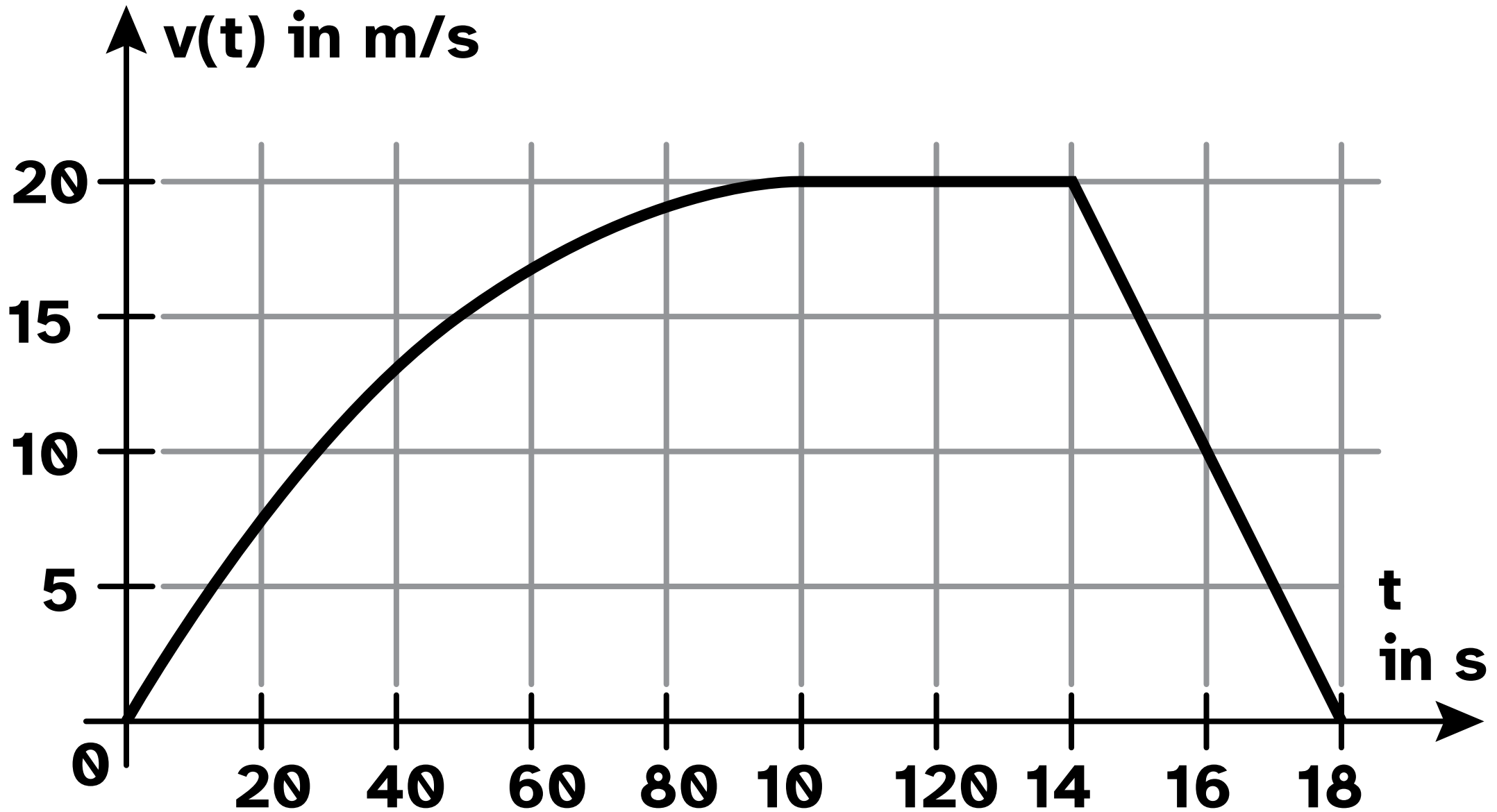
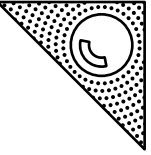
 $\int_0^{300} v(t) dt$

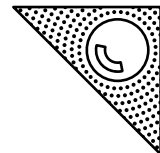










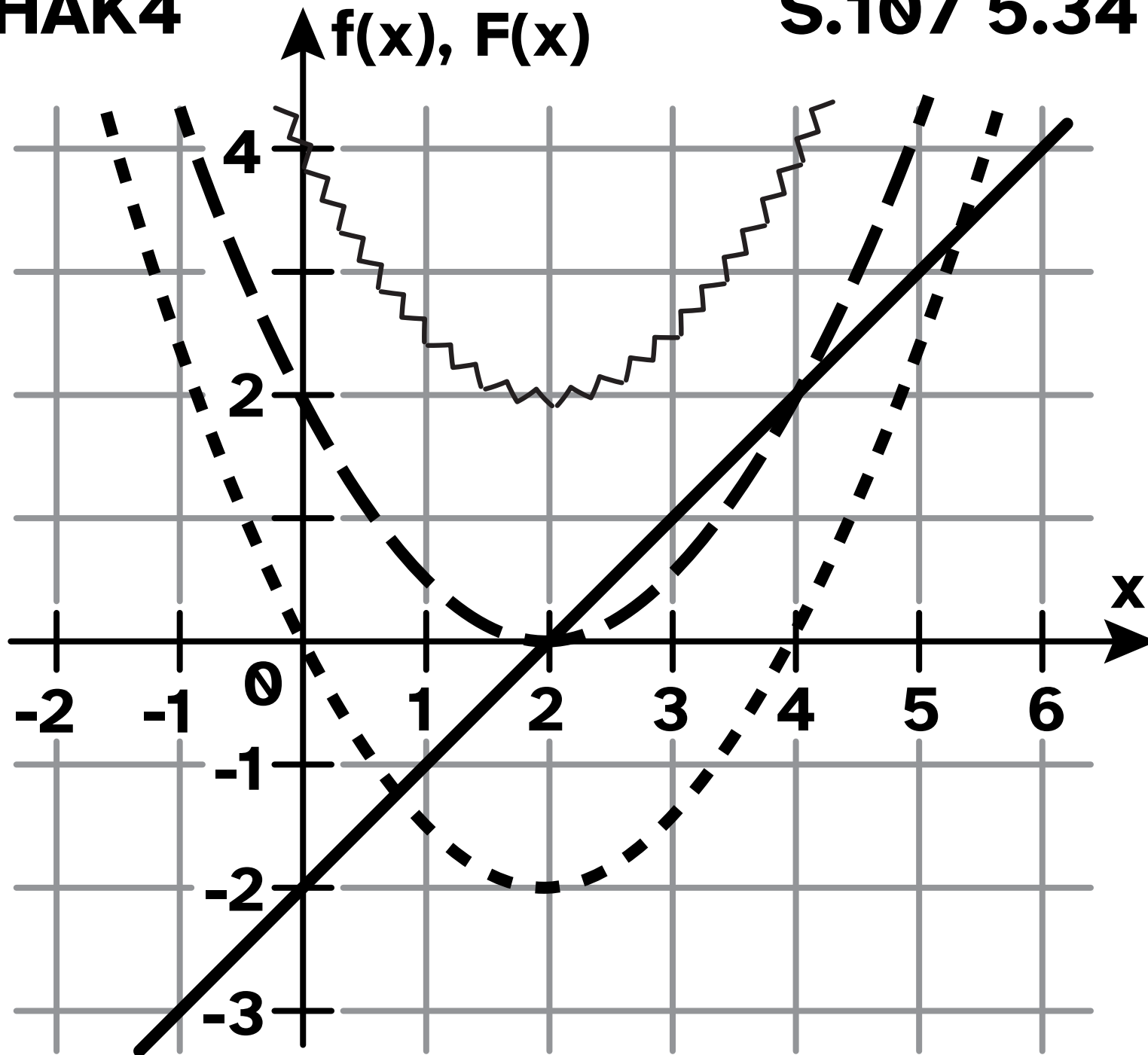


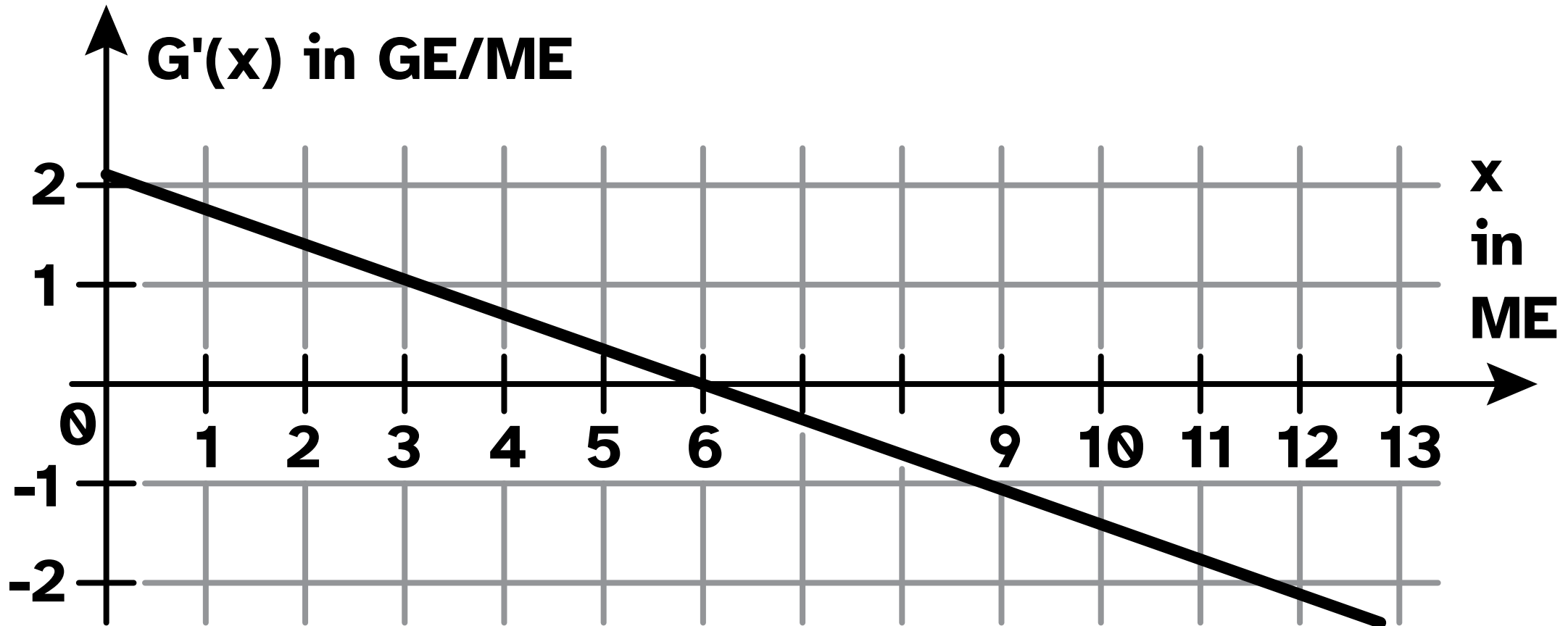
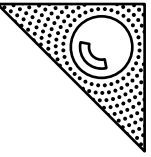


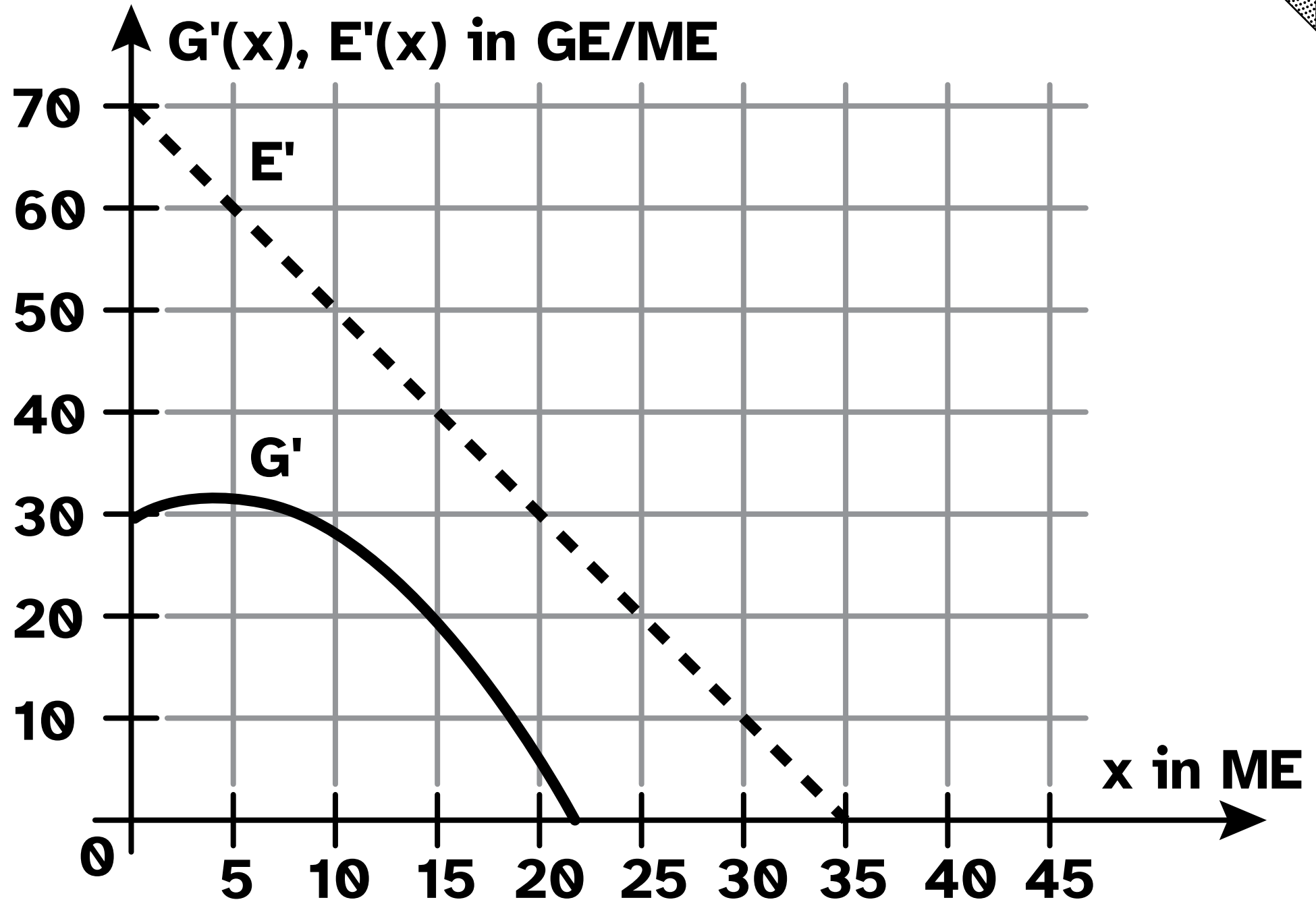
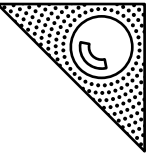


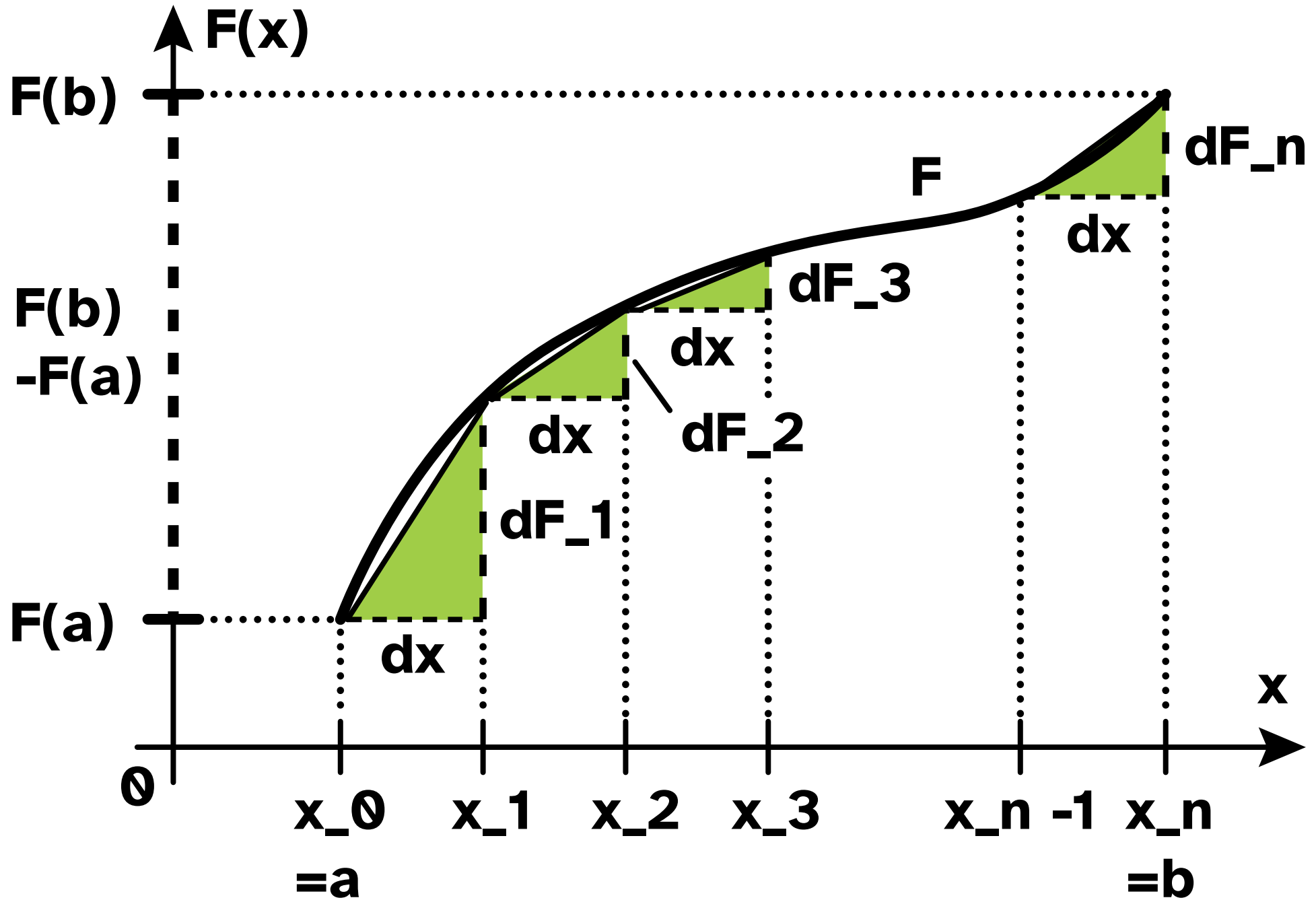
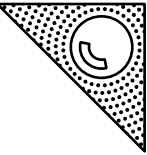
Legende:

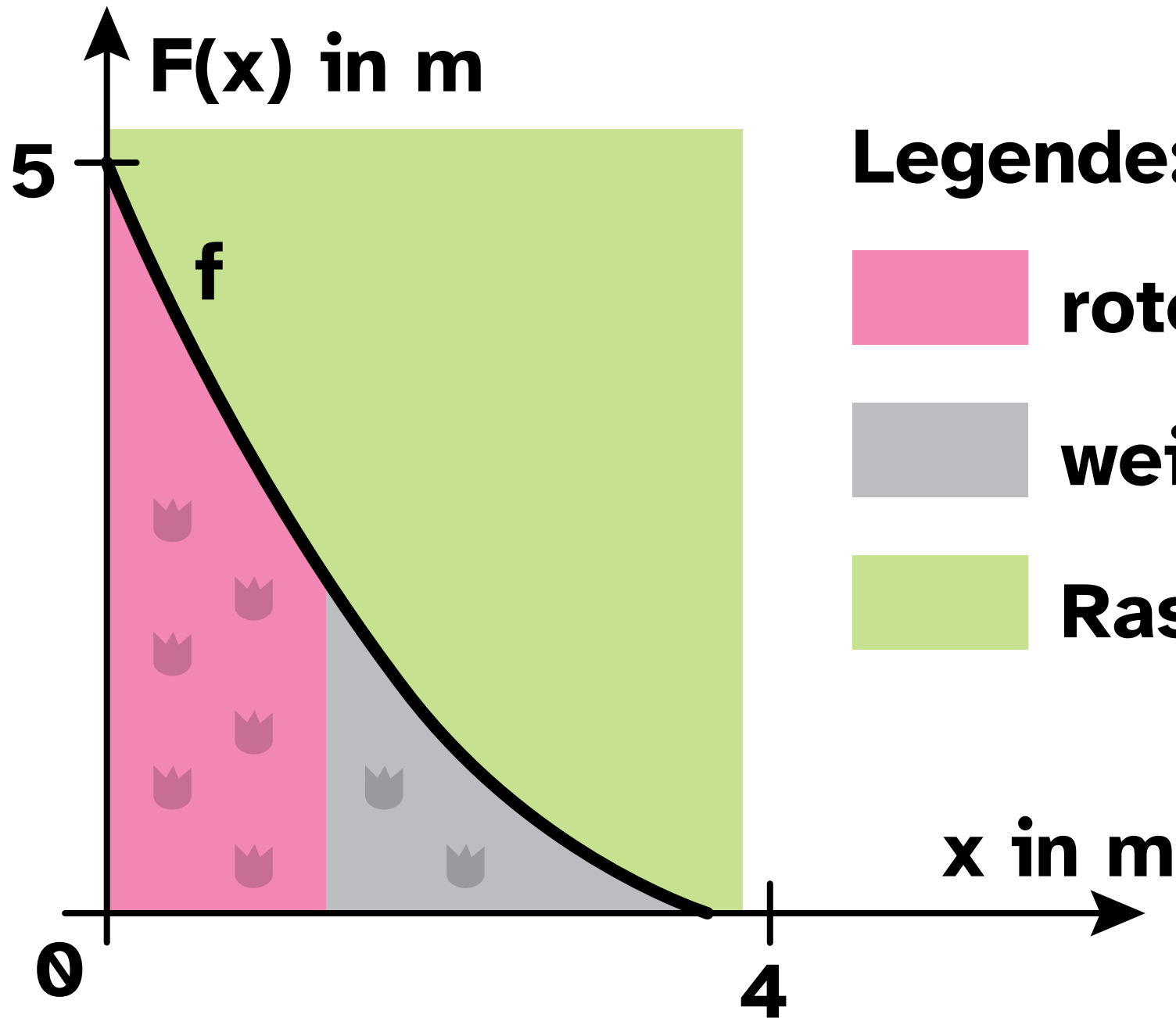
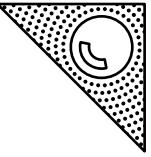
-  **f**
-  **F_1**
-  **F_2**
-  **F_3**











Legende:



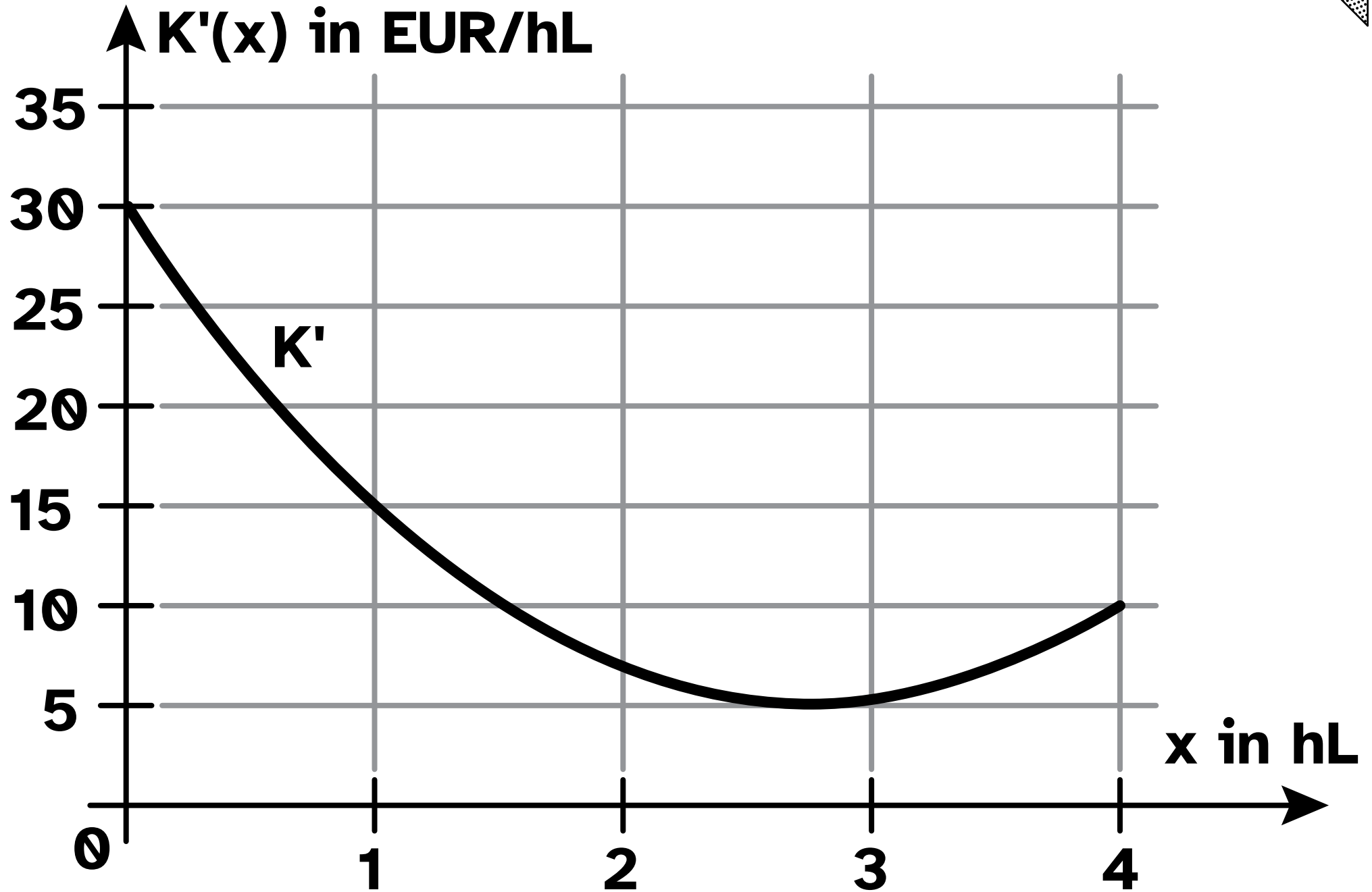
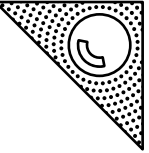
rote Tulpen

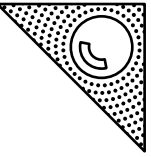


weiße Tulpen

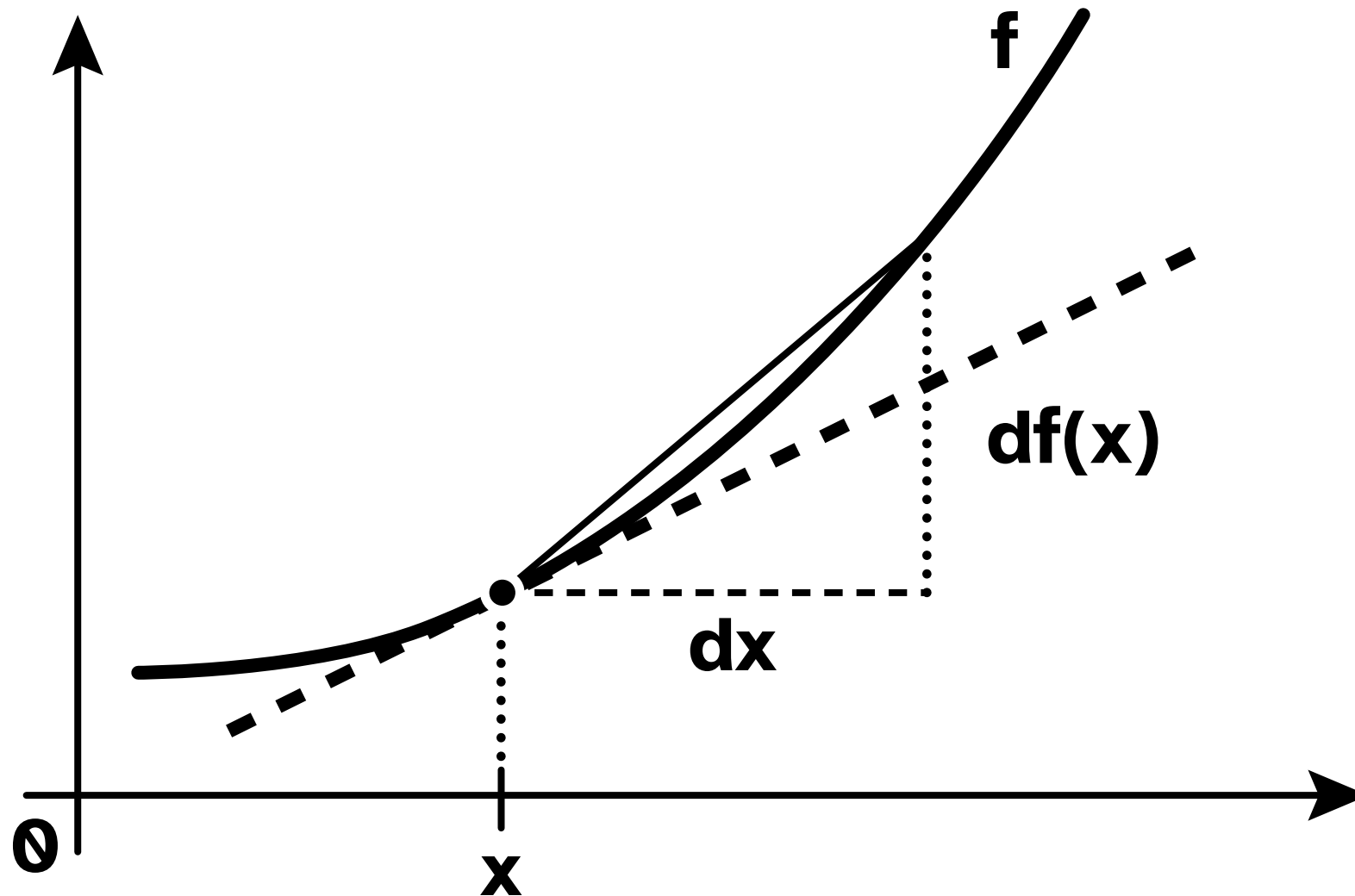


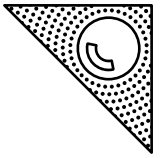
Rasen





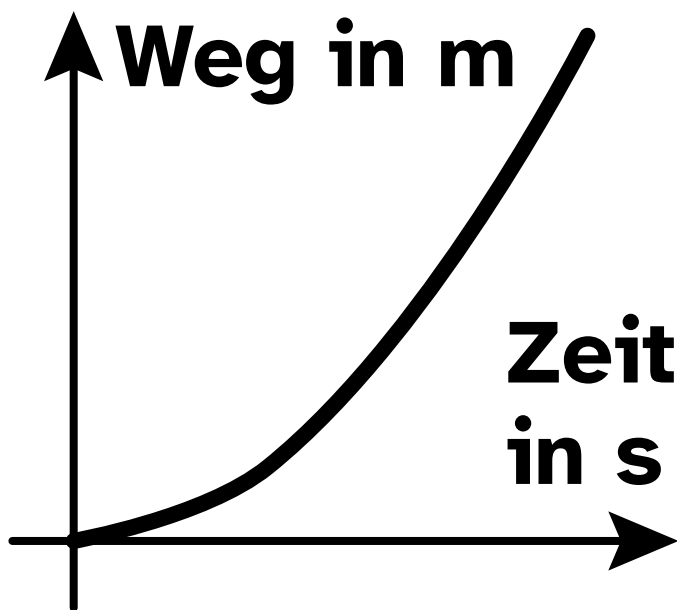
Differentialrechnung - Grundvorstellung: Steigung an der Stelle x





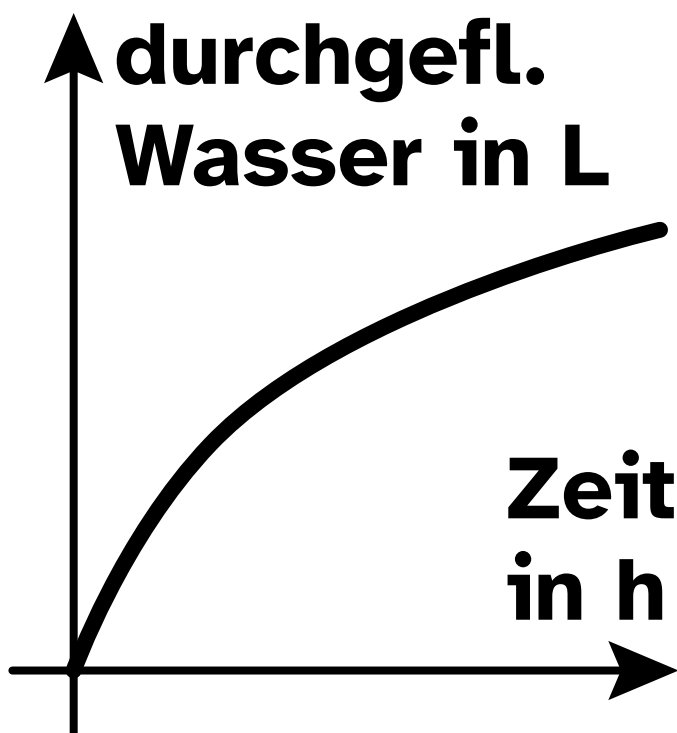
Differentialrechnung - Anwendungsbeispiele:

1) Weg -> Geschwindigkeit

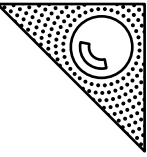


Weg in m
/Zeit in s
=Geschwindigkeit
in m/s

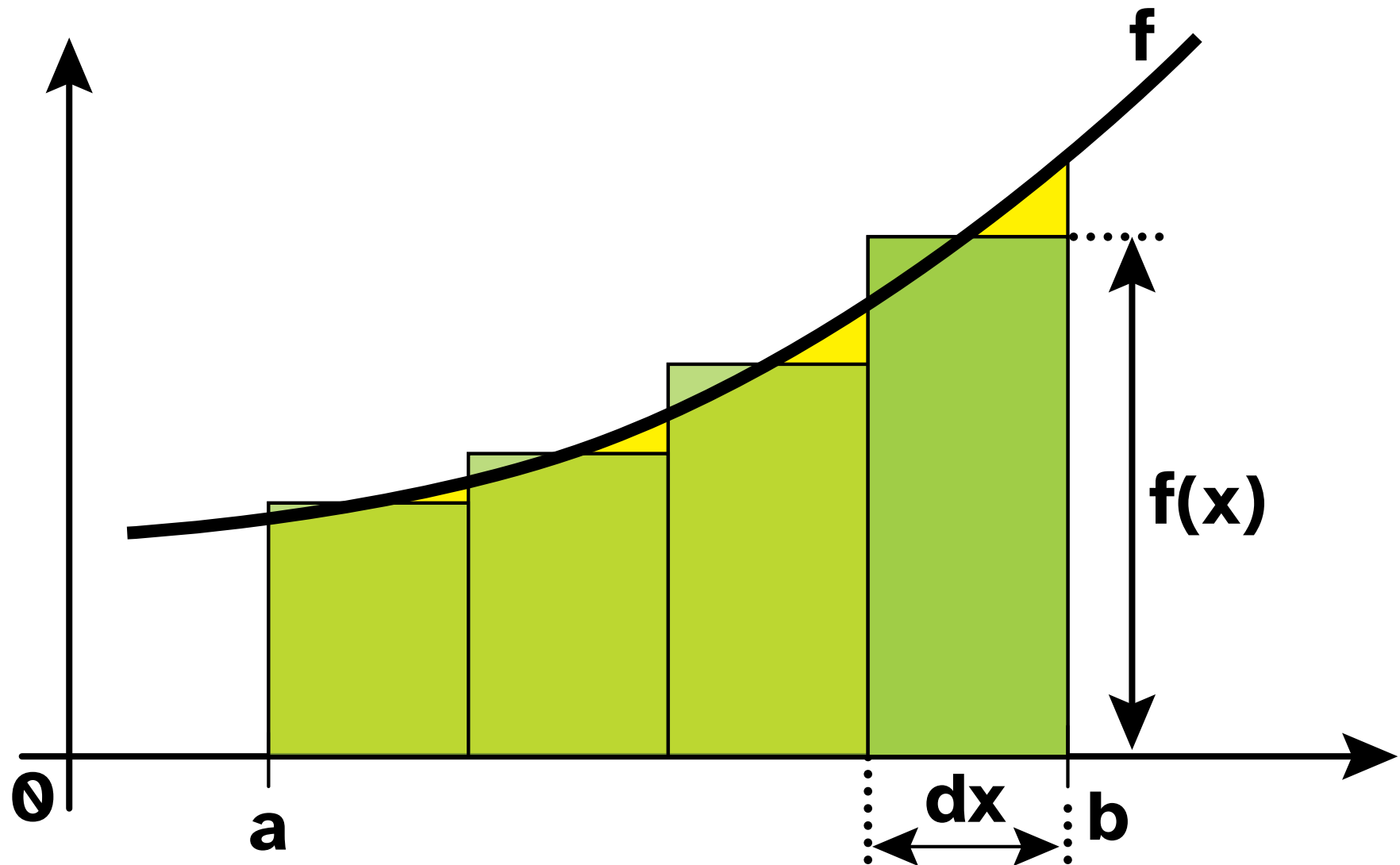
2) Bestand -> Änderungsrate

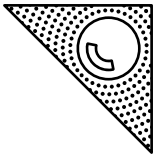


durchgefl.
Wasser Volu-
men in L /Zeit
in h =Durch-
flussrate in
L/h



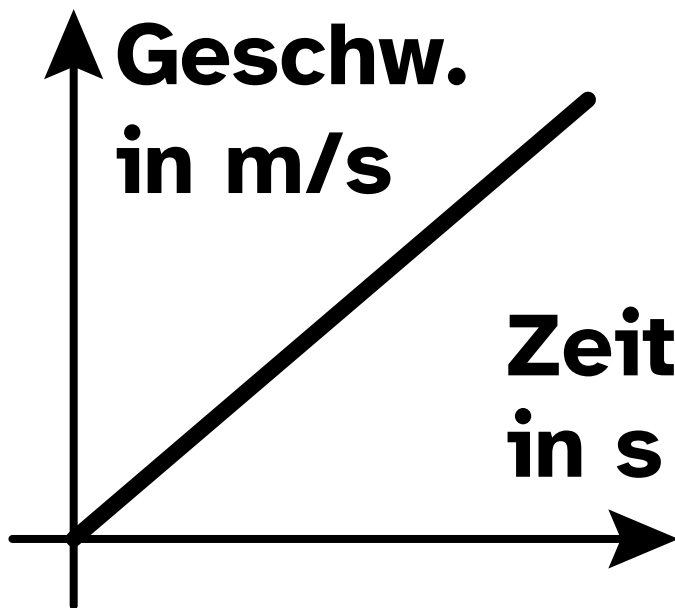
Integralrechnung - Grundvorstellung: Flächeninhalt im Intervall $[a; b]$





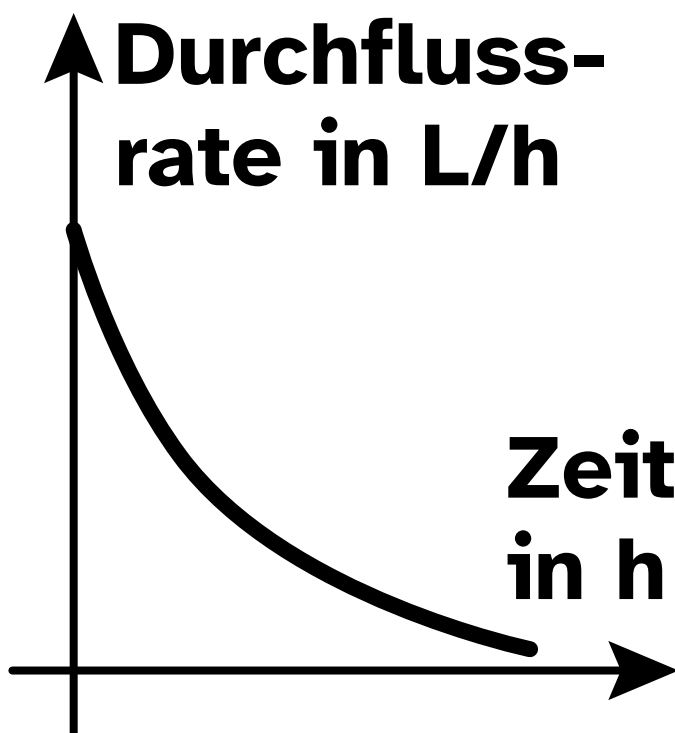
Integralrechnung - Anwendungsbeispiele:

1) Geschwindigkeit -> Weg

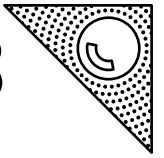


Geschwindigkeit * Zeit in
 $m/s * s = \text{Weg}$
in m

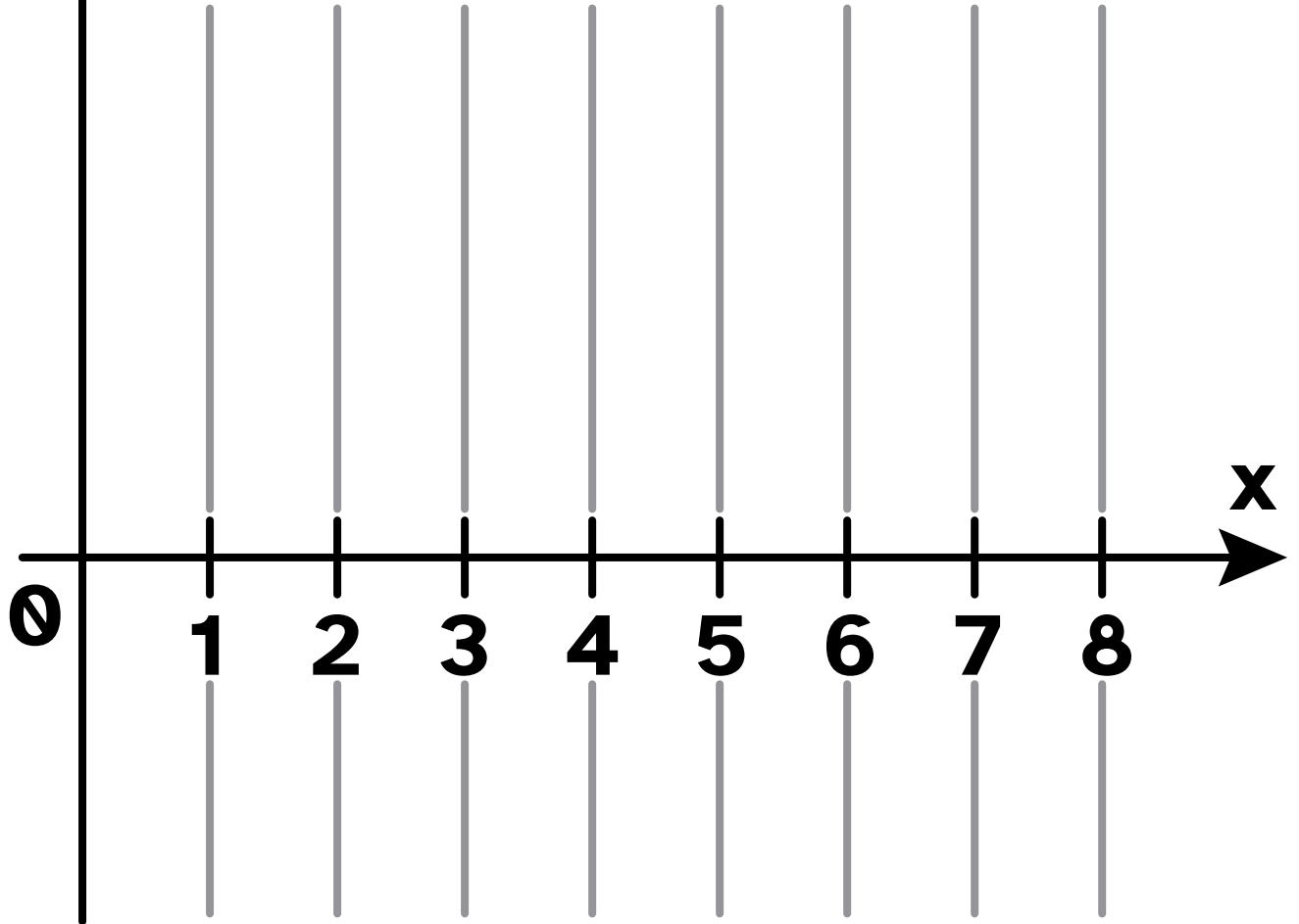
2) Änderungsrate -> Bestand



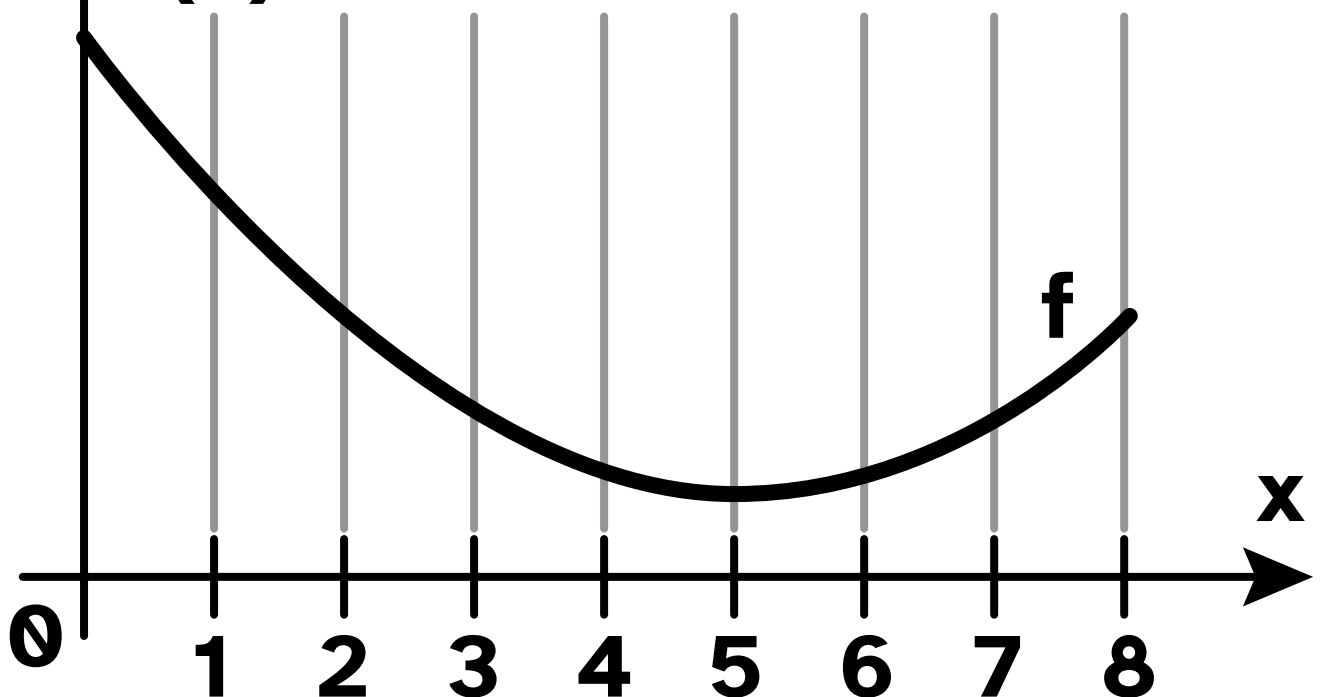
Durchflussrate * Zeit in $L/h * h$
 $= \text{durchgeflossenes Volumen}$
in L

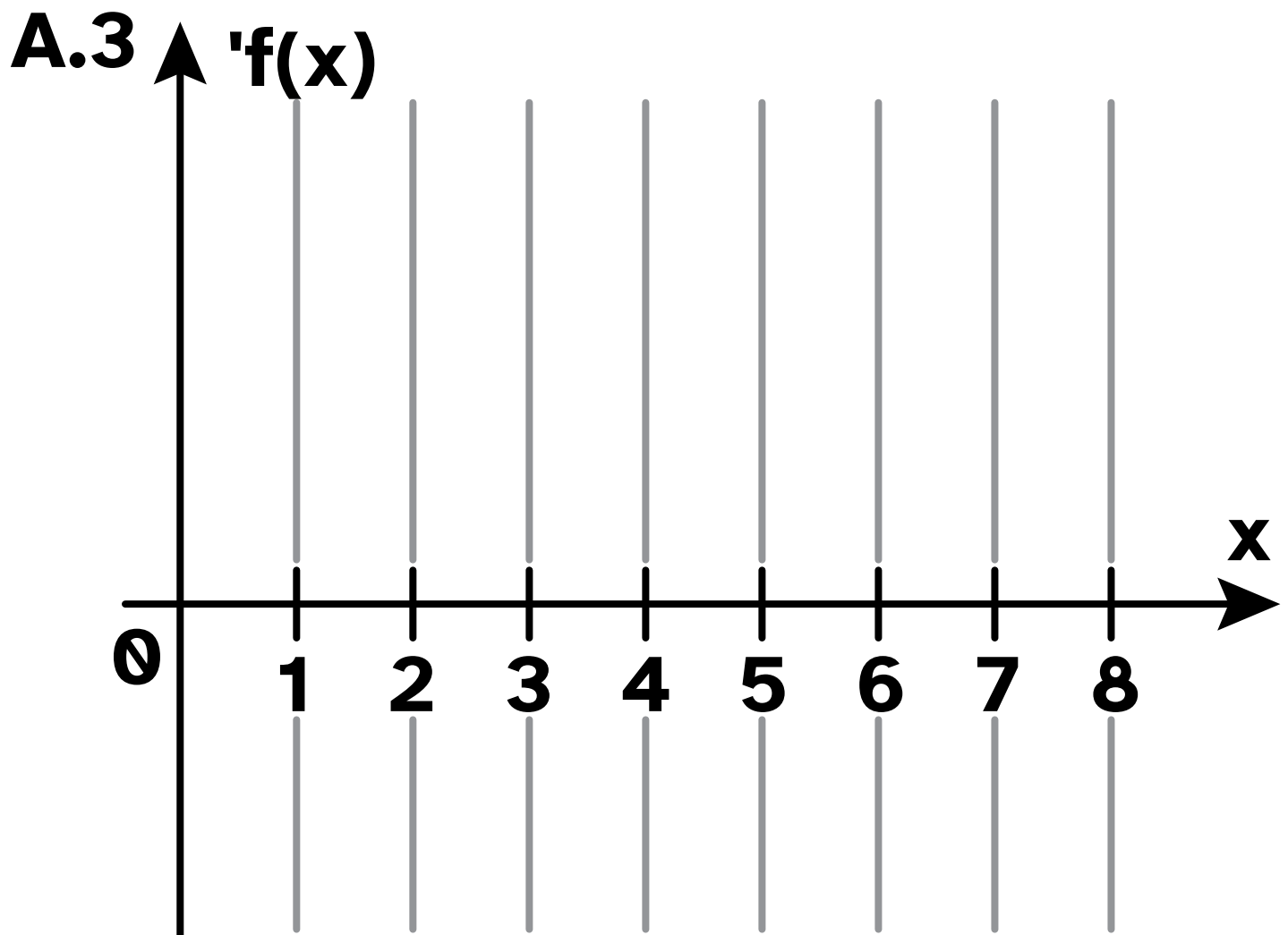
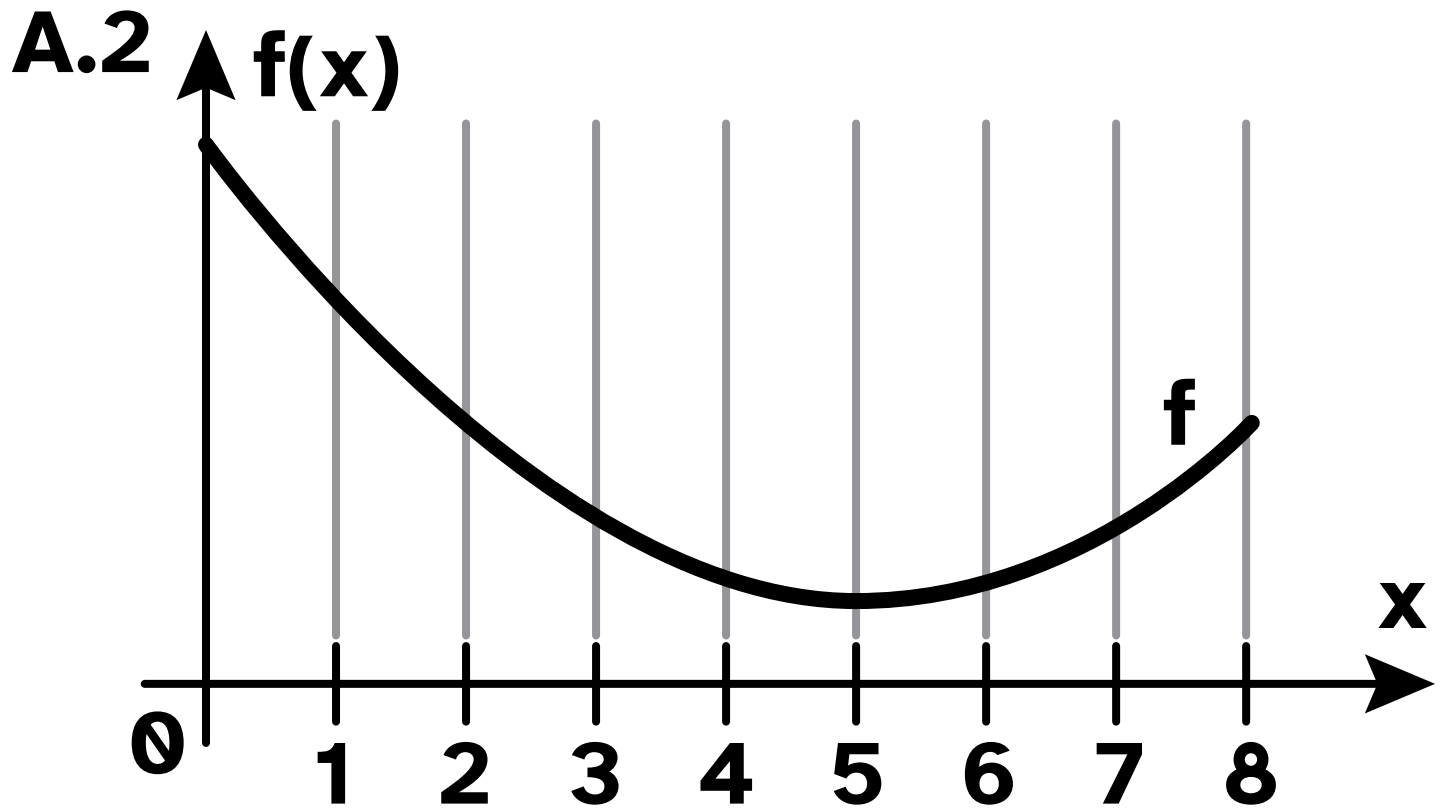
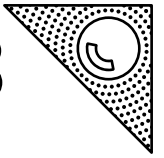


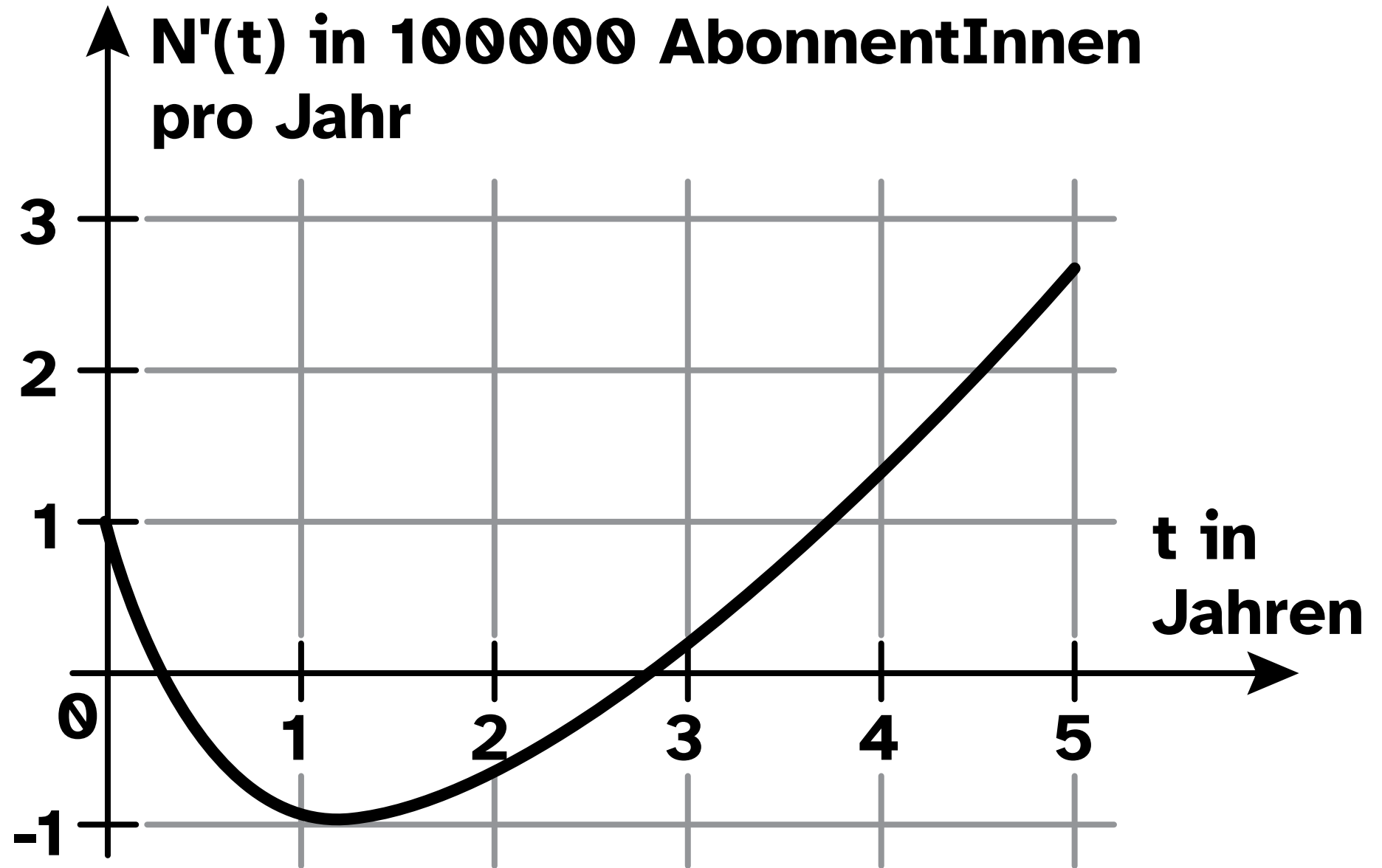
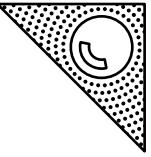
A.1 \uparrow $F_1(x), F_2(x)$

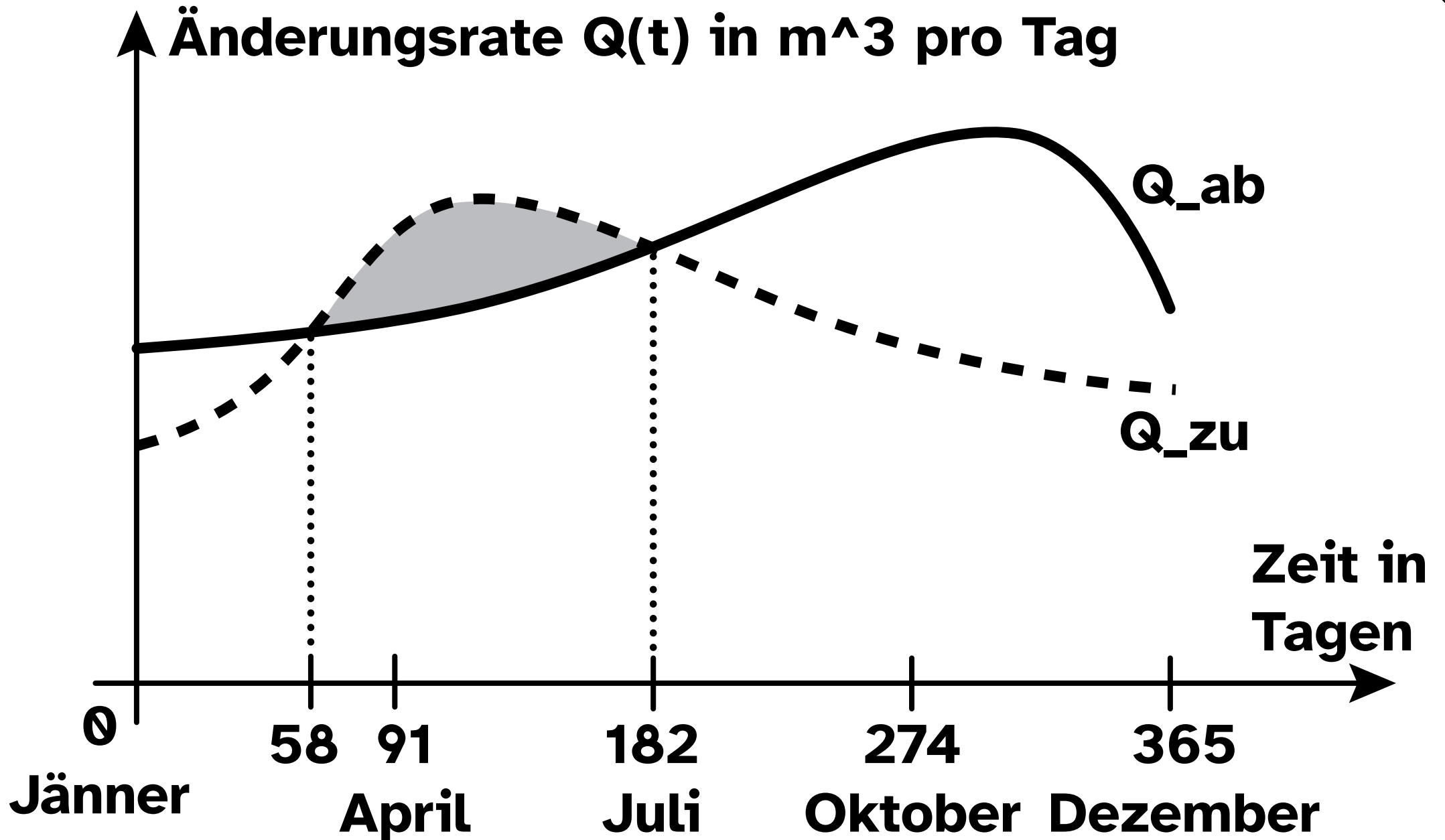
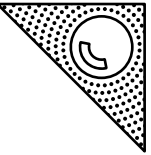


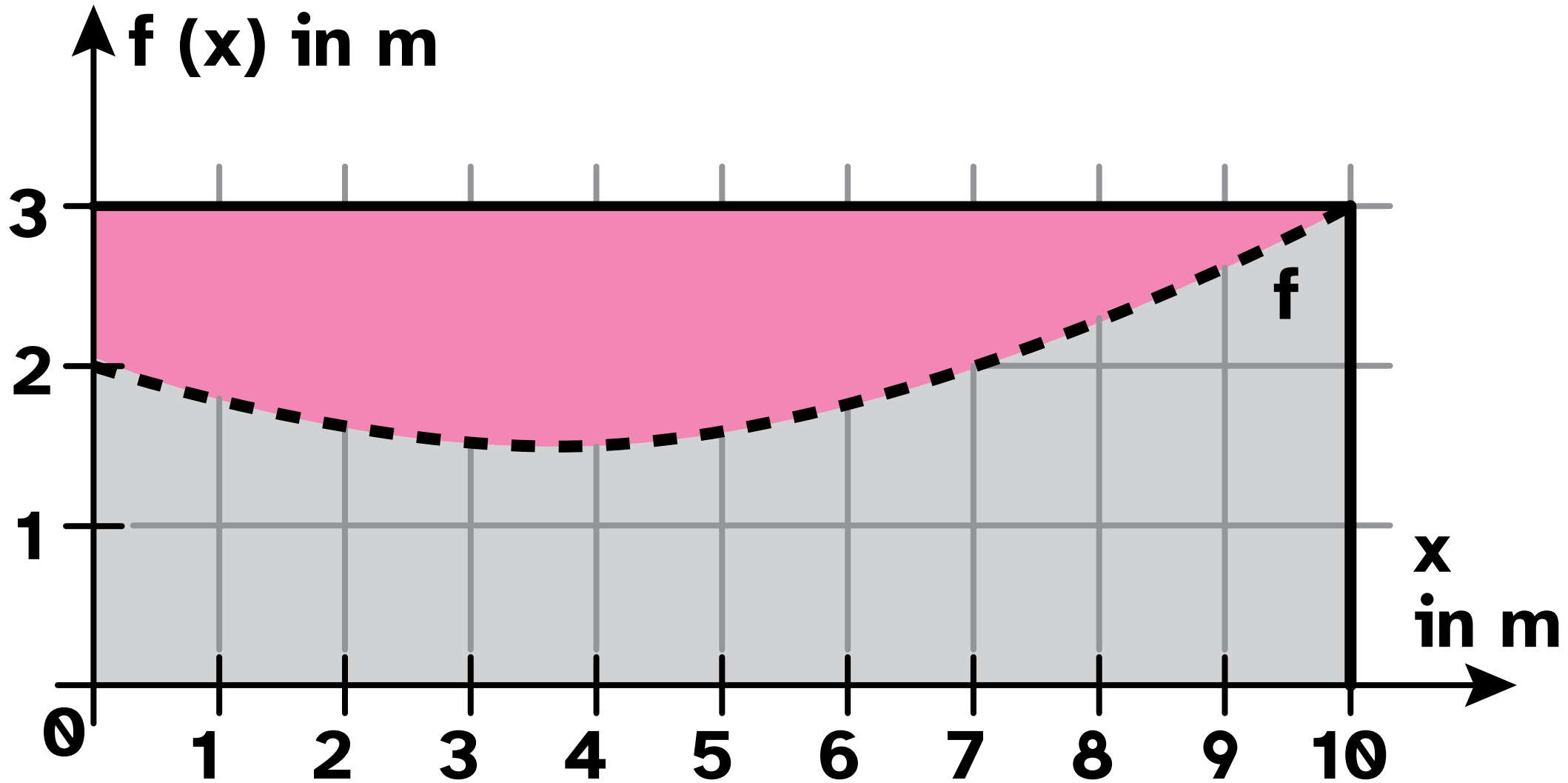
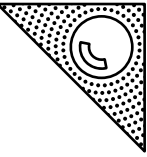
A.2 \uparrow $f(x)$

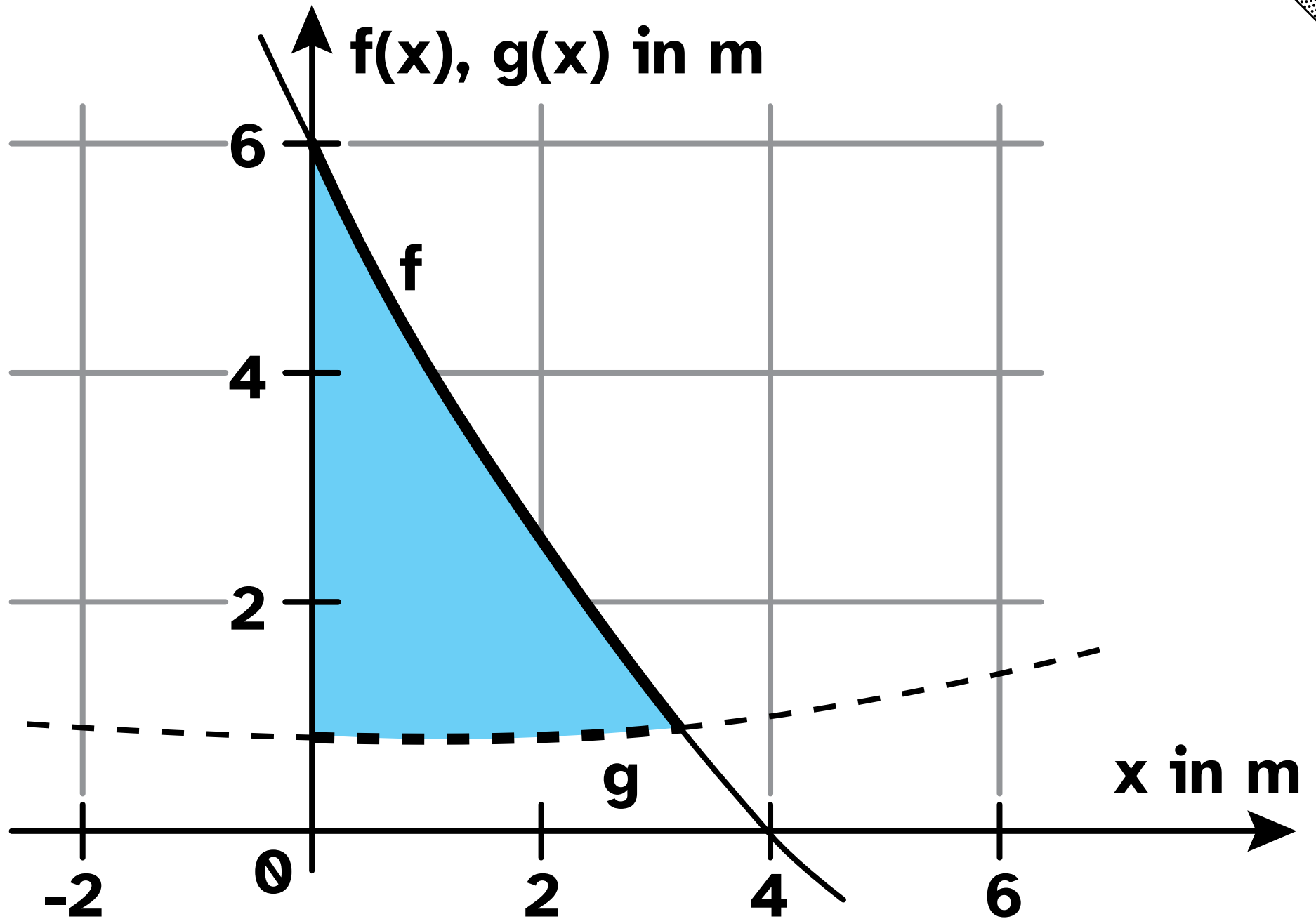
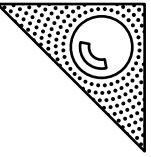


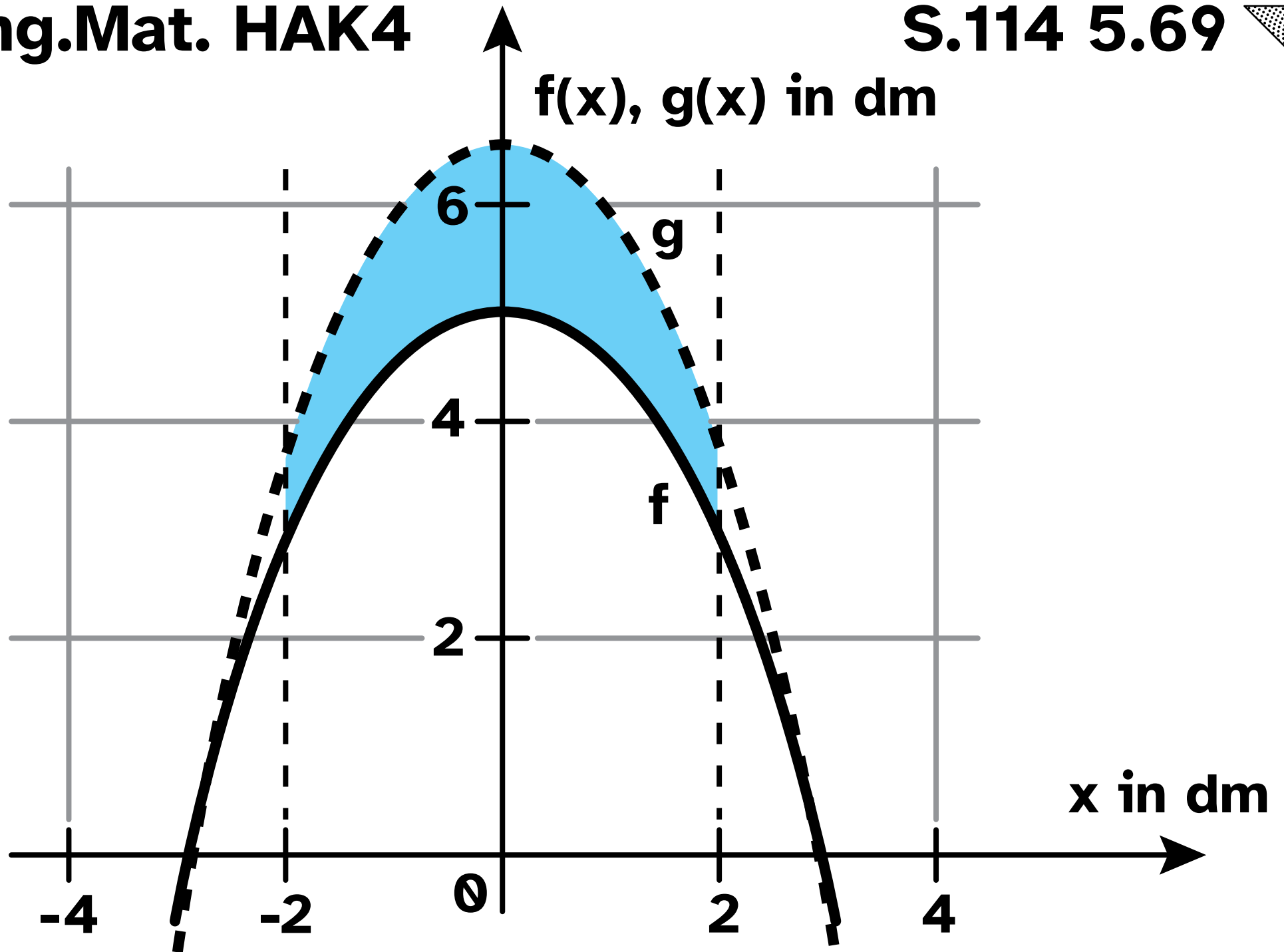
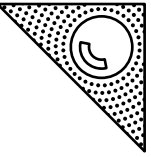


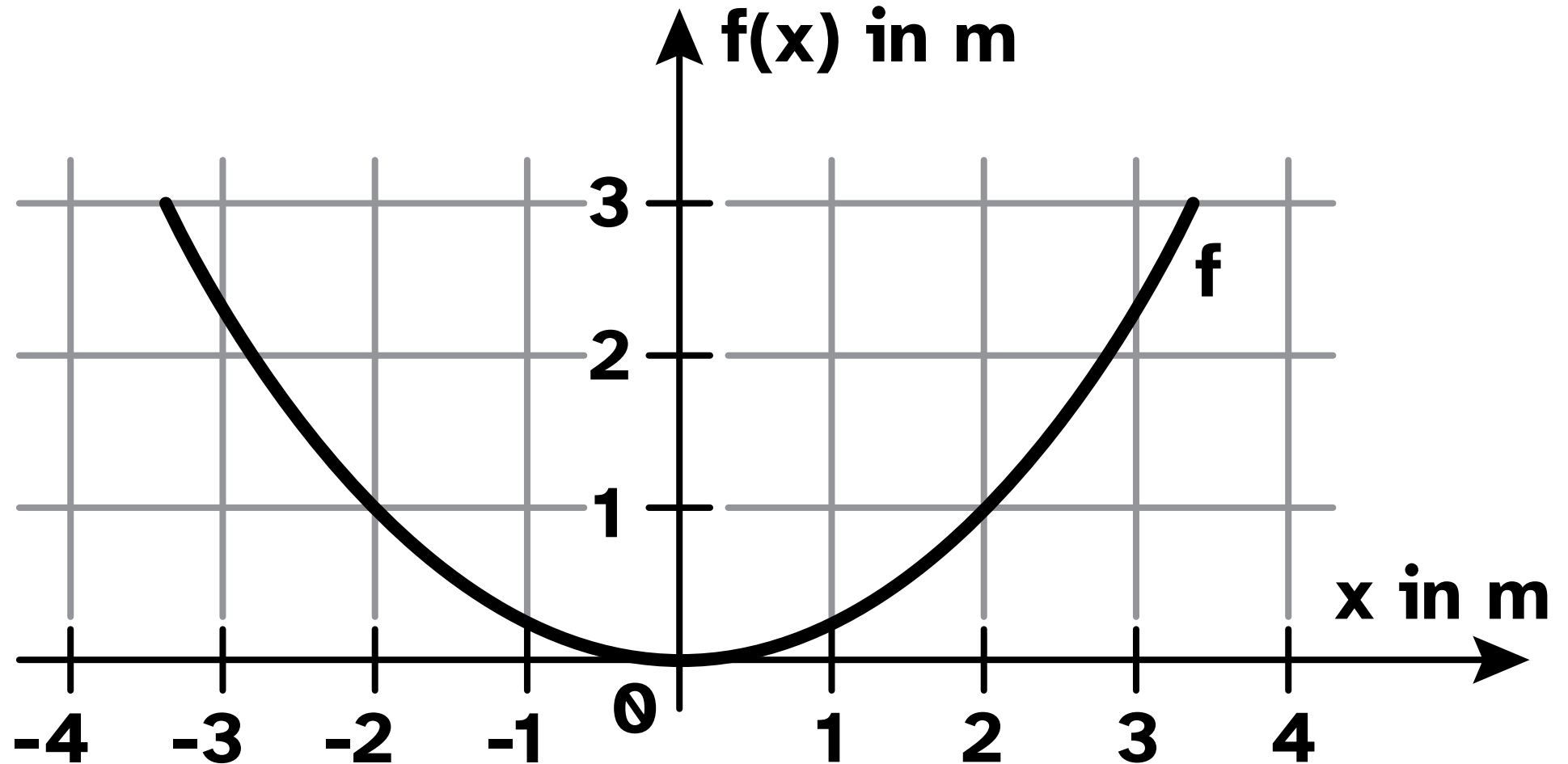
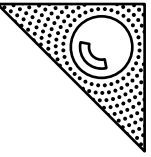


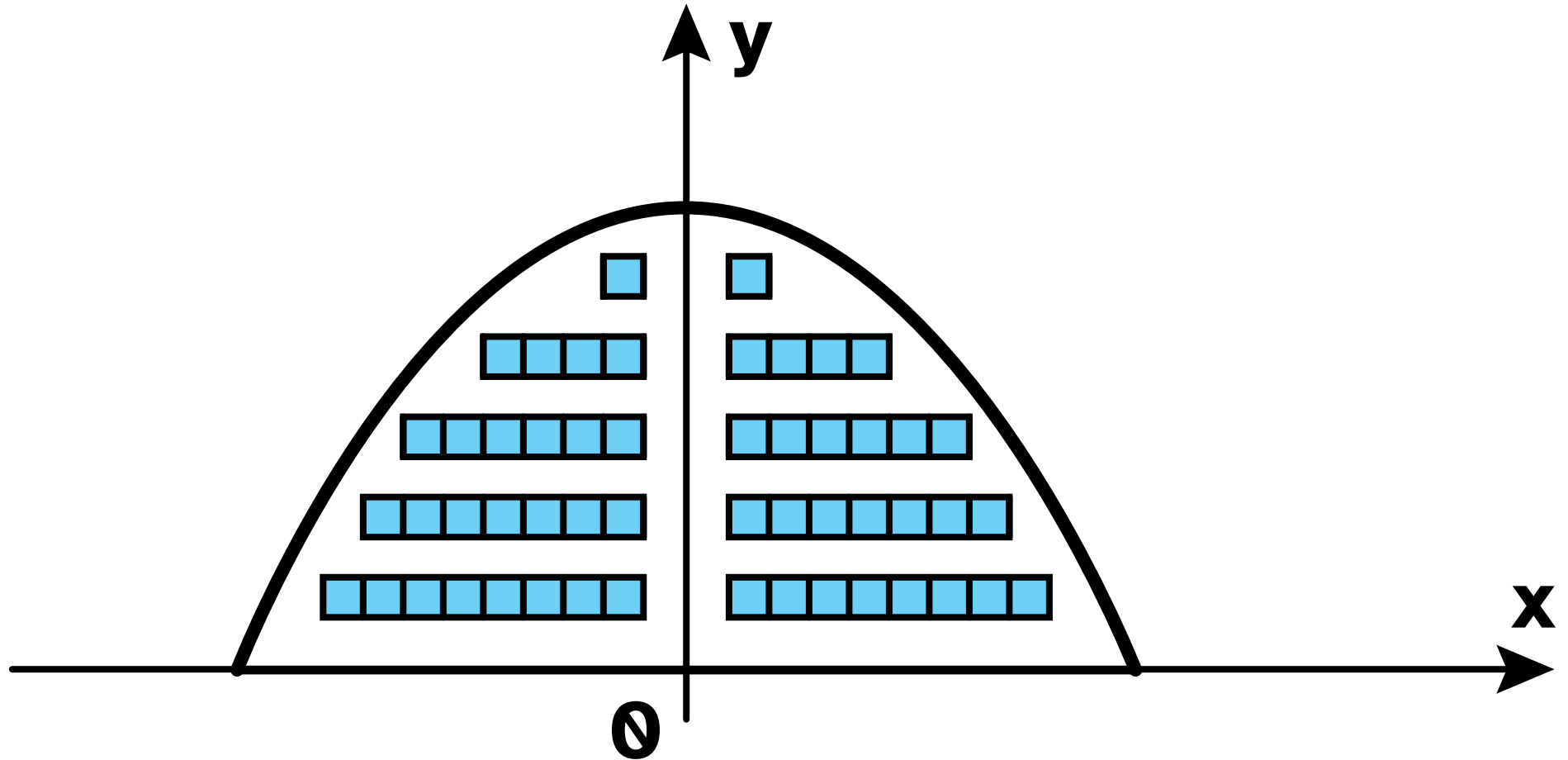
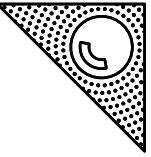


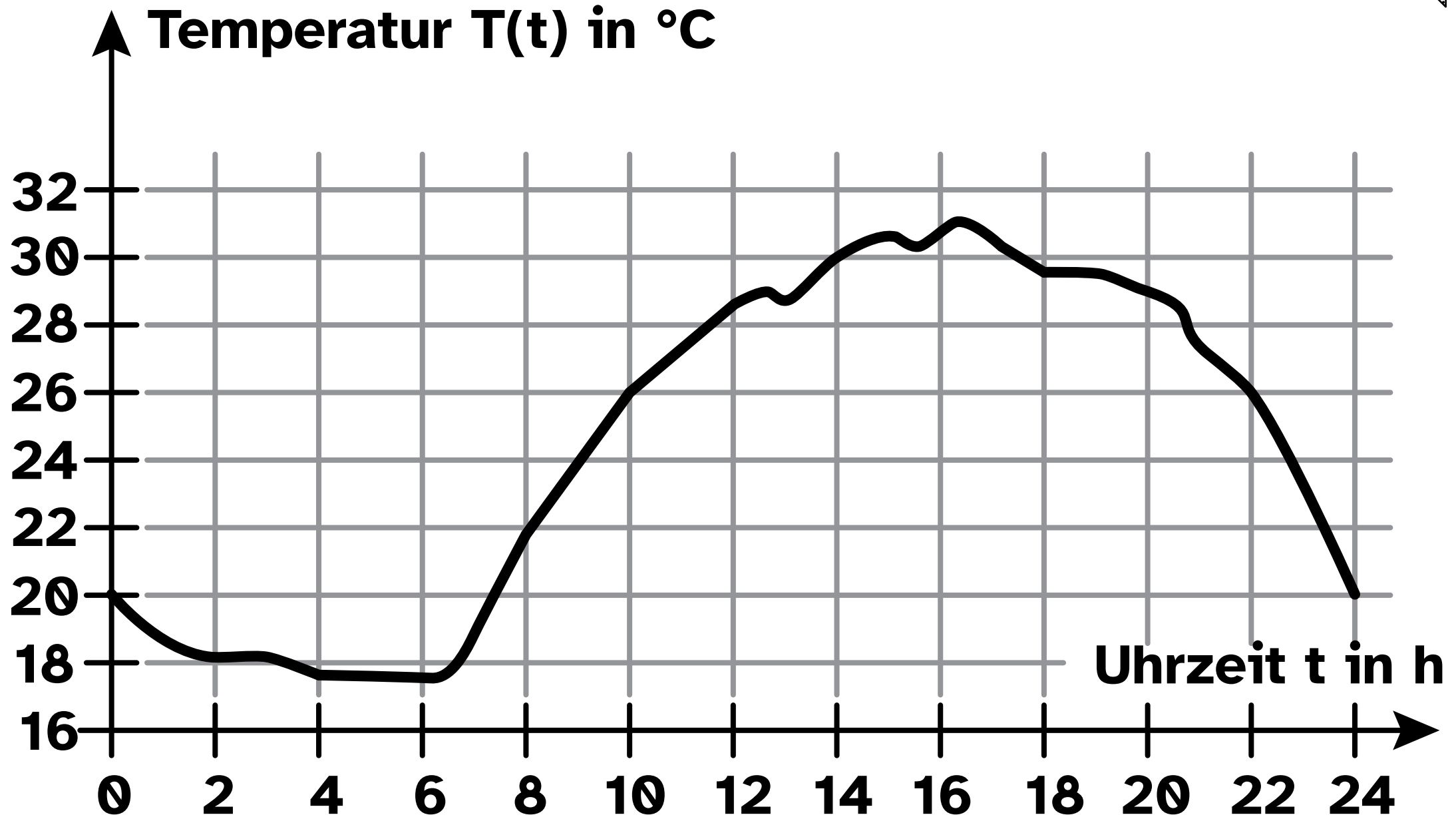
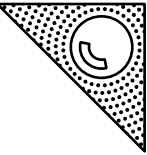


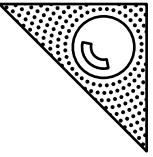




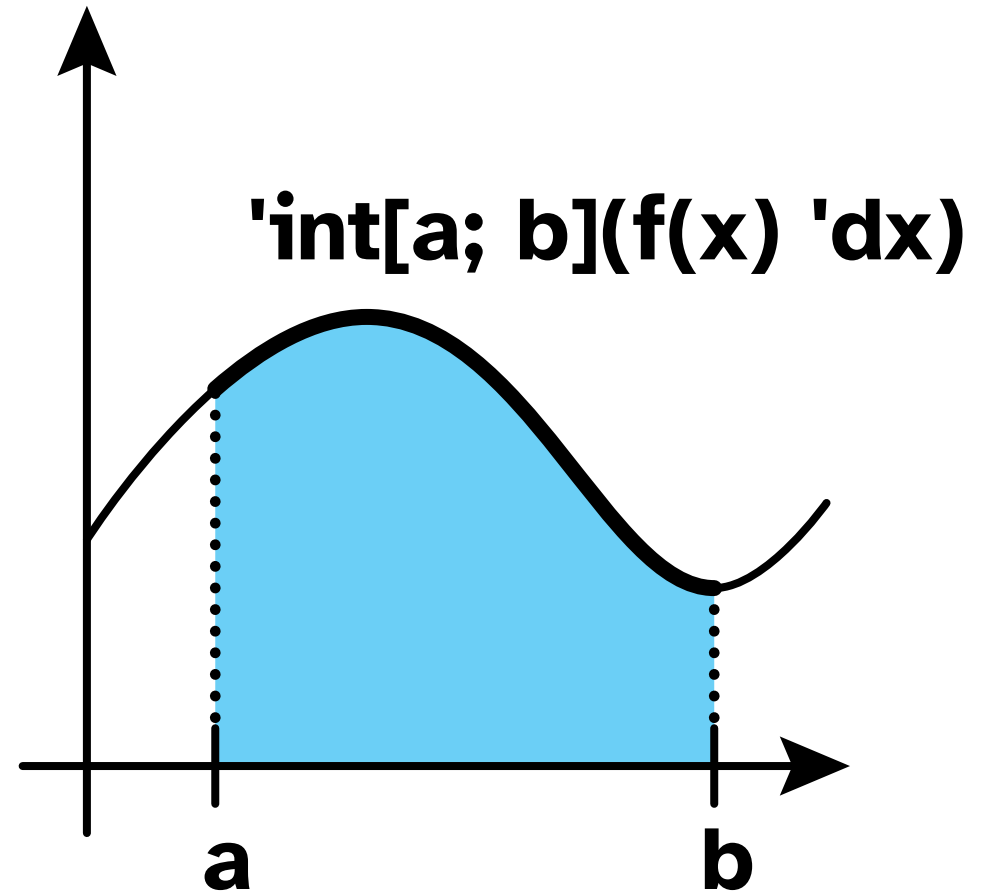
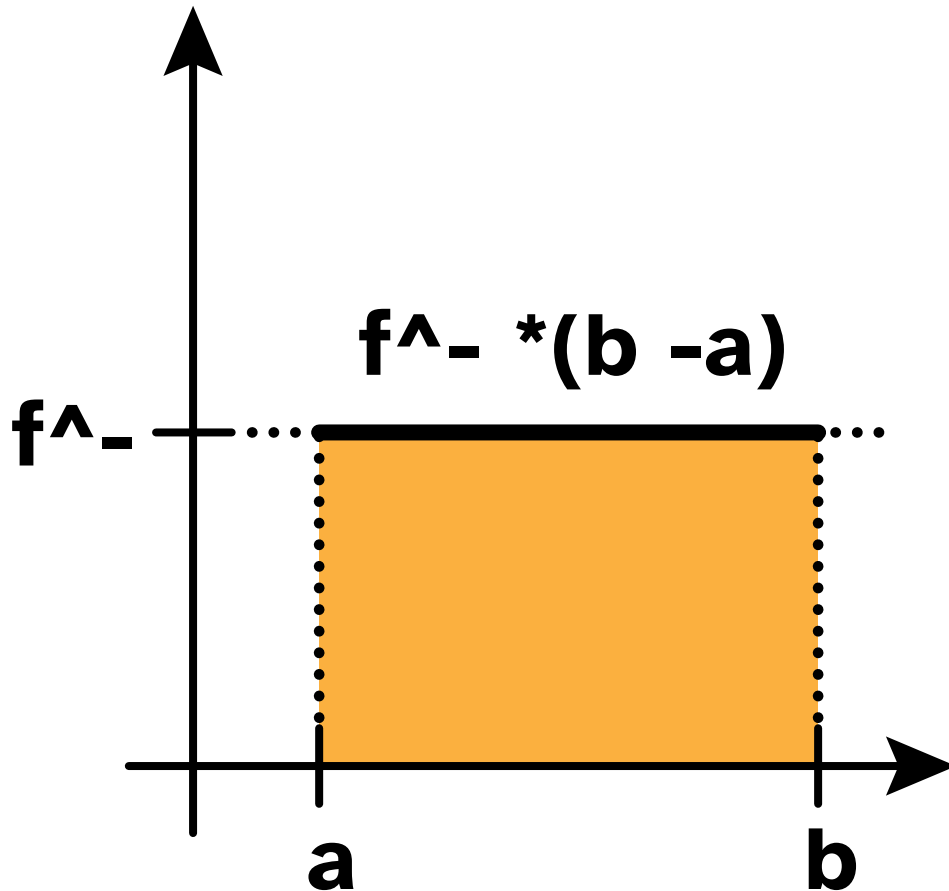




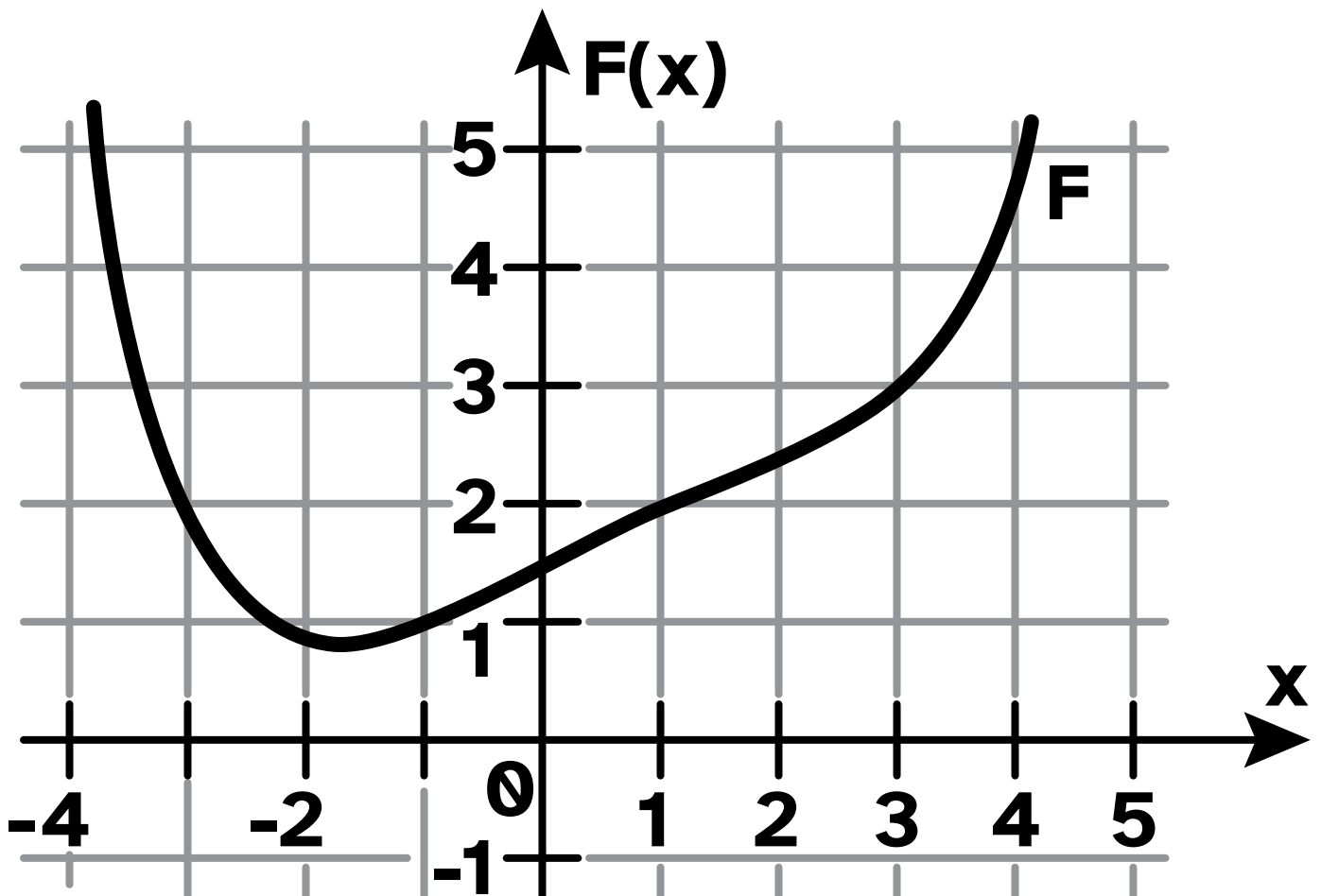
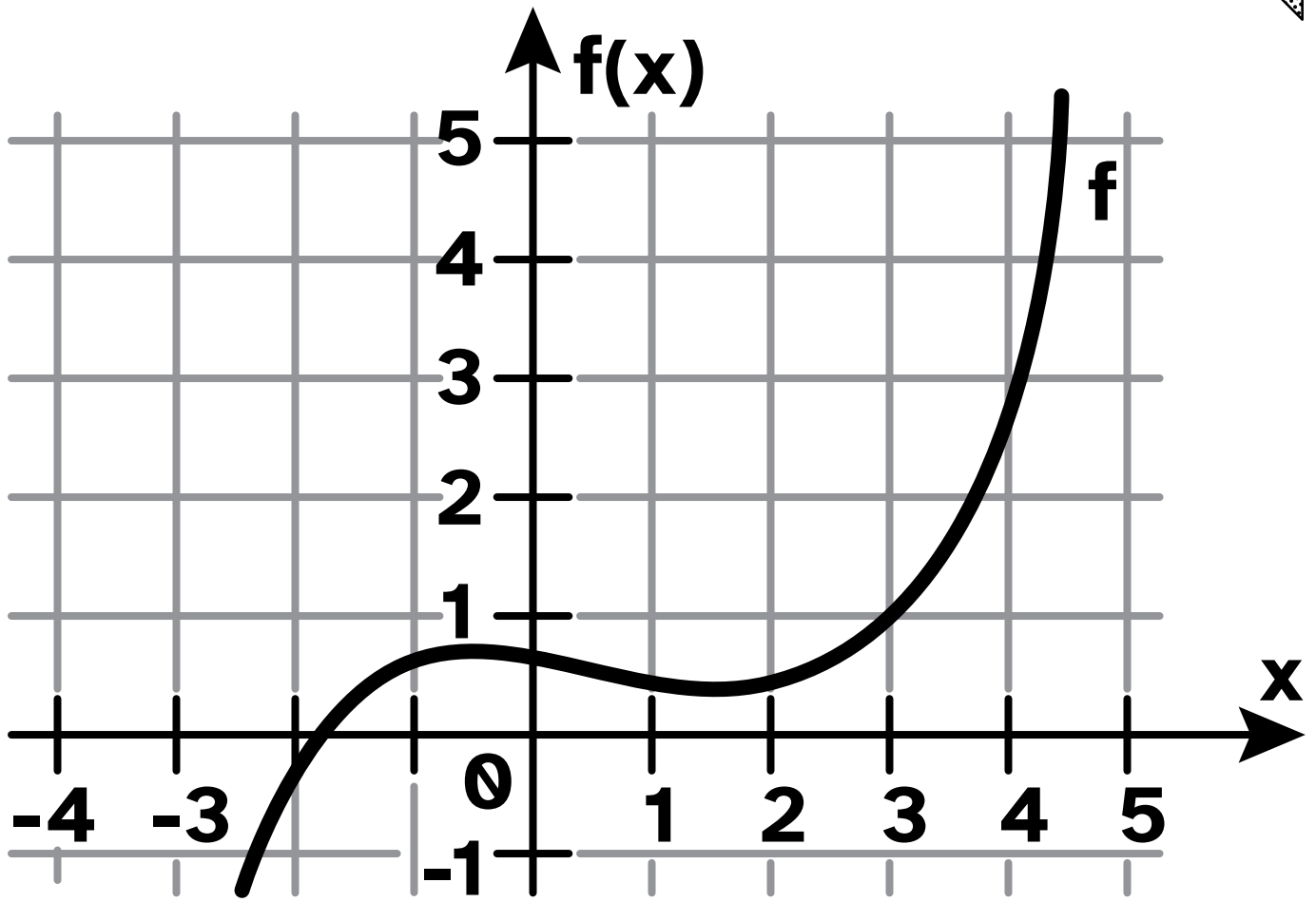
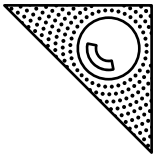


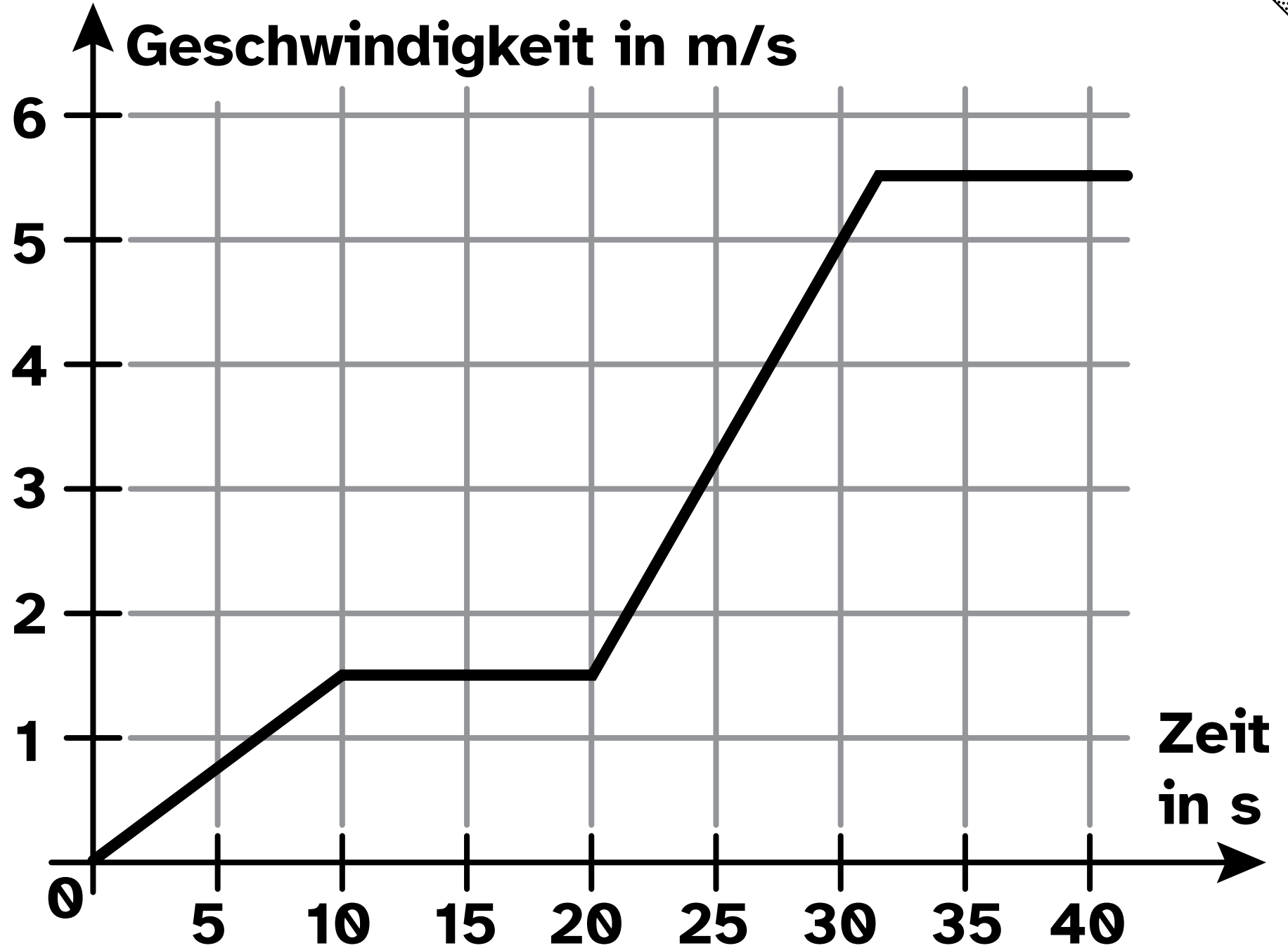
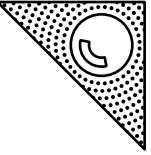


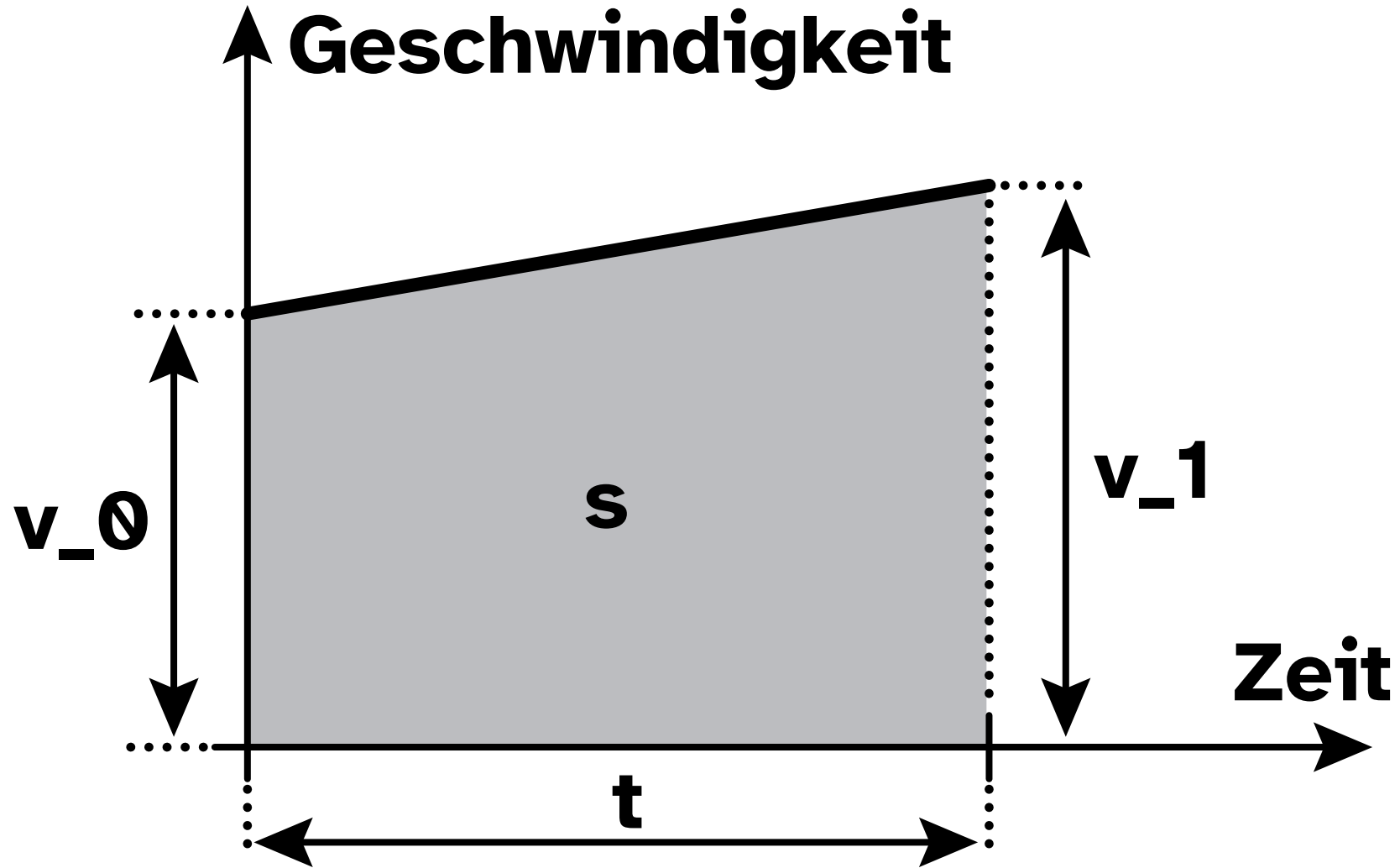
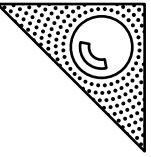
$$f^{\wedge} \cdot (b - a) = \int_a^b f(x) \, dx$$

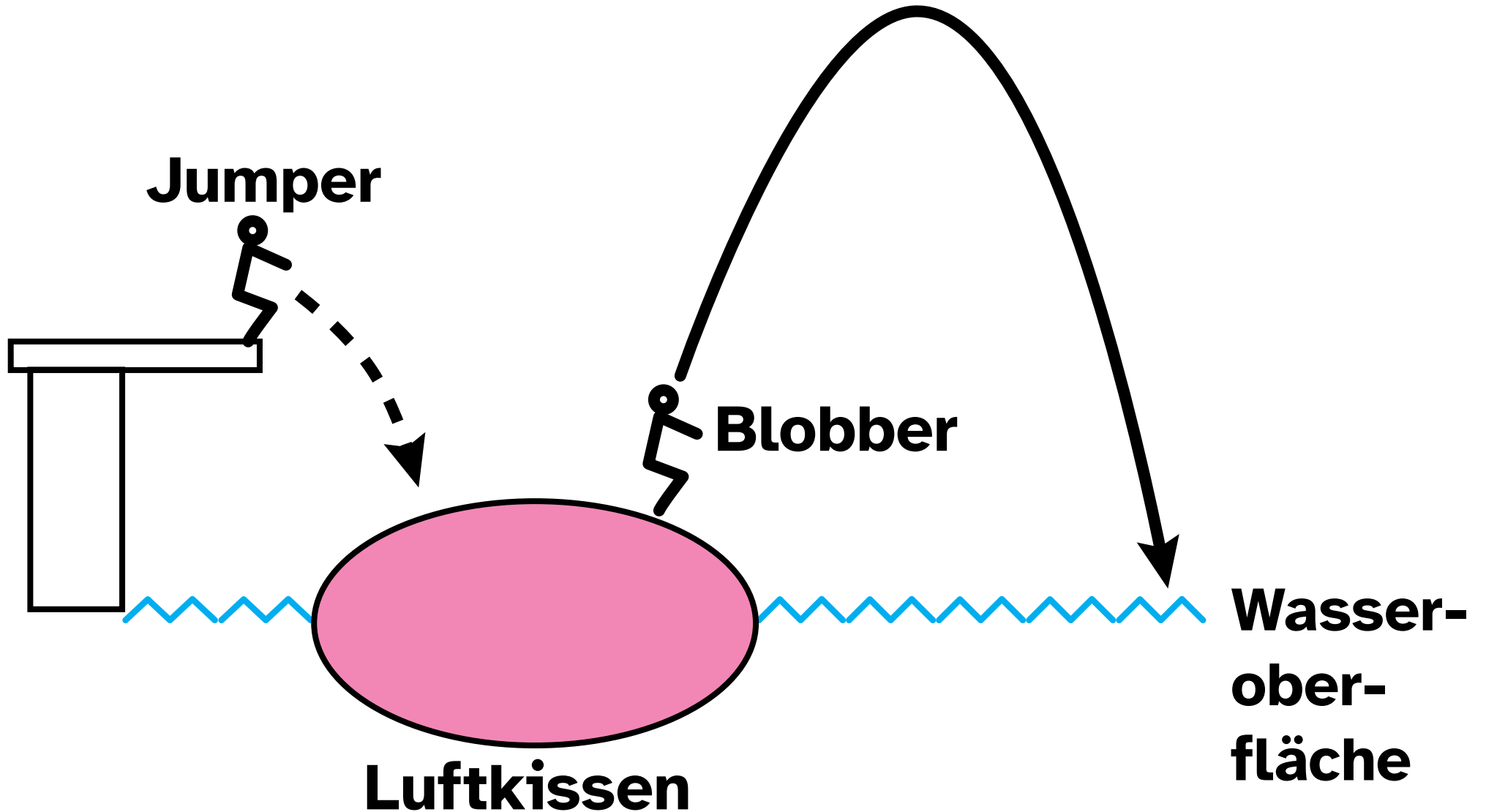
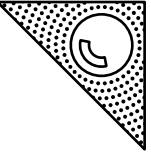


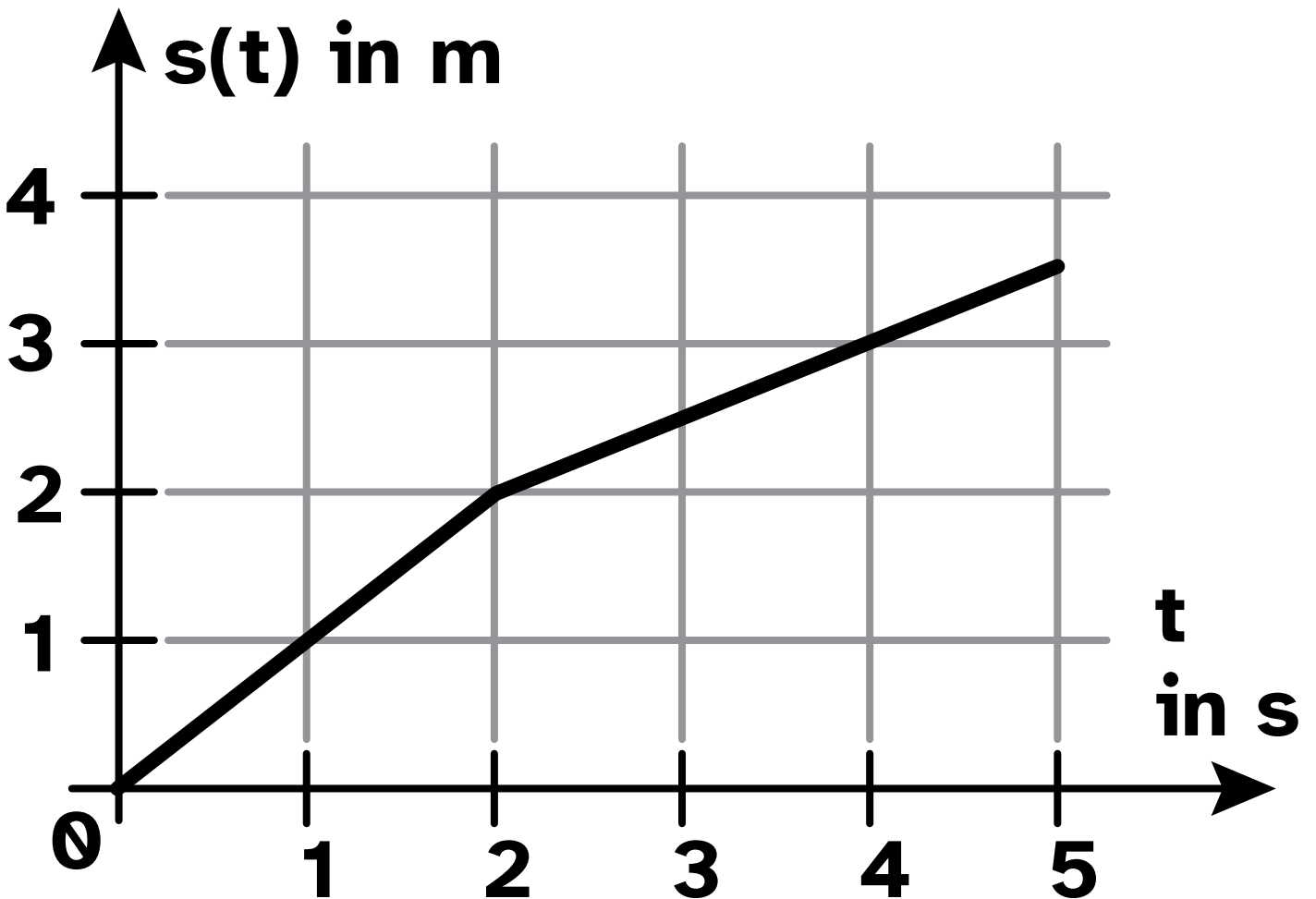
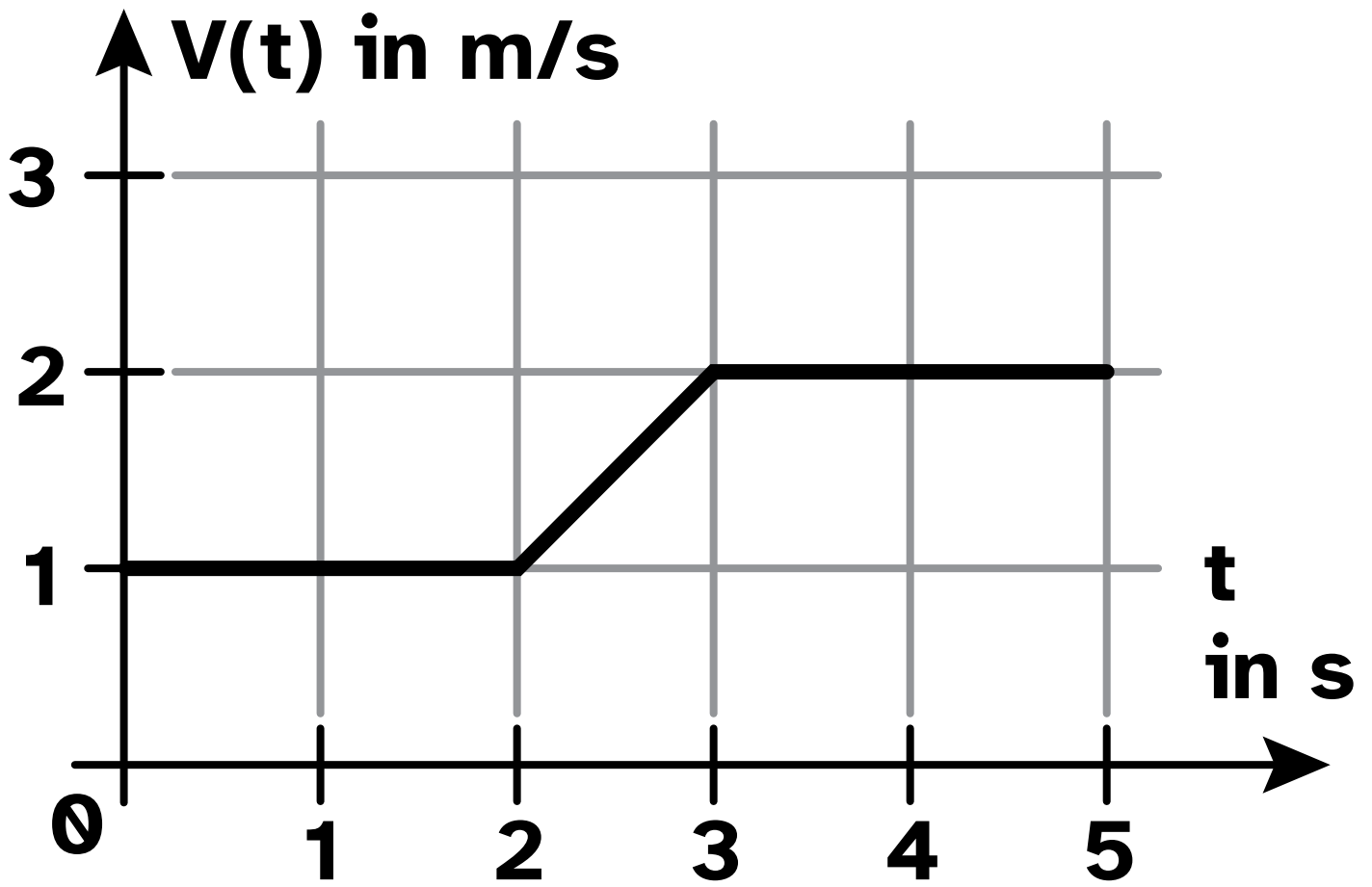
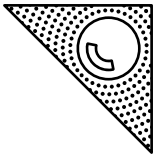
Ang.Mat. HAK4 S.116 5.74

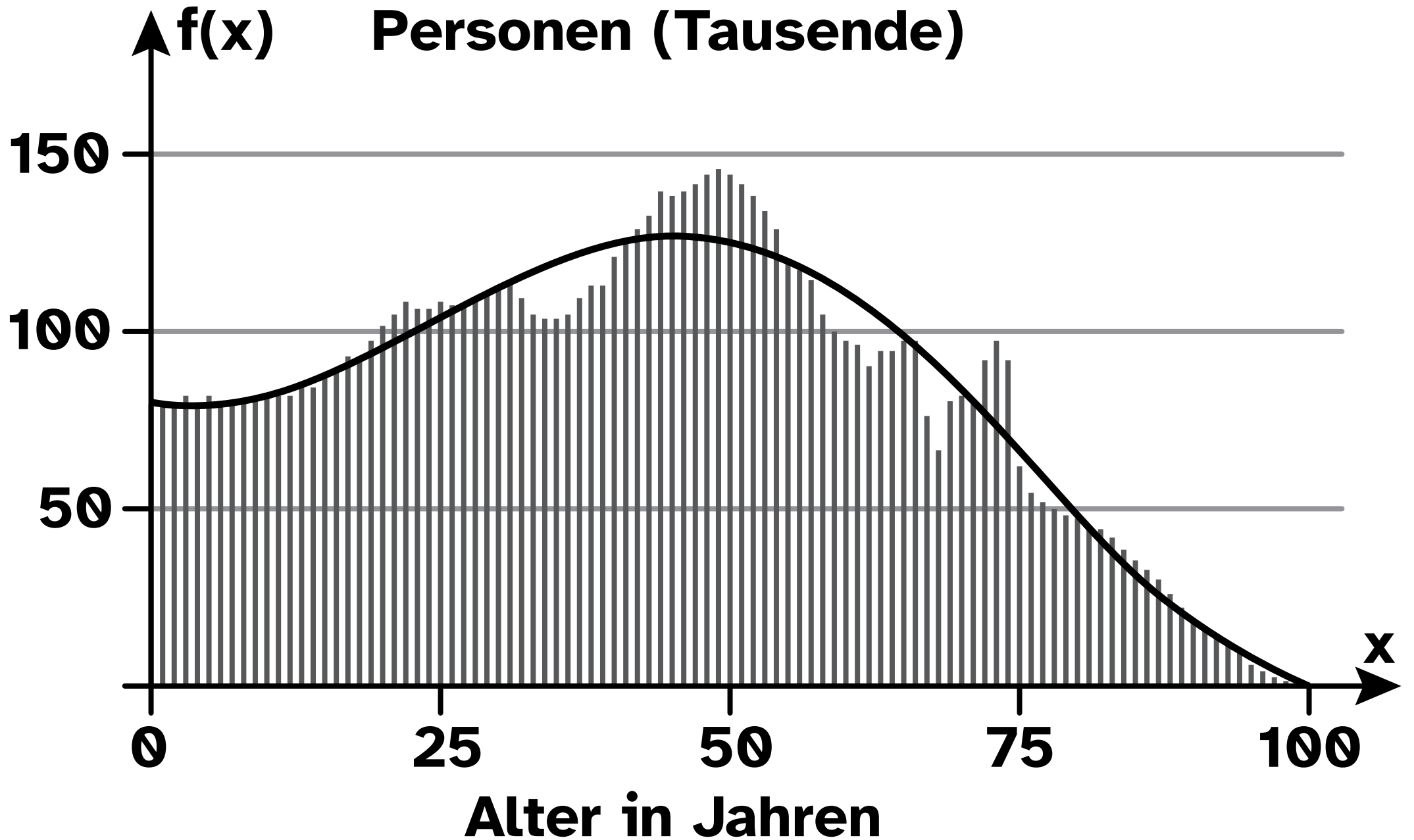
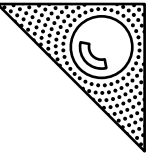


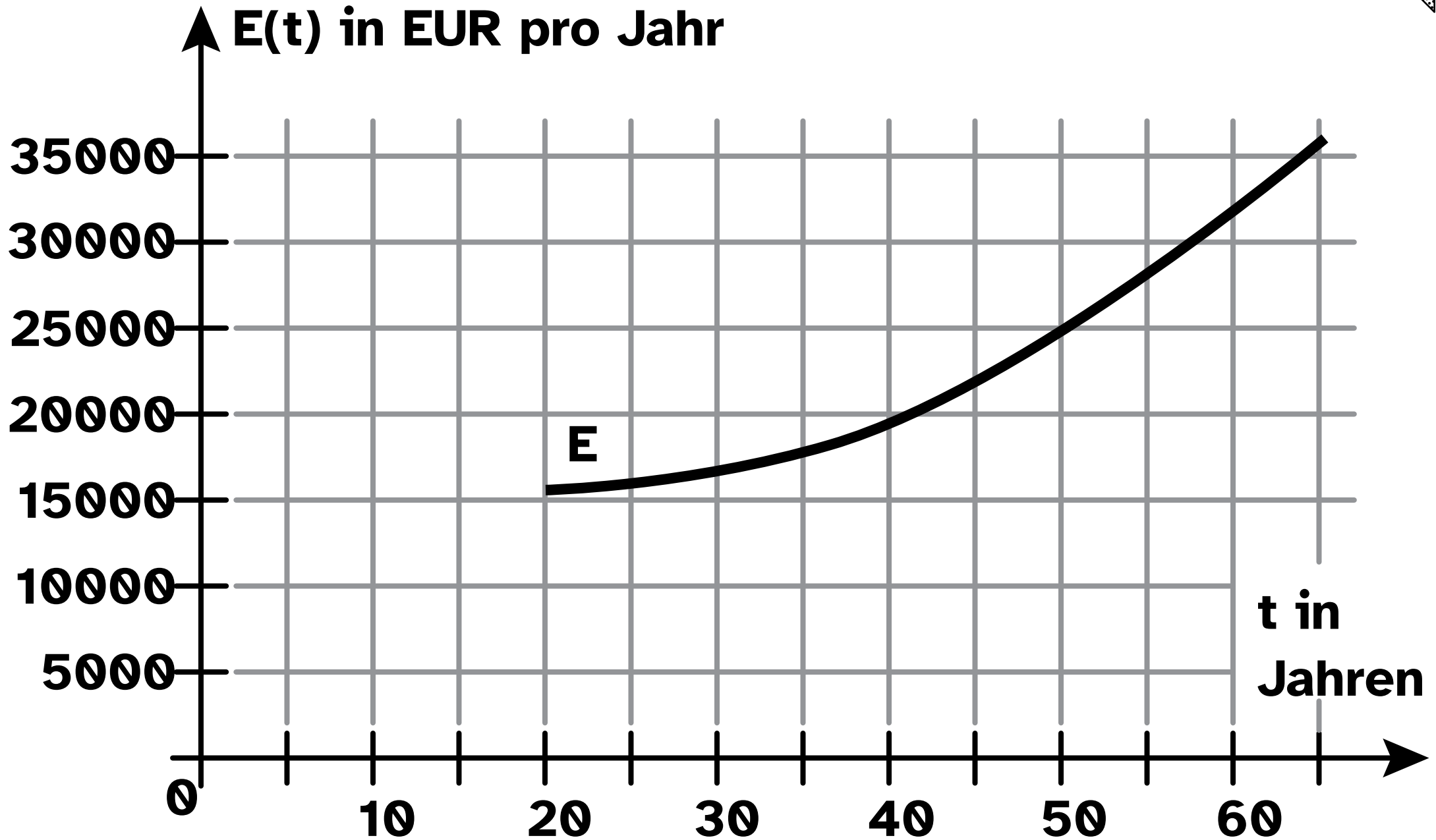
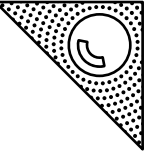


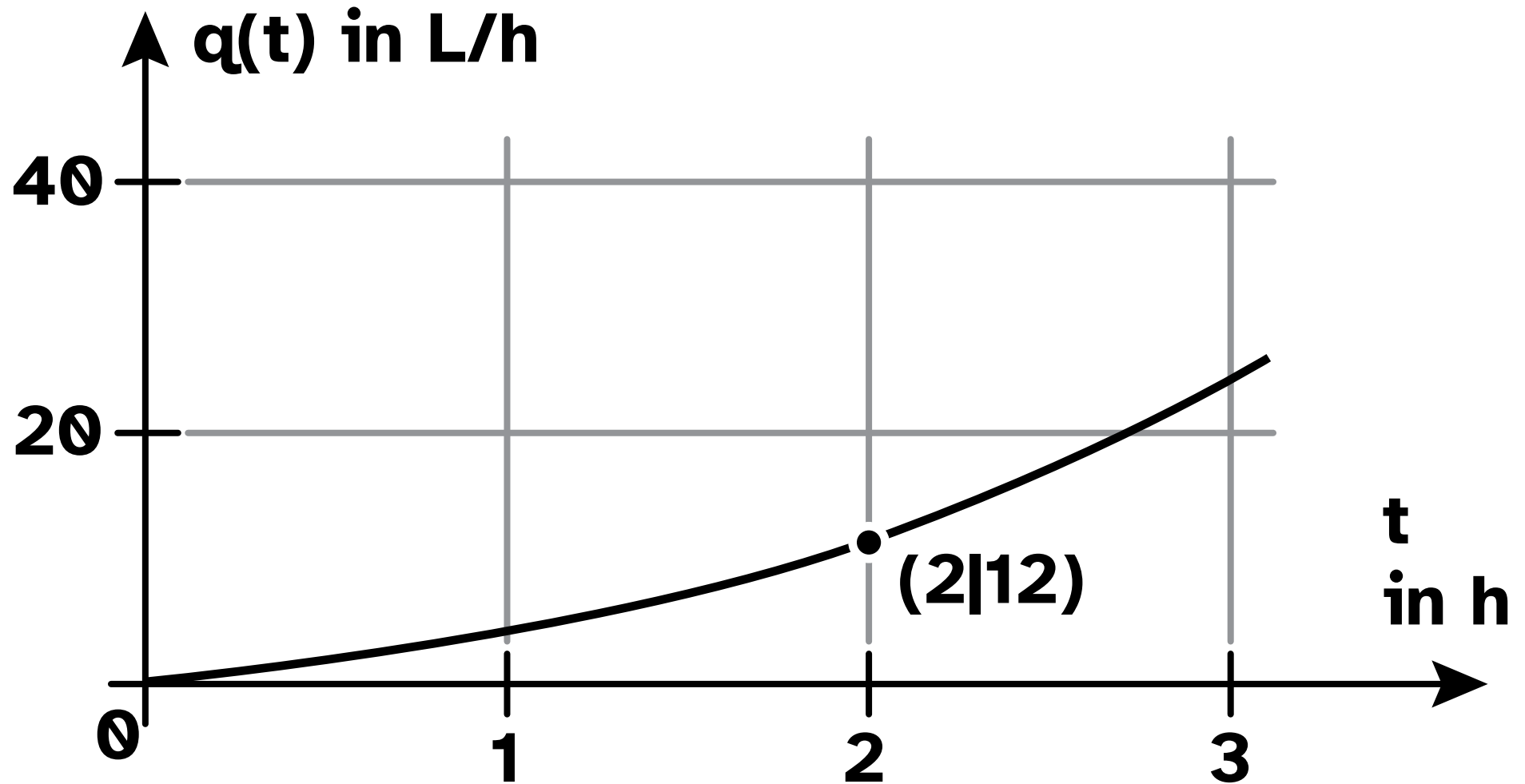
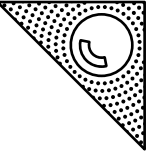


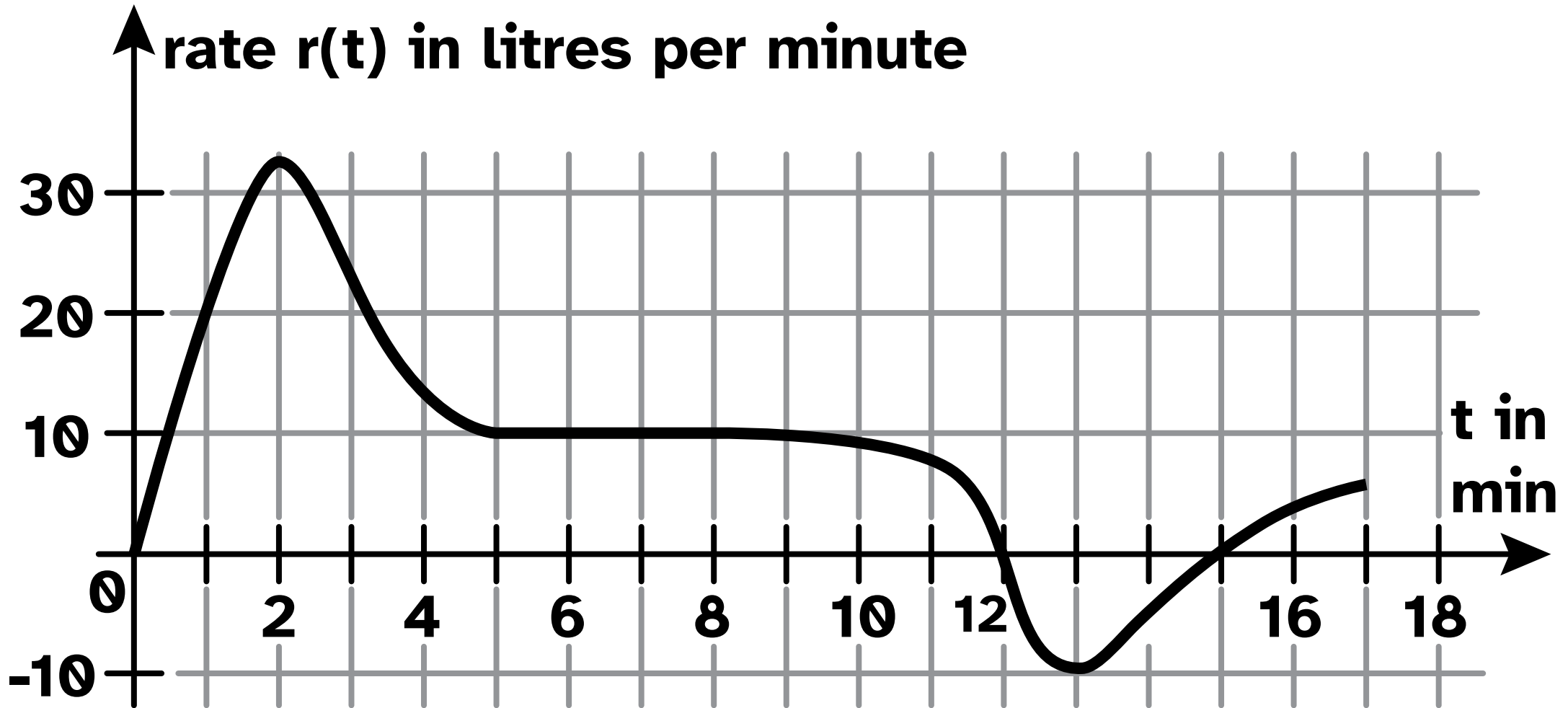
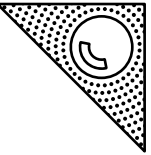


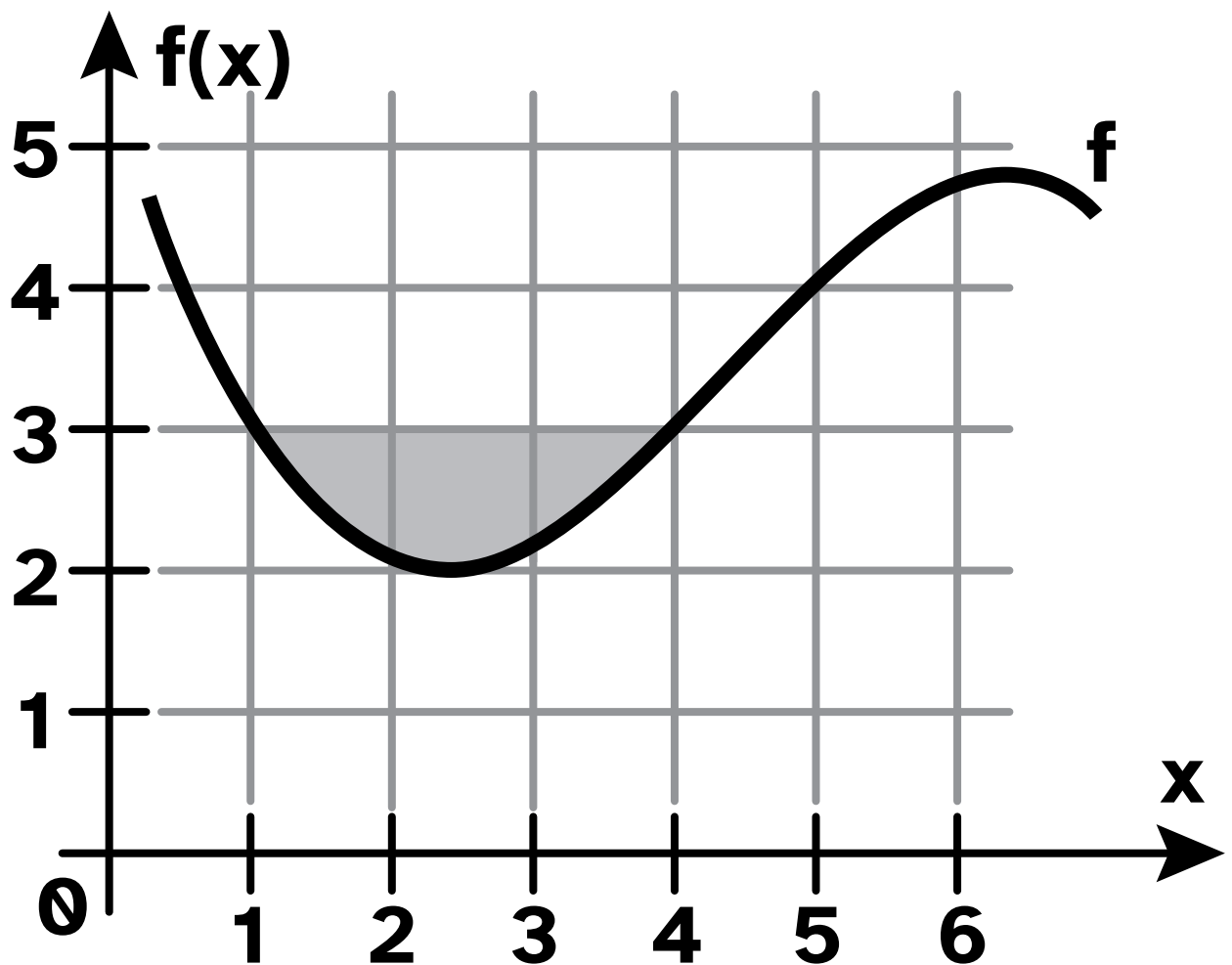
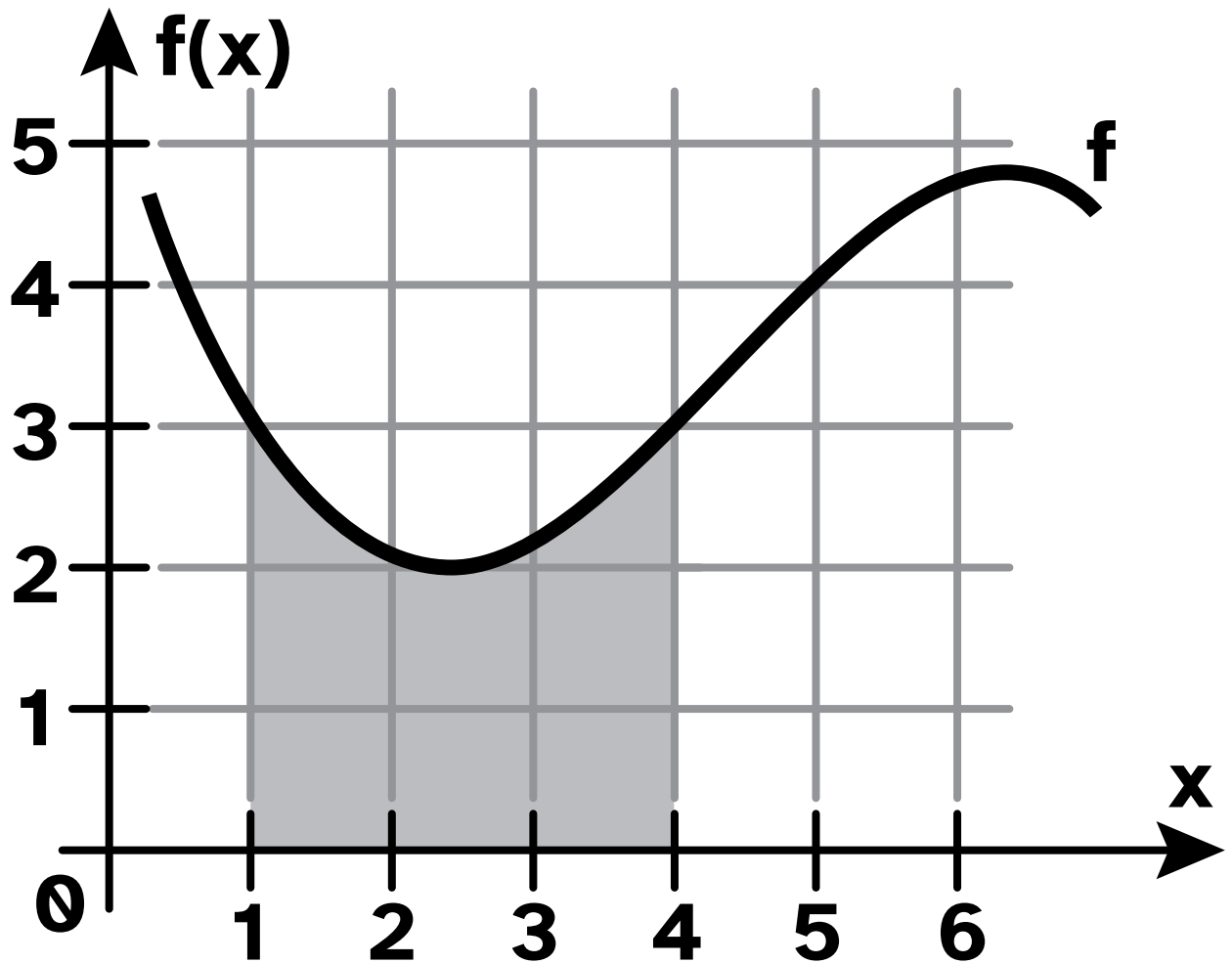
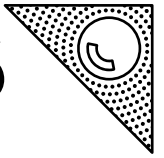


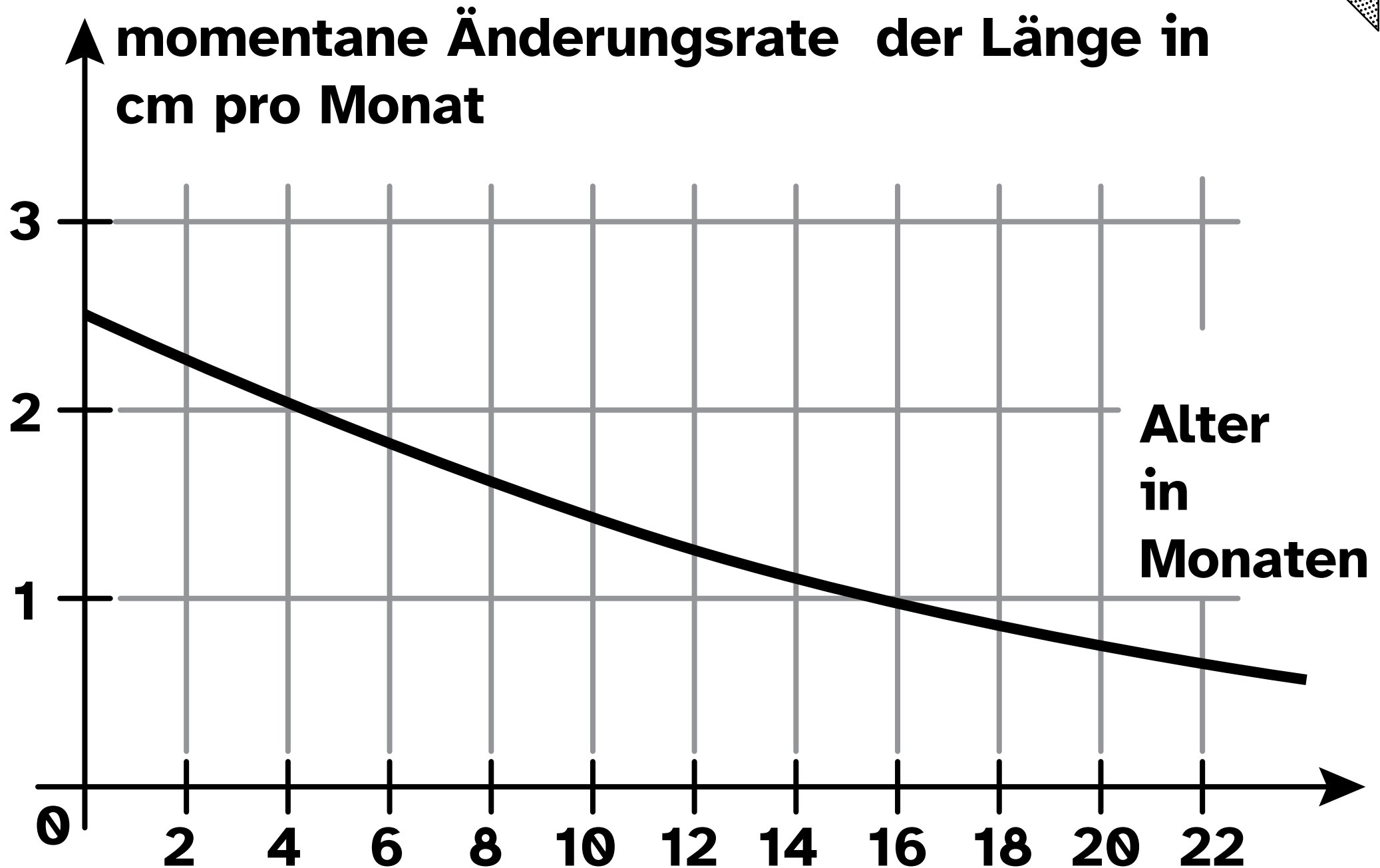
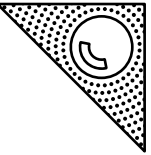


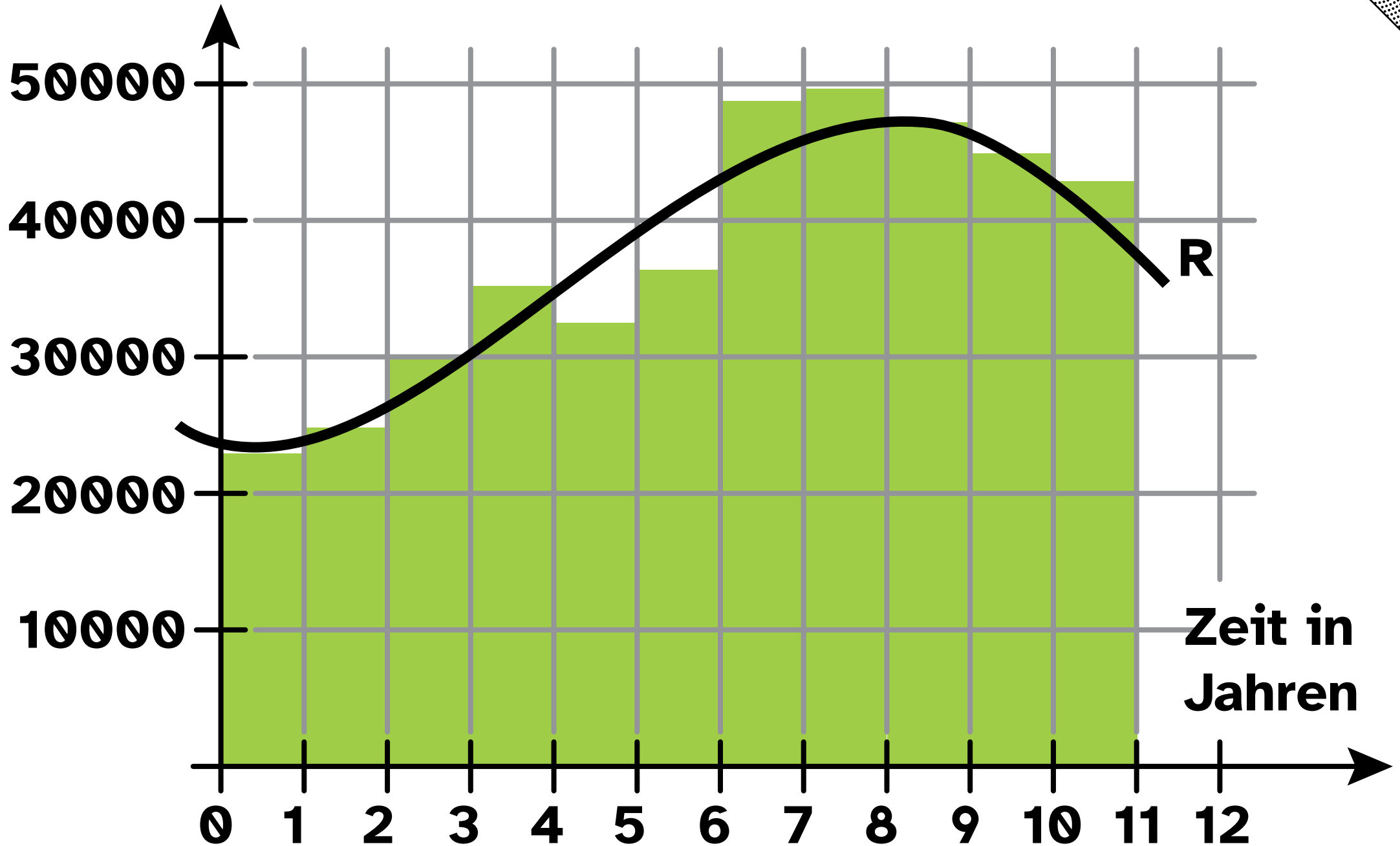
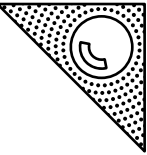


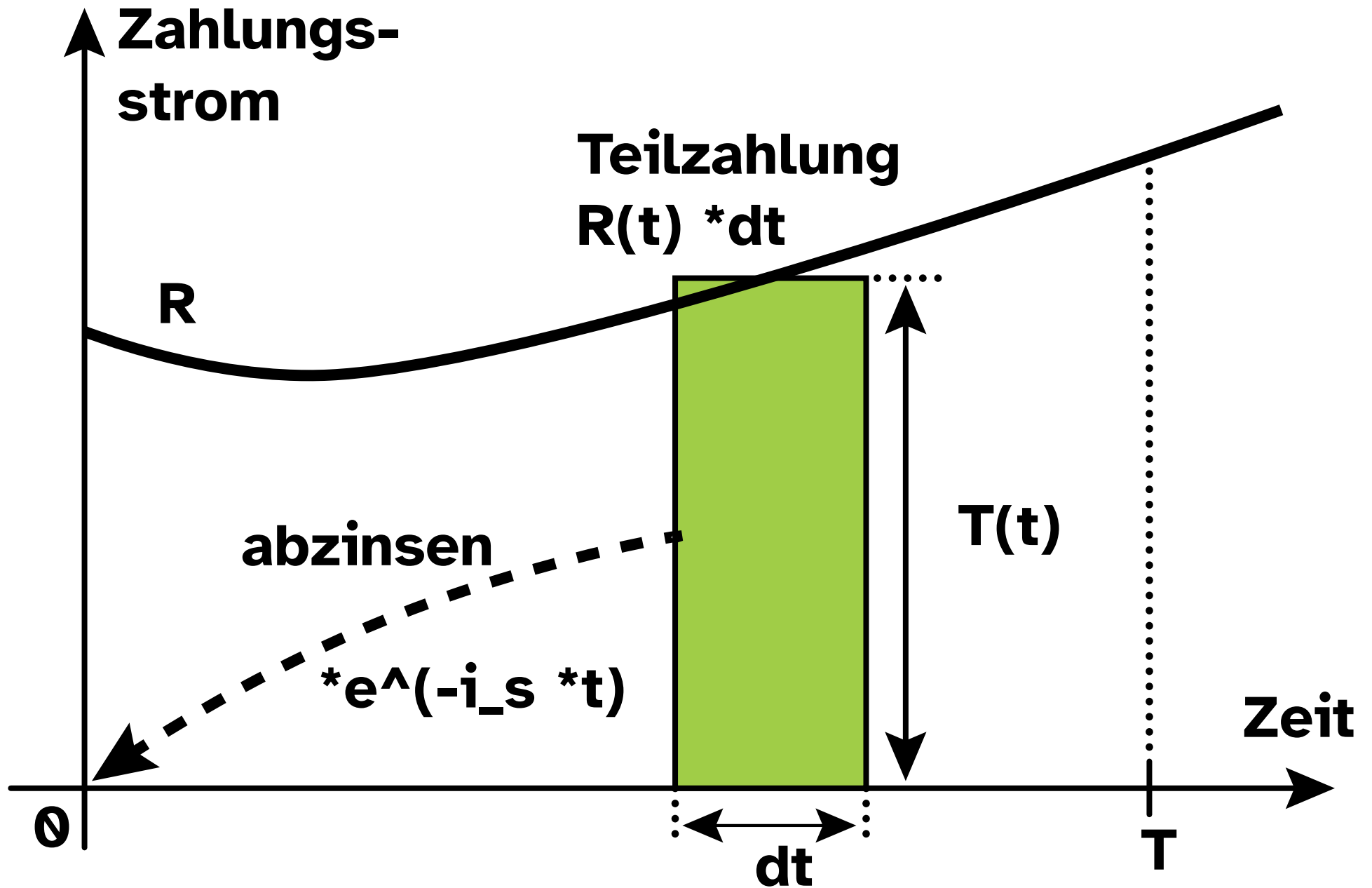
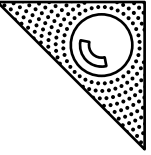


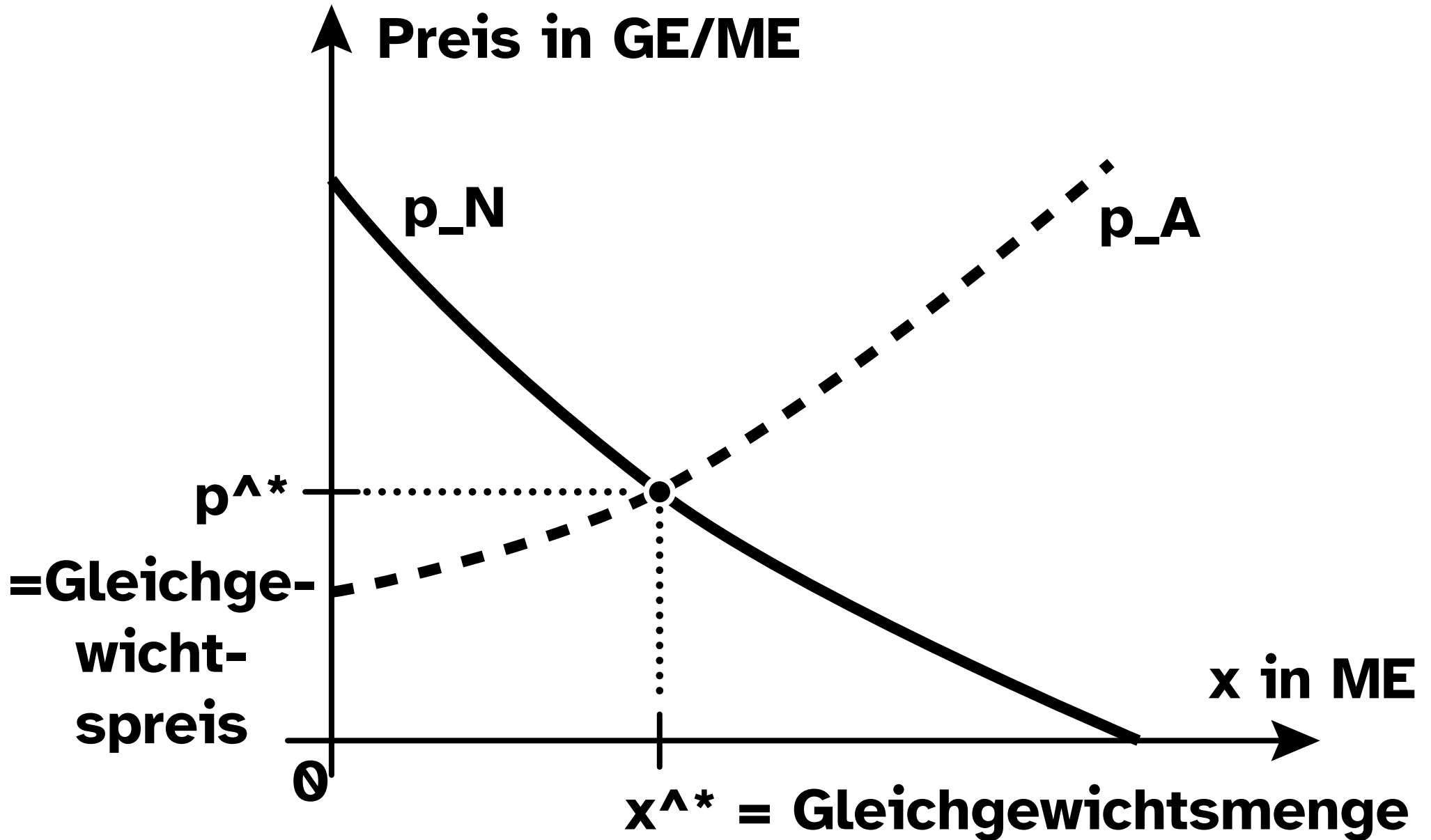
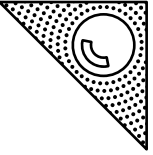




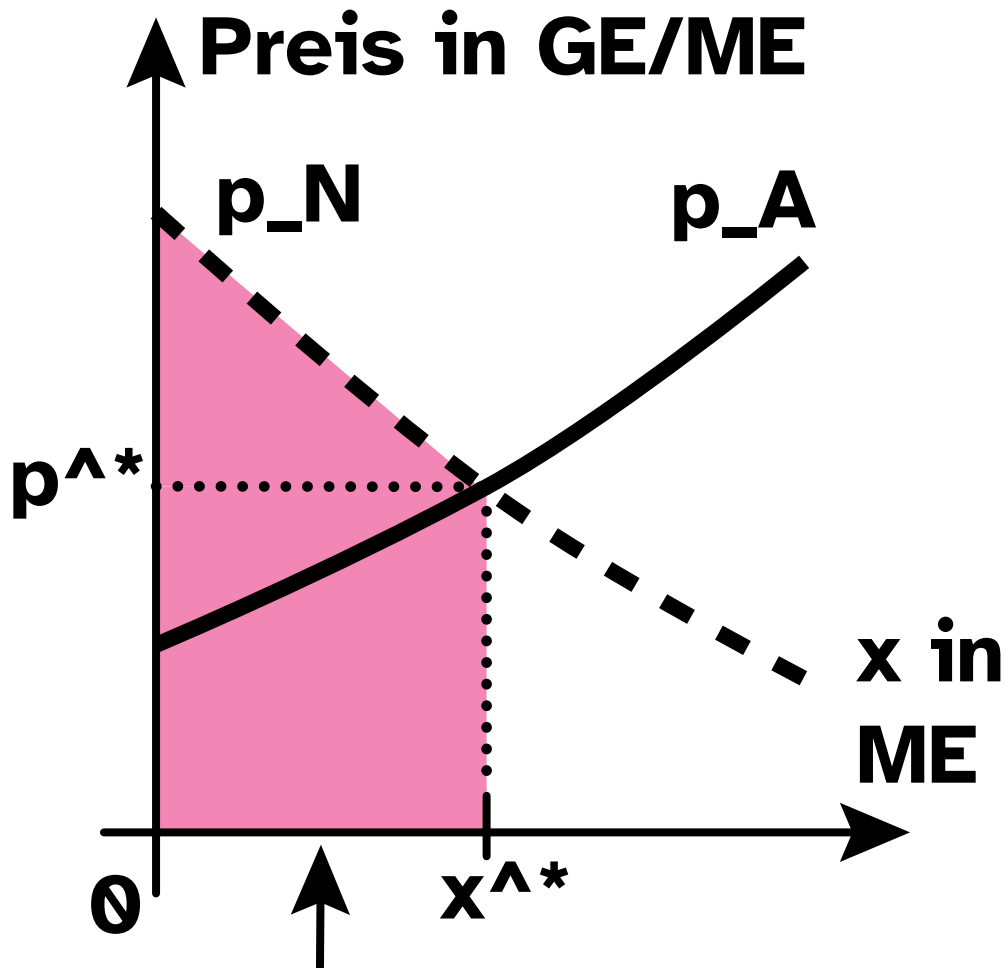
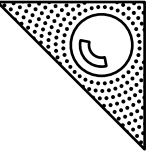




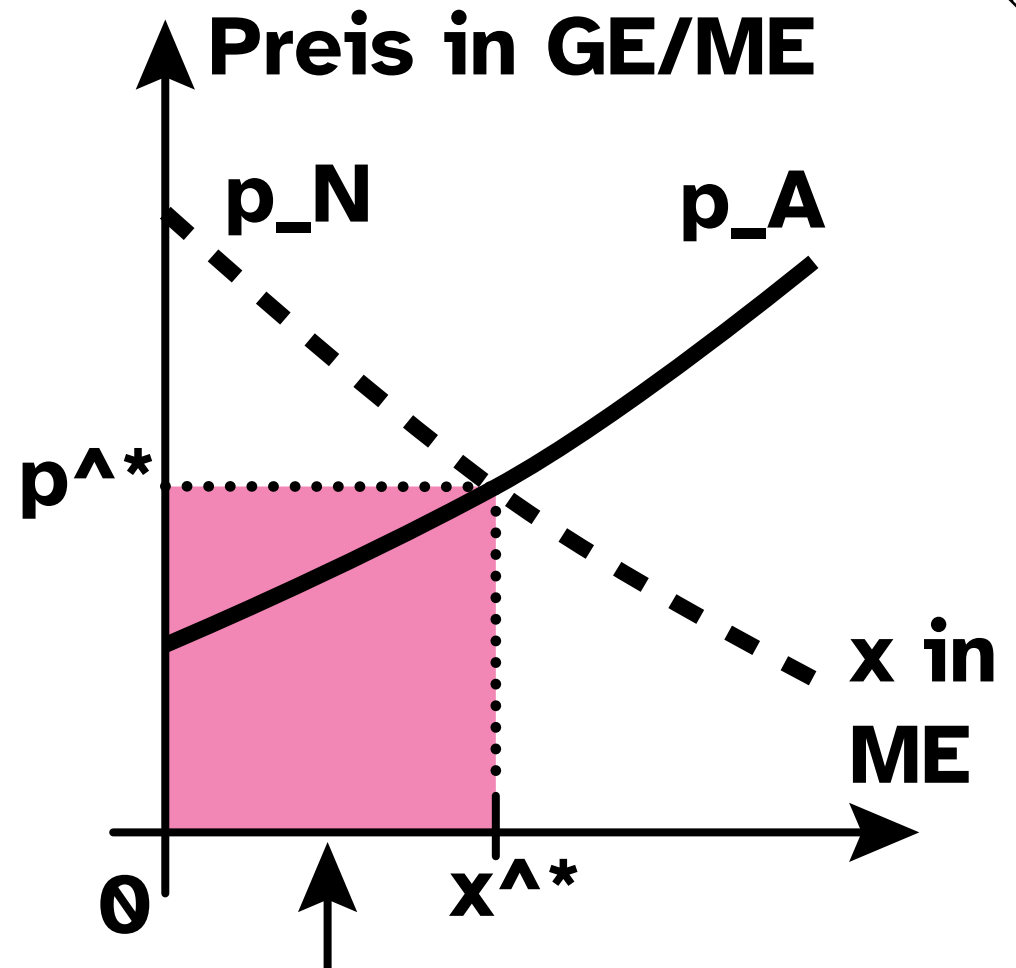




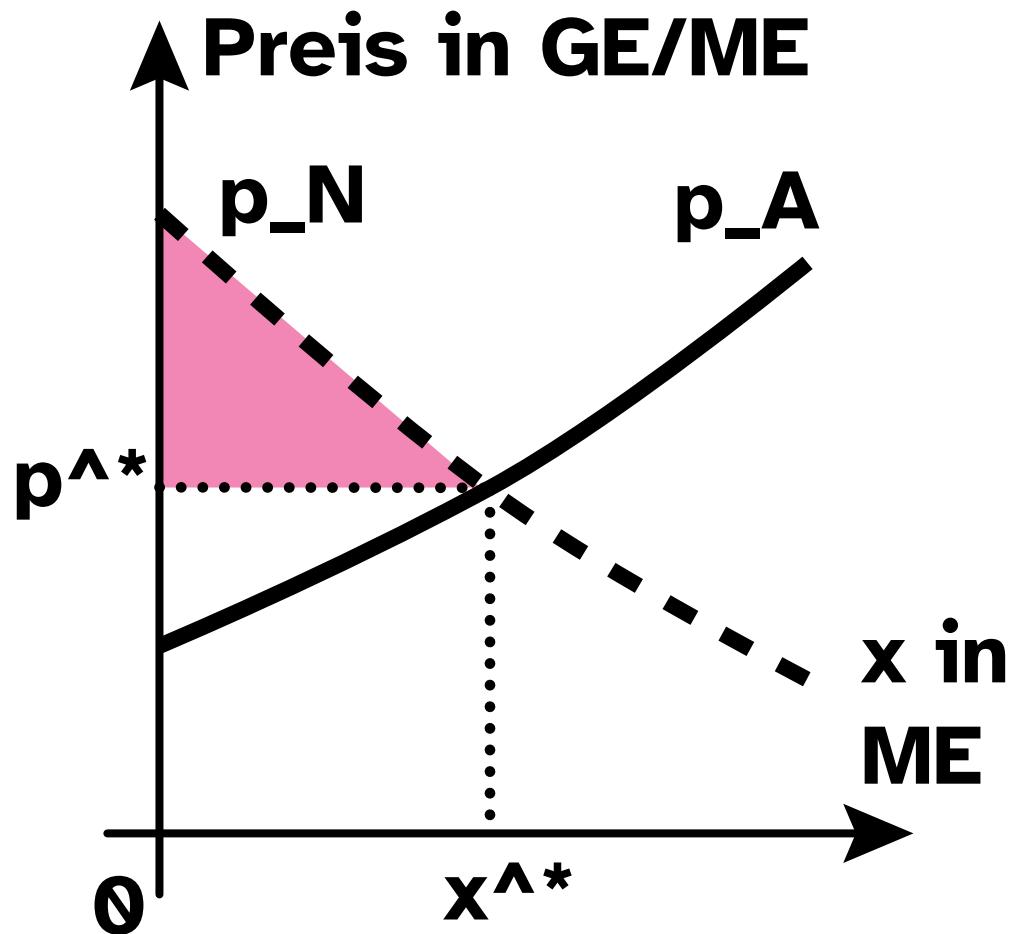
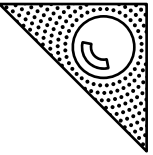
Ang.Mat. HAK4 S.121 Konsumentenrente 1/2



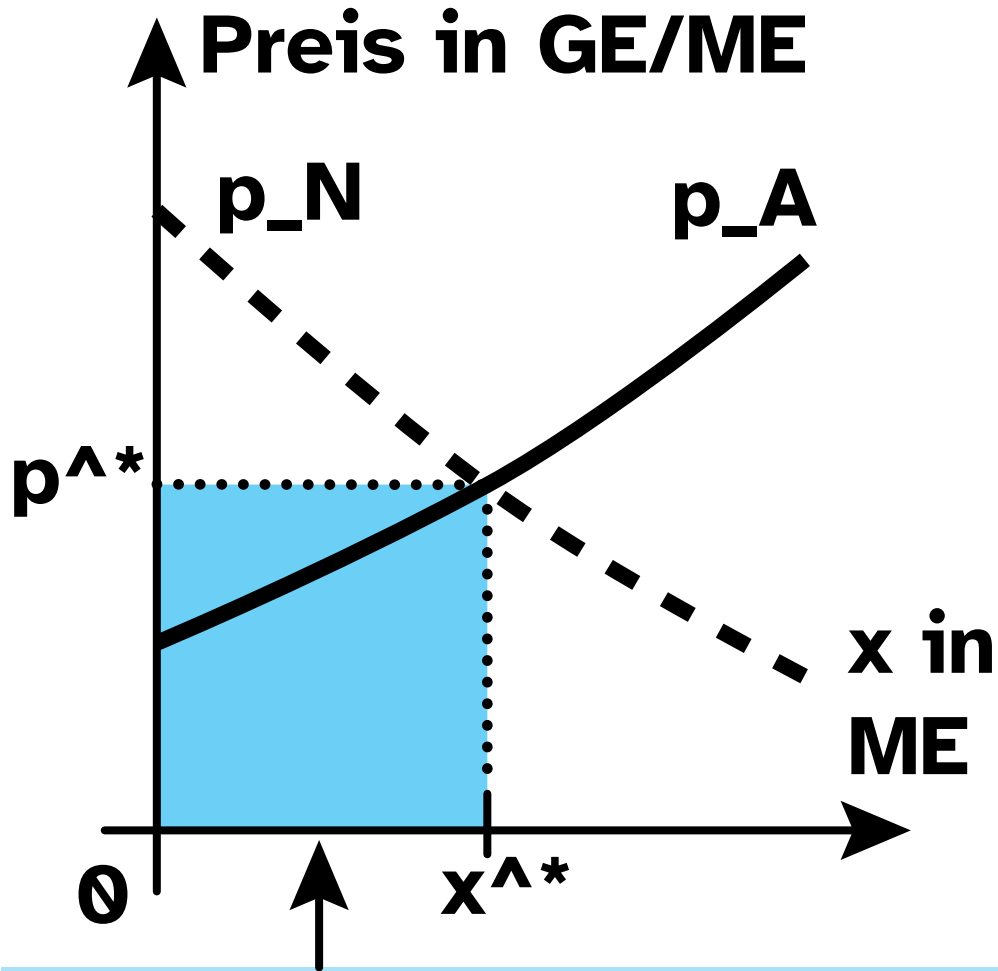
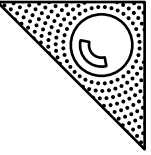
Konsumenten sind bereit
' $\int_0^{x^*} p_N(x)$ ' zu zahlen



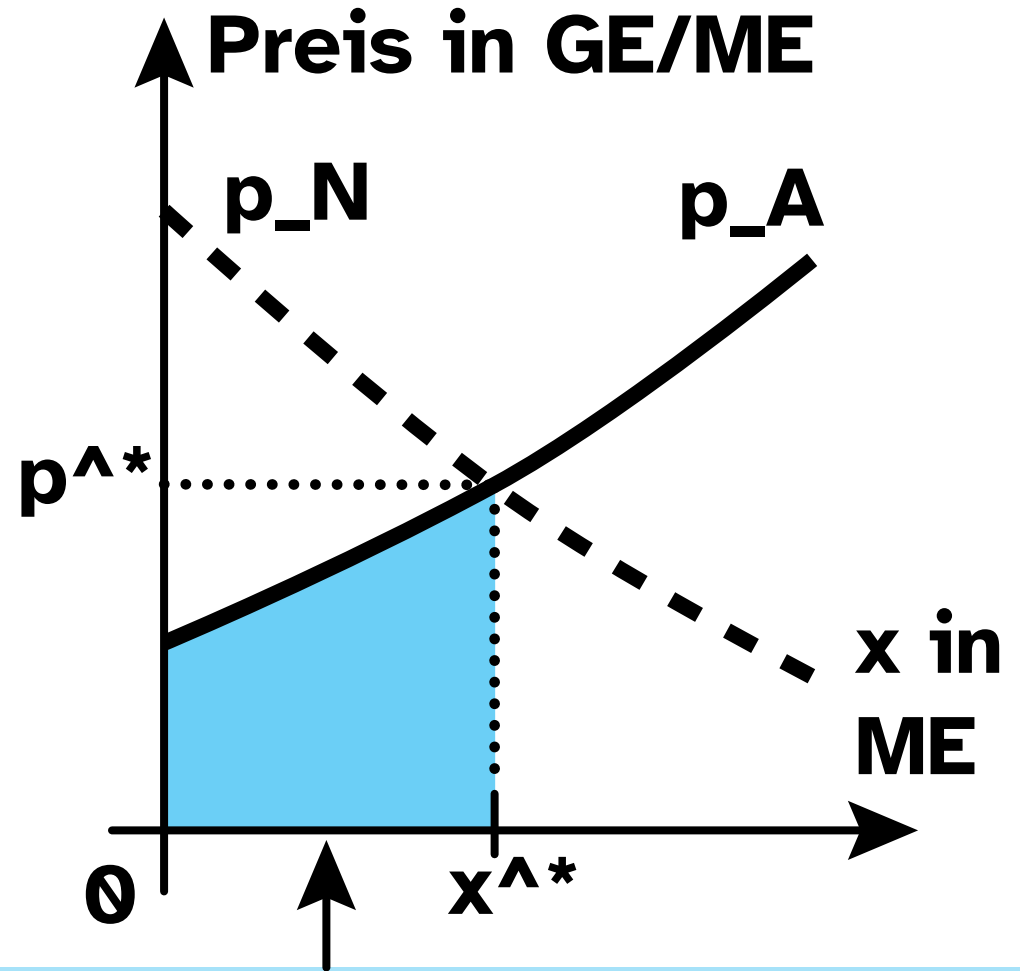
Tatsächlich zahlen
sie aber nur
 $p^* \cdot x^*$



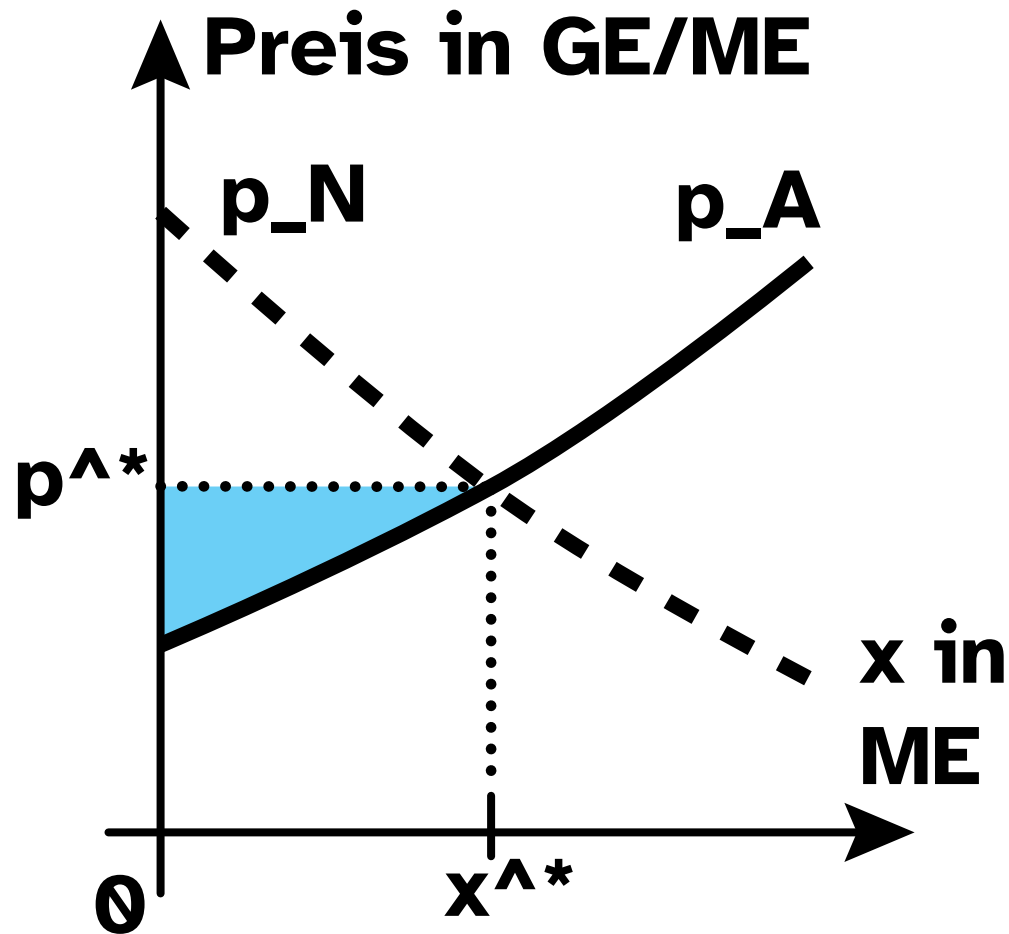
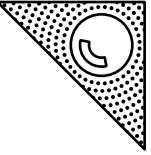
Konsumenten sparen also: $\int_0^{x^*} (p_N(x) - p^*) dx$. Das nennt man Konsumentenrente



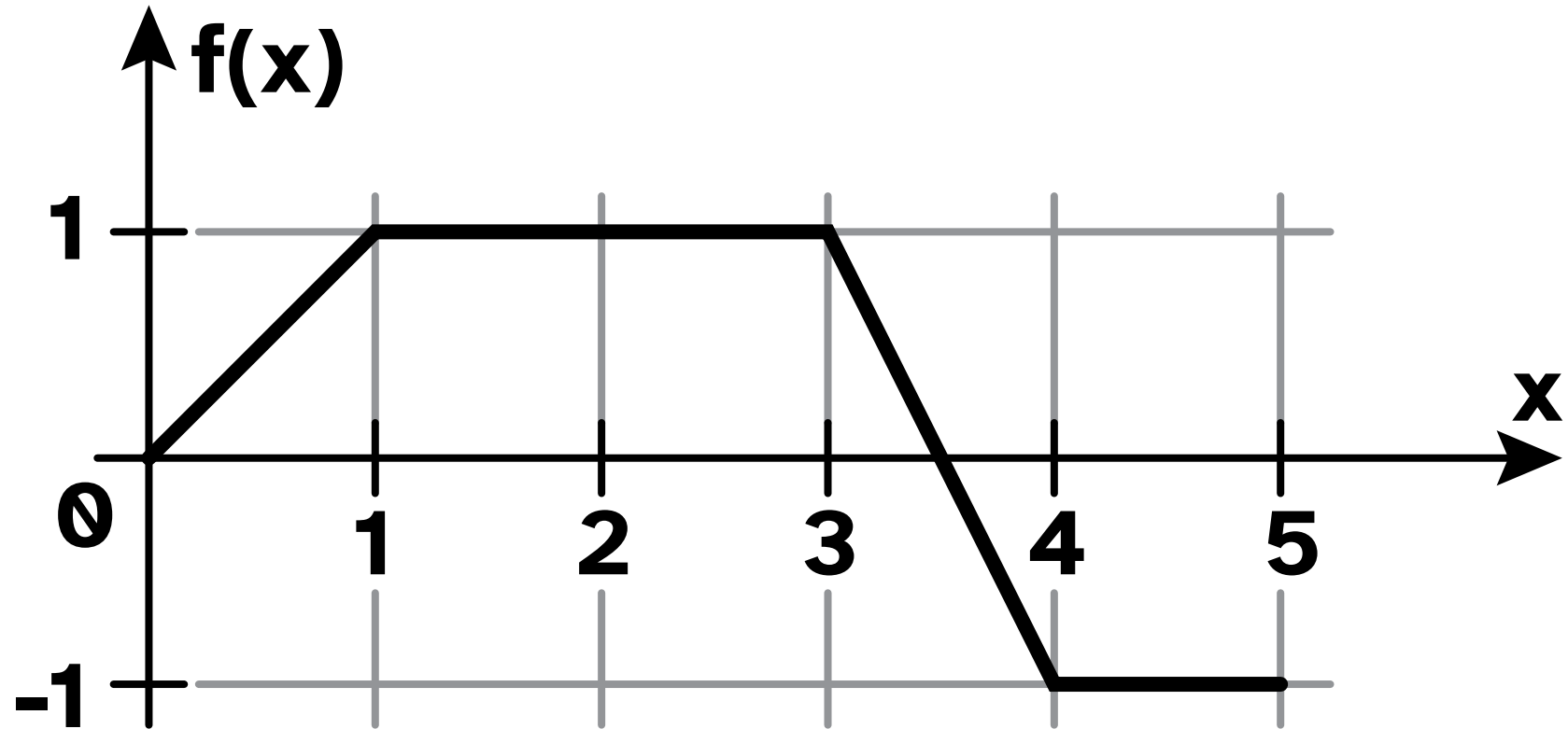
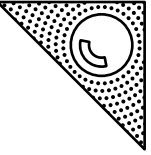
Produzenten erzielen einen Erlös von $p^* \cdot x^*$

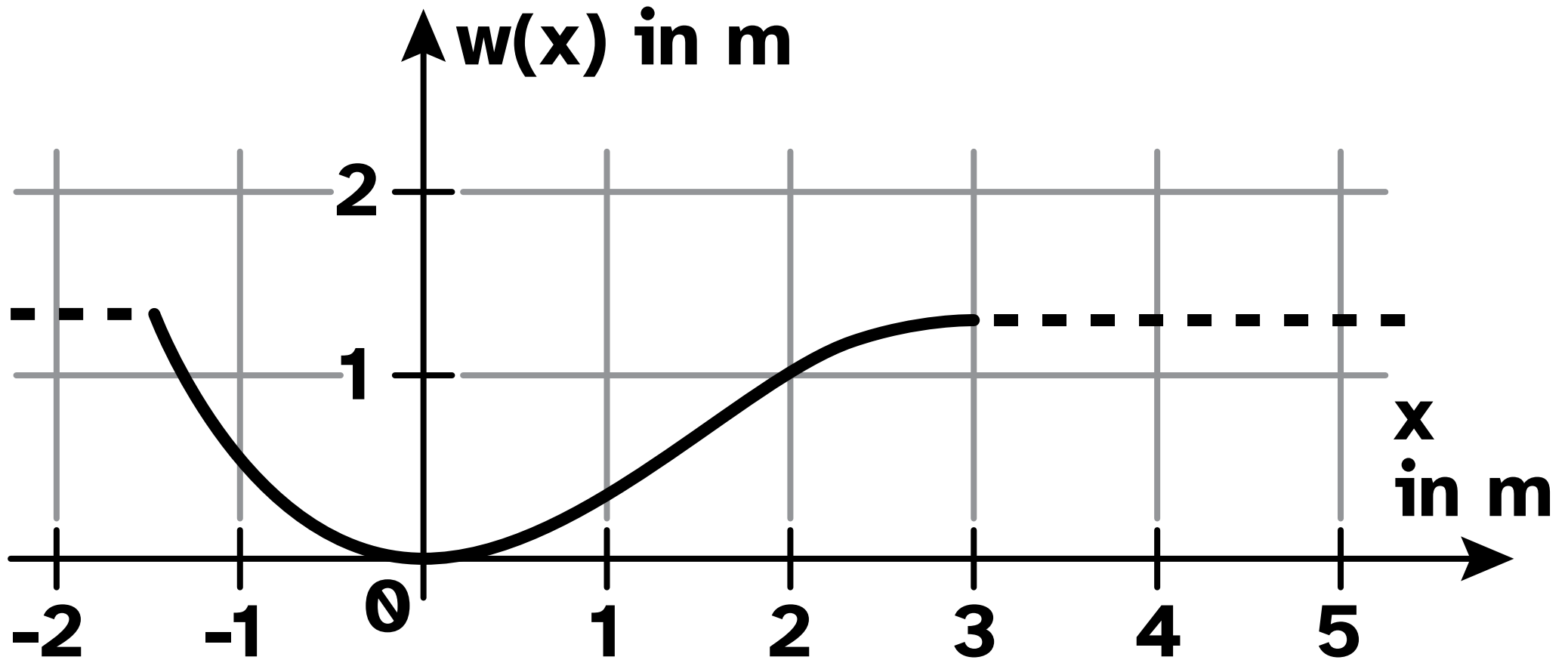
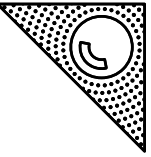


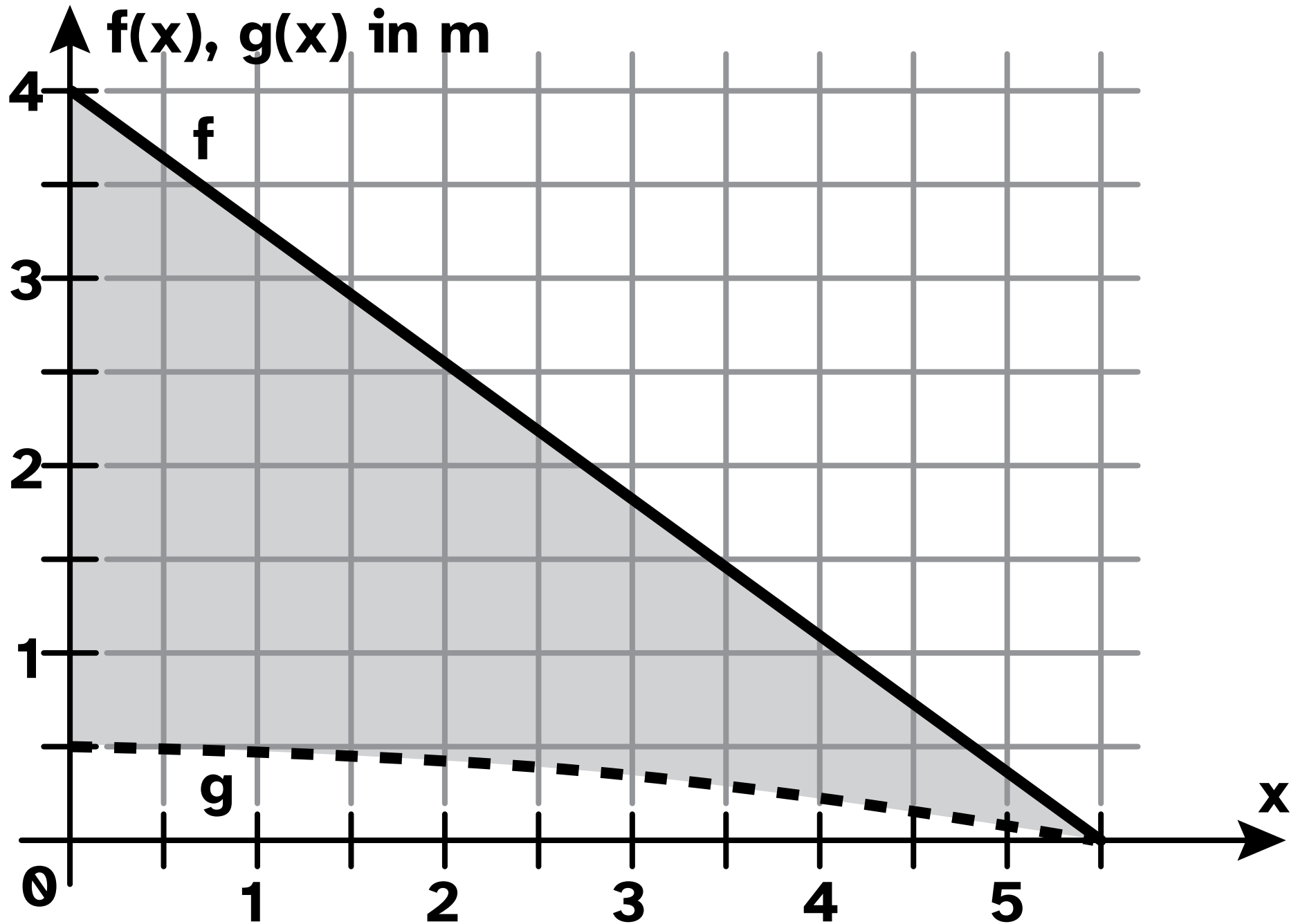
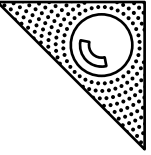
Würden aber Erlös von $\int_0^{x^*} p_N(x) dx$ akzeptieren



Produzenten haben somit
 $p^* \cdot x^* - \int_0^{x^*} p_N(x) dx$ gespart.
Das nennt man Produzentenrente.







Ang.Mat. HAK4

S.123 5.92 b)

