Bundes-
Blindeninstitut
Wien

LeMa
Lehr- und Lernmaterialien

Tastbare Versionen ausgewählter Grafiken aus dem Schulbuch

Angewandte Mathematik HAK 4

Grafiken: Tomáš Baťha

LEMA - Lehr- und Lernmaterialien
Hergestellt von der Abteilung für Inklusion und Lehrmittel am Bundes-Blindeninstitut Wien,
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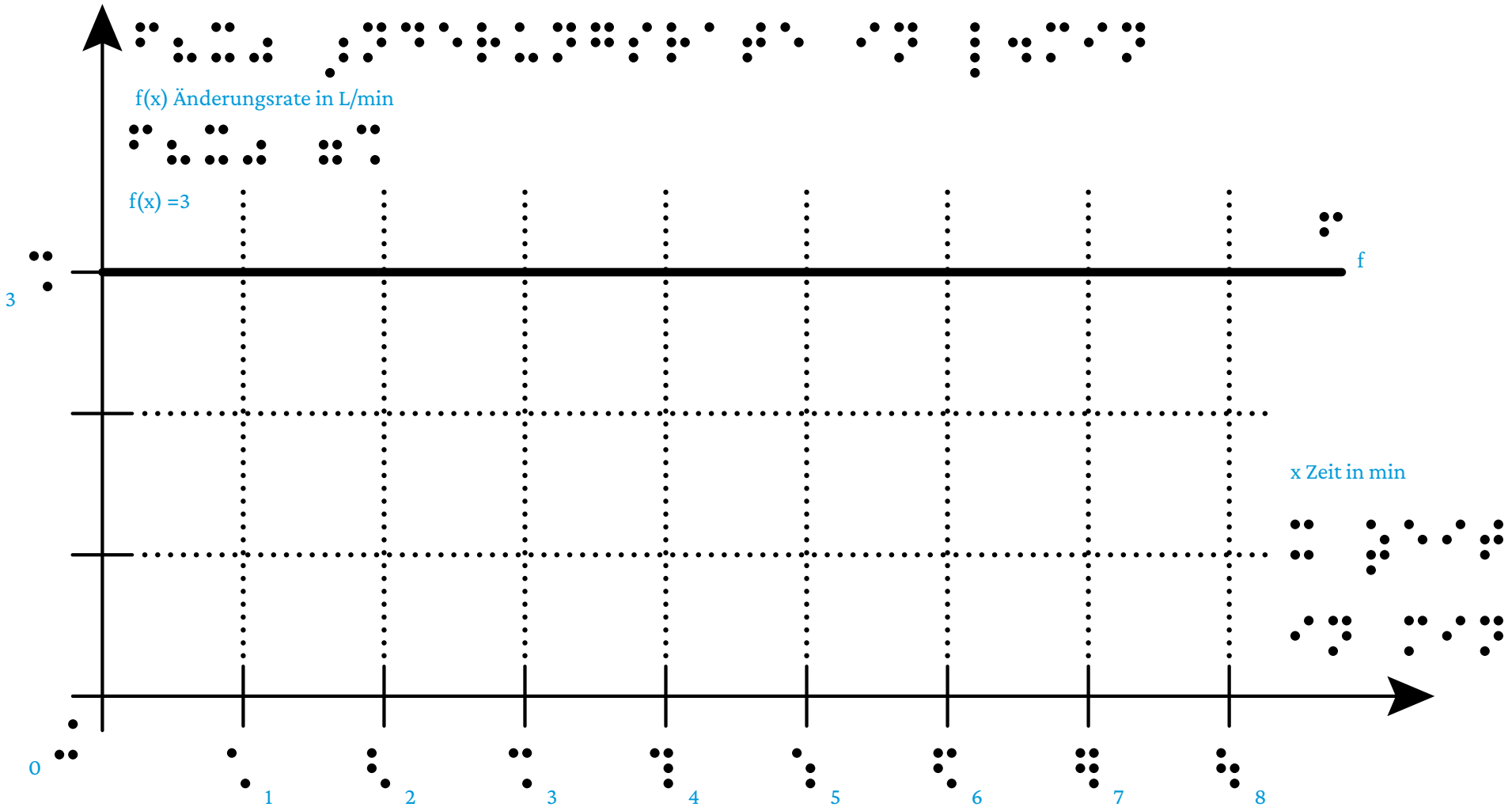
Angewandte Mathematik
HAK Band 4

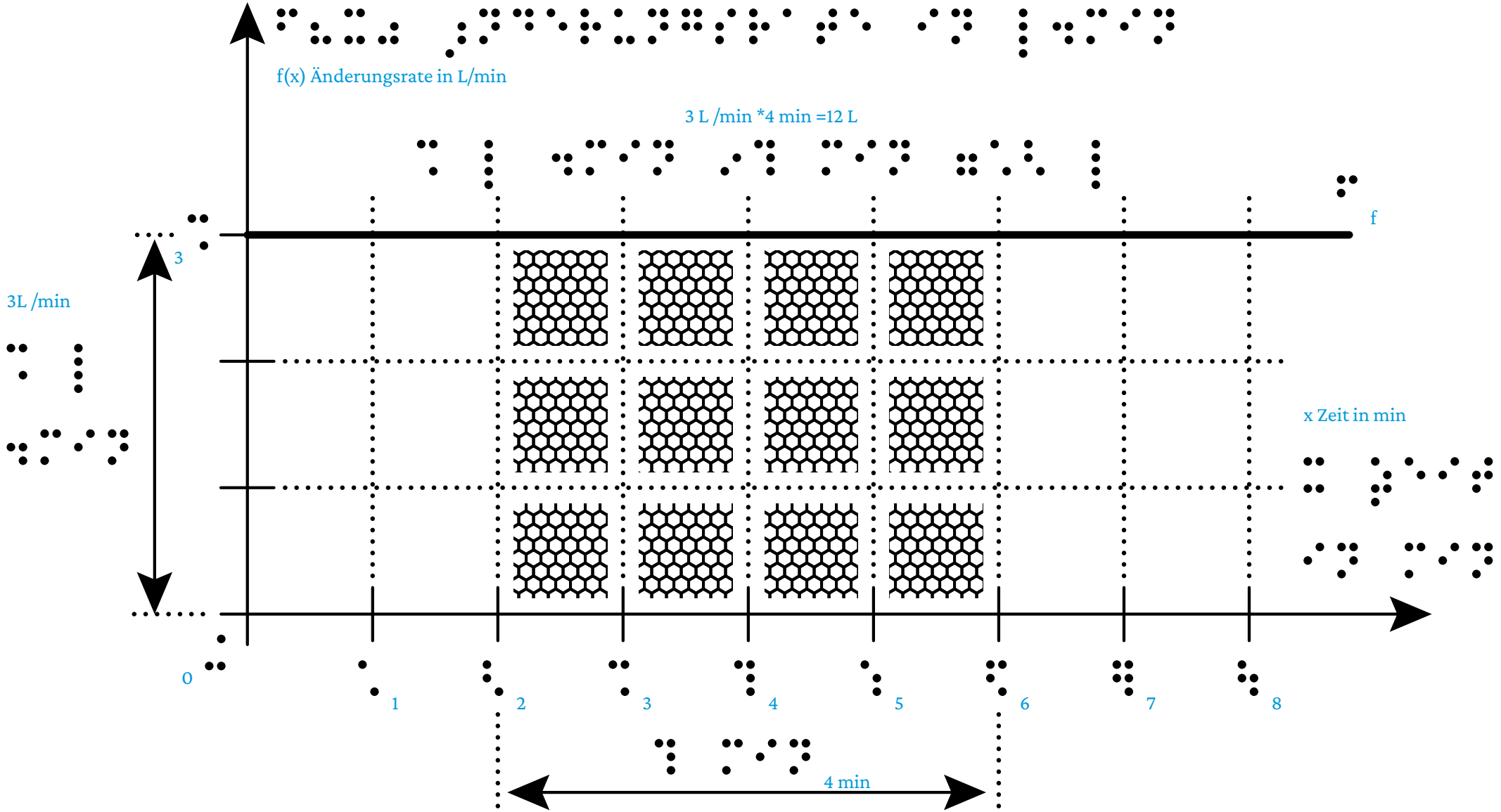
5.

Integralrechnung

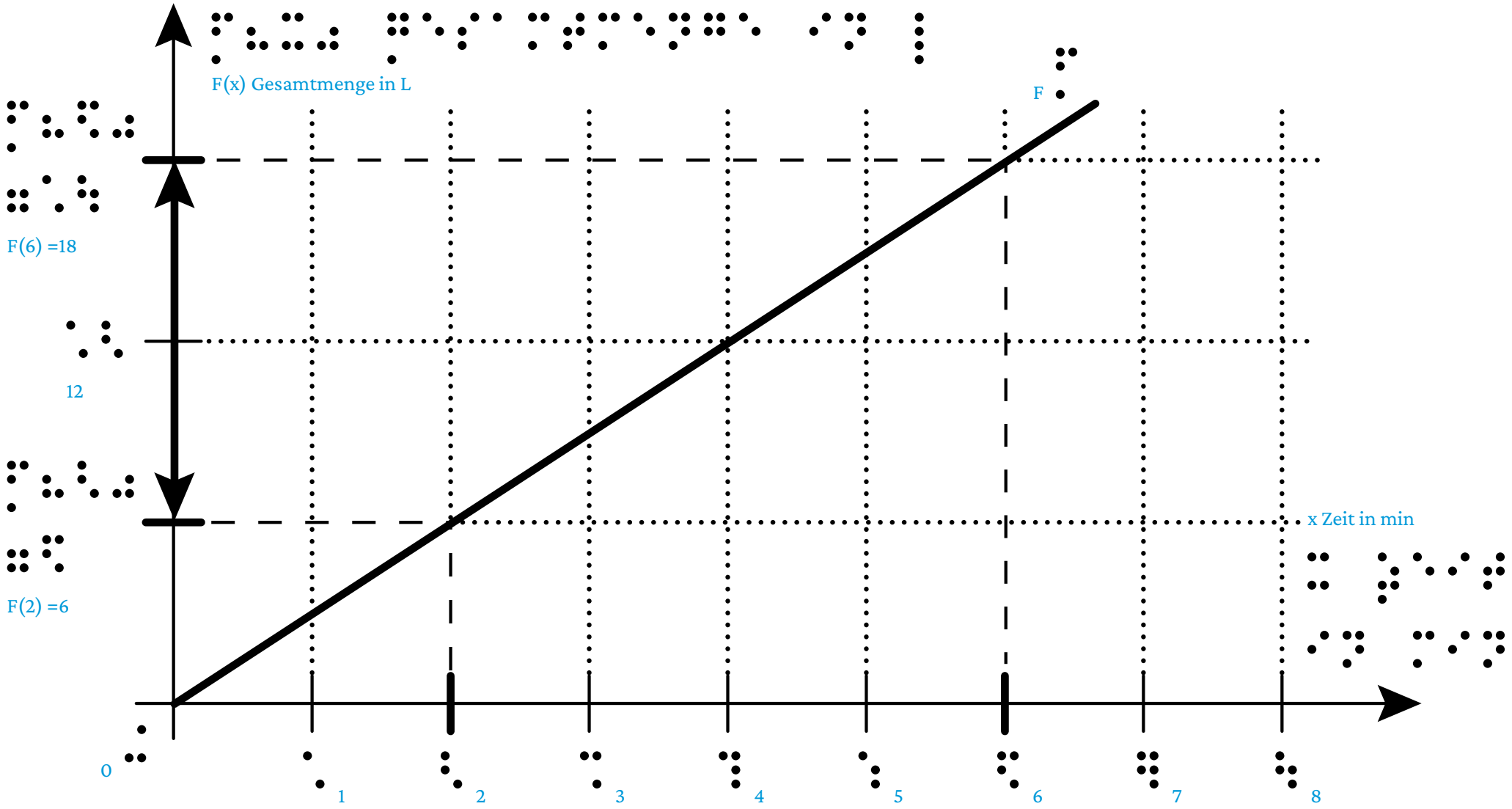
Beispiel mit konstanter Zuflussrate:



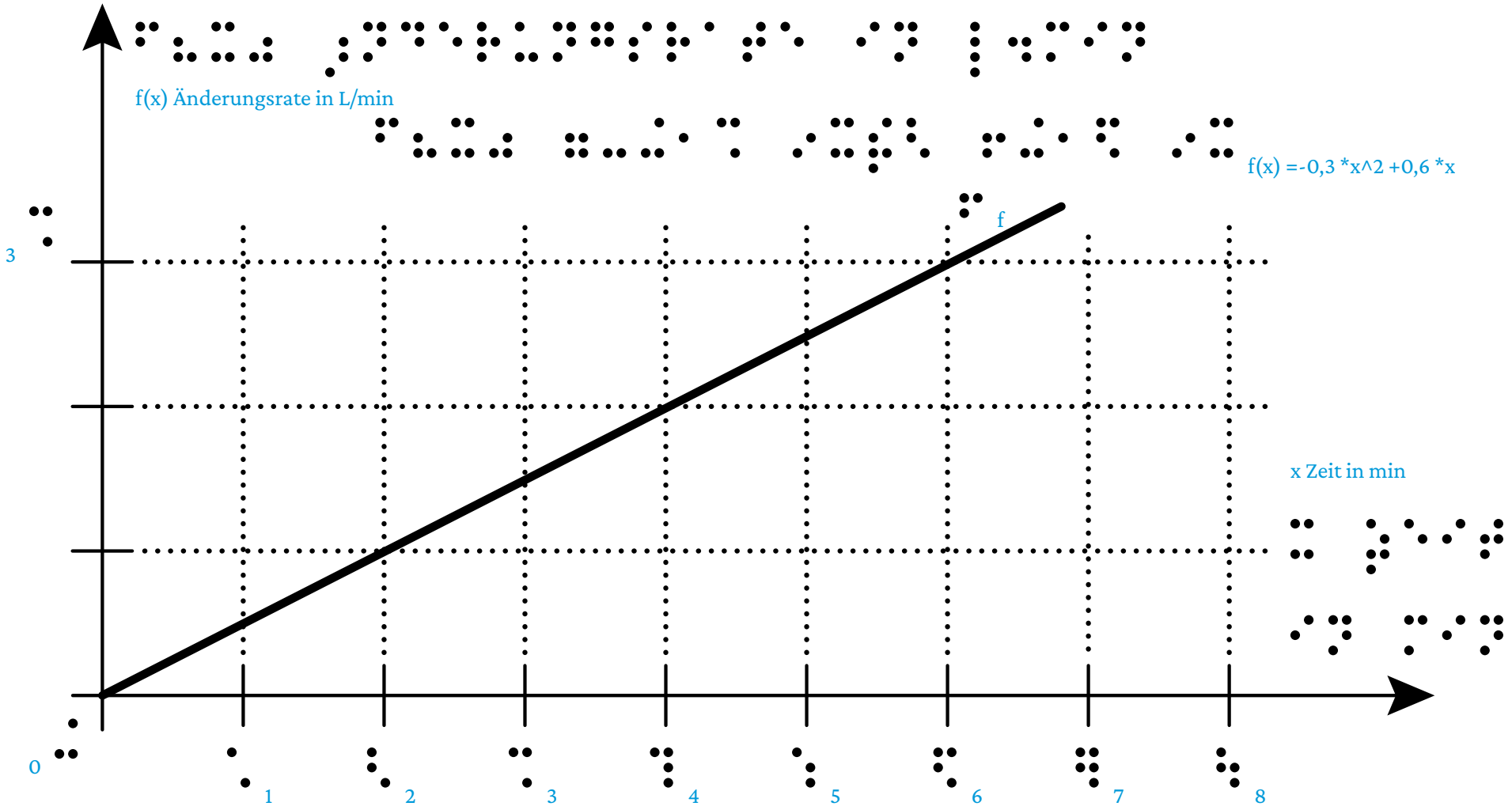
Zuflussmenge als Flächeninhalt:



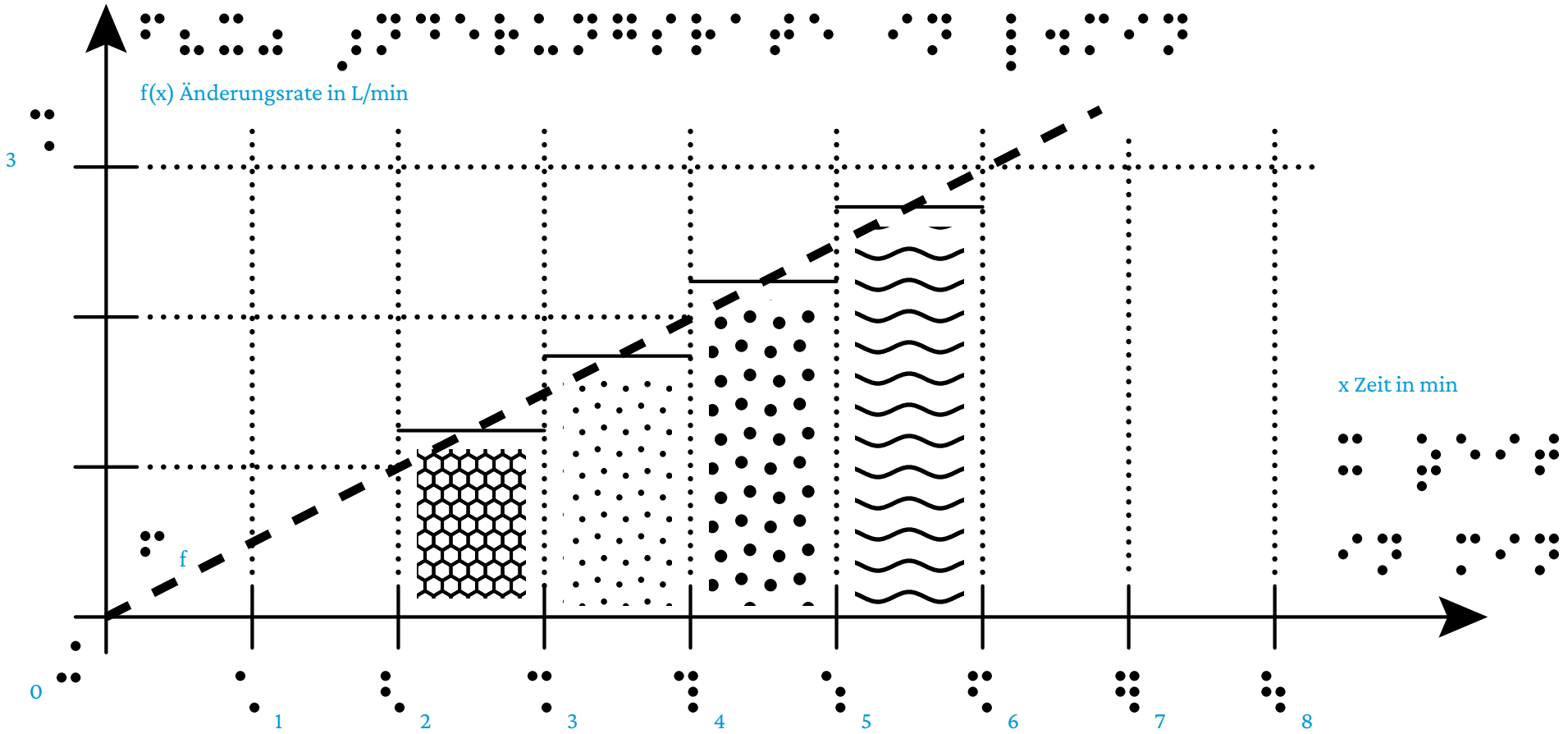
Zuflussmenge mithilfe der Bestandsfunktion



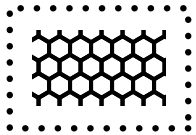
Beispiel mit nicht konstanter Zuflussrate:



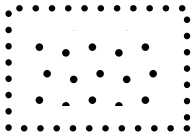
Zuflussmenge als Flächeninhalt:



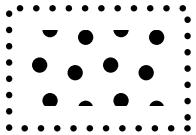
Legende:



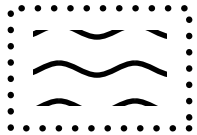
1,3 *1



1,7 *1

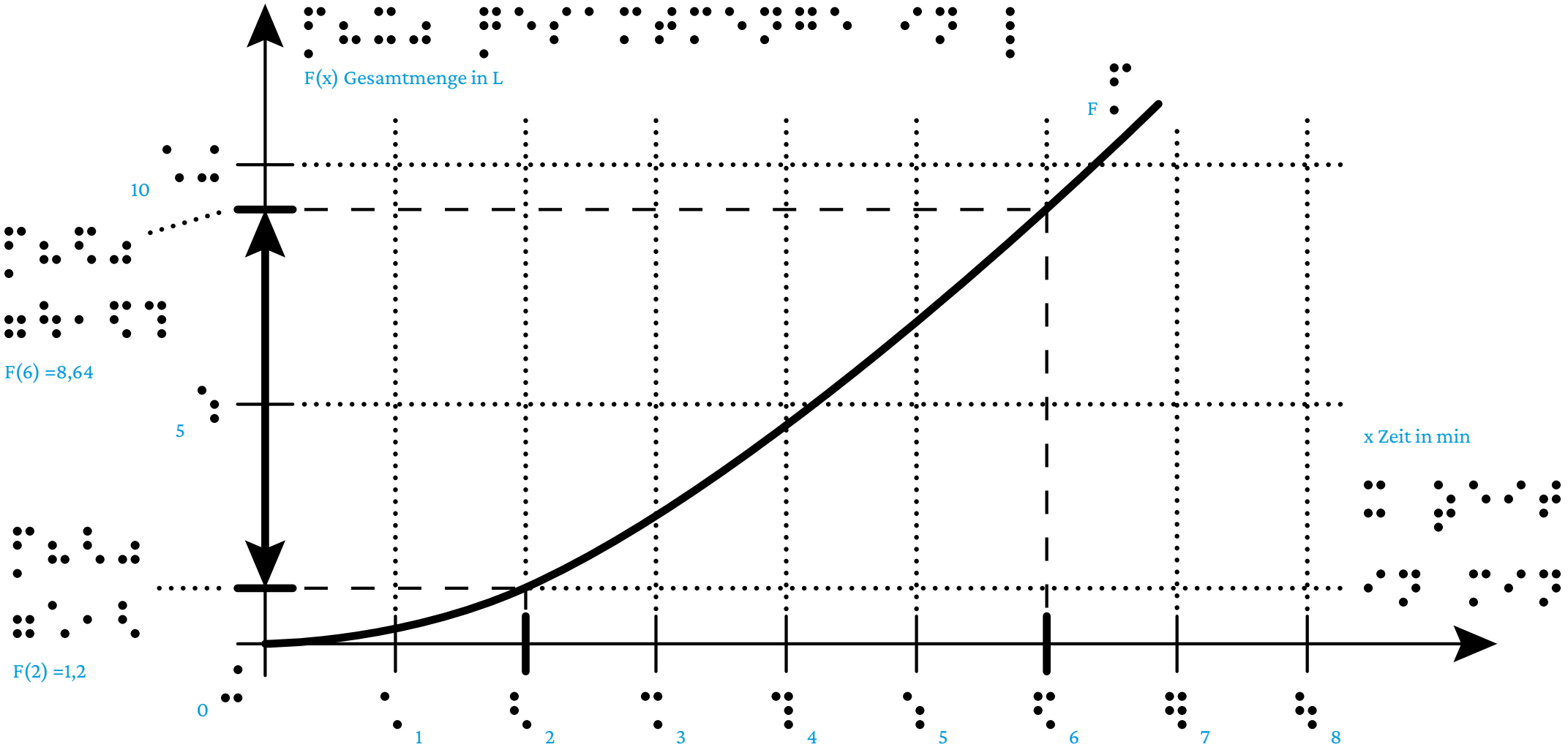


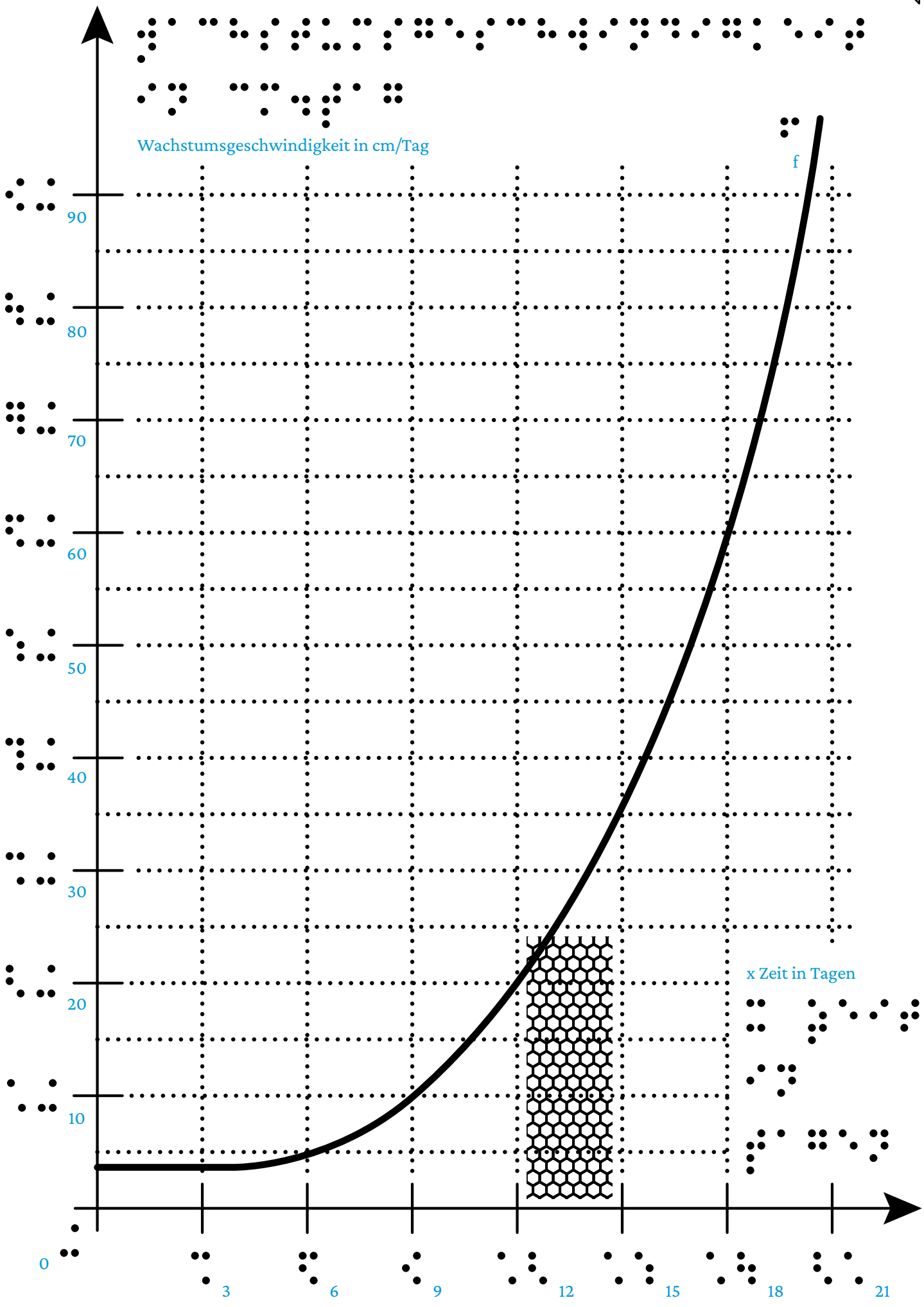
2,1 *1

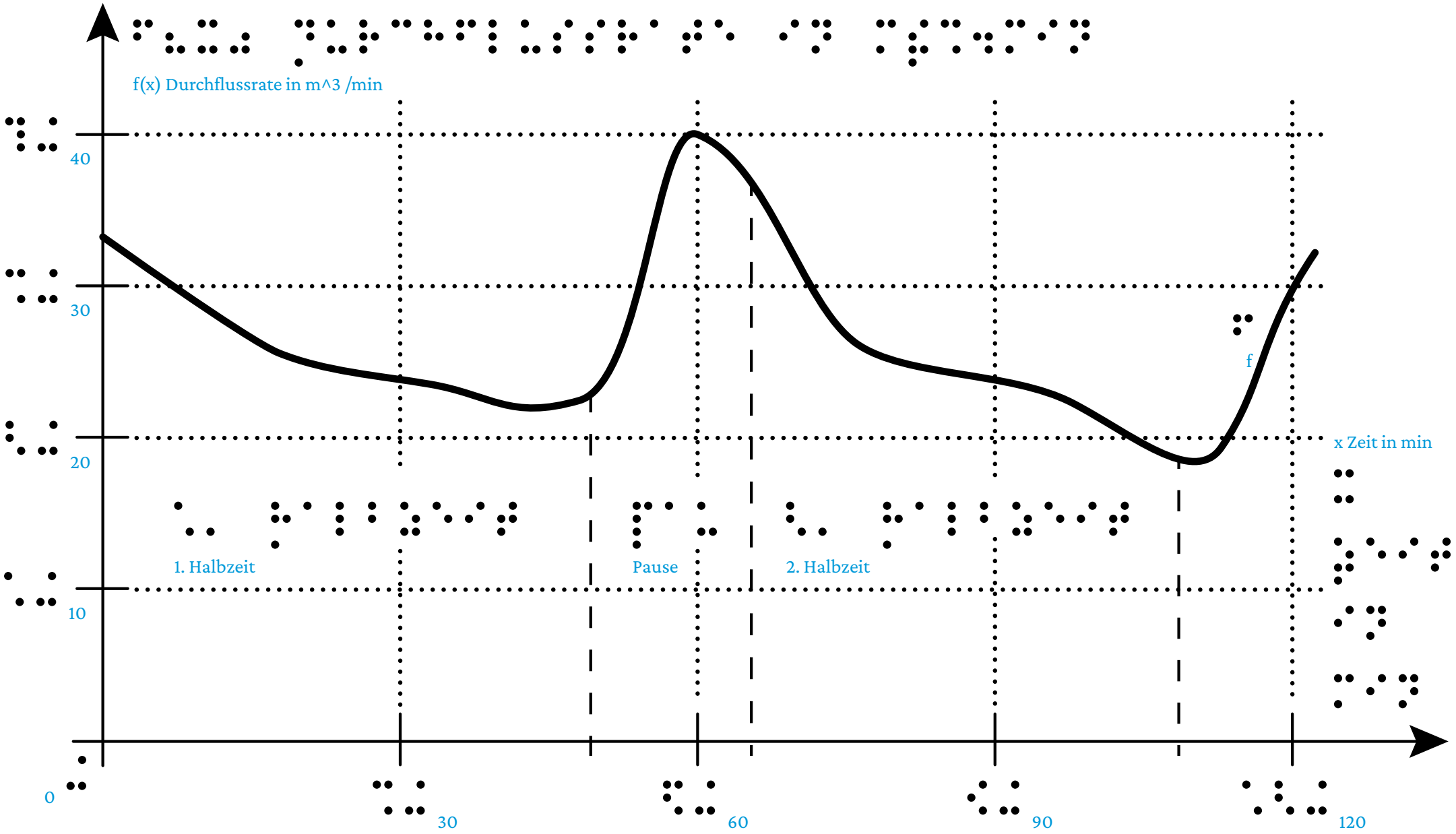


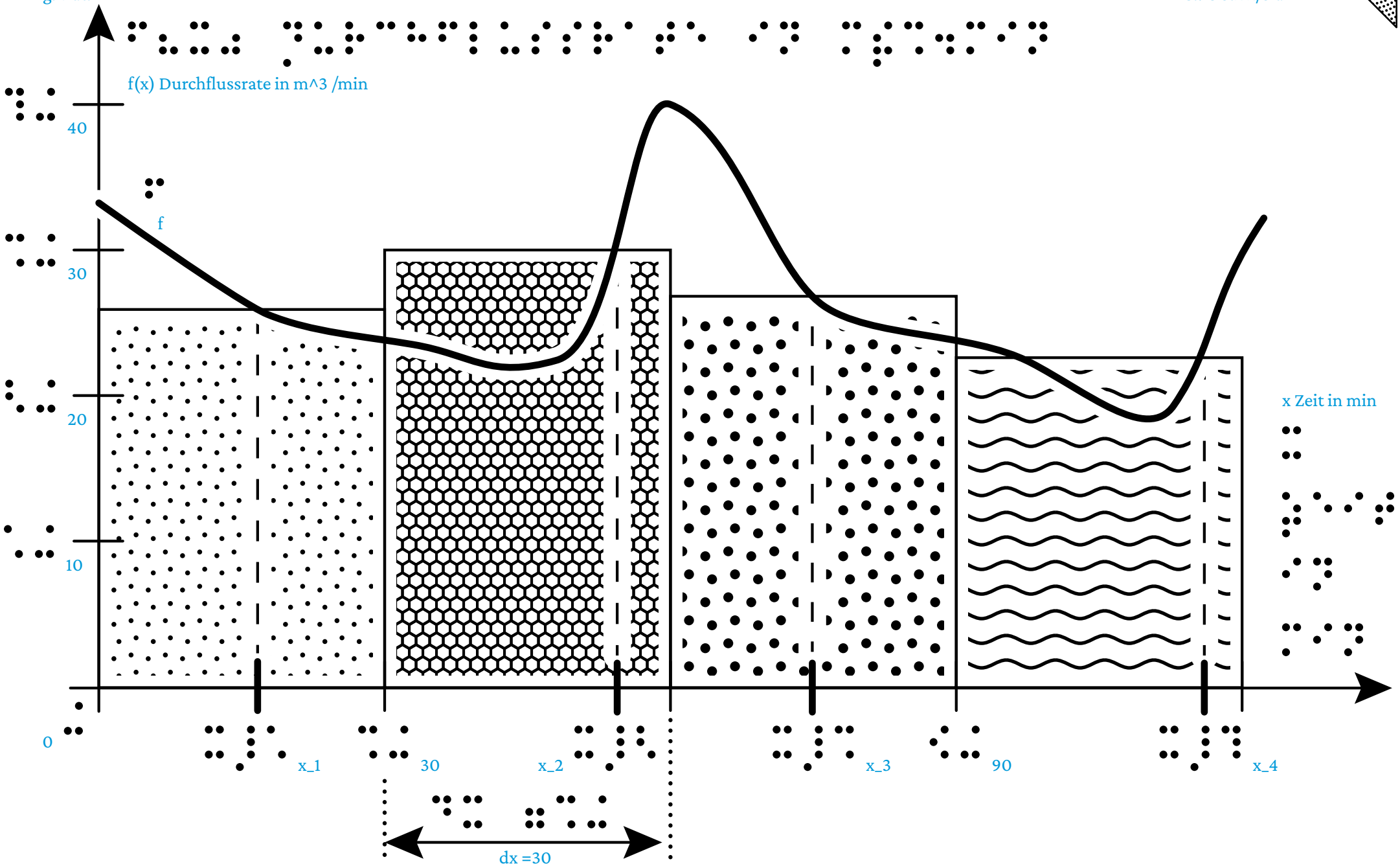
2,4*1

Zuflussmenge mithilfe der Bestandsfunktion

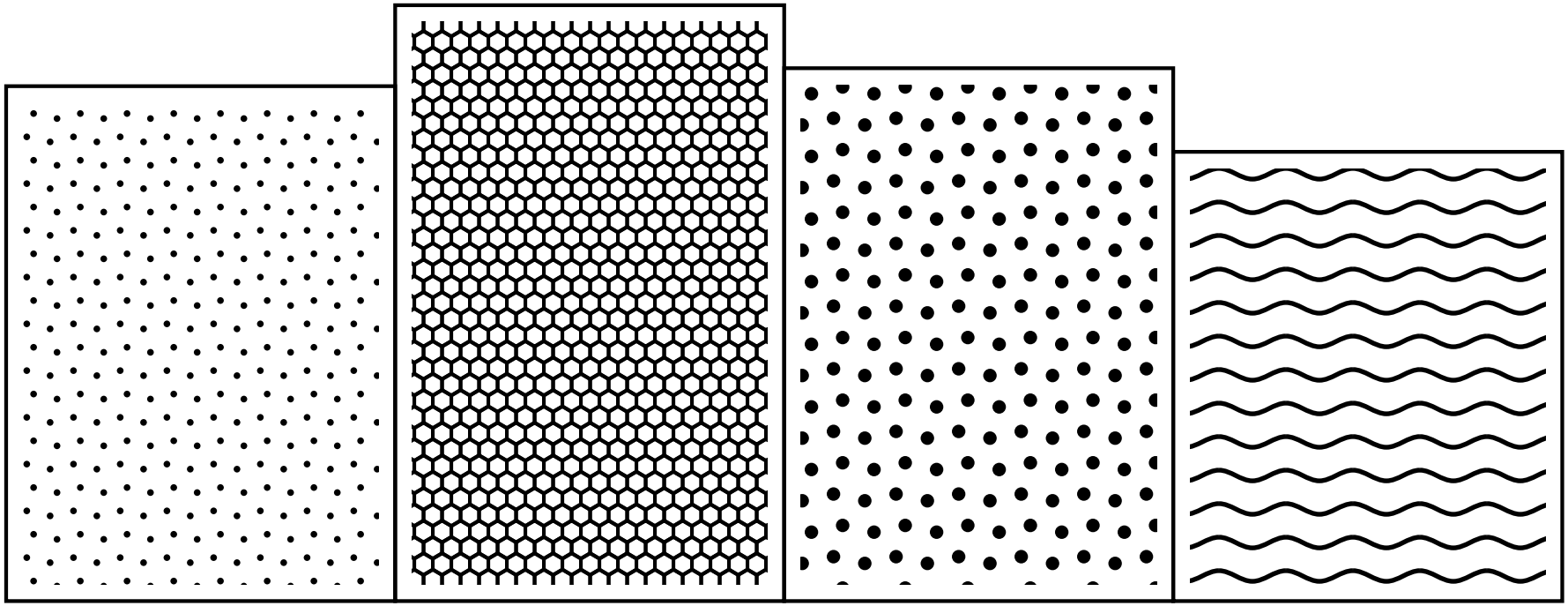



Wachstumsgeschwindigkeit in cm/Tag
f
90
80
70
60
50
40
30
20
10
0
3
6
9
12
15
18
21
x Zeit in Tagen


f(x) Durchflussrate in m^3 /min
40
30
20
10
0
f
x Zeit in min
1. Halbzeit
Pause
2. Halbzeit
30
60
90
120


f(x) Durchflussrate in m^3 /min
40
30
20
10
f
0
x_1
30
x_2
x_3
90
x_4
x Zeit in min
dx =30

Legende zum Graph:

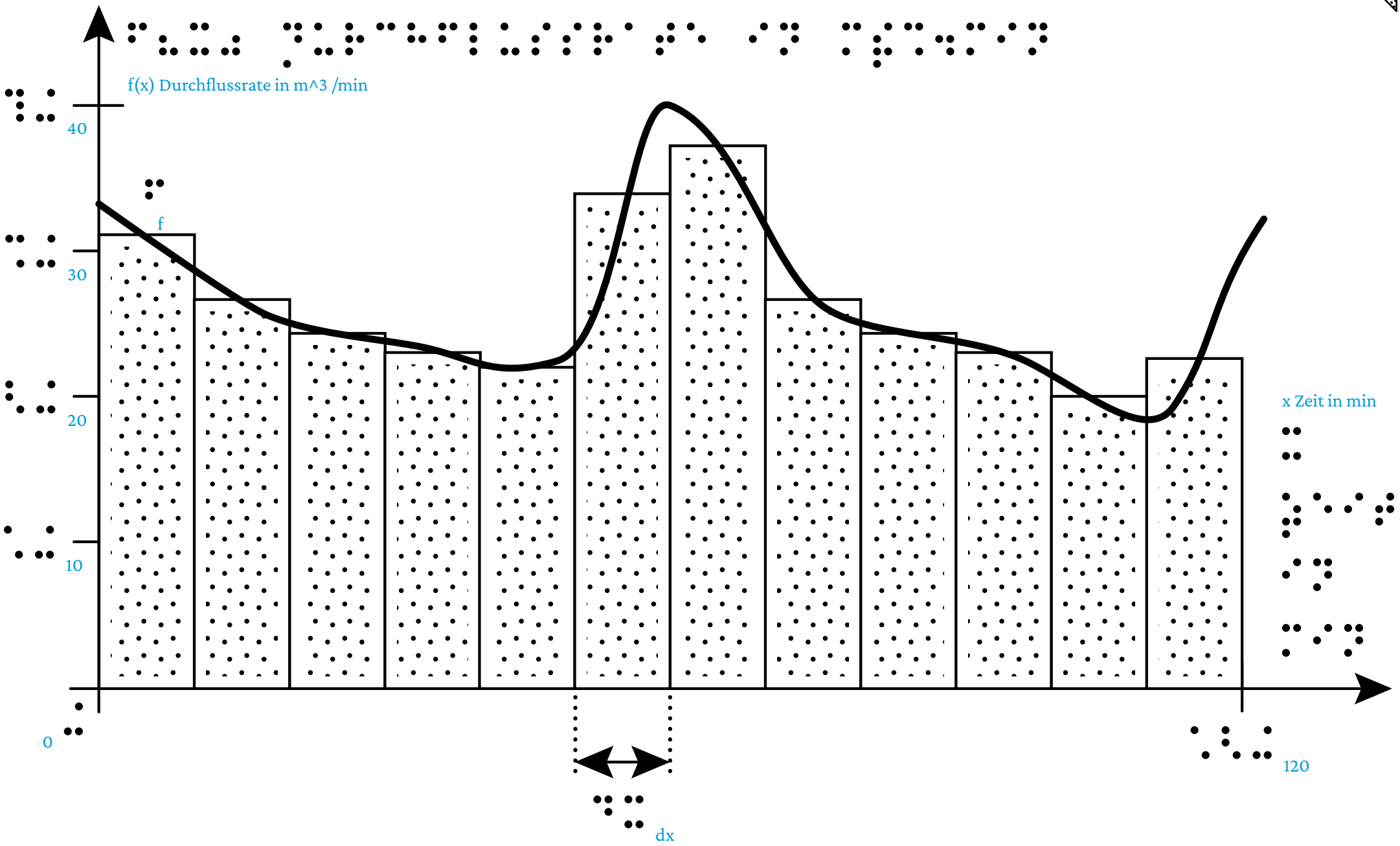


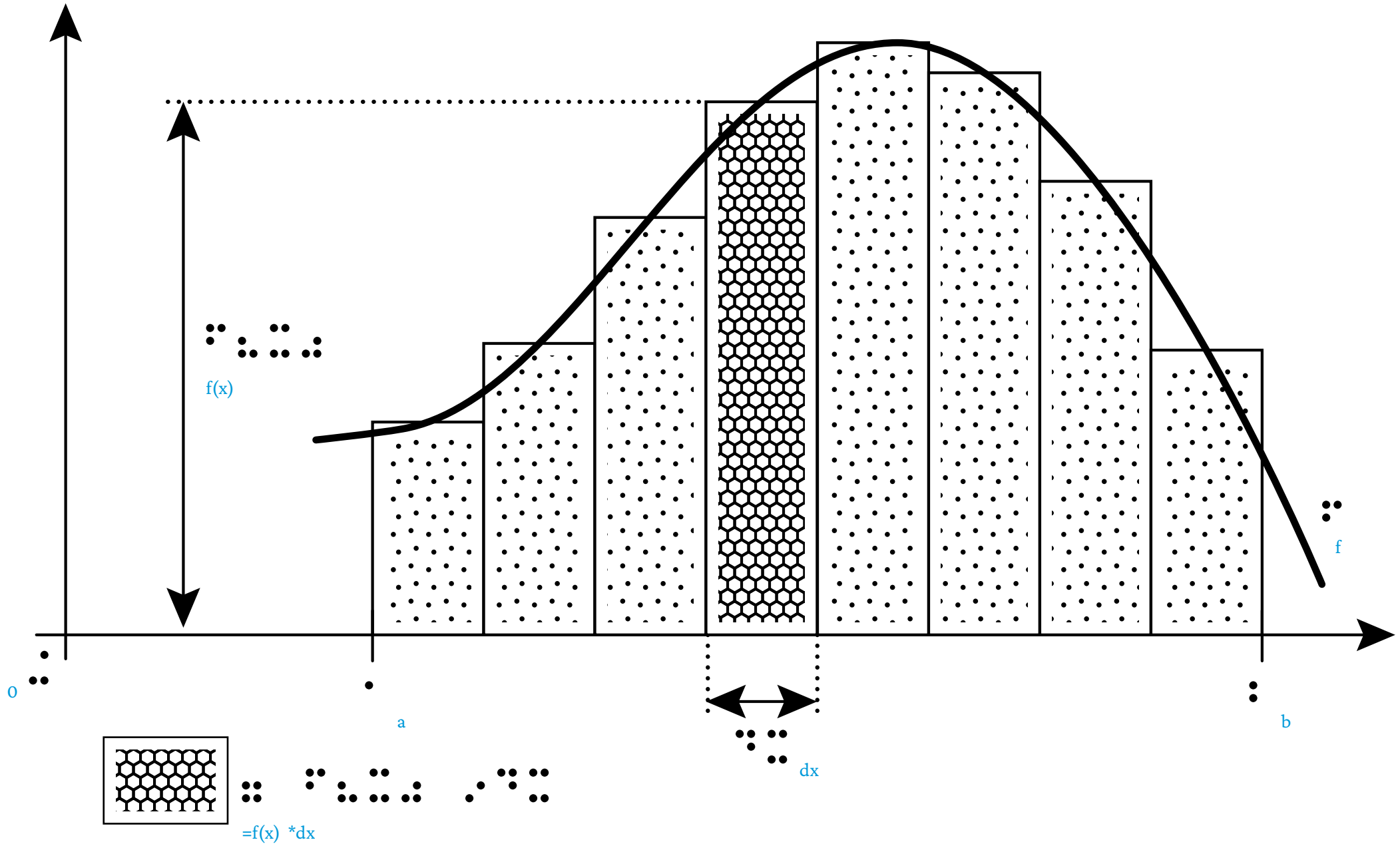
Fläche =f(x_1) *30

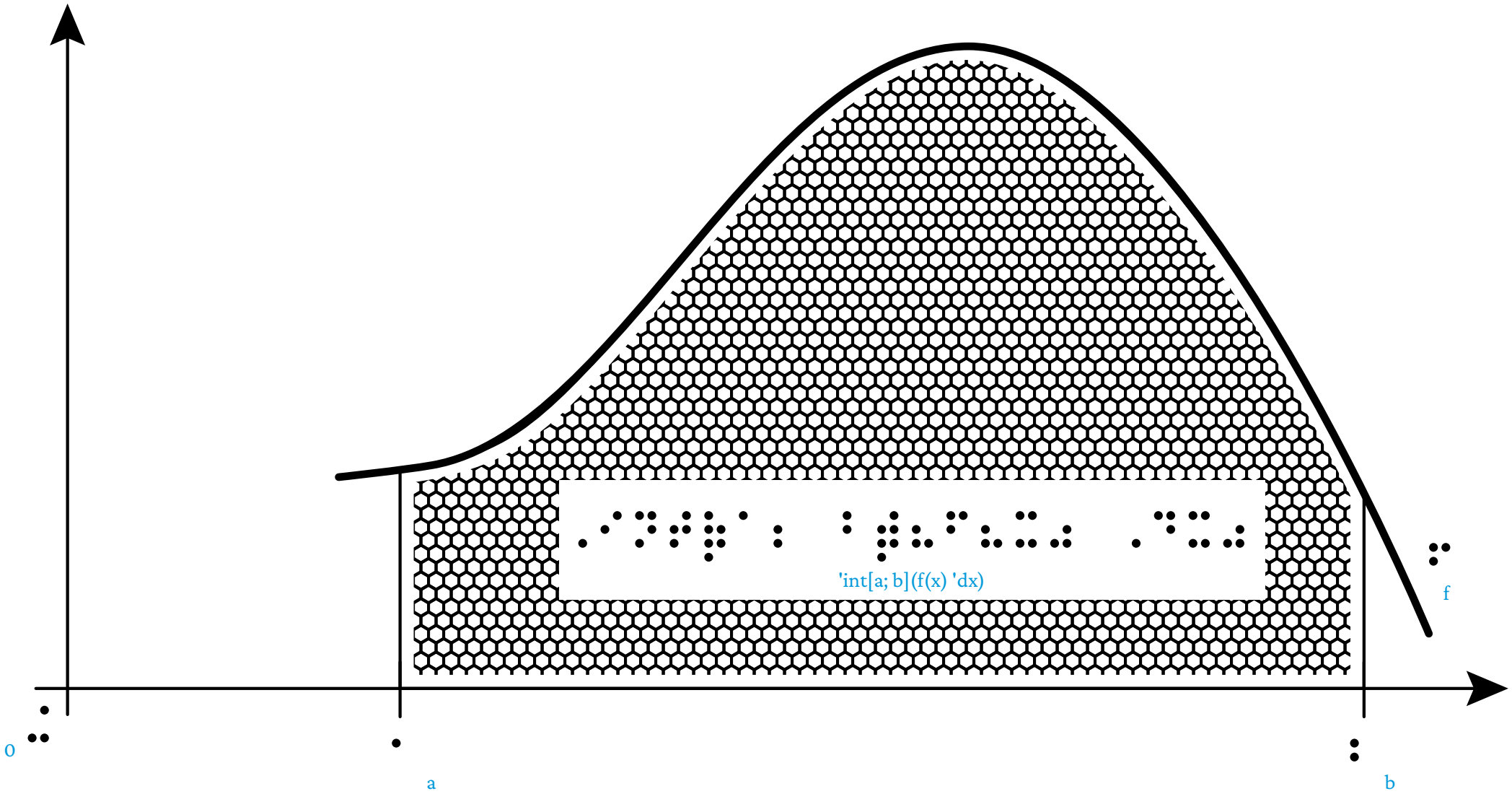
Fläche =f(x_2) *30

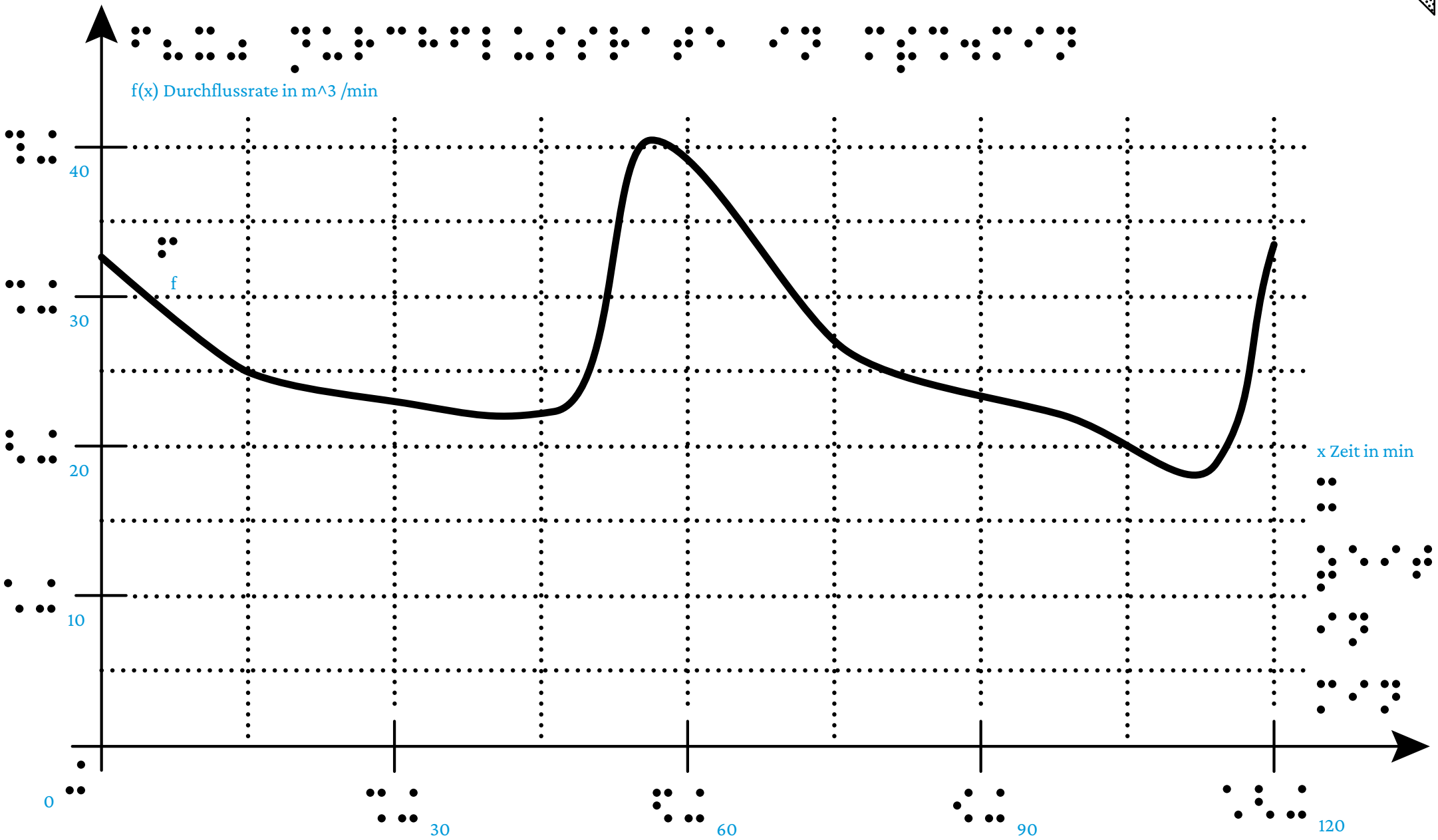
Fläche =f(x_3) *30

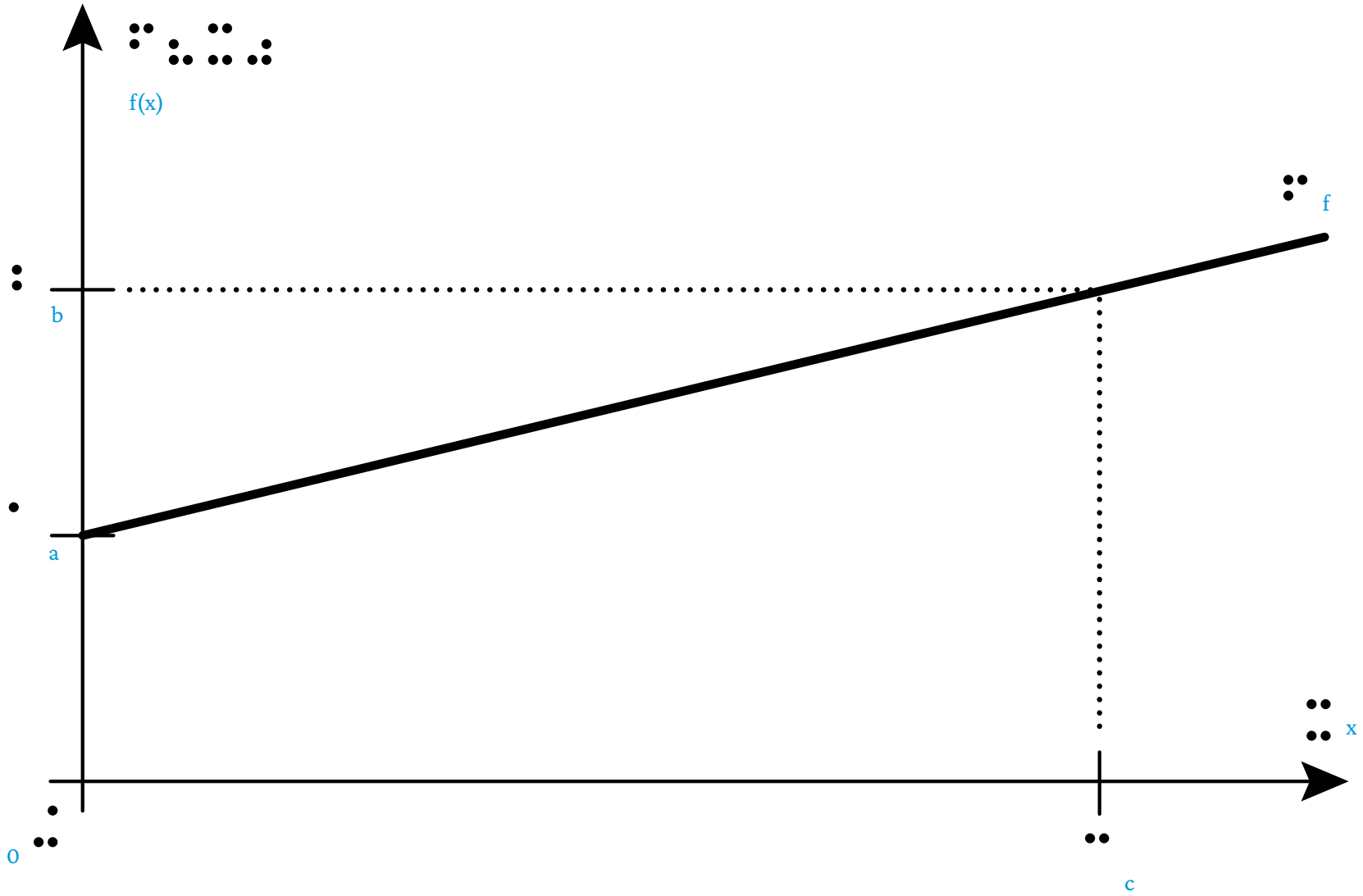
Fläche =f(x_4) *30


f(x) Durchflussrate in m^3 /min
40
30
20
10
0
f
x Zeit in min
120
dx

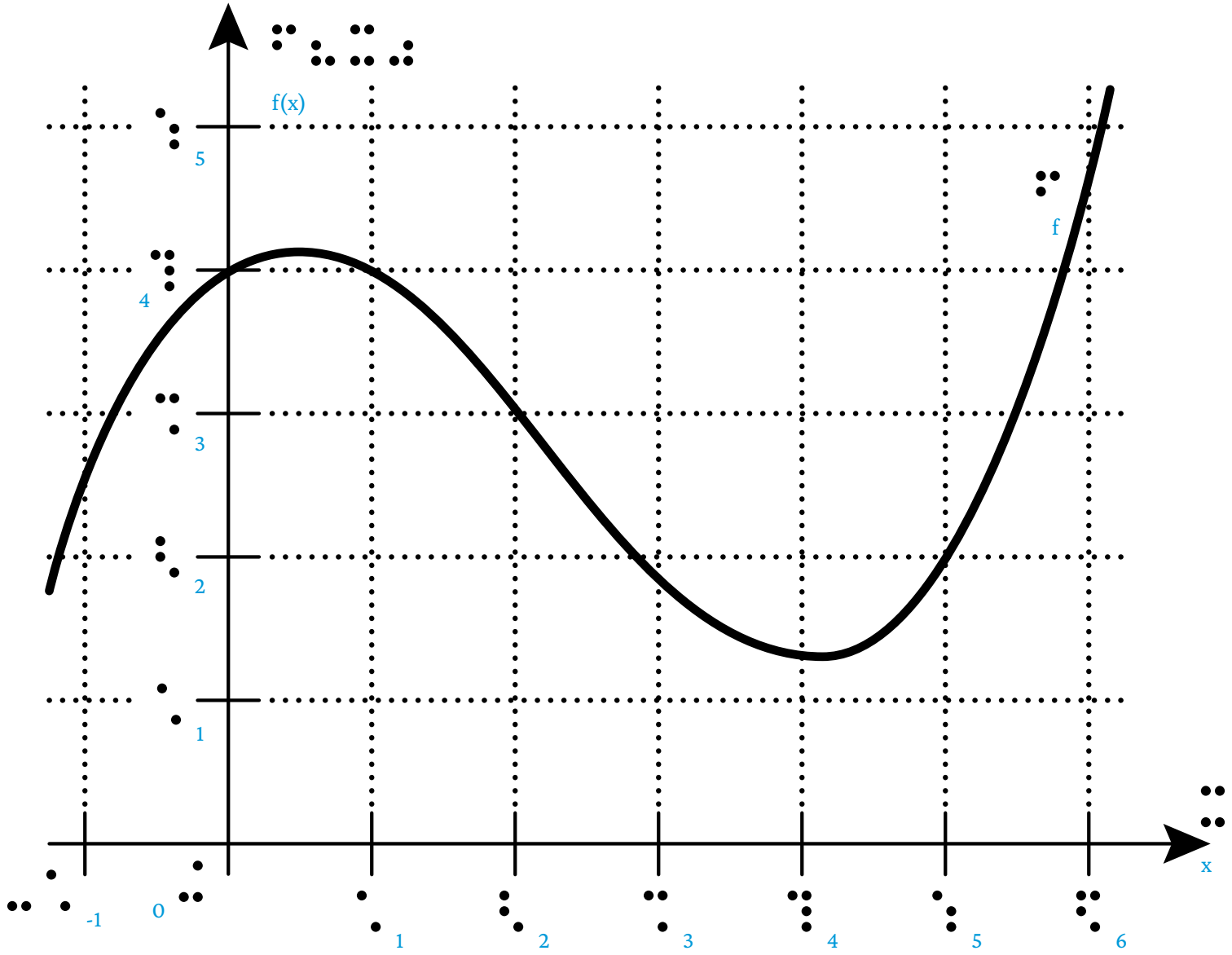

f(x)
f
0
a
b
dx
=f(x) *dx


'int[a; b](f(x) 'dx)
f
0
a
b

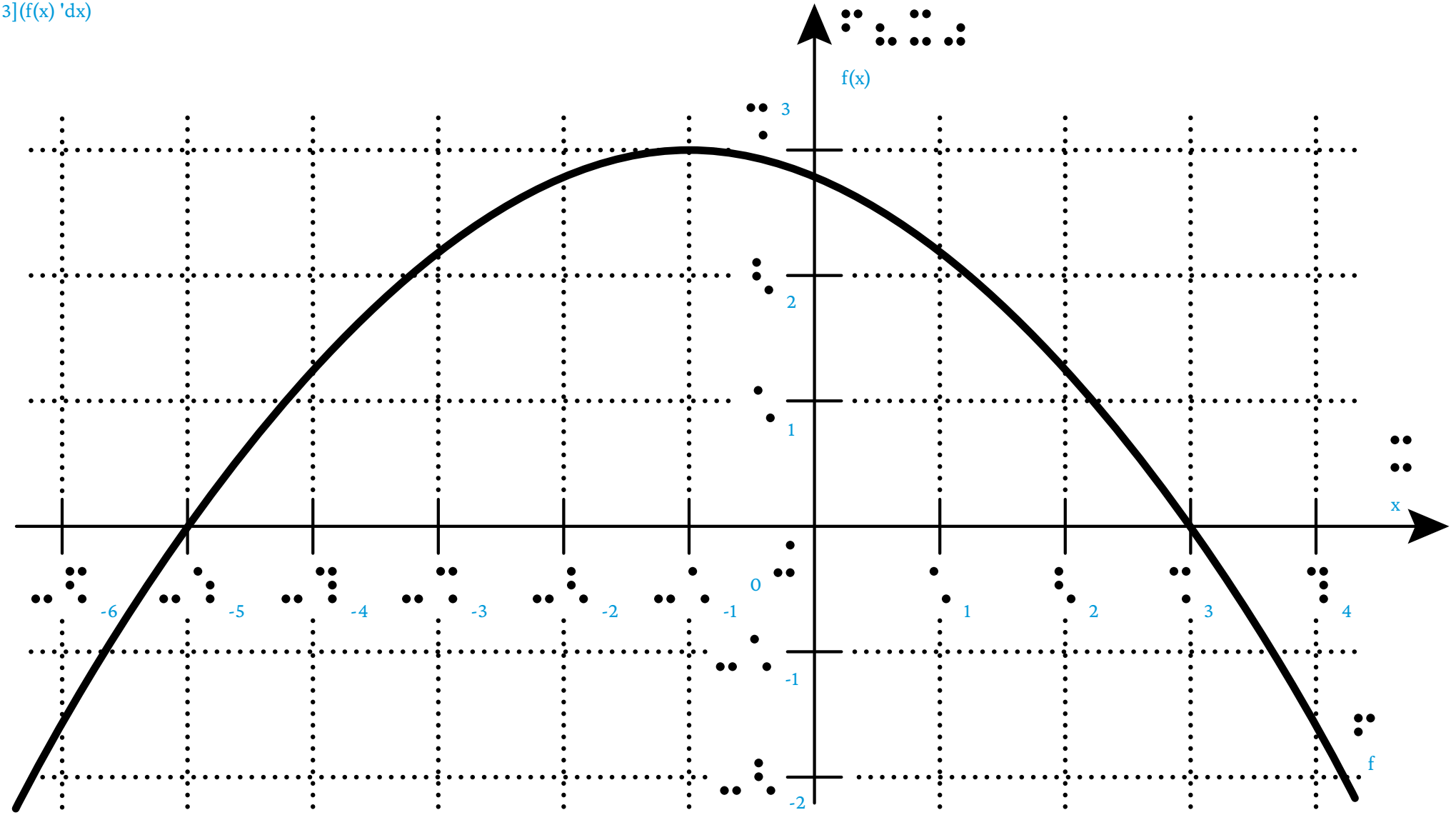

f(x) Durchflussrate in m^3 /min
f
40
30
20
10
0
30
60
90
120
x Zeit in min


f(x)
f
b
a
x
0
c

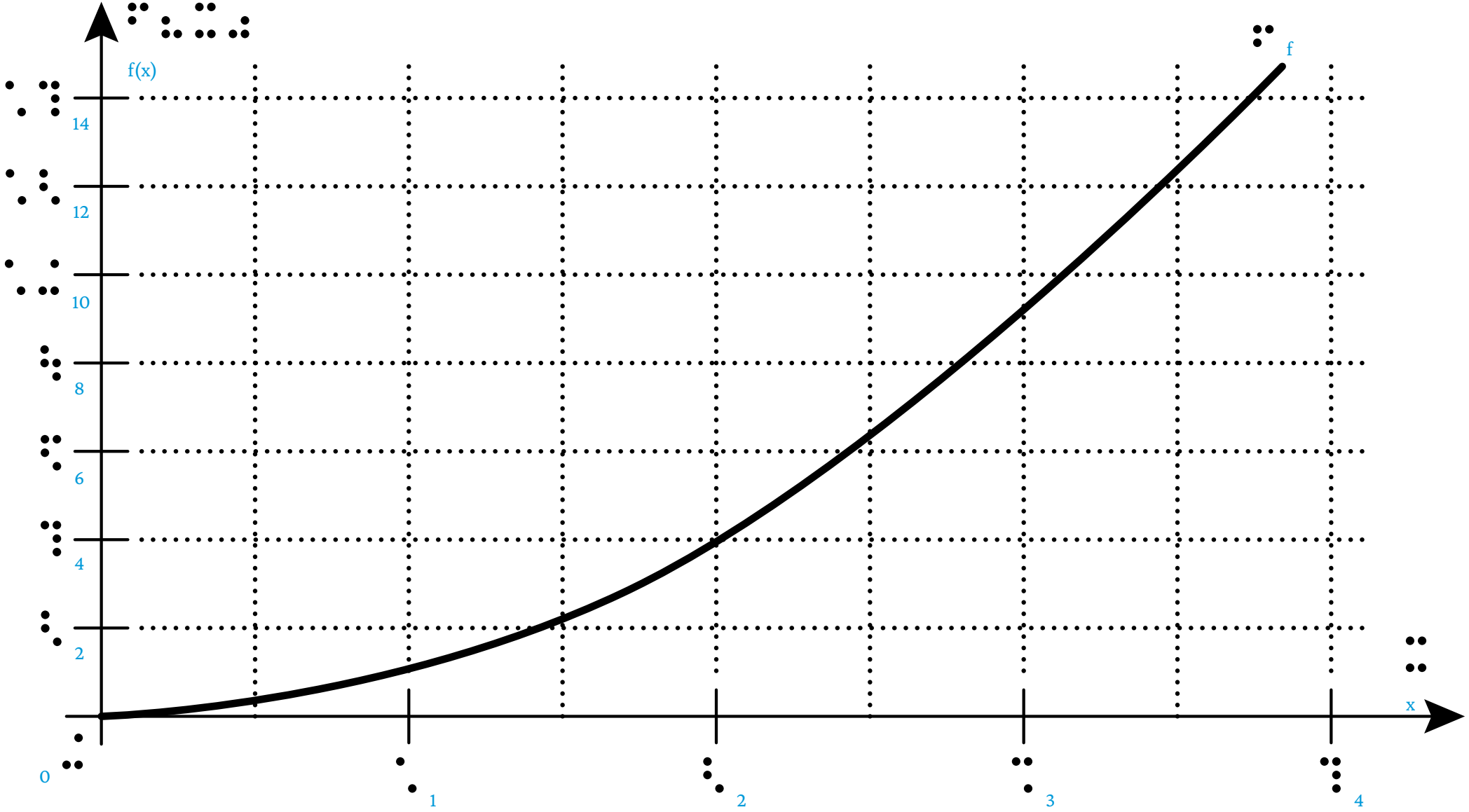
'int[0; 5](f(x) 'dx)

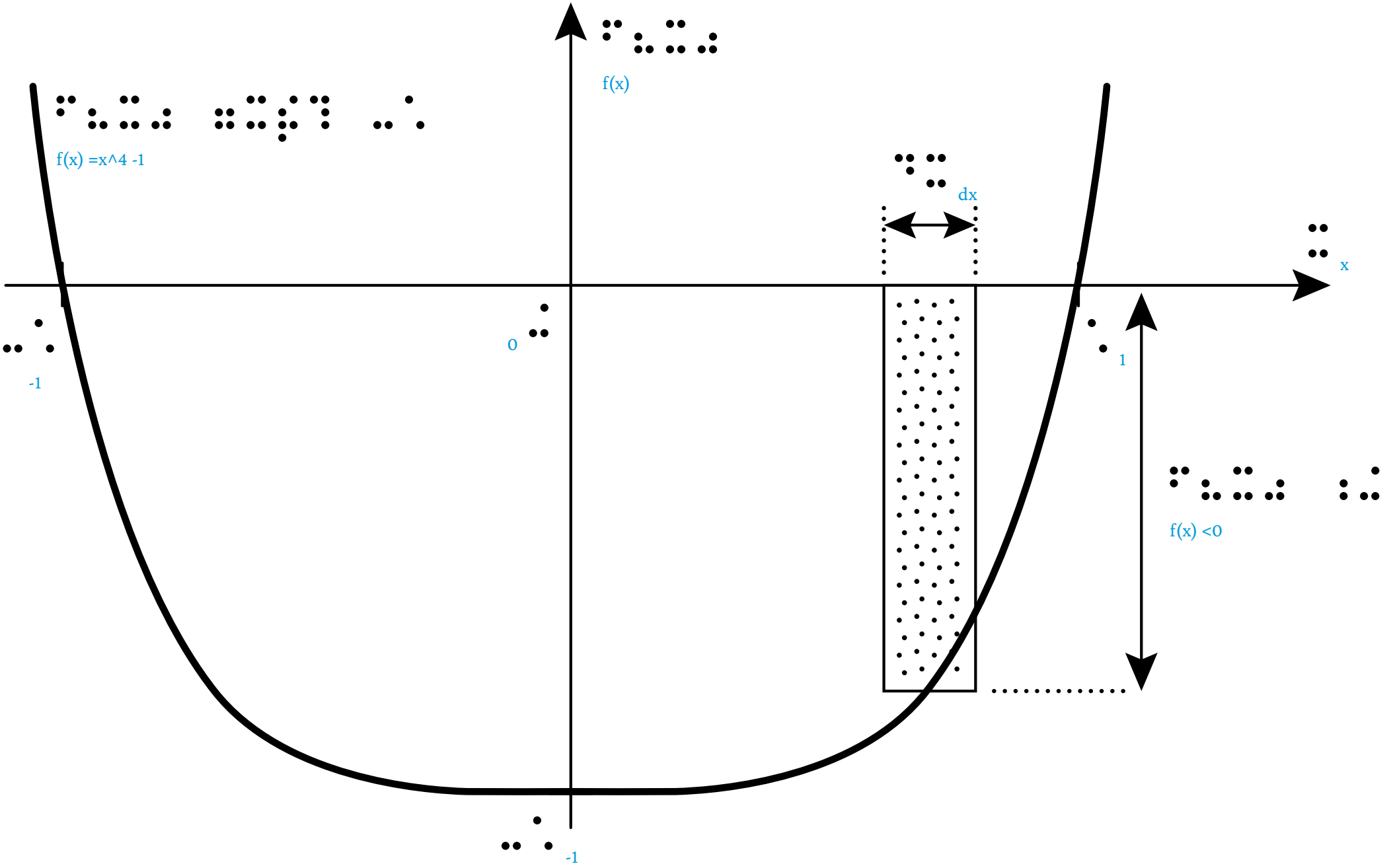


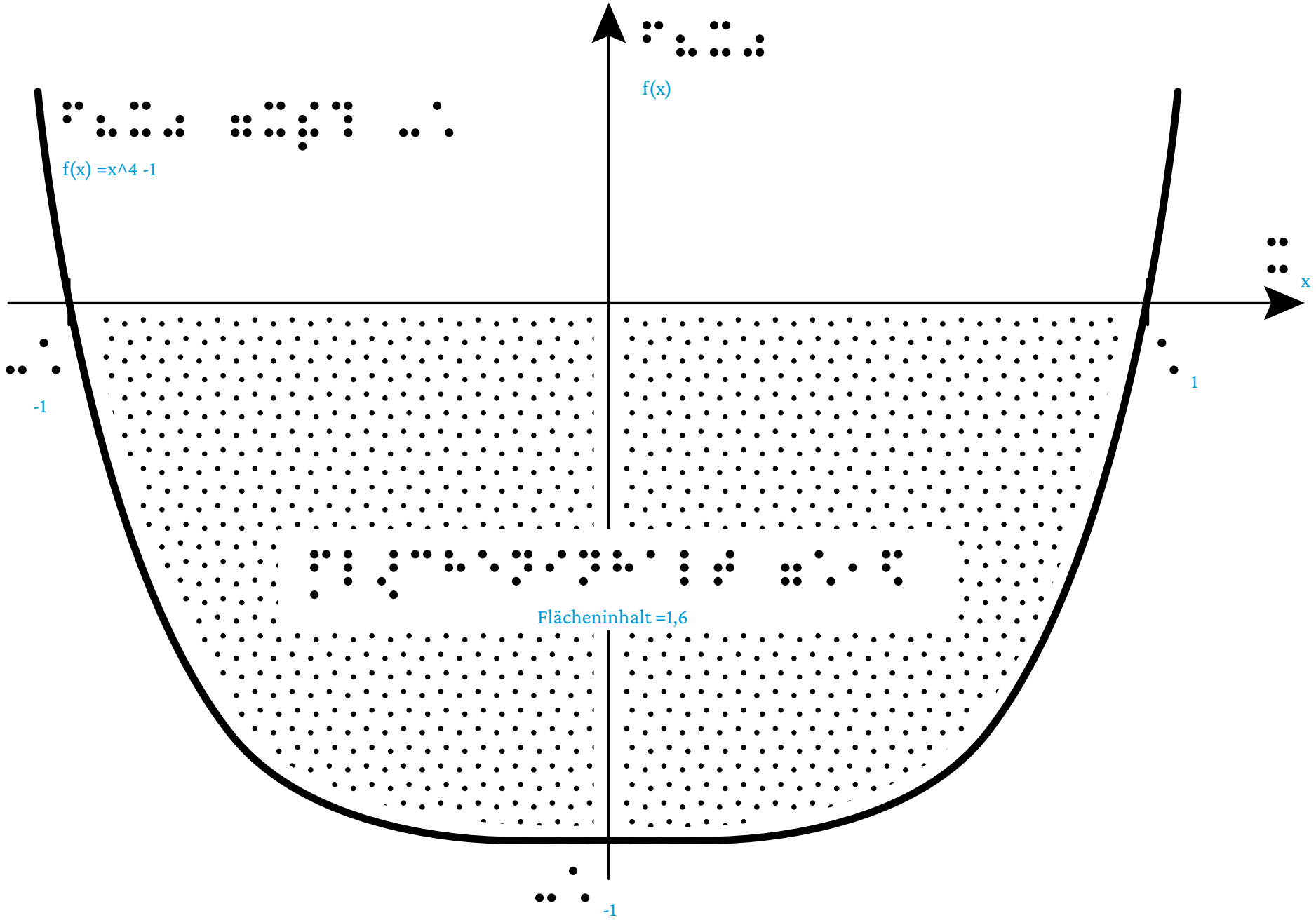
'int[-2; 3](f(x) 'dx)

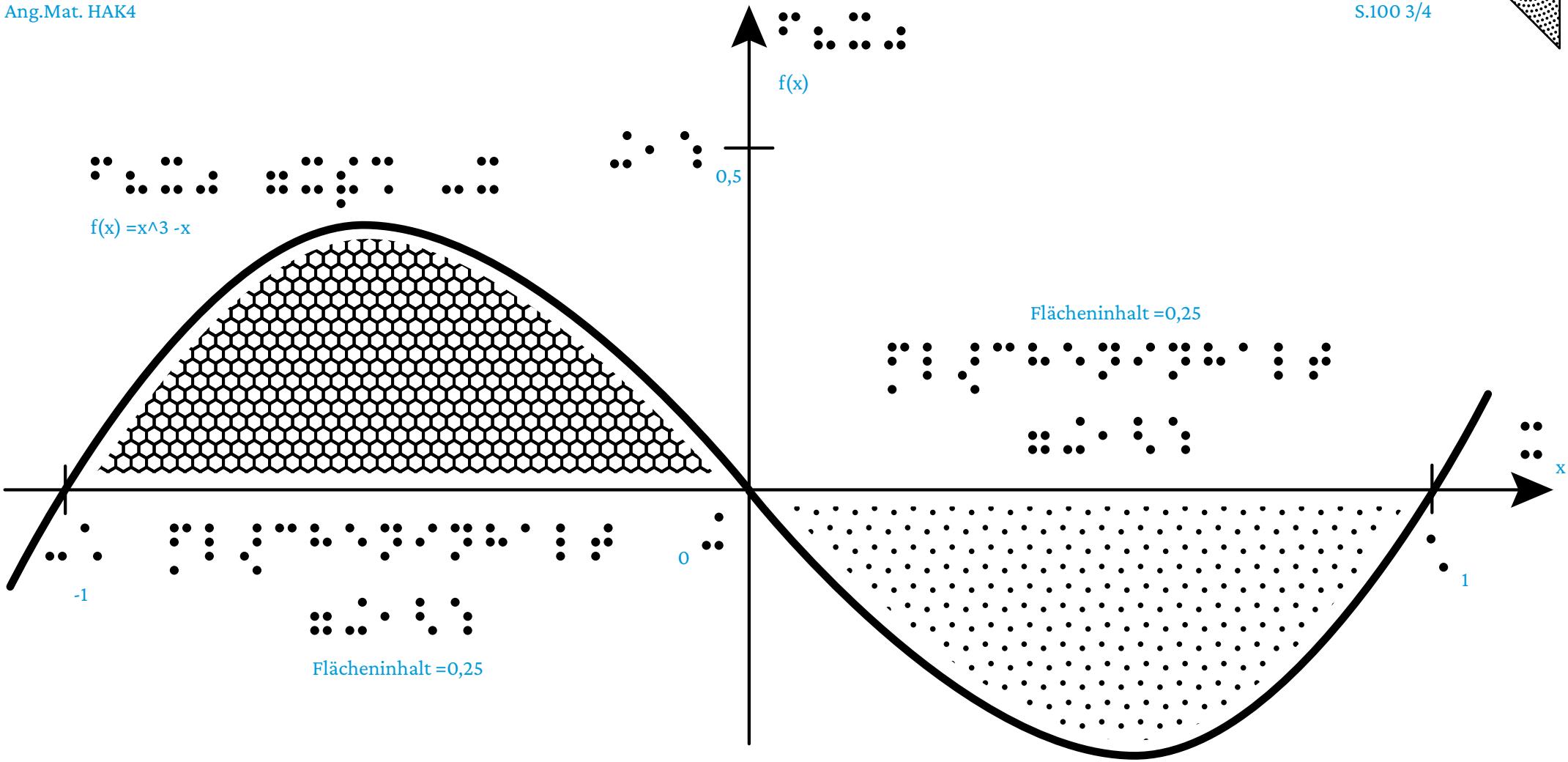


'int[1; 3](f(x) 'dx)

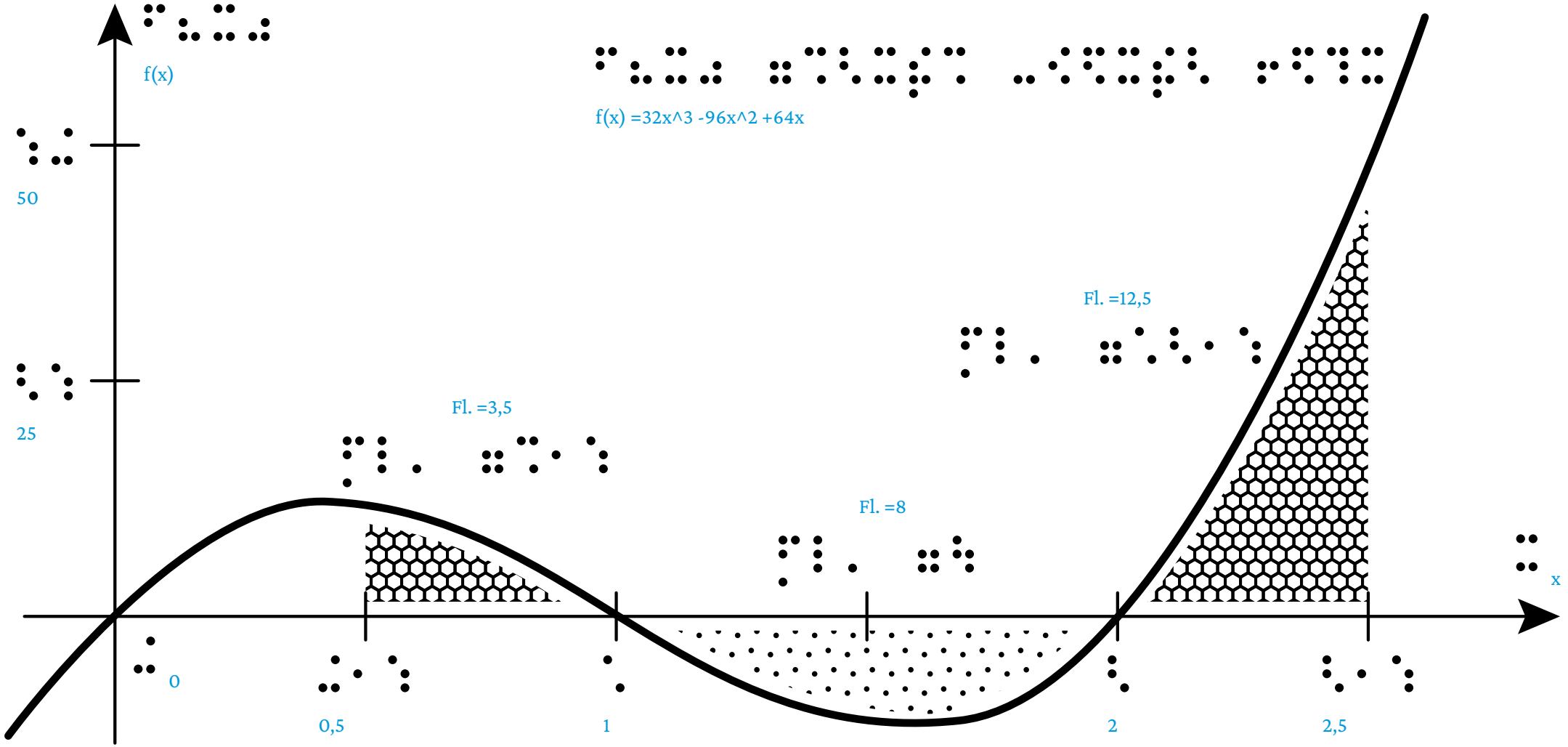



f(x)
f(x) =x^4 -1
dx
x
0
-1
1
-1
f(x) <0


f(x)
f(x) =x^4 -1
x
-1
1
Flächeninhalt =1,6
-1

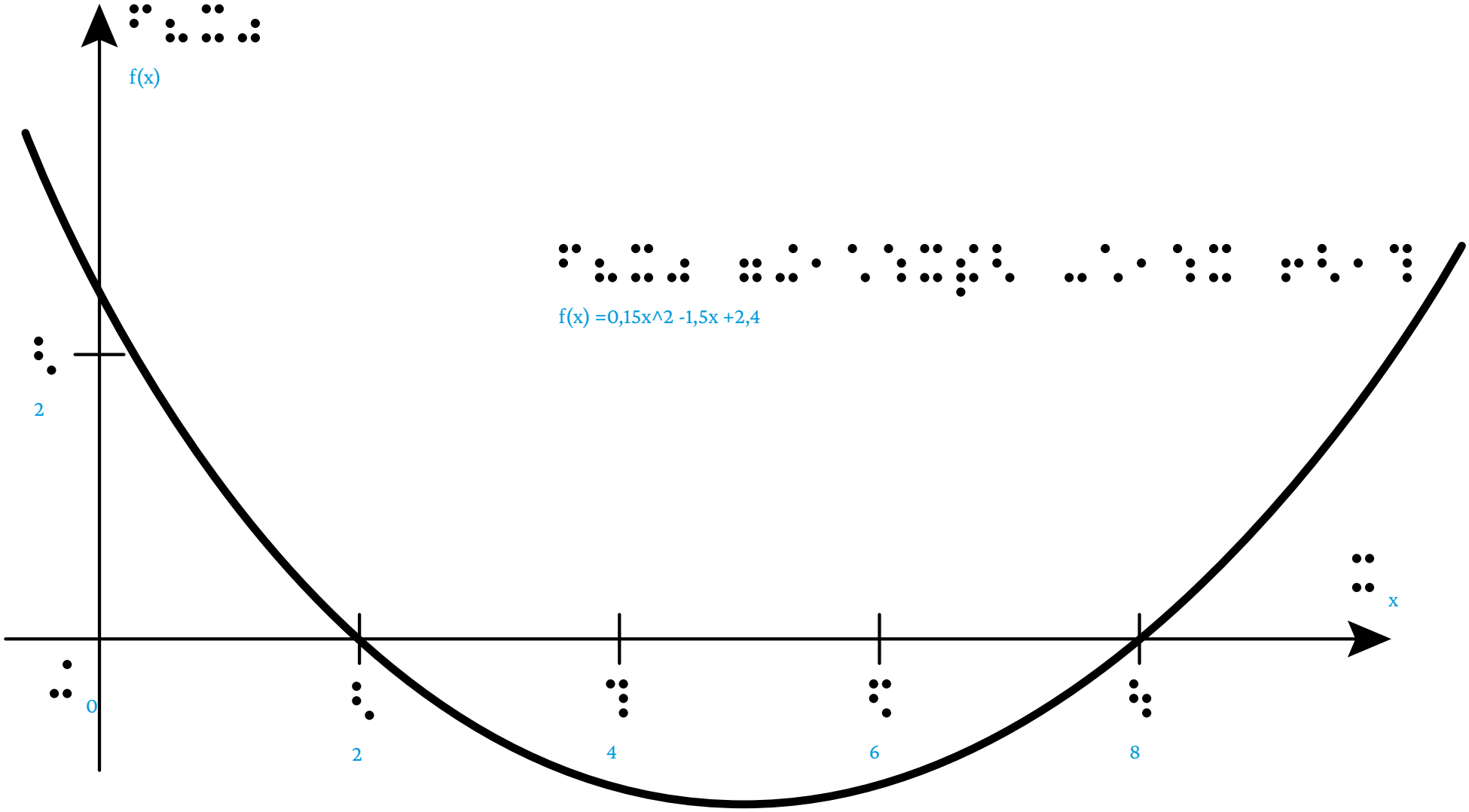


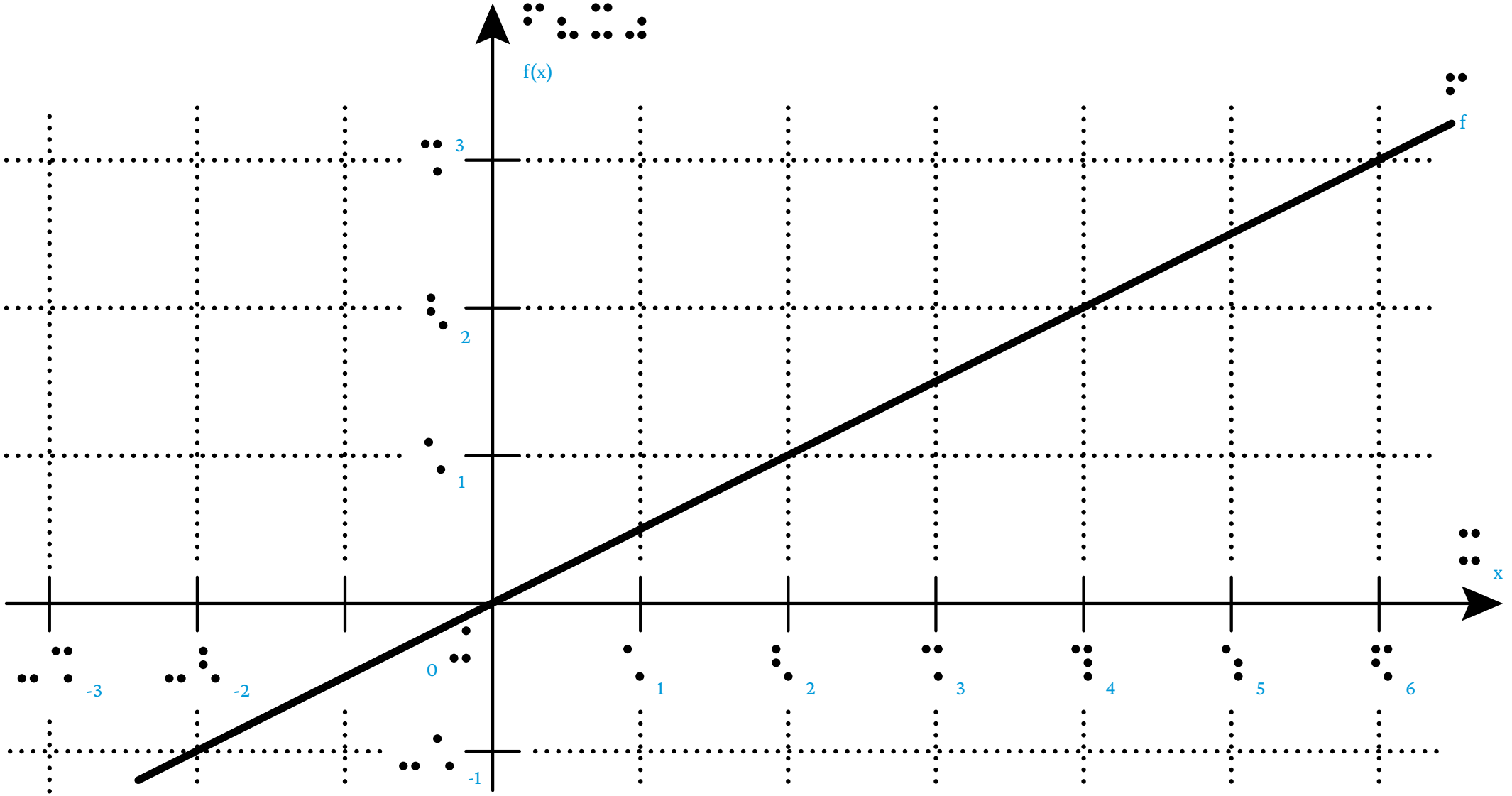
'int[-1; 1](f(x) 'dx) =0,25 -0,25 =0

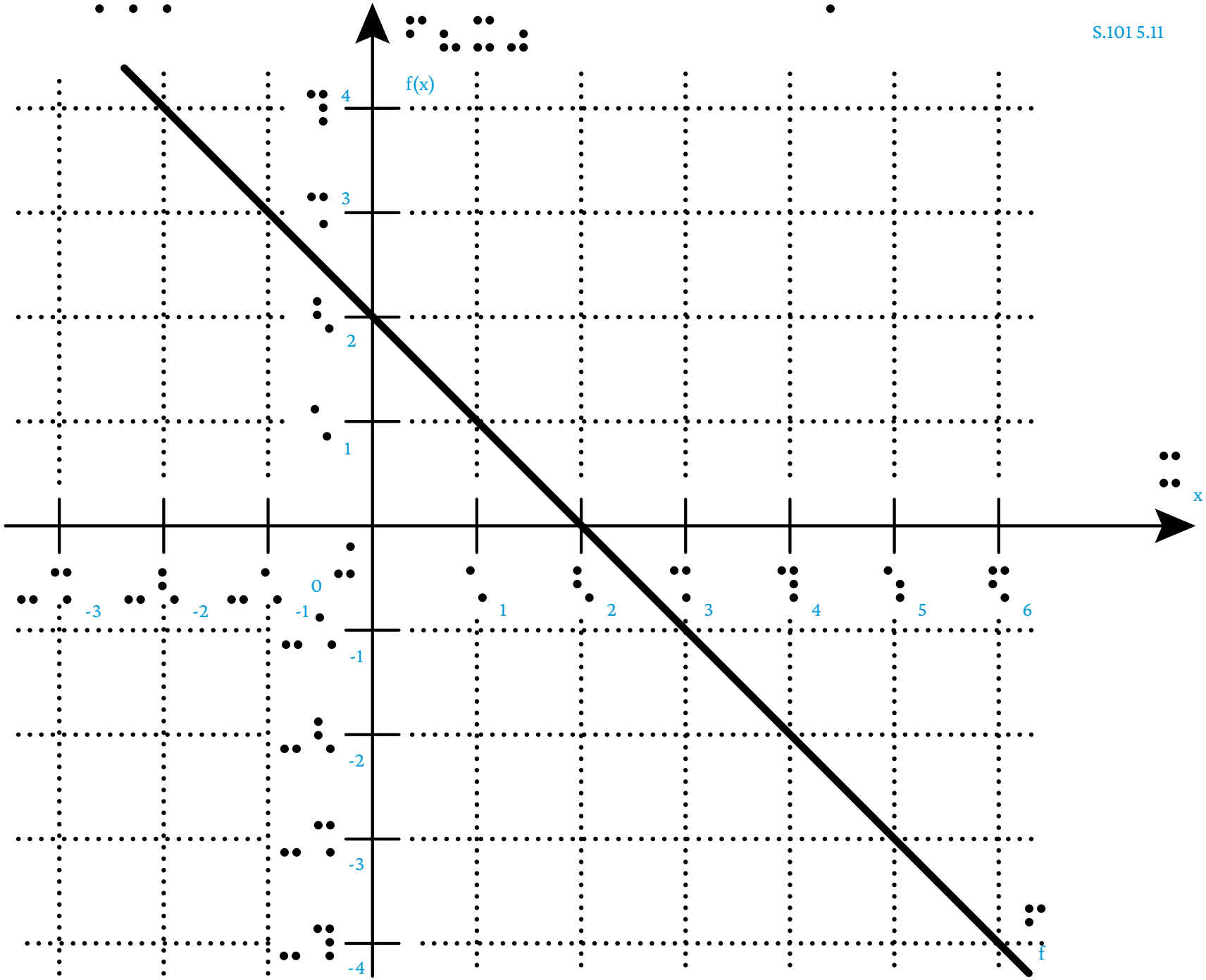


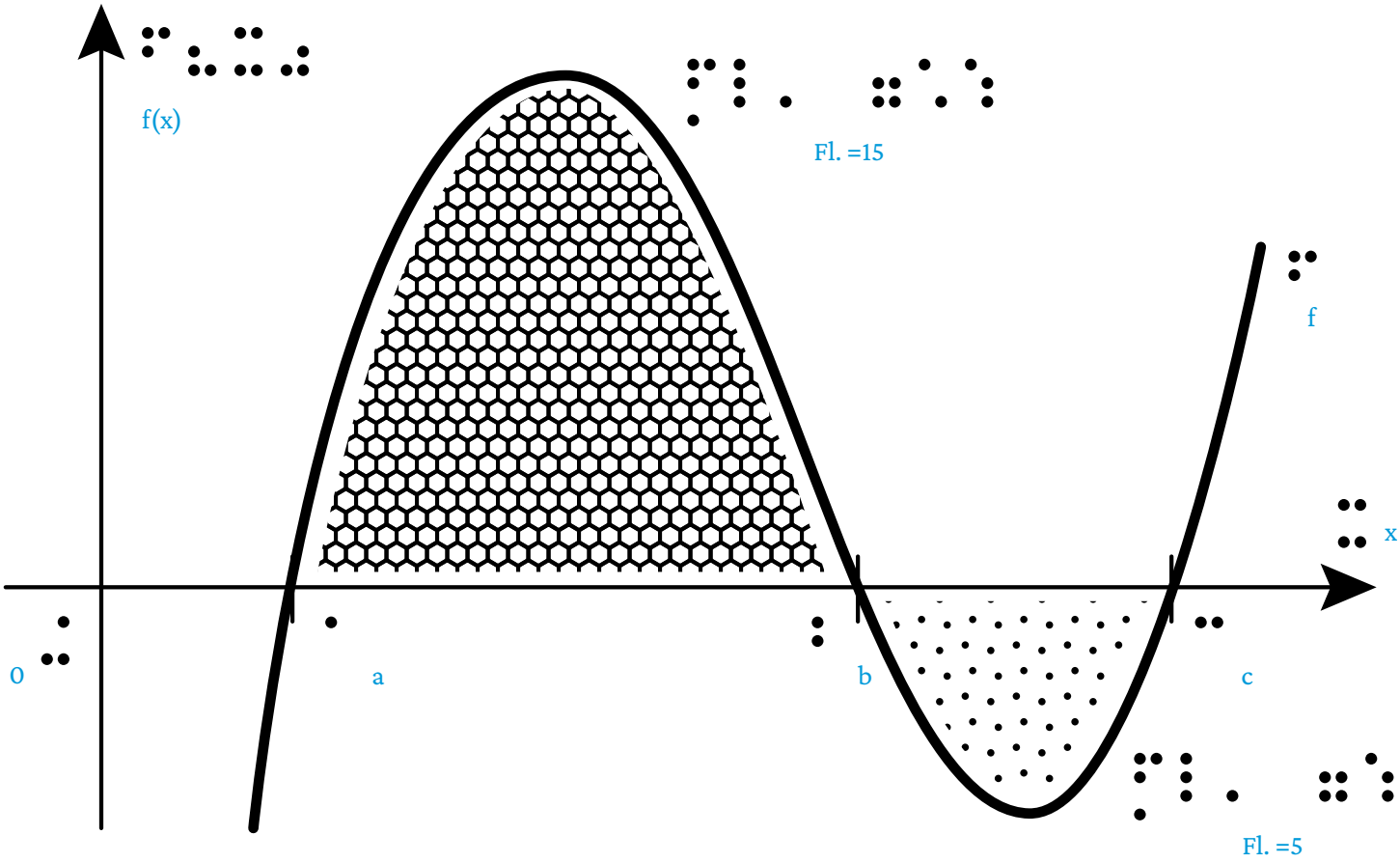
f(x) =32x^3 -96x^2 +64x

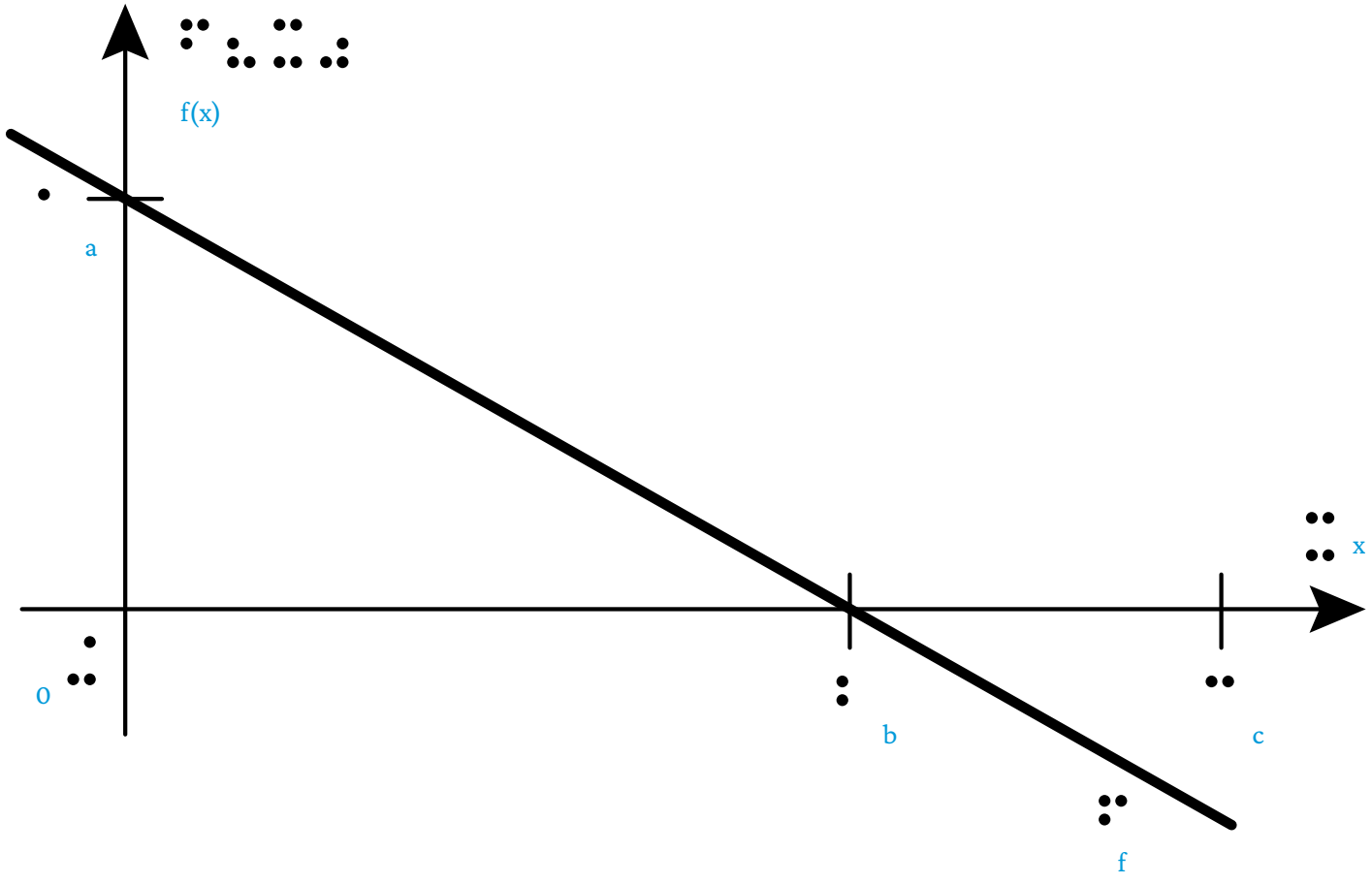
'int[0,5; 2,5](f(x) 'dx) =3,5 -8 +12,5 =8

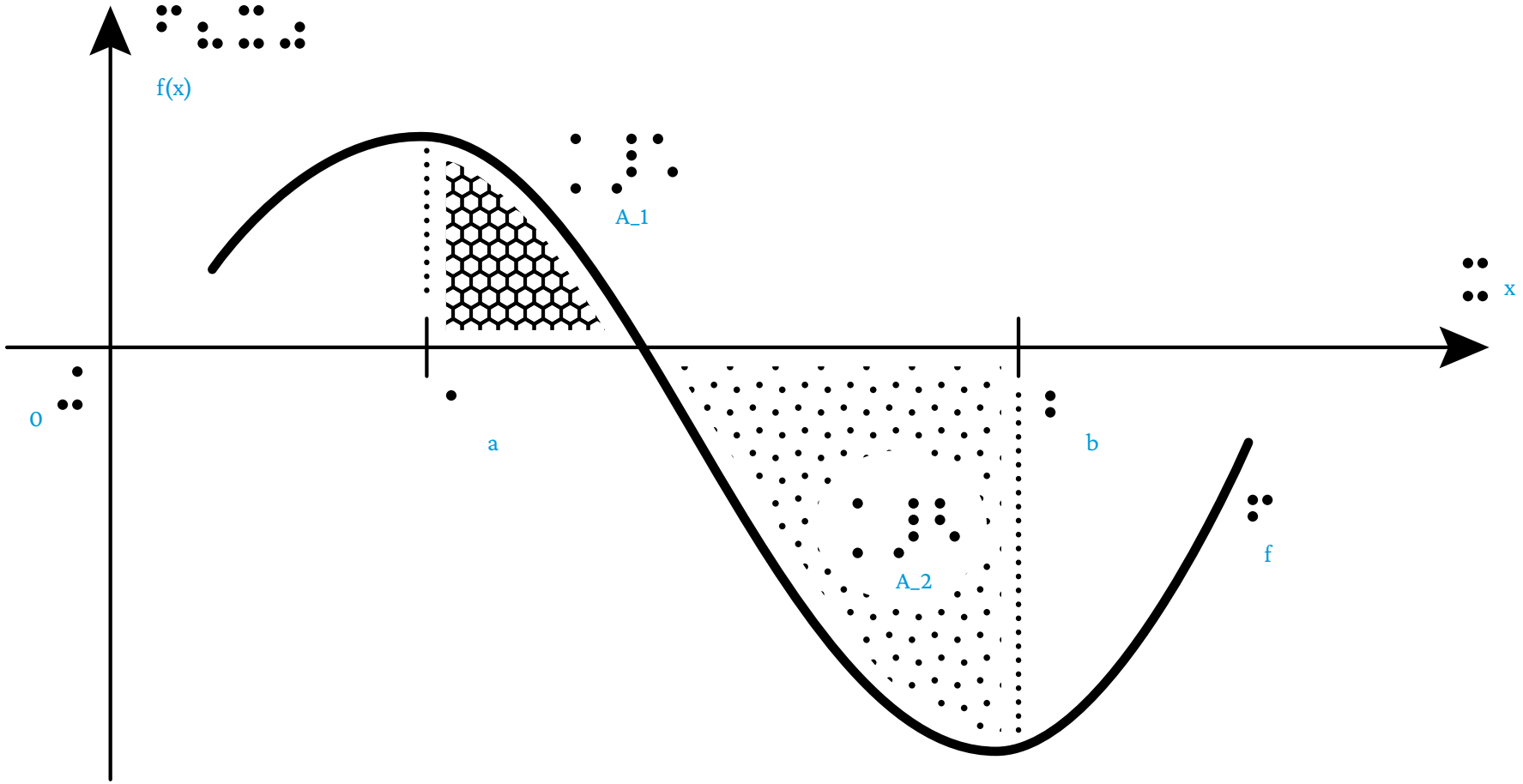

f(x)
f(x) =0,15x^2 -1,5x +2,4
2
x
0
2
4
6
8

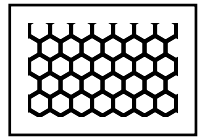

f(x)
f
x
3
2
1
-1
0
-3
-2
1
2
3
4
5
6


f(x)
x
4
3
2
1
0
-1
-2
-3
-4
-3
-2
-1
1
2
3
4
5
6
f

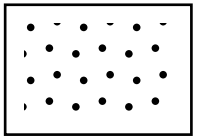

f(x)
Fl. =15
f
x
0
a
b
c
Fl. =5


f(x)
a
x
0
b
c
f

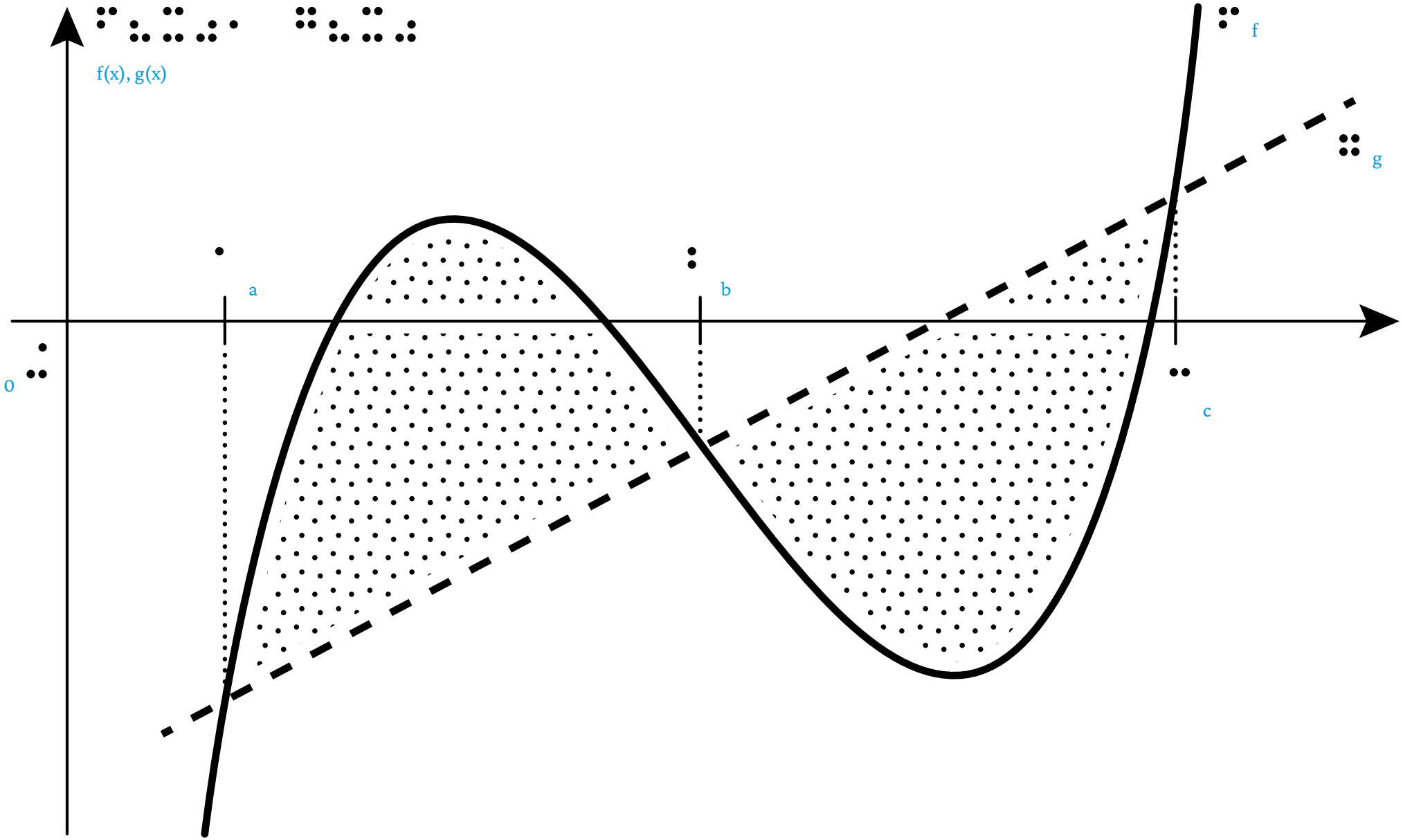

f(x)
A_1
x
0
a
b
A_2
f

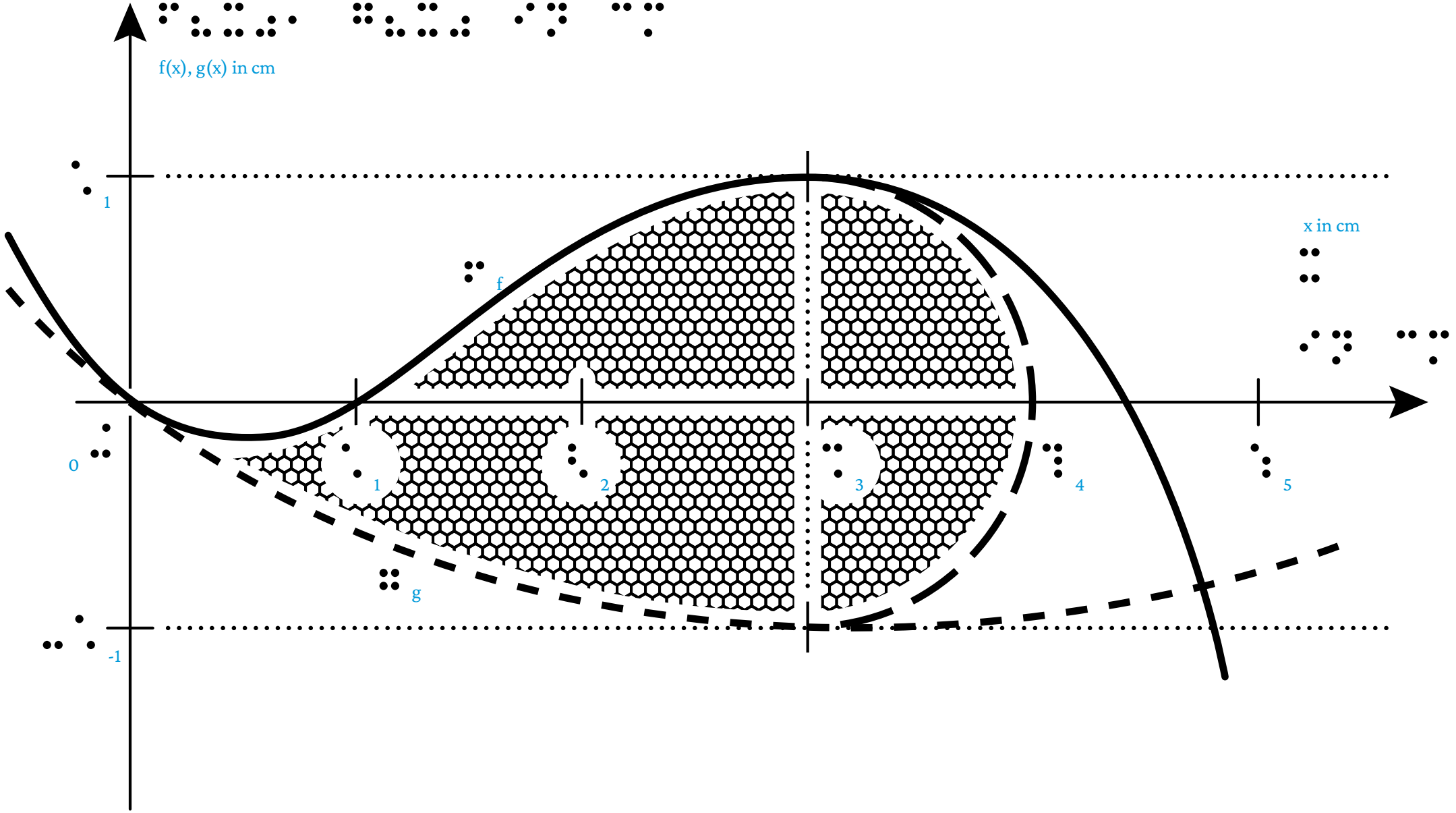


(f(x) -g(x)) *dx

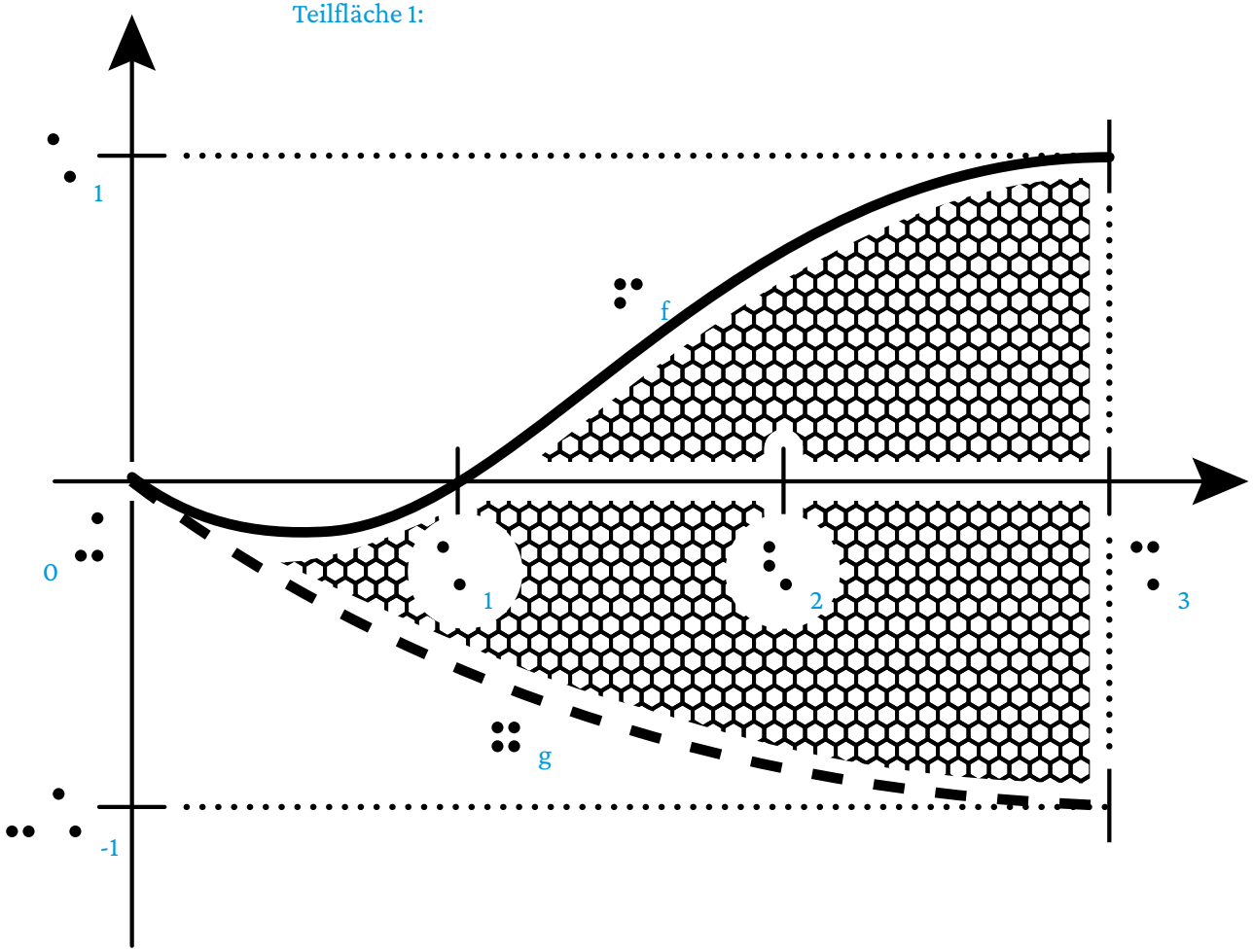


'int[a; b](f(x) -g(x) 'dx)

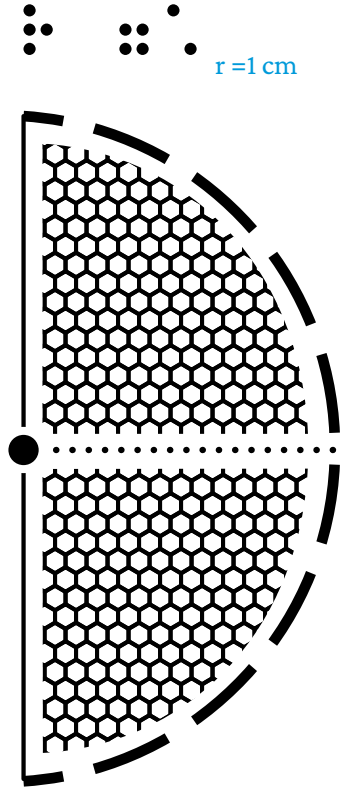

f(x), g(x)
f
g
a
b
c
0


f(x), g(x) in cm
1
x in cm
f
0
1
2
3
4
5
g
-1

Teilfläche 1:

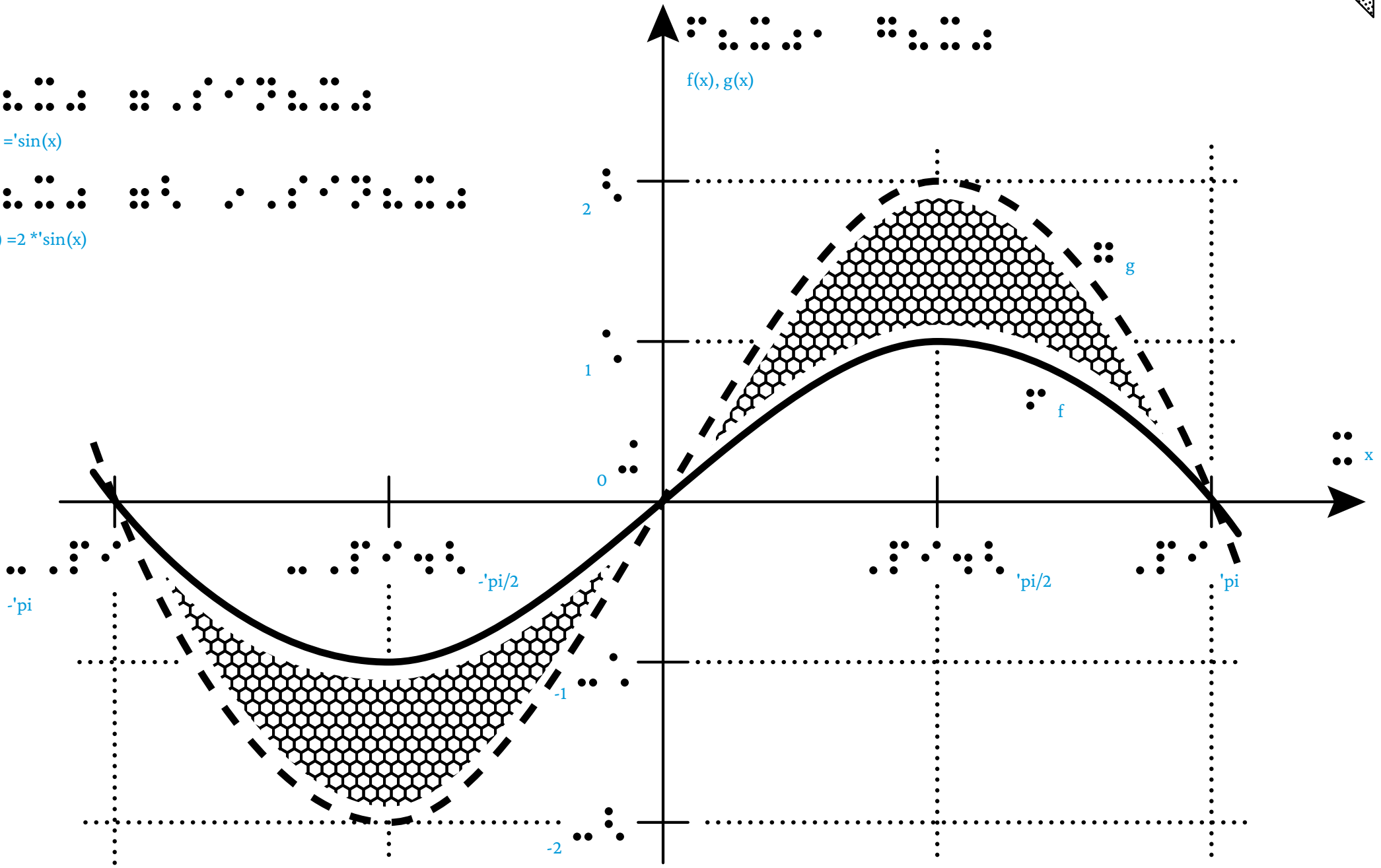


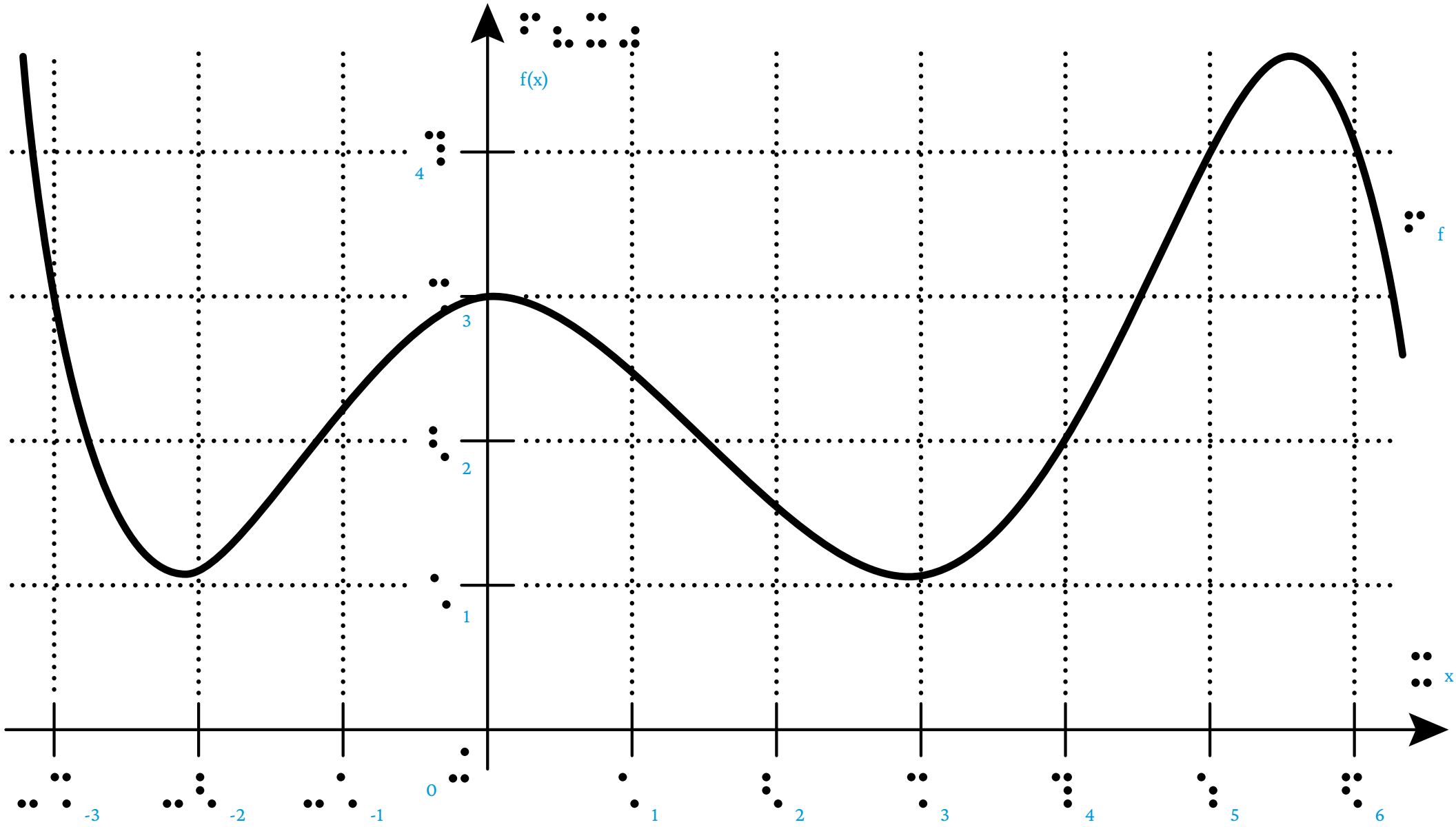
Teilfläche 2:

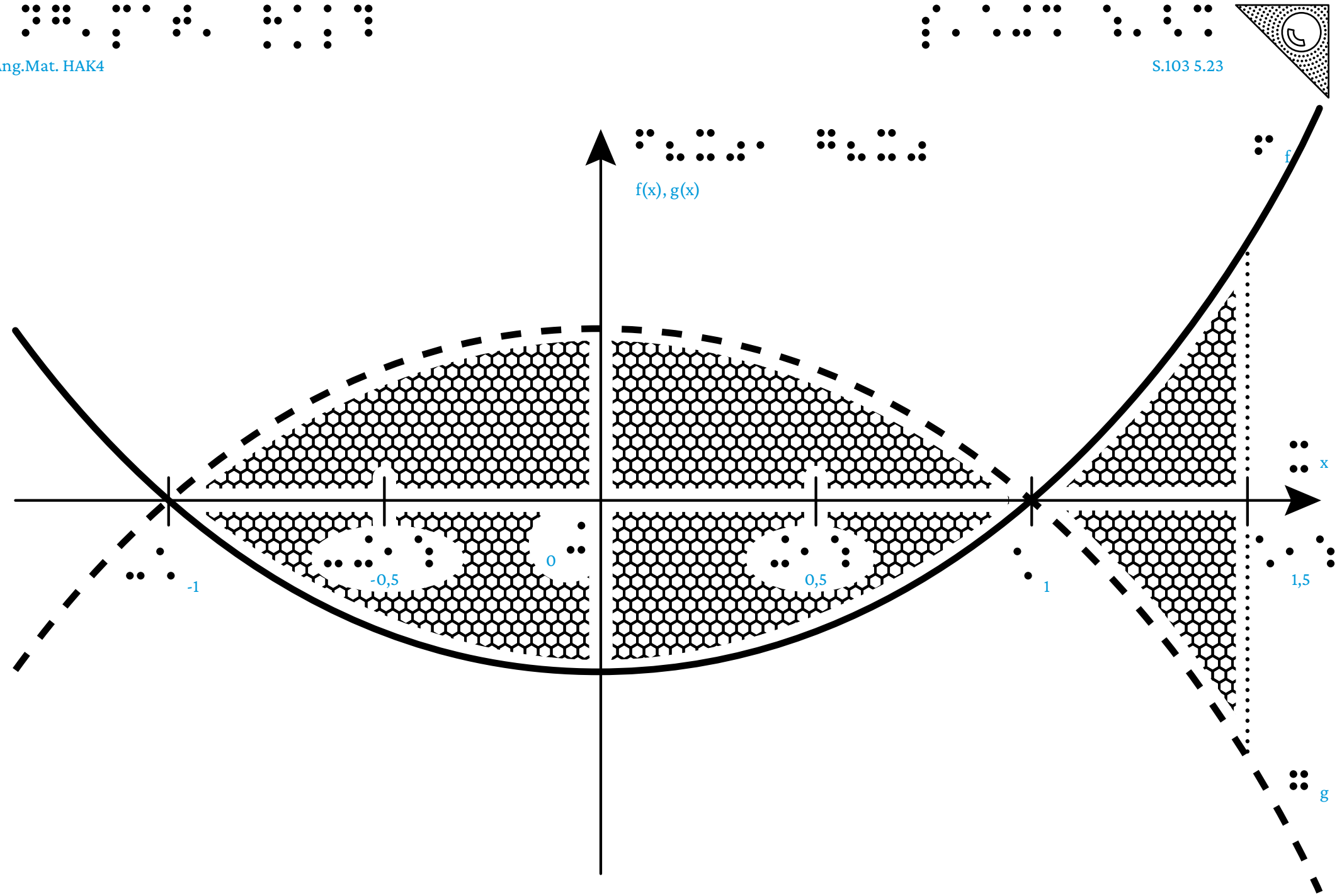


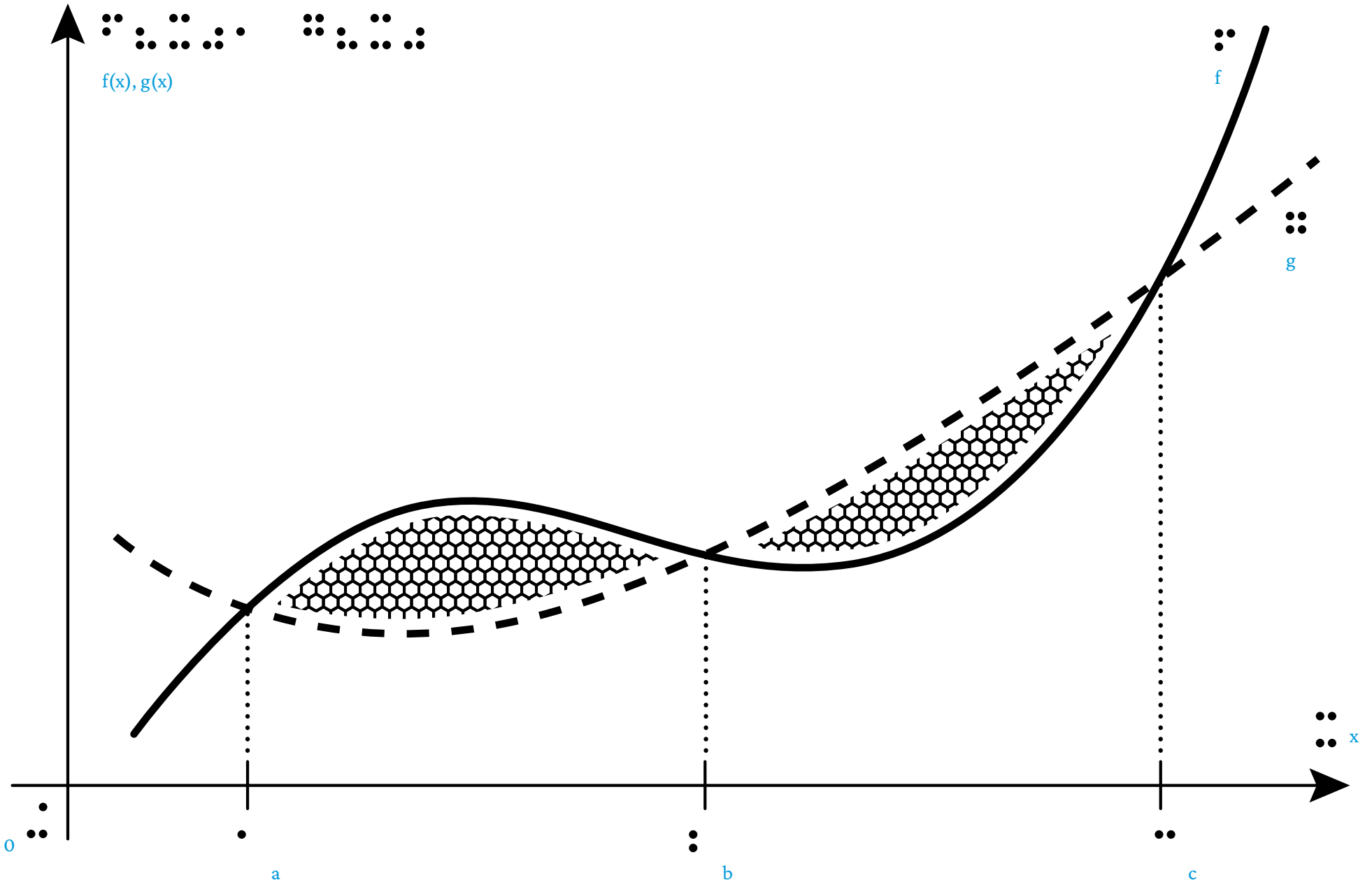
f(x) ='sin(x)

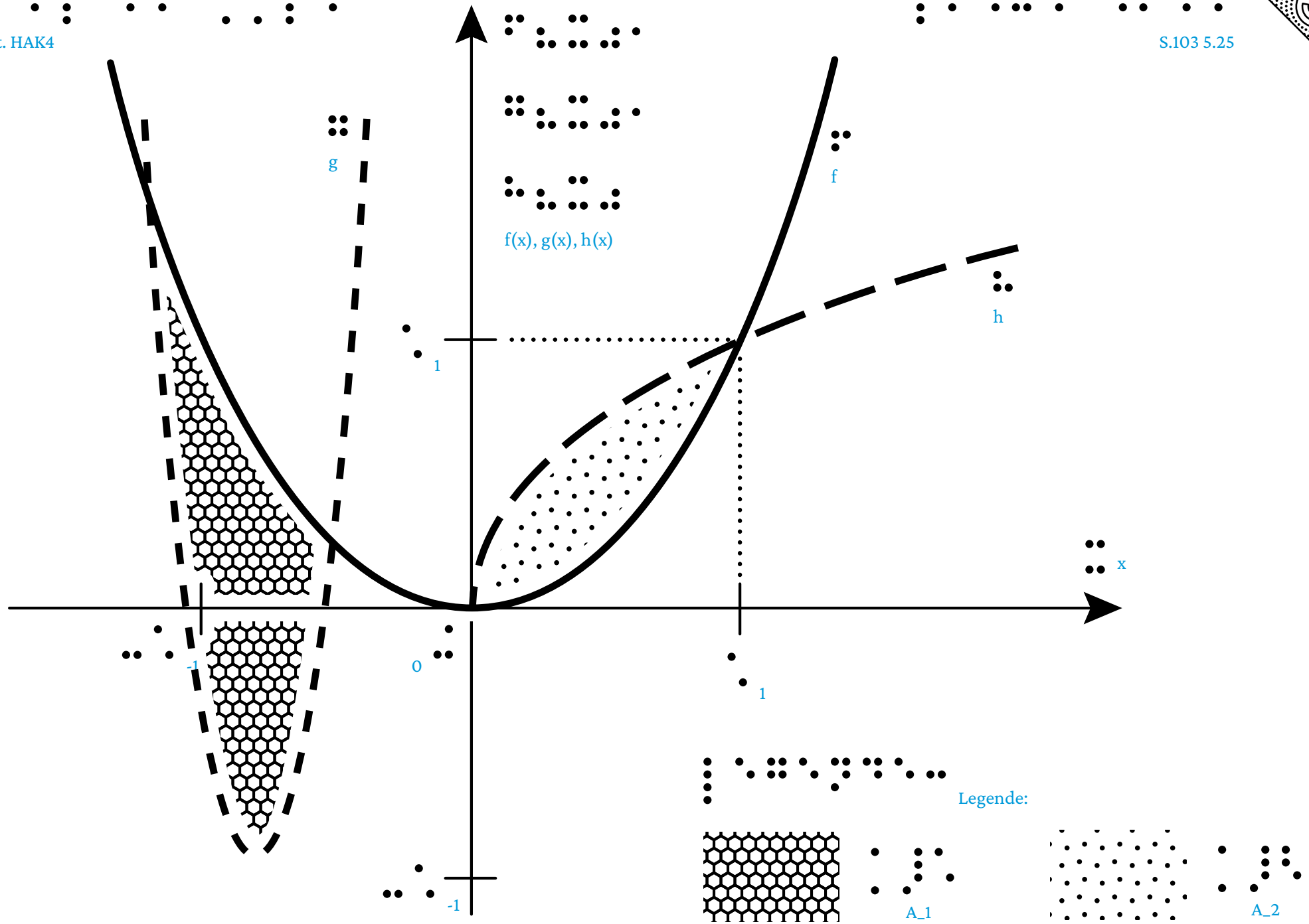
g(x) =2 *'sin(x)

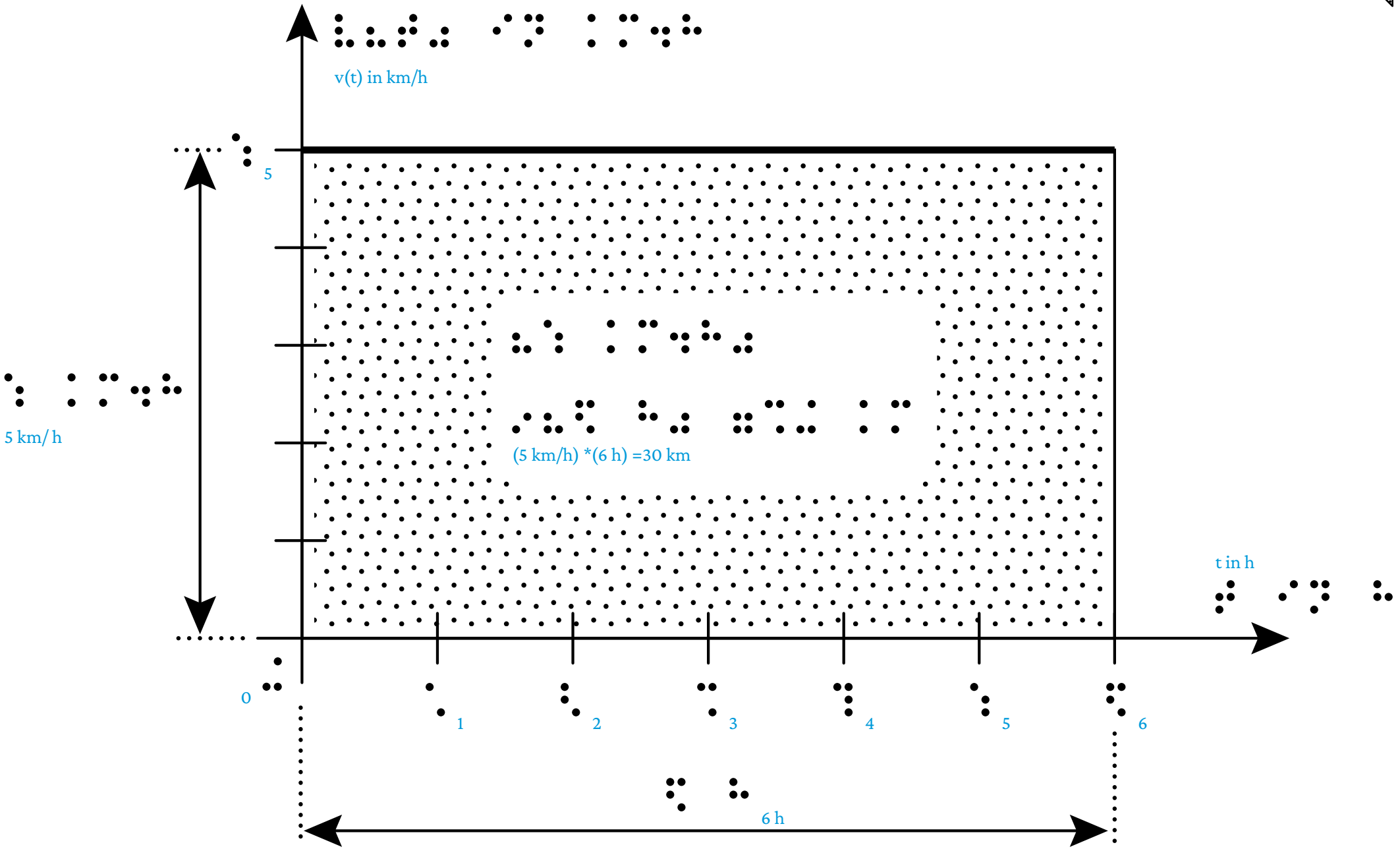


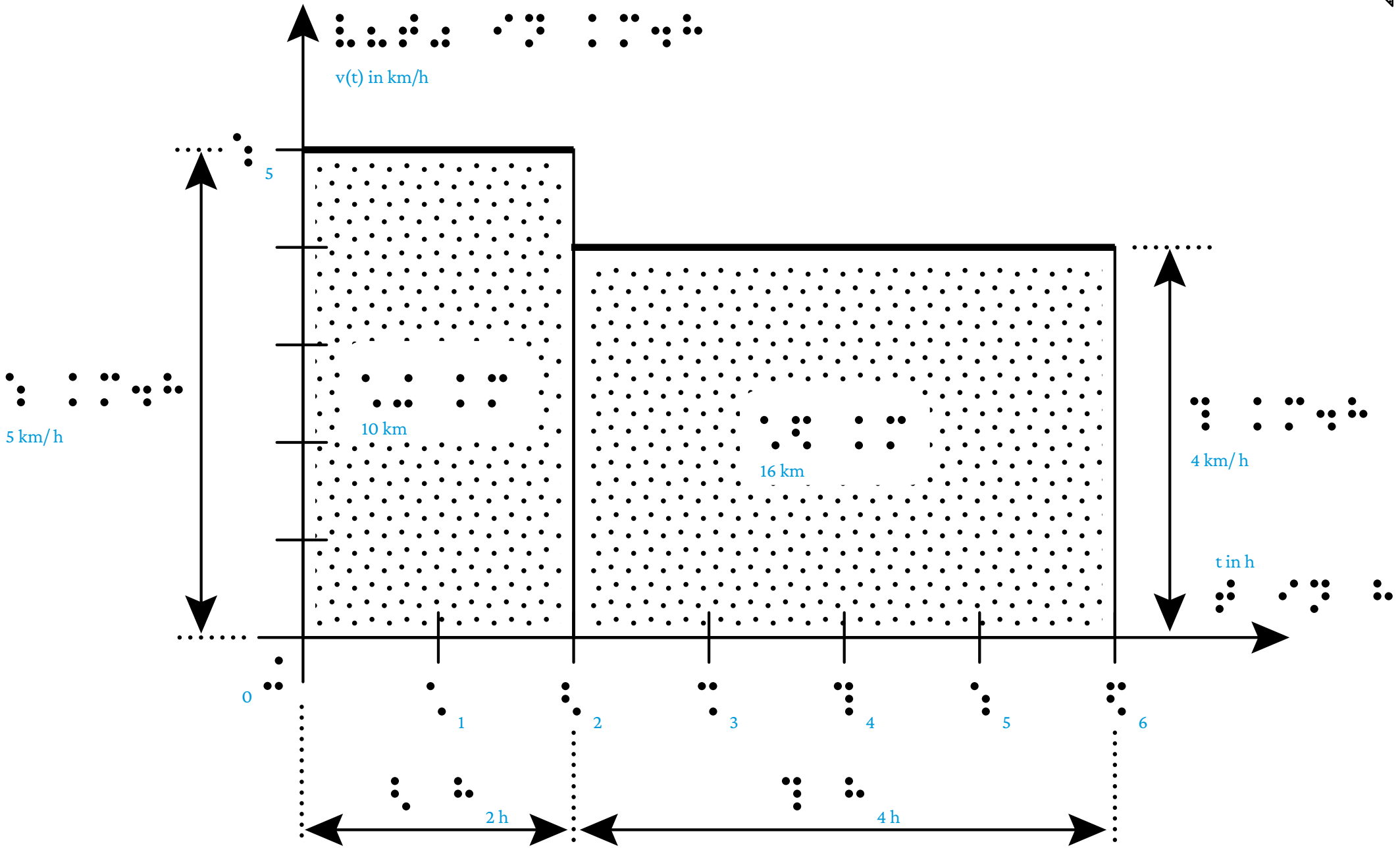

f(x)
f
x
4
3
2
1
0
-3
-2
-1
1
2
3
4
5
6

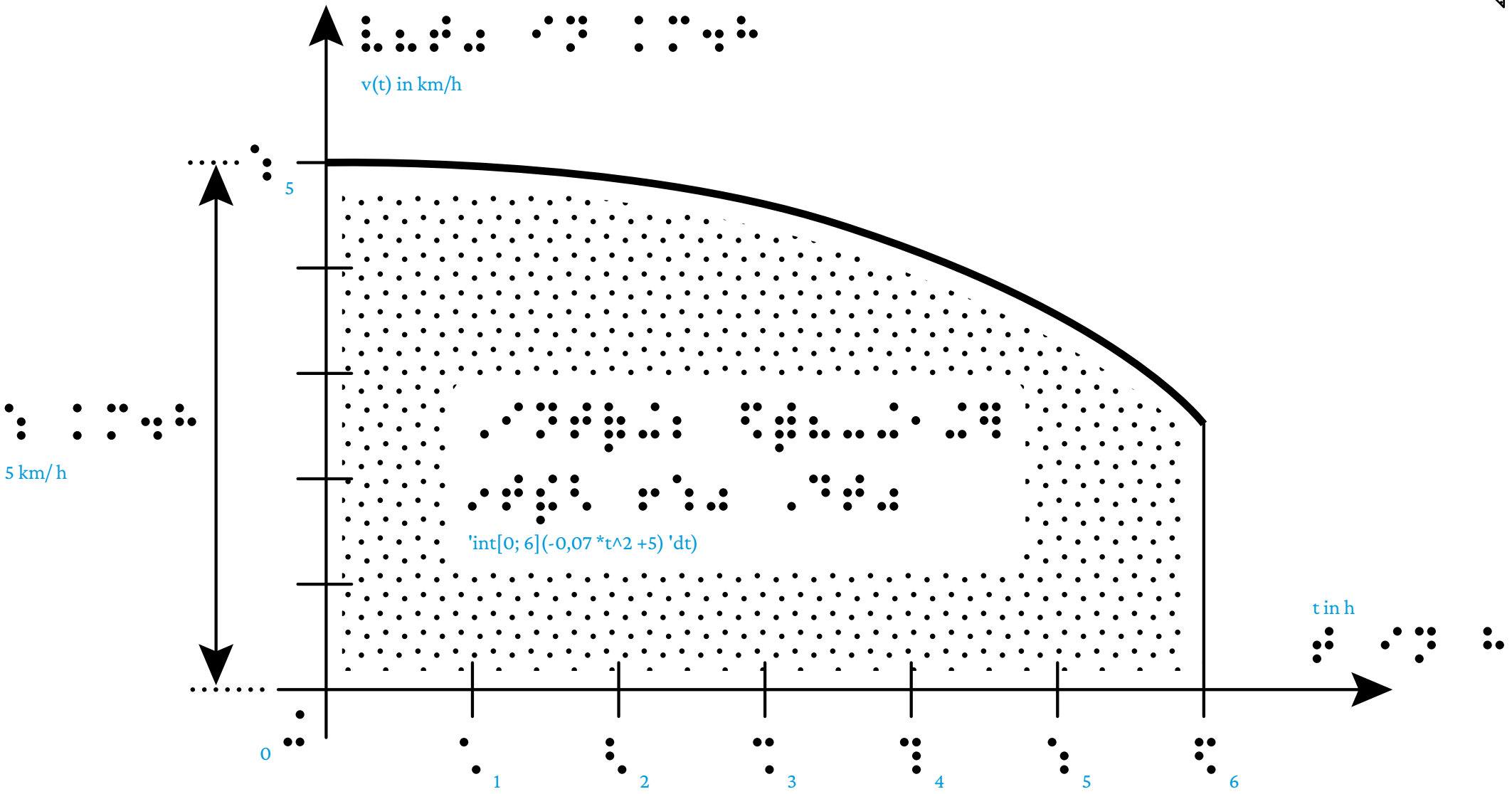

f(x), g(x)
f
x
g
-1
-0,5
0
0,5
1
1,5

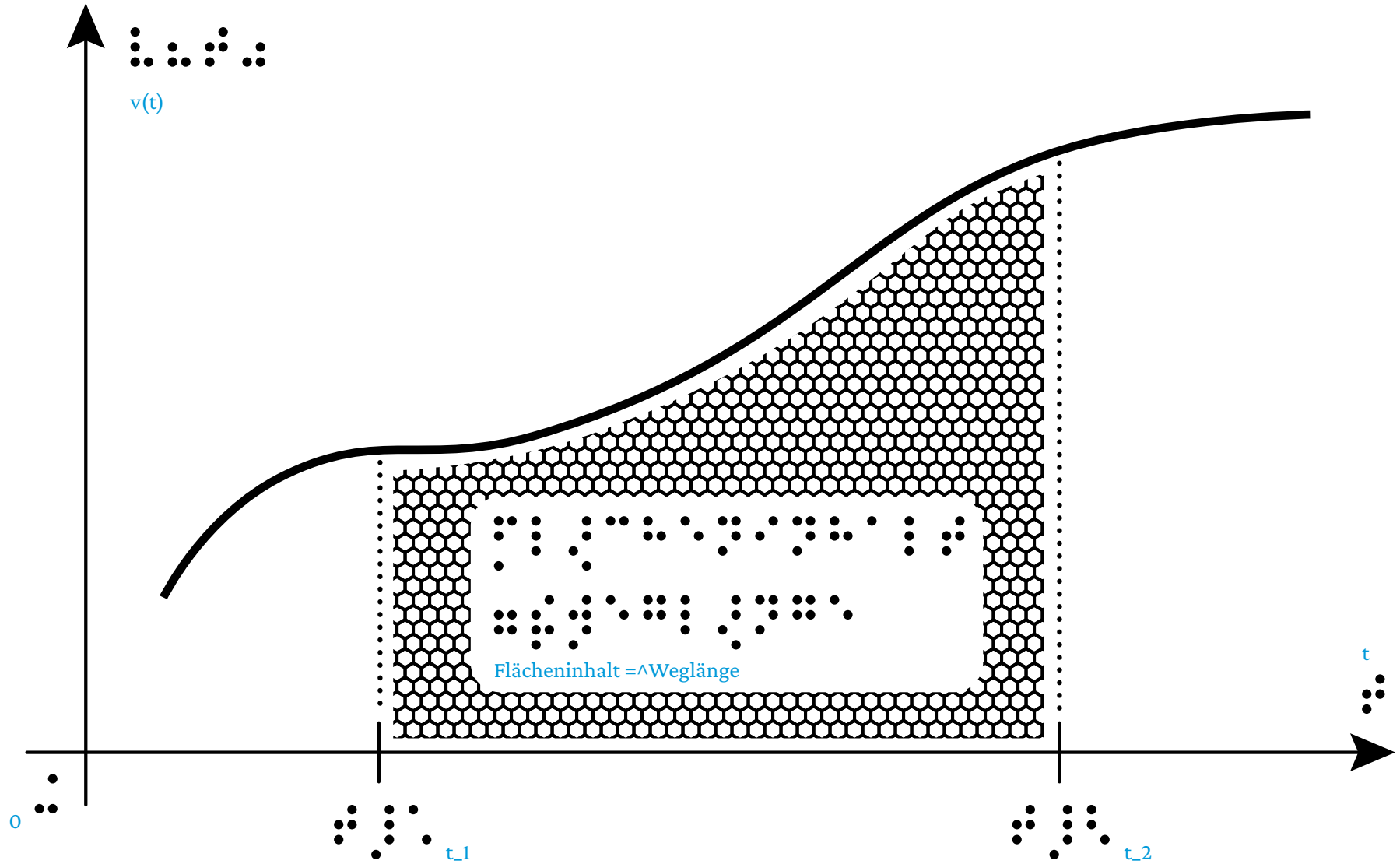

f(x), g(x)
f
g
x
0
a
b
c

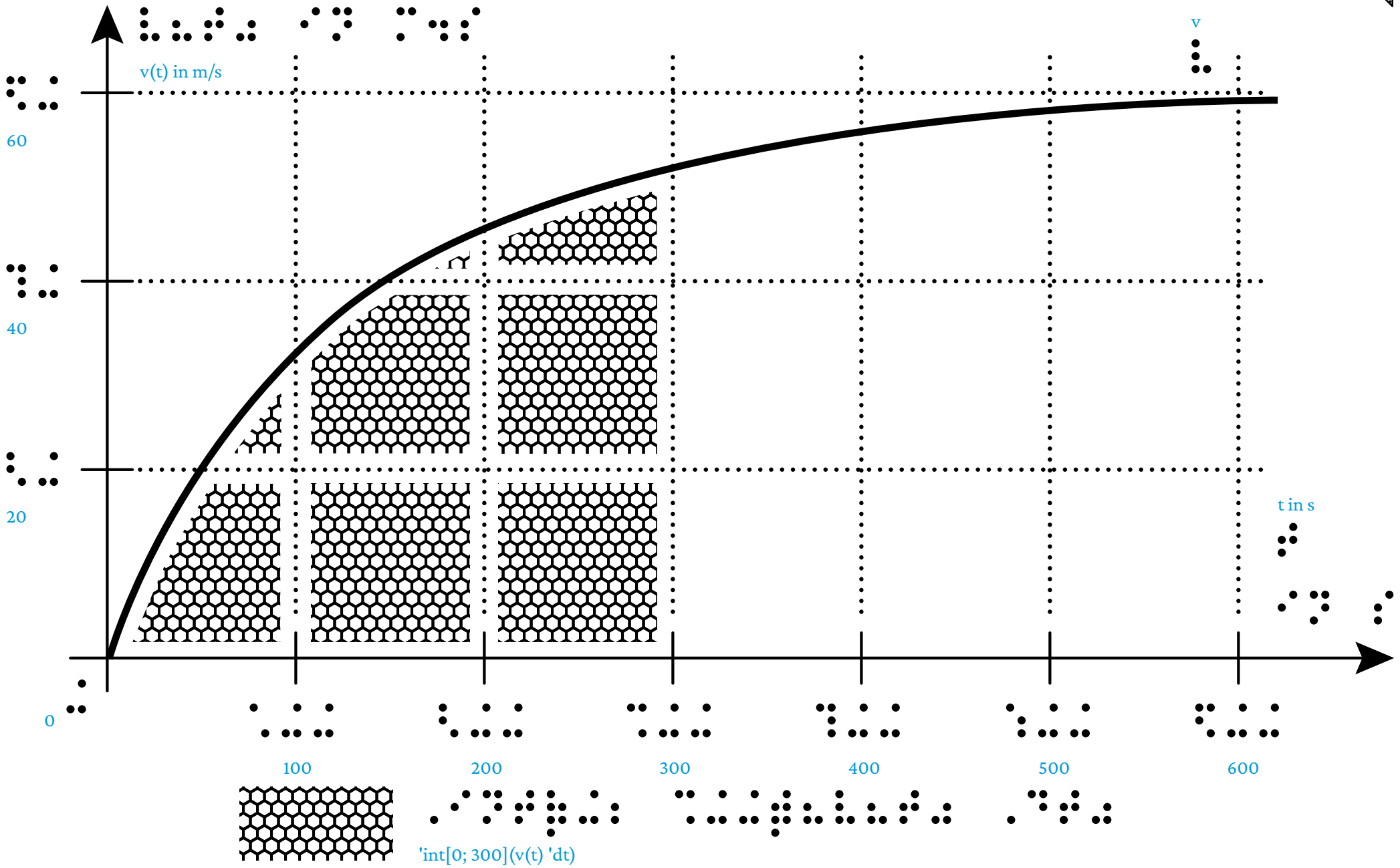

g
f
f(x), g(x), h(x)
h
1
x
-1
0
1
-1
Legende:
A_1
A_2

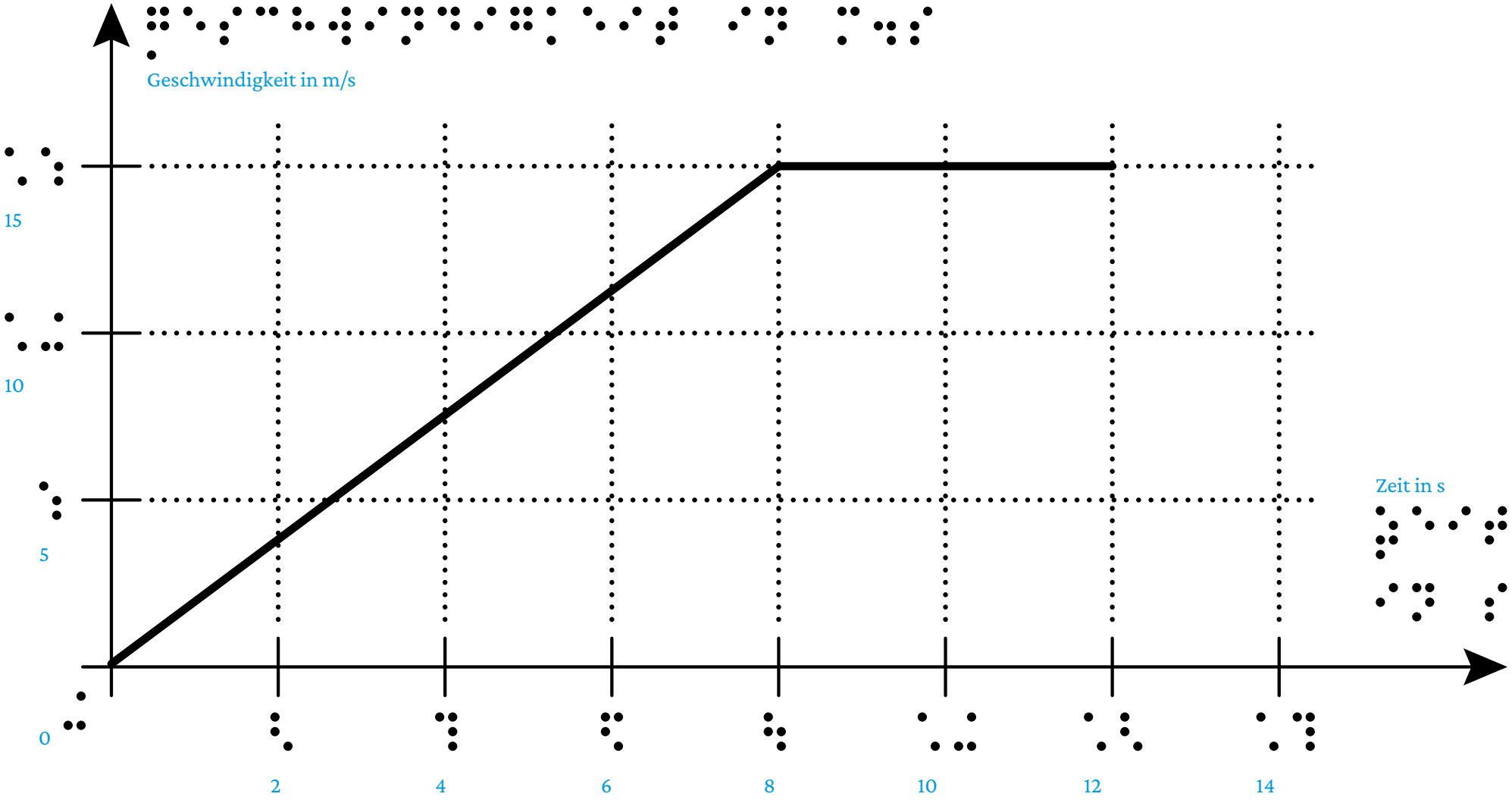

v(t) in km/h
5
5 km/ h
(5 km/h) *(6 h) =30 km
t in h
0
1
2
3
4
5
6
6 h

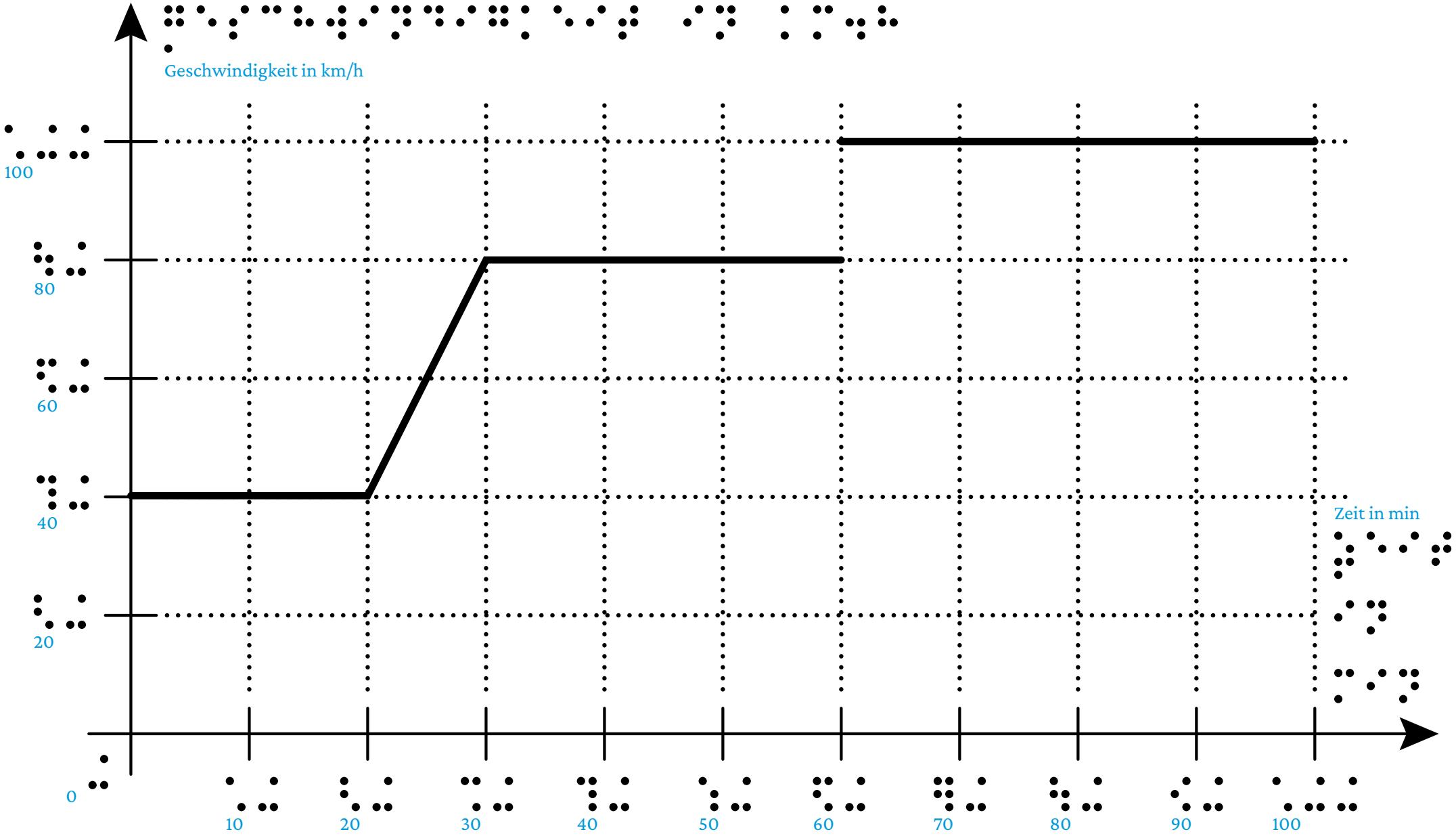

v(t) in km/h
5
5 km/h
10 km
16 km
4 km/h
t in h
0
1
2
3
4
5
6
2 h
4 h

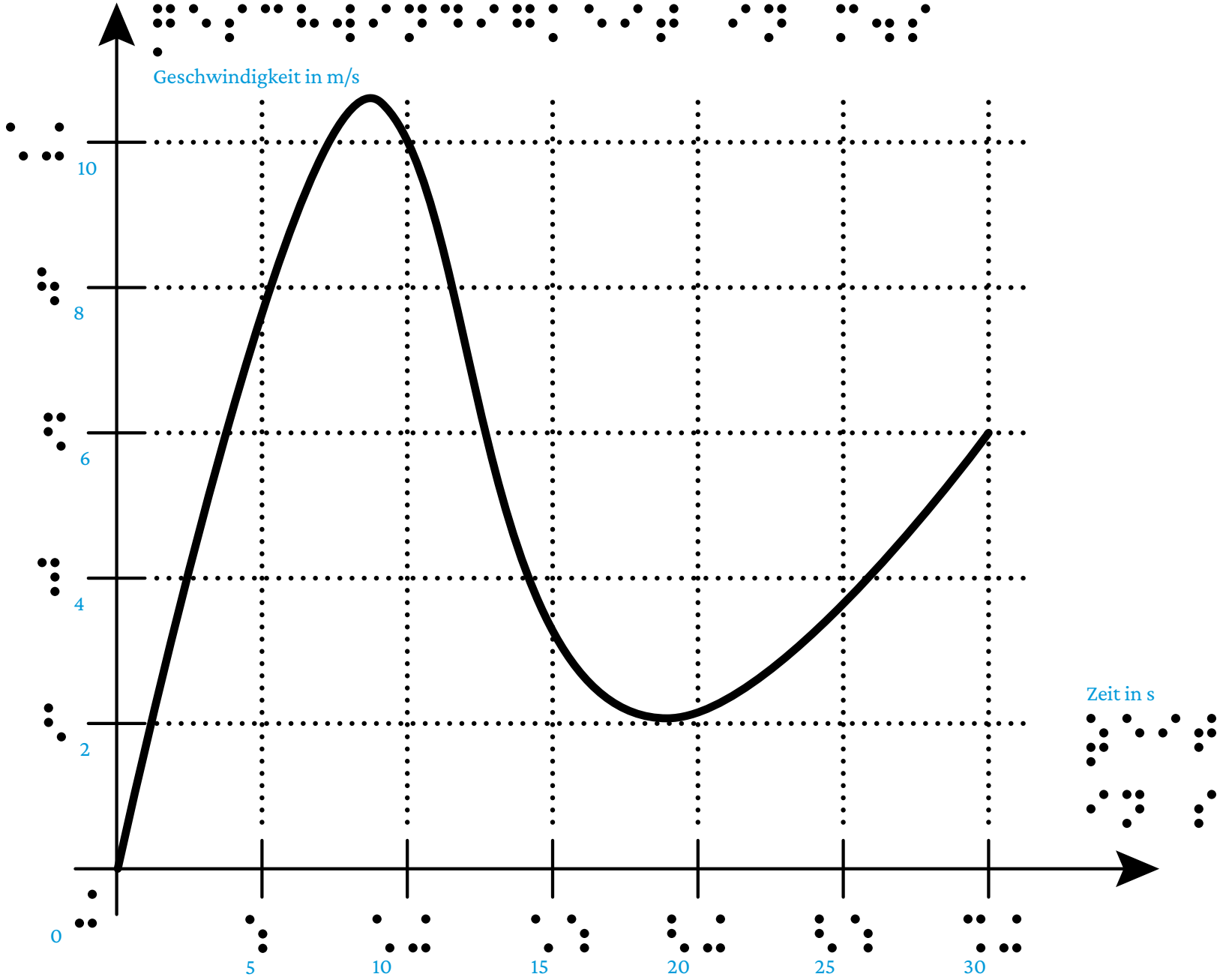

v(t) in km/h
5
5 km/ h
'int[0; 6](-0,07 *t^2 +5) 'dt)
t in h
0
1
2
3
4
5
6

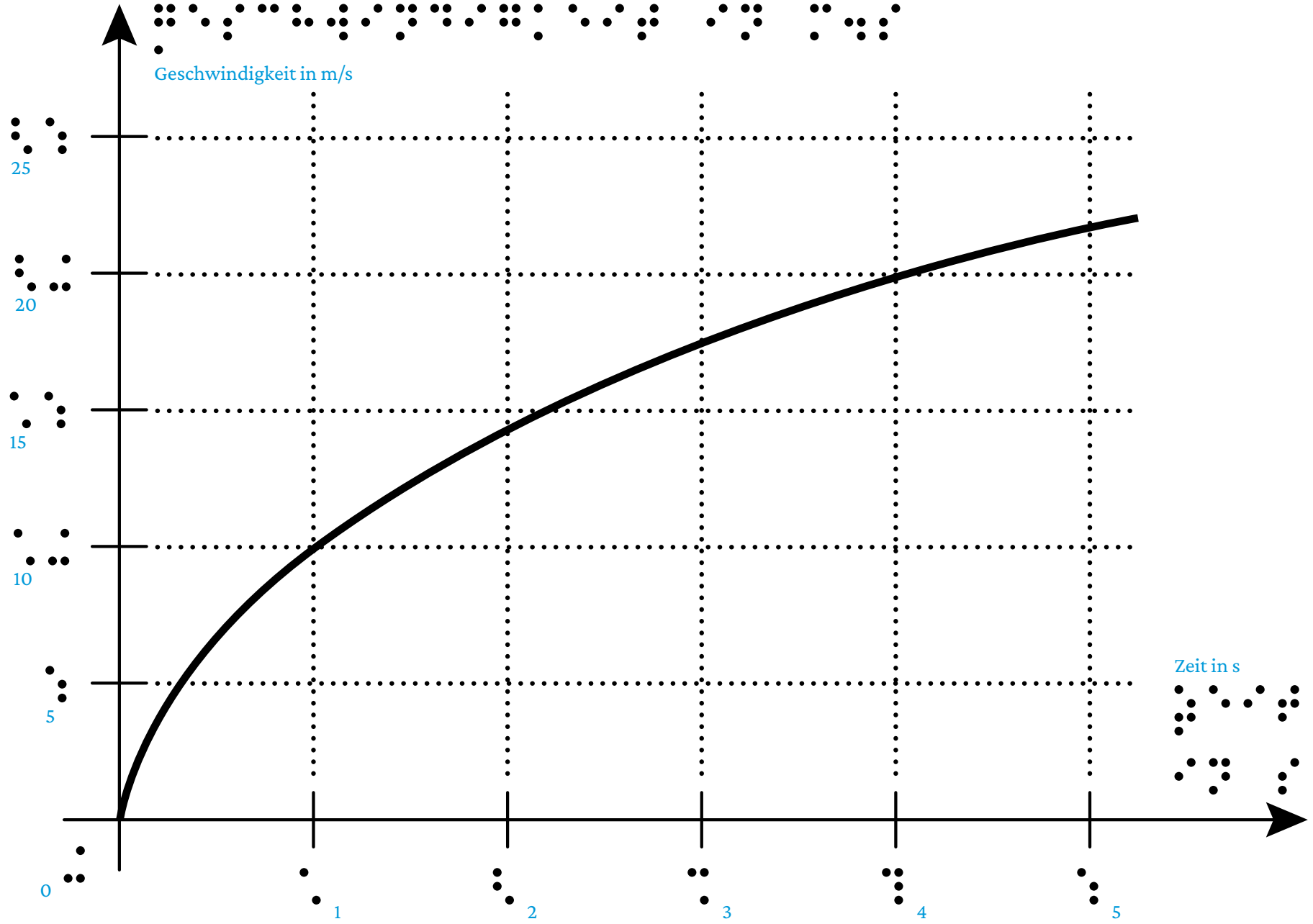

v(t)
Flächeninhalt =^Weglänge
t
0
t_1
t_2

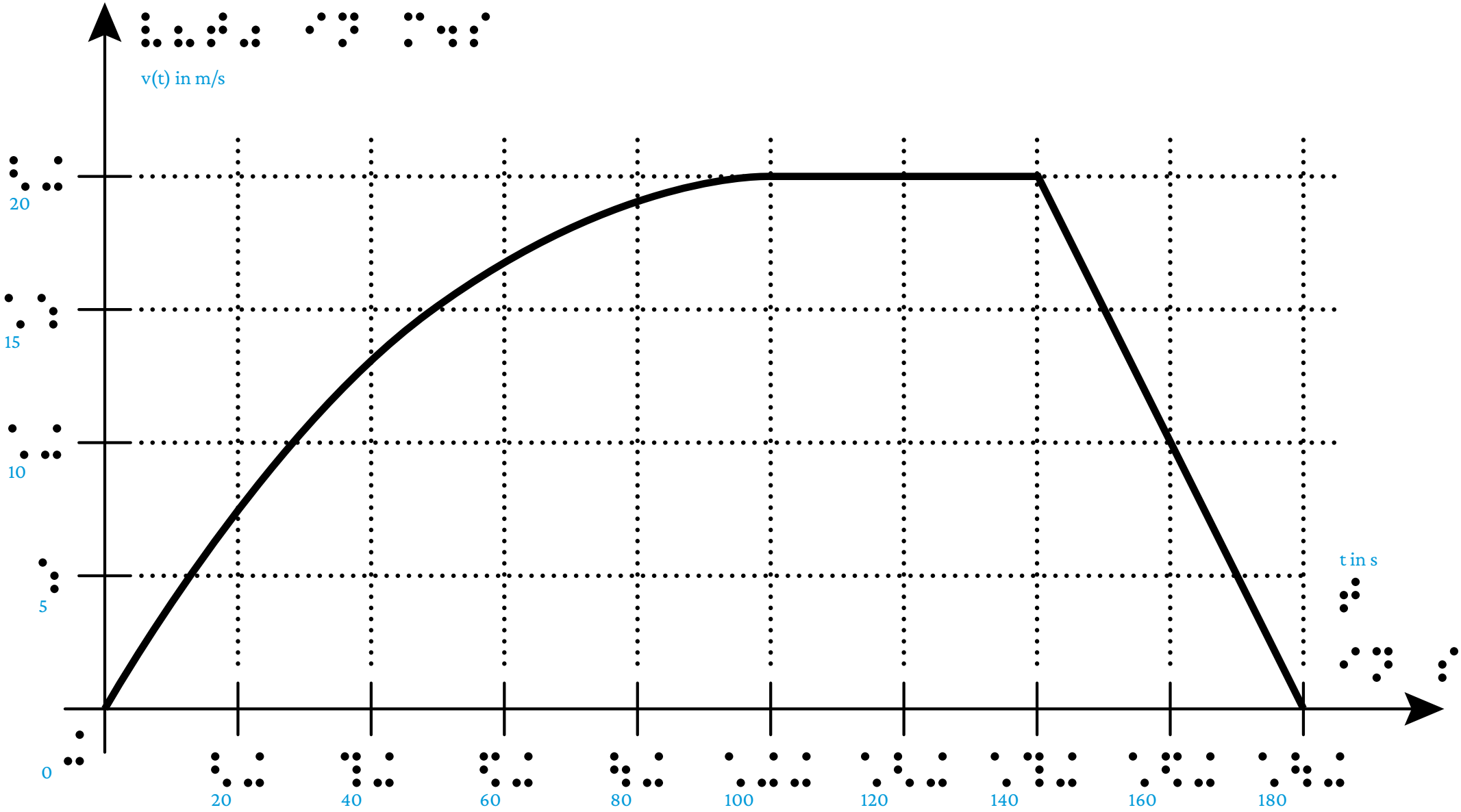

v
v(t) in m/s
60
40
20
0
t in s
100
200
300
400
500
600
'int[0; 300](v(t) 'dt)


Geschwindigkeit in m/s
15
10
5
0
2
4
6
8
10
12
14
Zeit in s


Geschwindigkeit in km/h
100
80
60
40
20
0
Zeit in min
10
20
30
40
50
60
70
80
90
100


Geschwindigkeit in m/s
10
8
6
4
2
0
5
10
15
20
25
30
Zeit in s


Geschwindigkeit in m/s
25
20
15
10
5
0
1
2
3
4
5
Zeit in s


v(t) in m/s
20
15
10
5
0
t in s
20
40
60
80
100
120
140
160
180

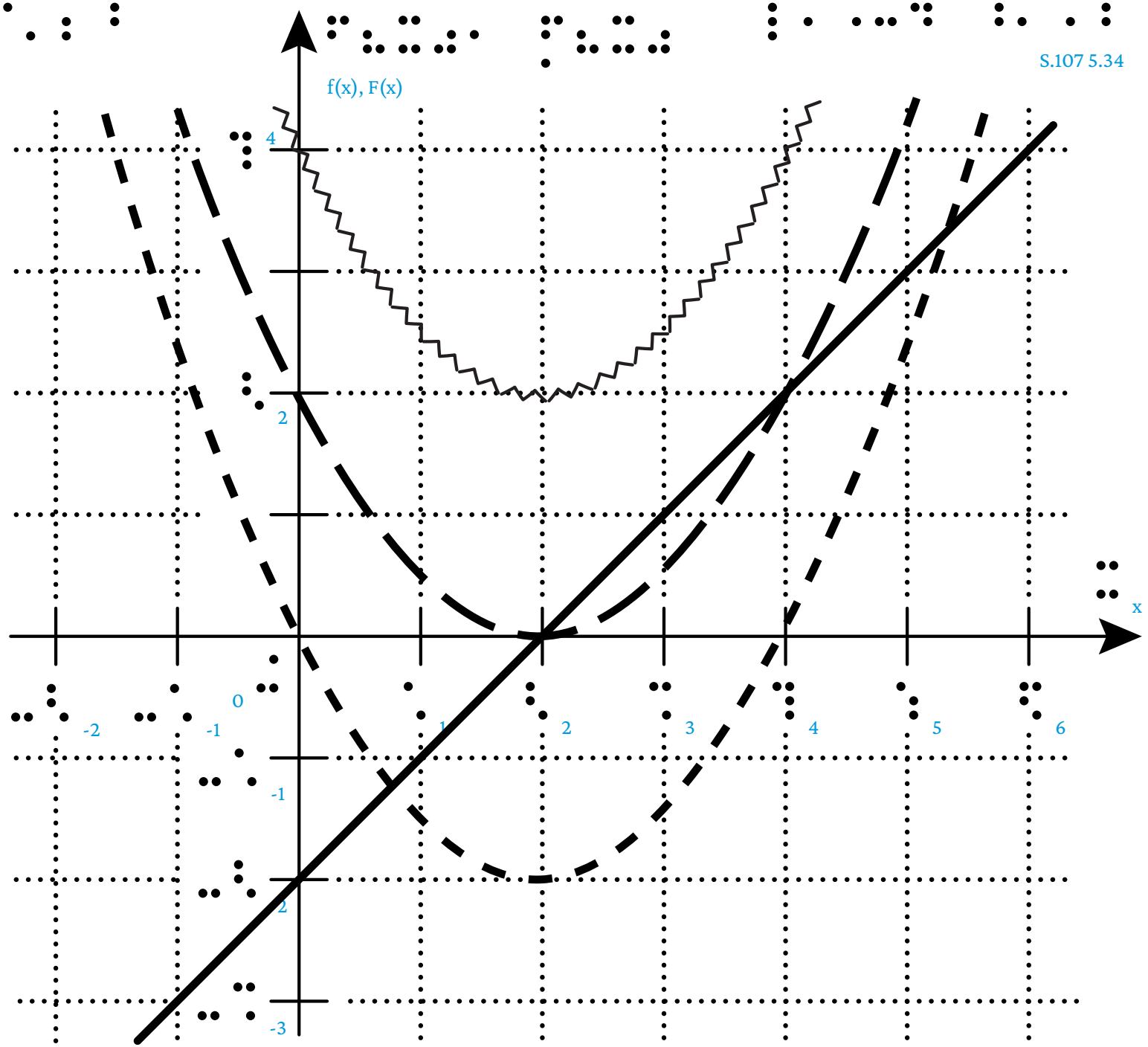
Legende:

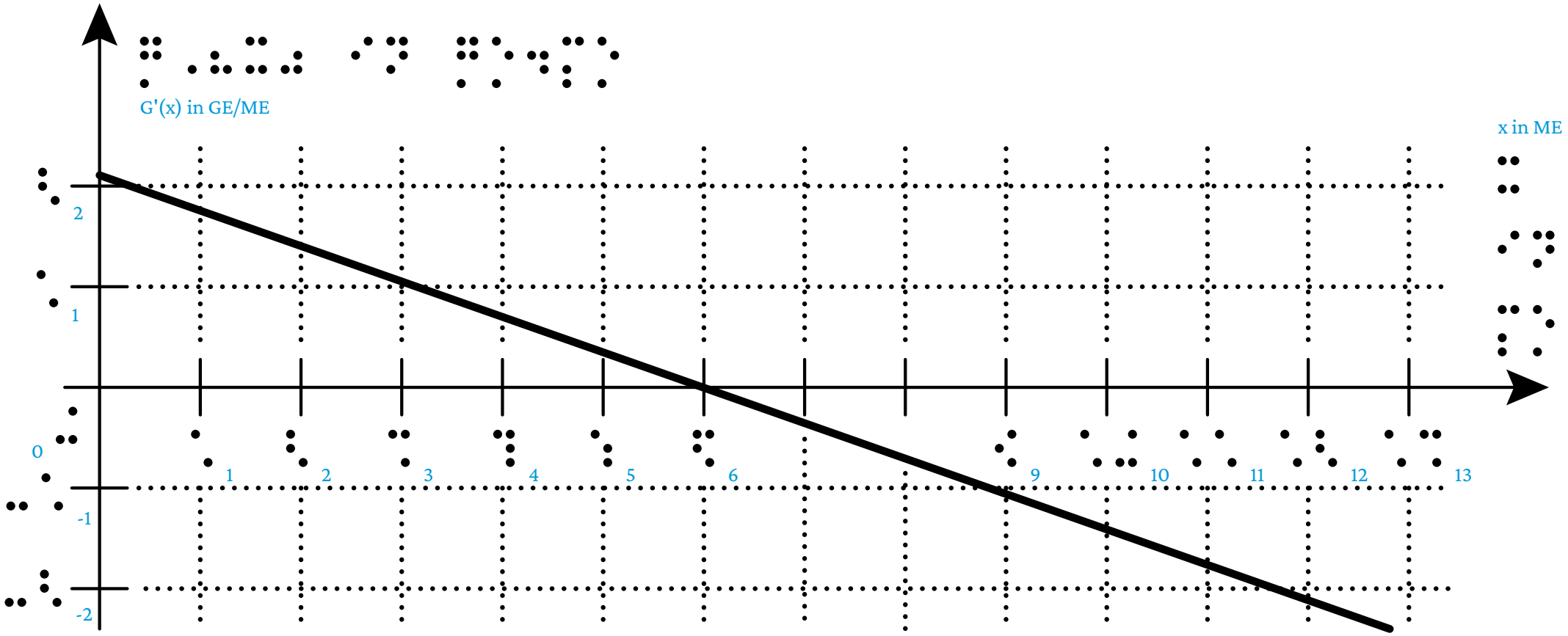
f

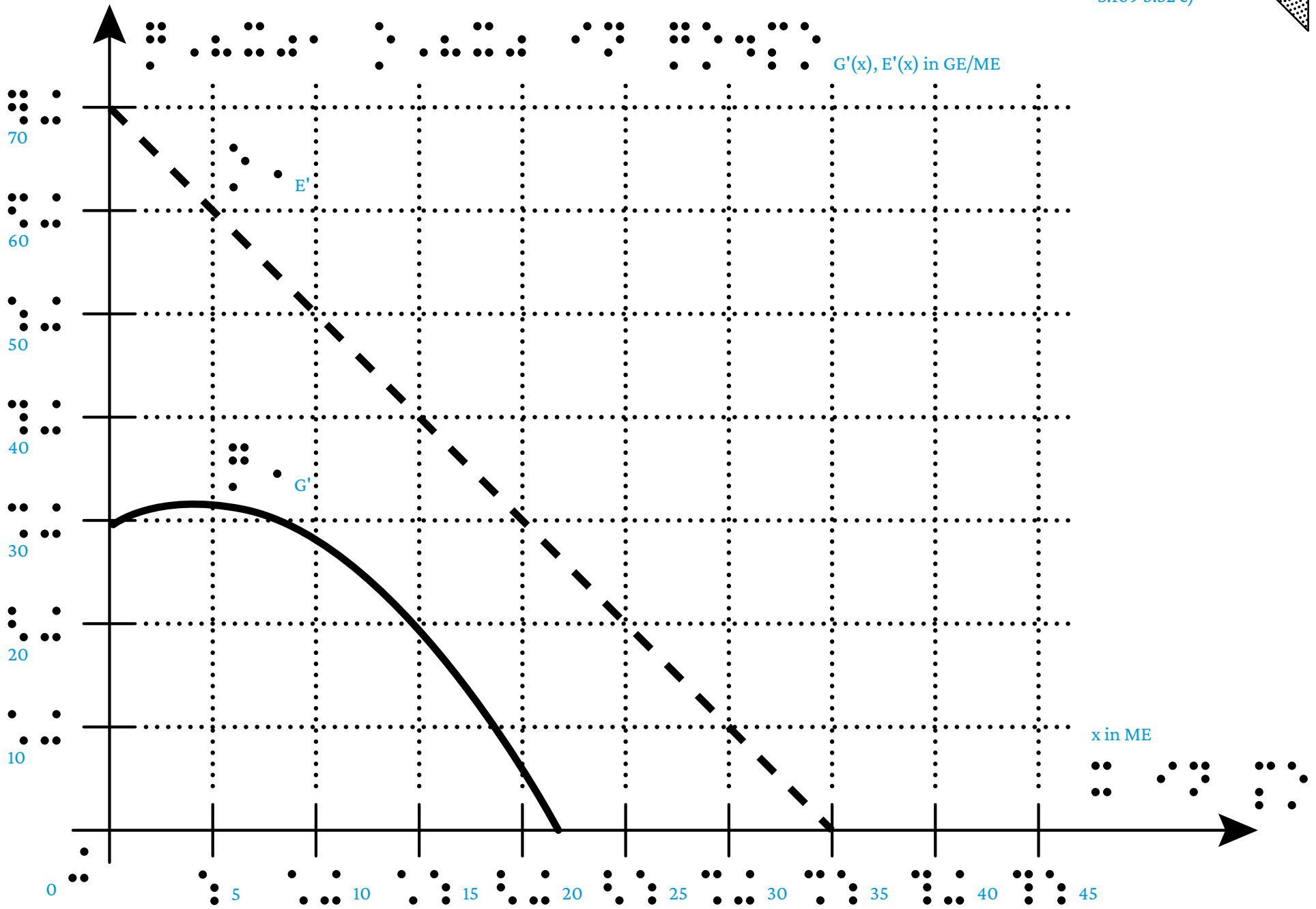
F_1

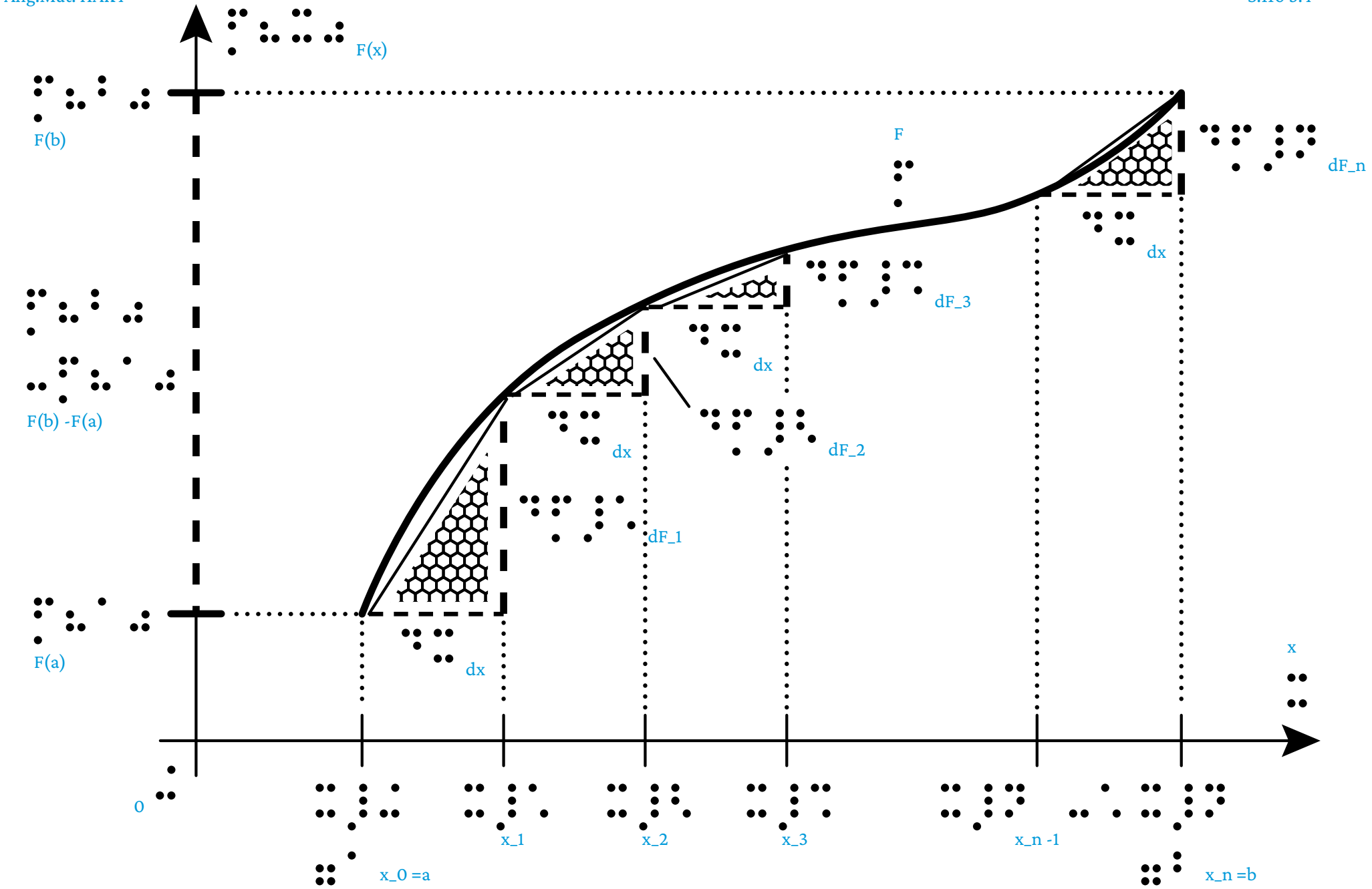
F_2

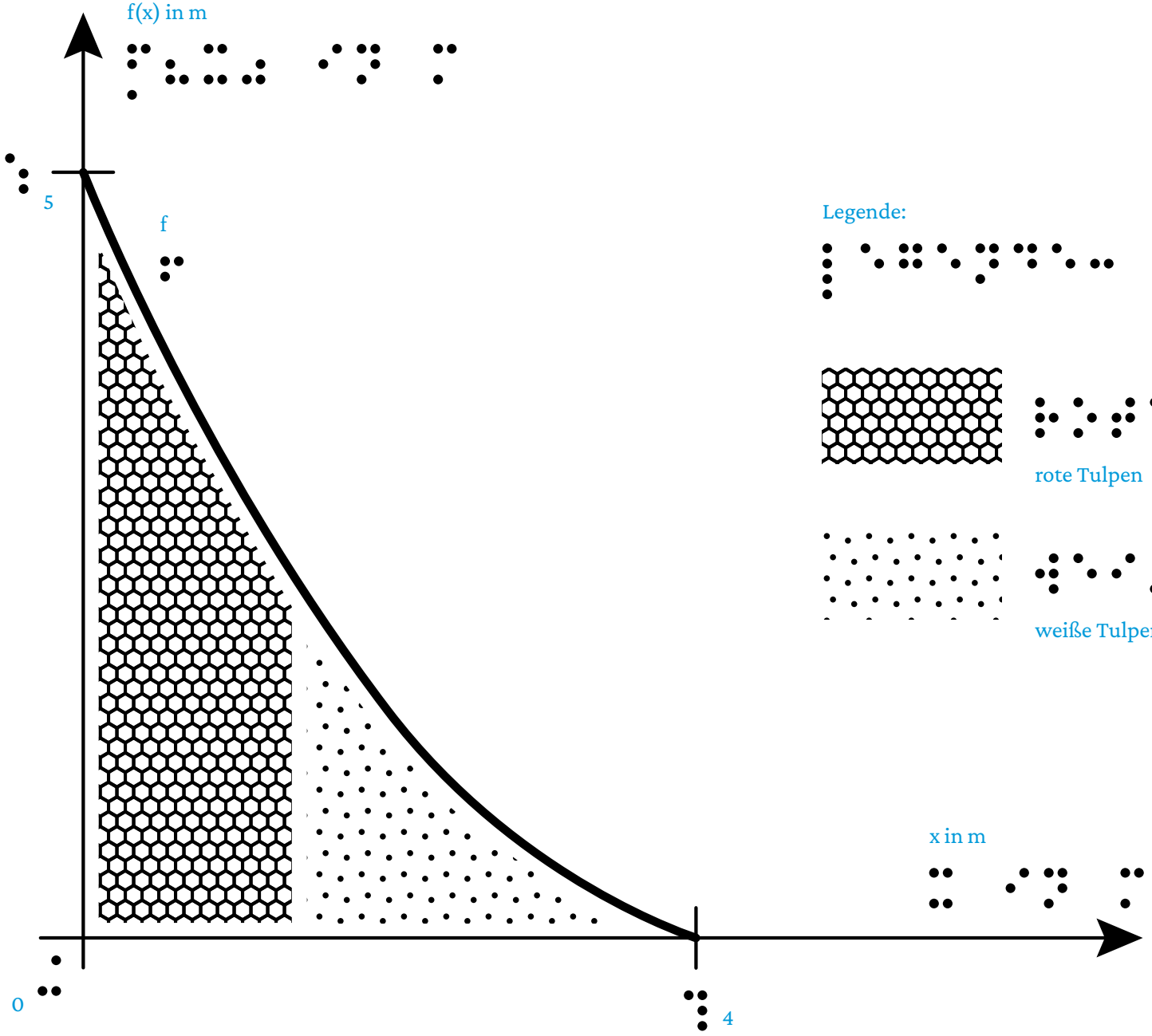
F_3




G'(x) in GE/ME
x in ME
2
1
0
-1
-2
1
2
3
4
5
6
9
10
11
12
13


G'(x), E'(x) in GE/ME
70
60
50
40
30
20
10
E'
G'
0
5
10
15
20
25
30
35
40
45
x in ME

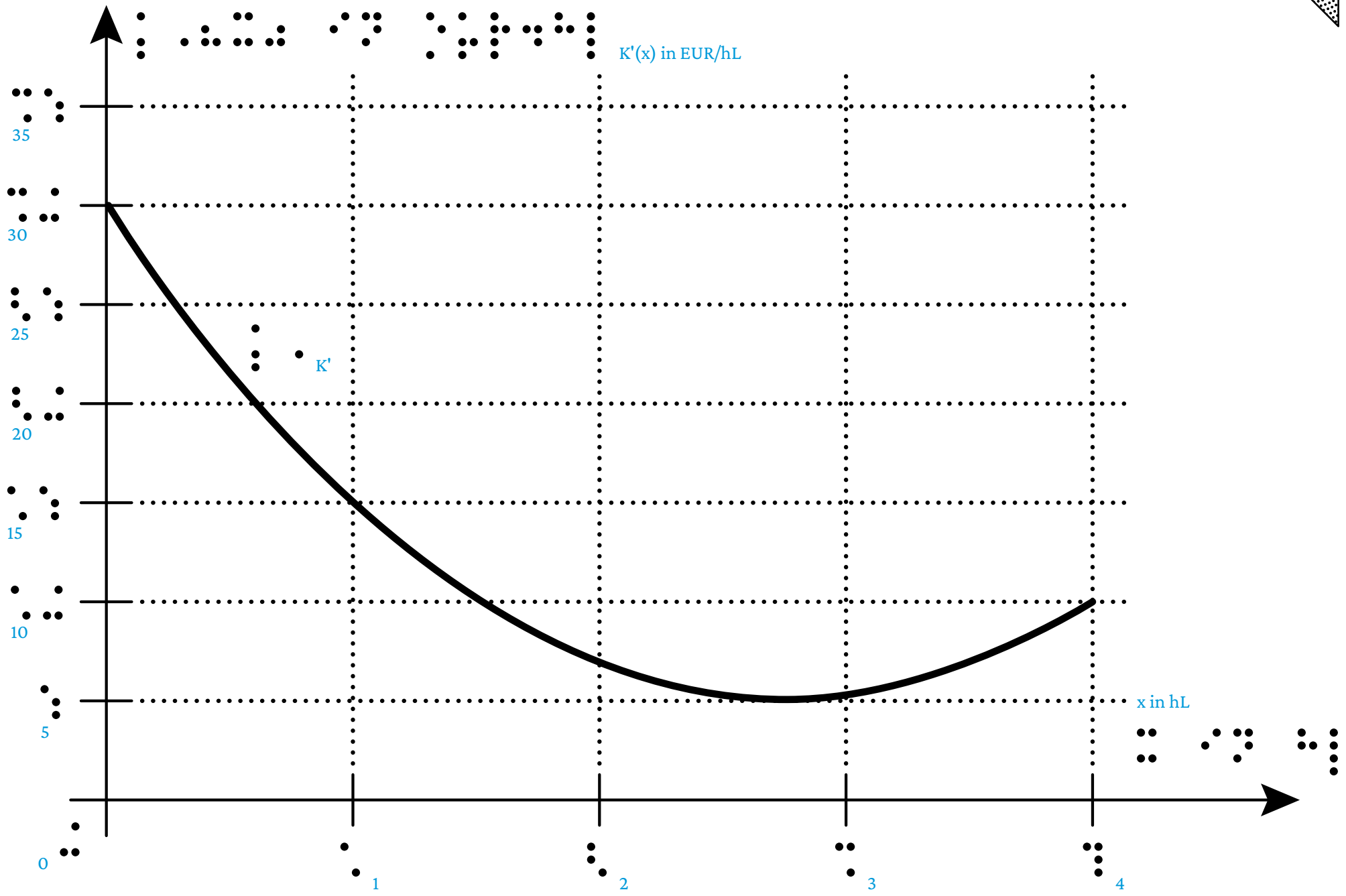

F(x)
F(b)
F
dF_n
dx
dF_3
dx
dx
dF_2
F(b) -F(a)
dF_1
F(a)
dx
x
0
x_1
x_2
x_3
x_n -1
x_0 =a
x_n =b



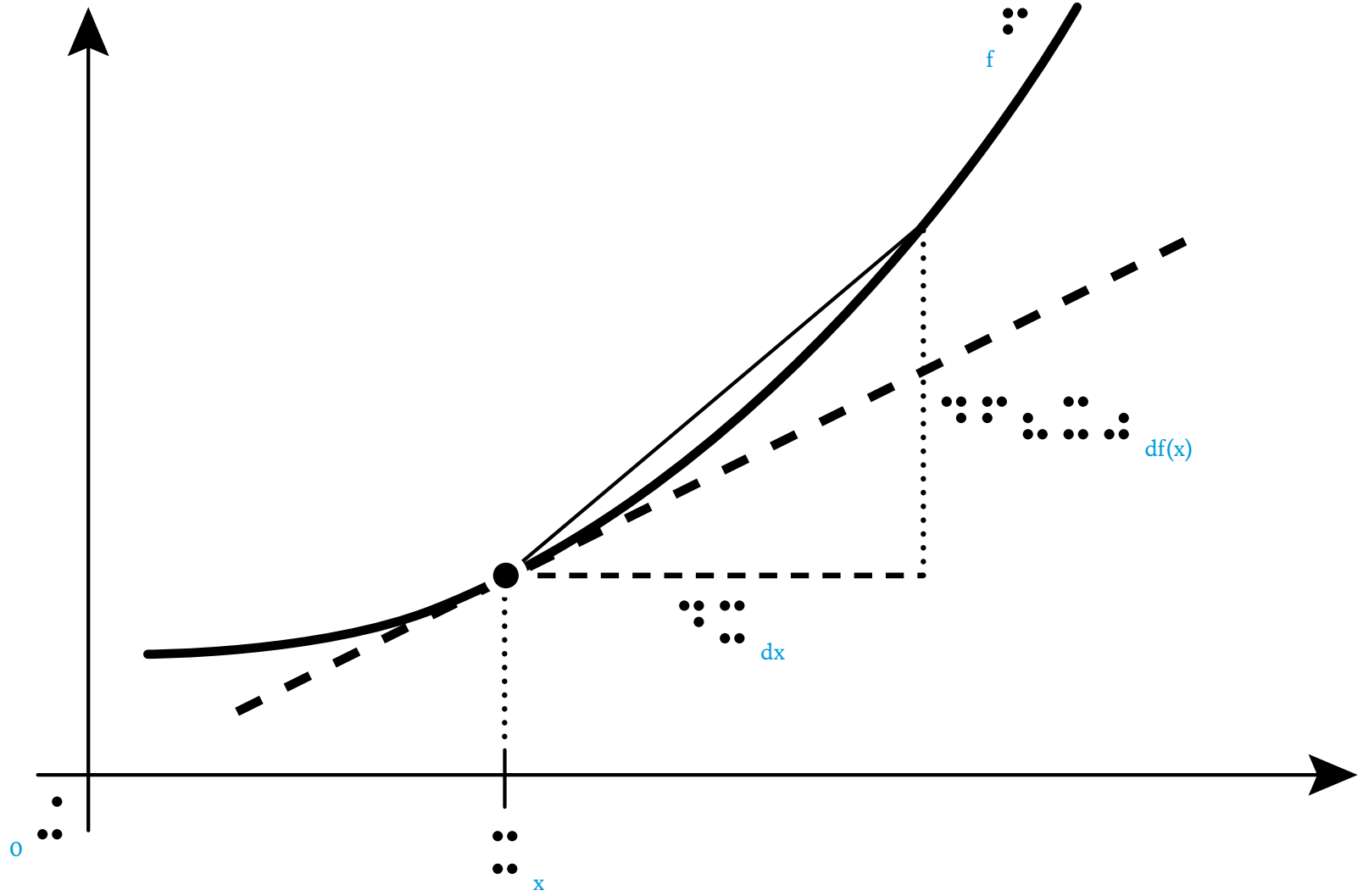
Legende:

rote Tulpen

weiße Tulpen

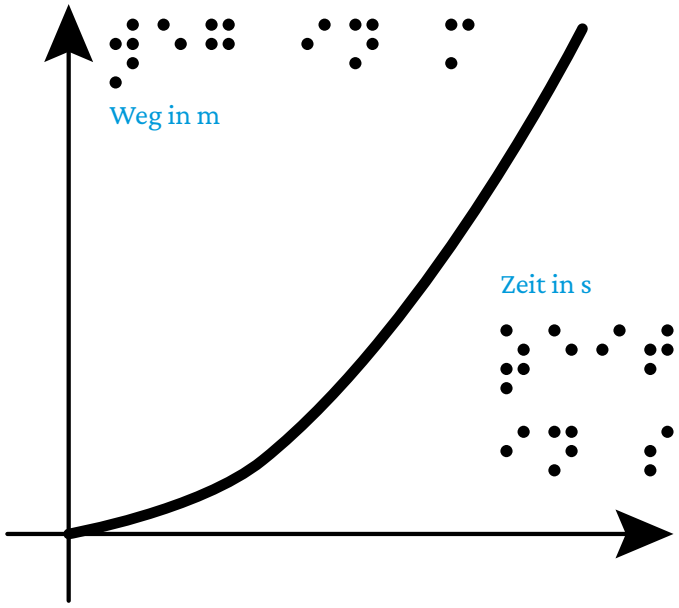

K'(x) in EUR/hL
35
30
25
K'
20
15
10
5
0
1
2
3
4
x in hL

Differentialrechnung - Grundvorstellung: Steigung an der Stelle x



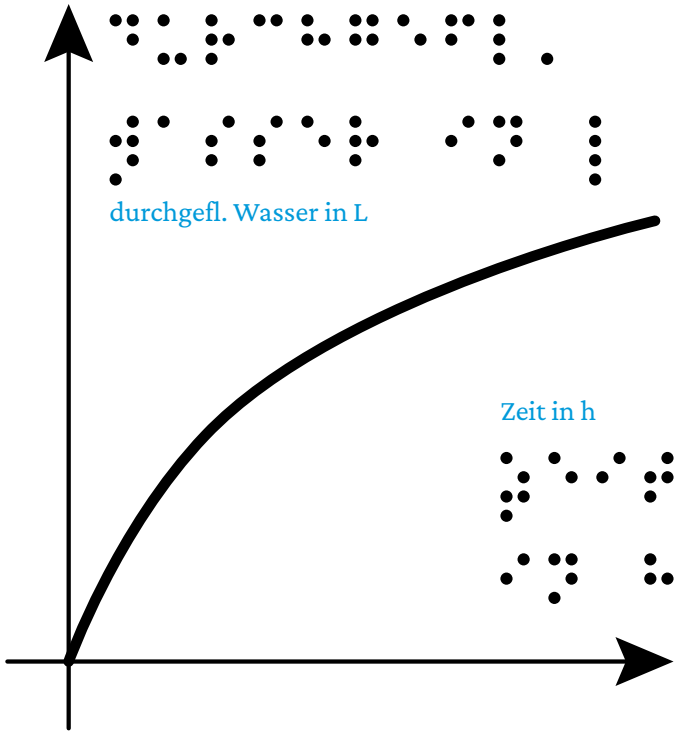
Differentialrechnung - Anwendungsbeispiele:

1) Weg -> Geschwindigkeit



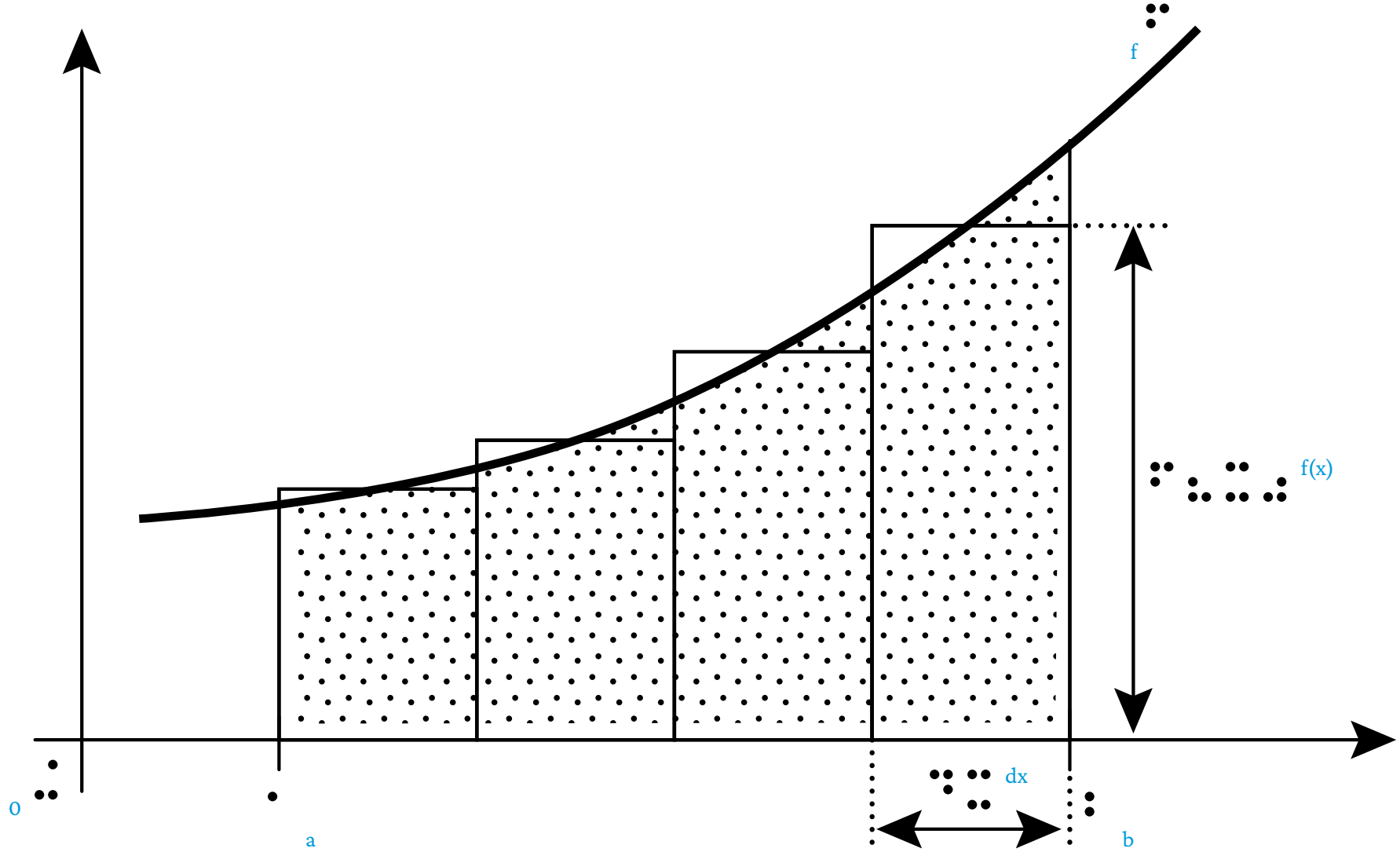
Weg in m /Zeit in s =Geschwindigkeit in m/s

2) Bestand -> Änderungsrate



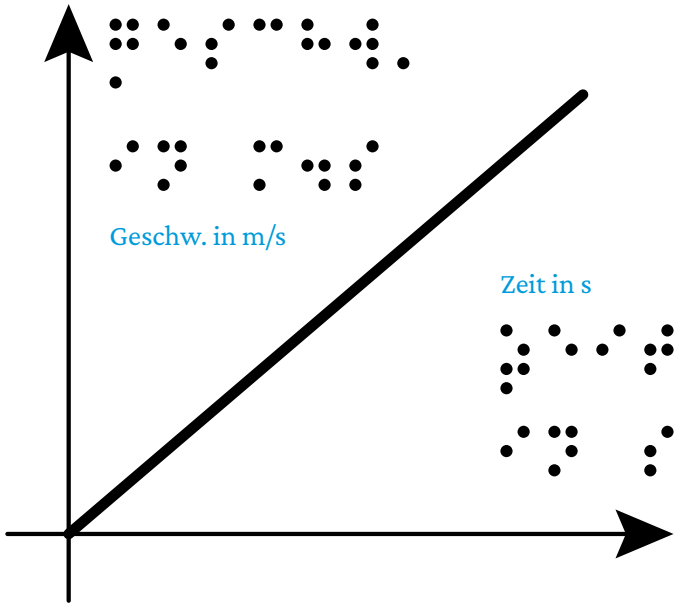
durchgefl. Wasser Volumen in L /Zeit in h
=Durchflussrate in L/h

Integralrechnung - Grundvorstellung: Flächeninhalt im Intervall [a; b]



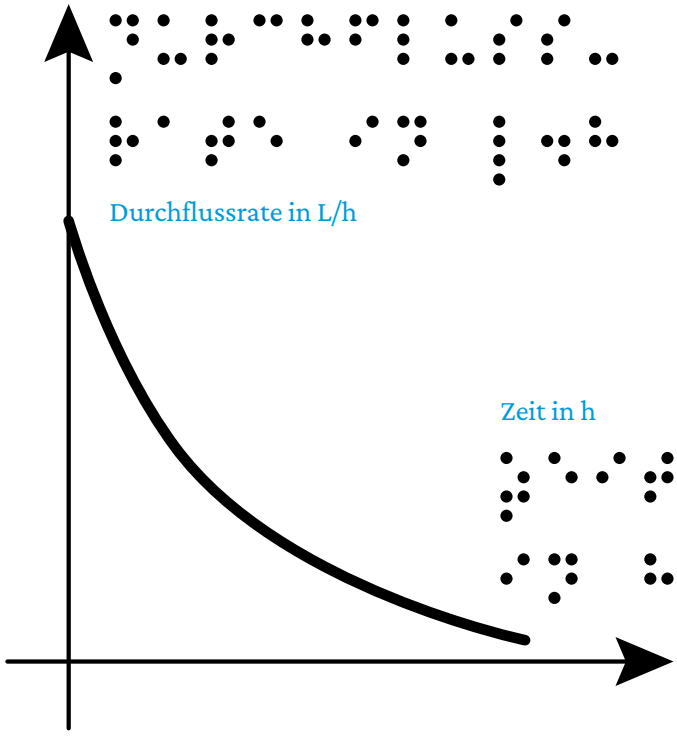
Integralrechnung - Anwendungsbeispiele:

1) Geschwindigkeit -> Weg



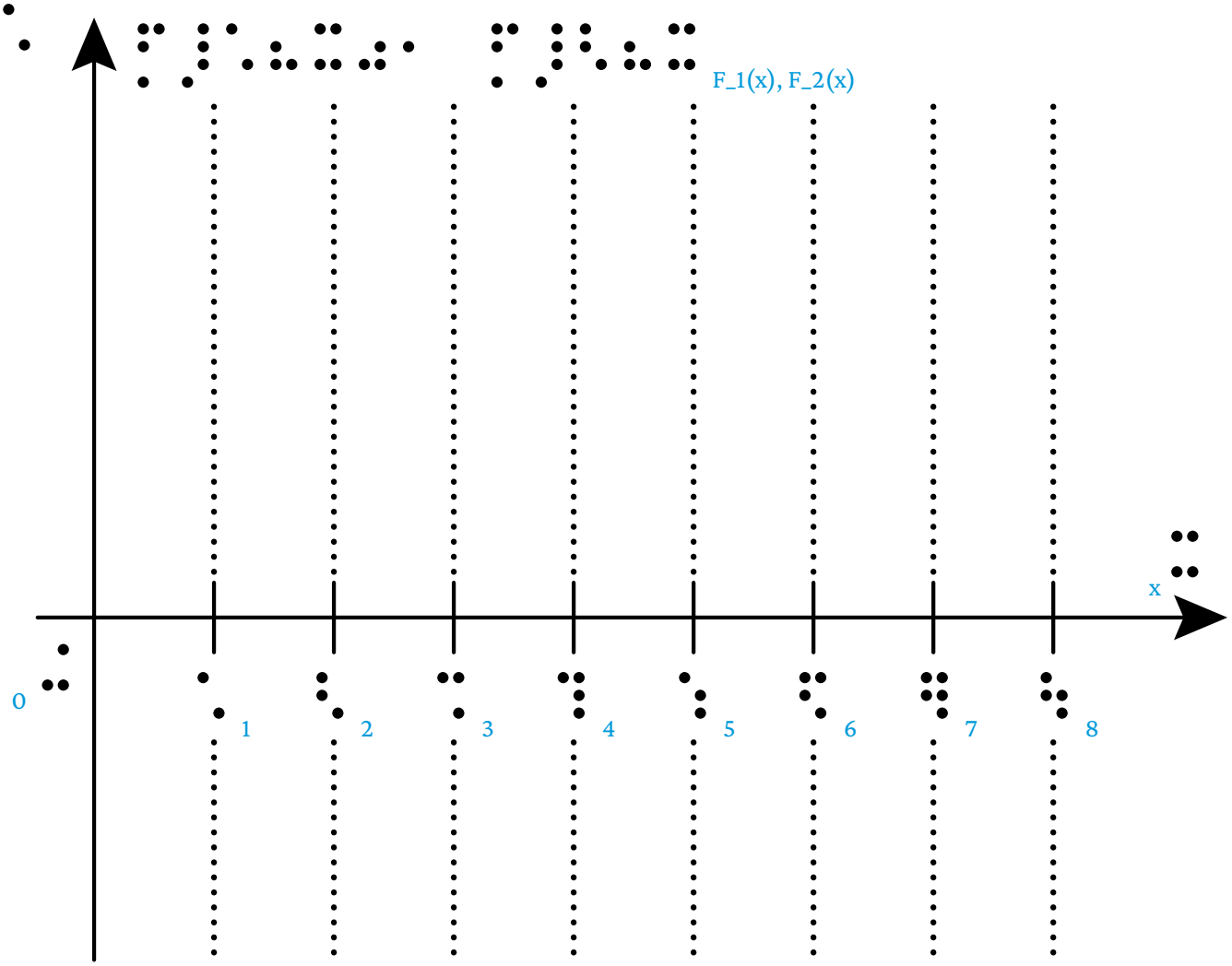
Geschwindigkeit *Zeit in m/s *s =Weg in m

2) Änderungsrate -> Bestand

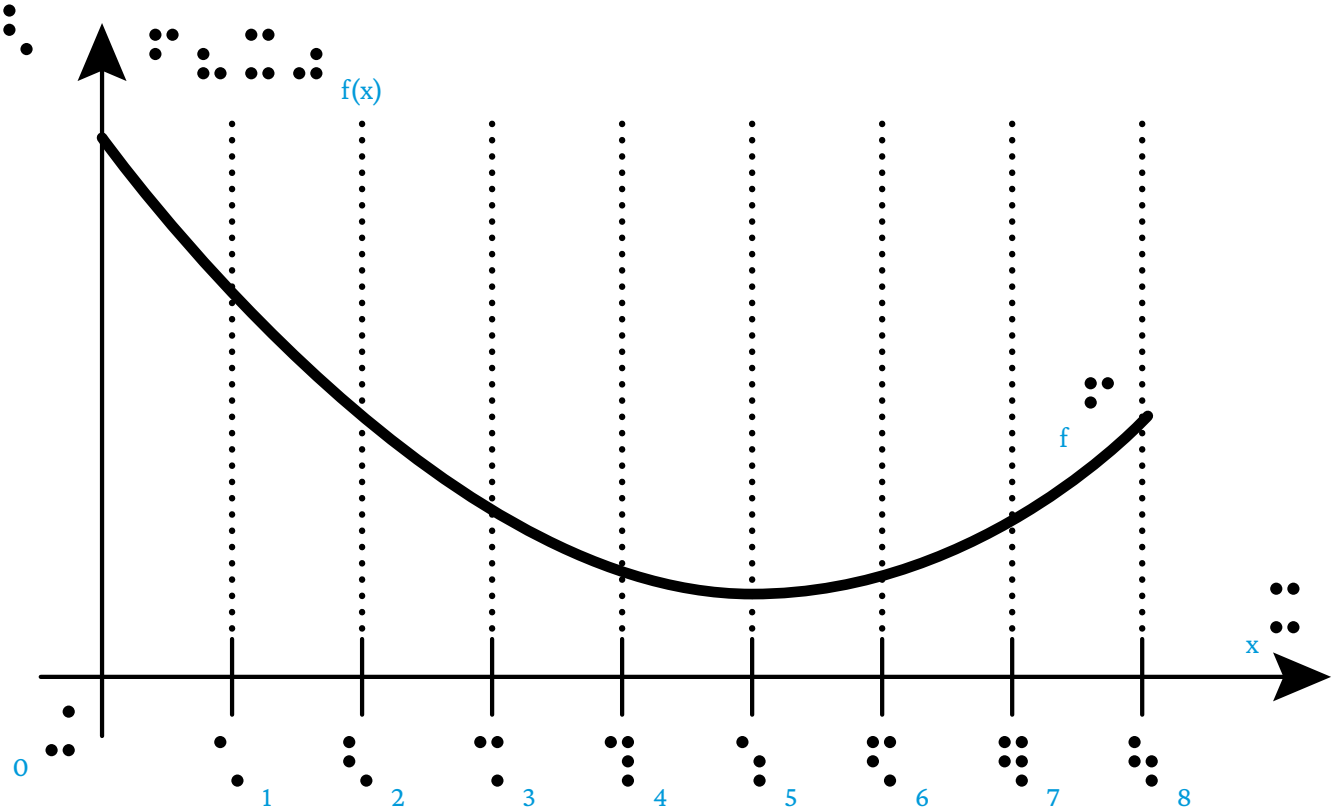


Durchflussrate *Zeit in L/h *h =durchgeflossenes Volumen in L

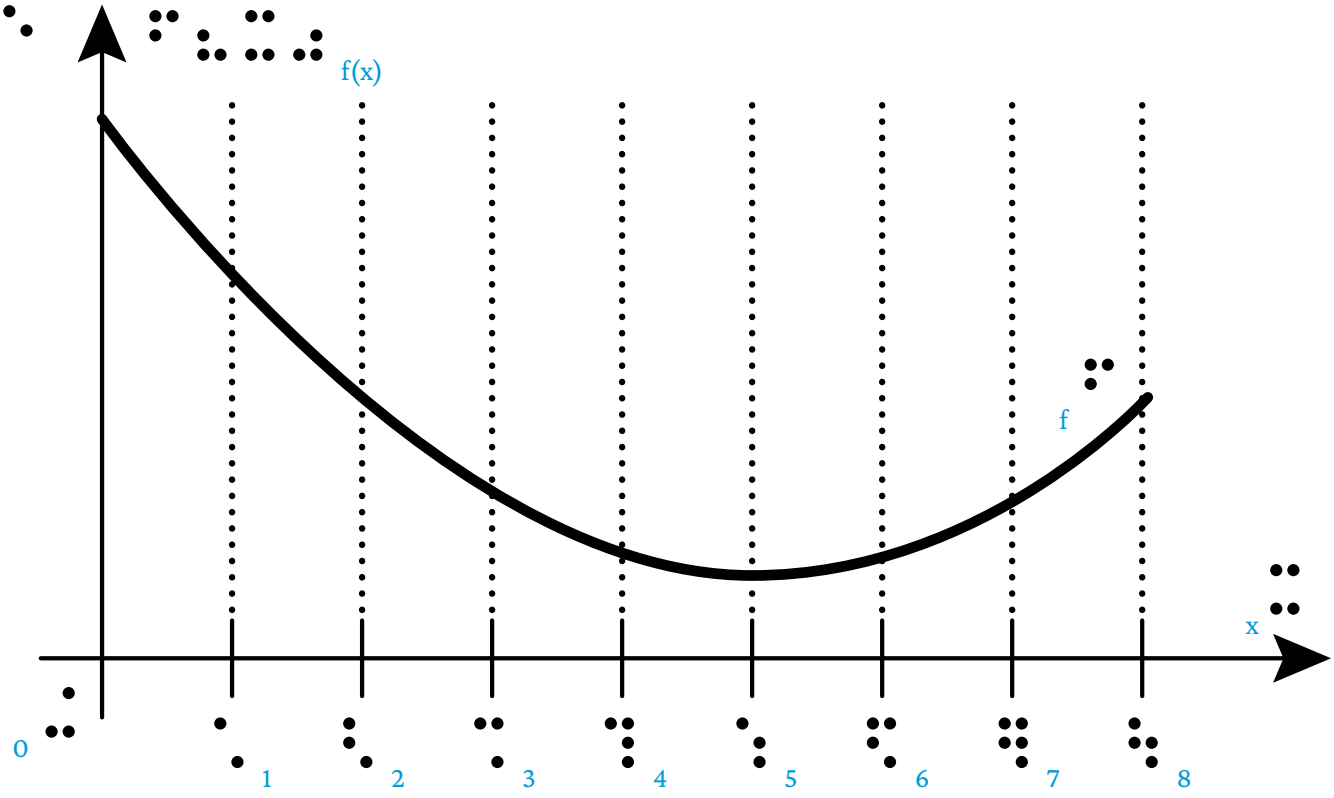
A. 1



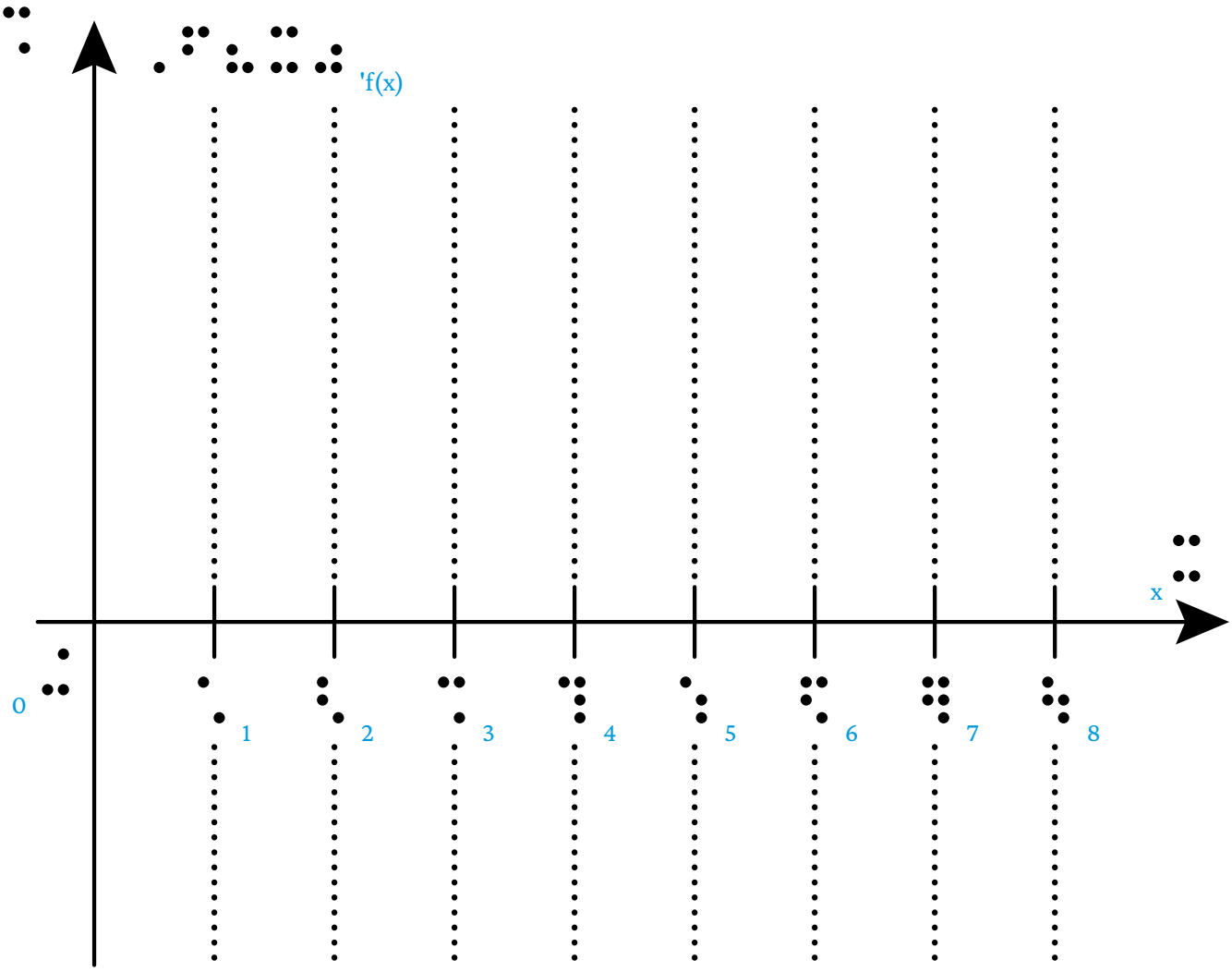
A. 2

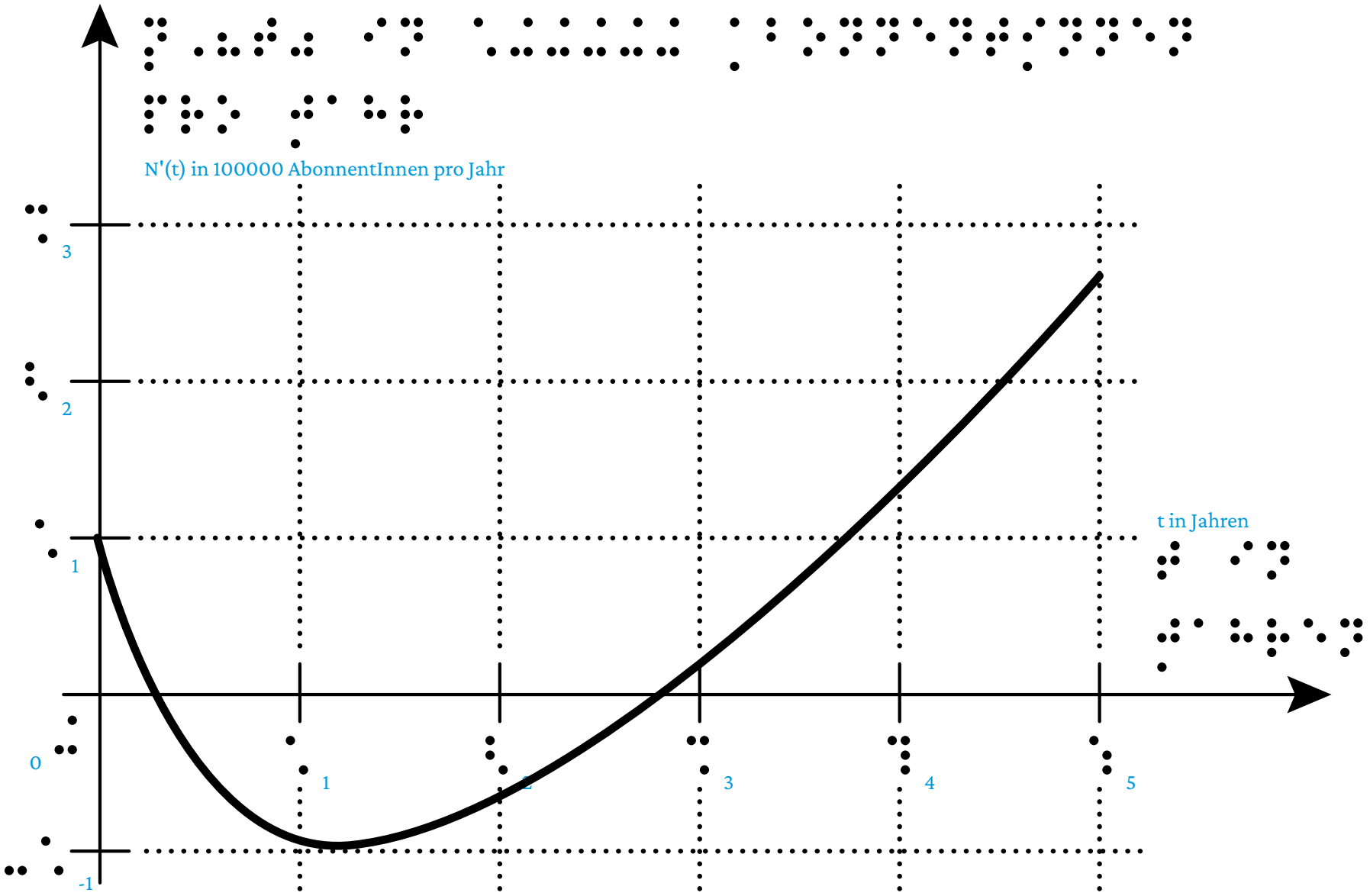


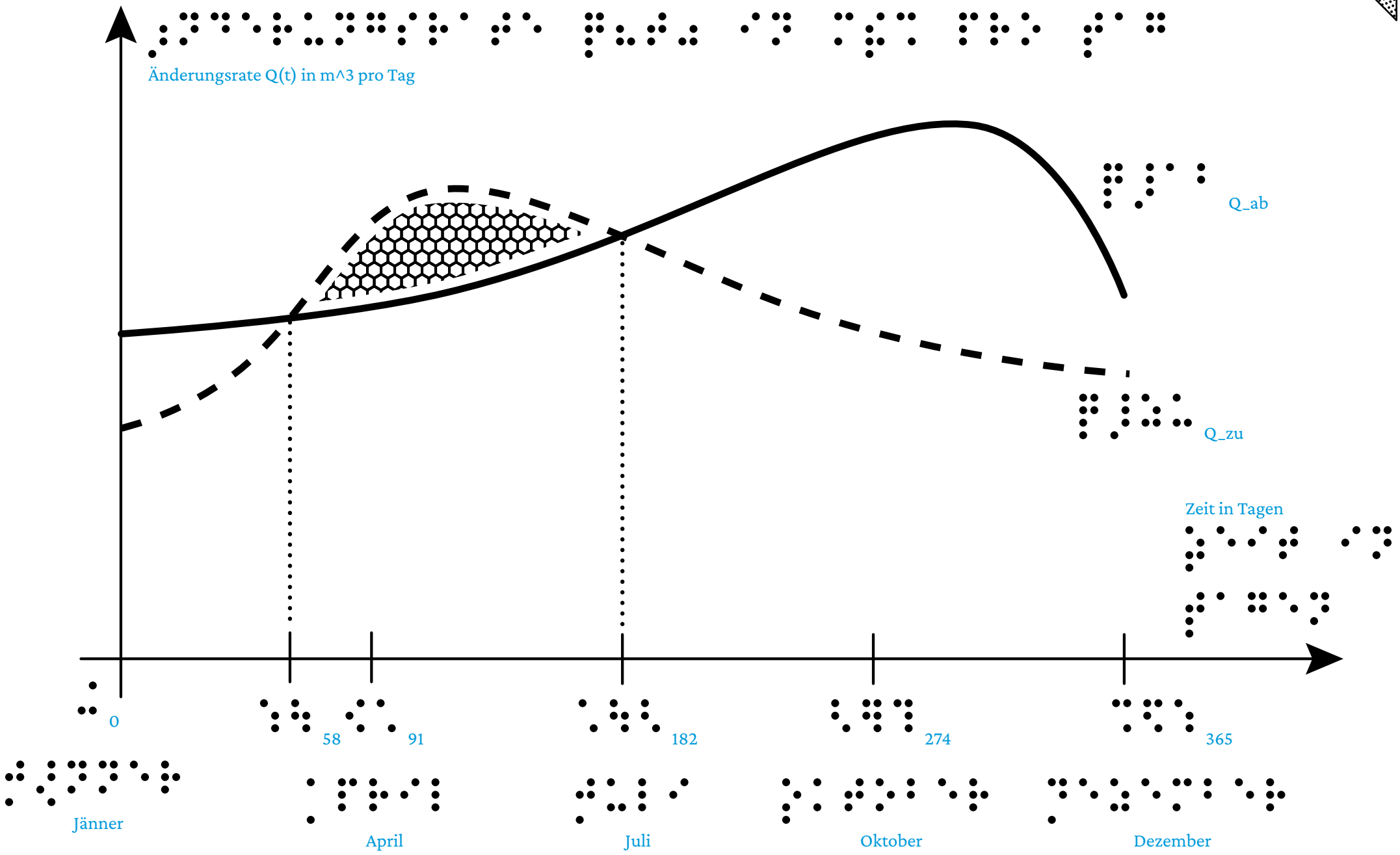
A. 2

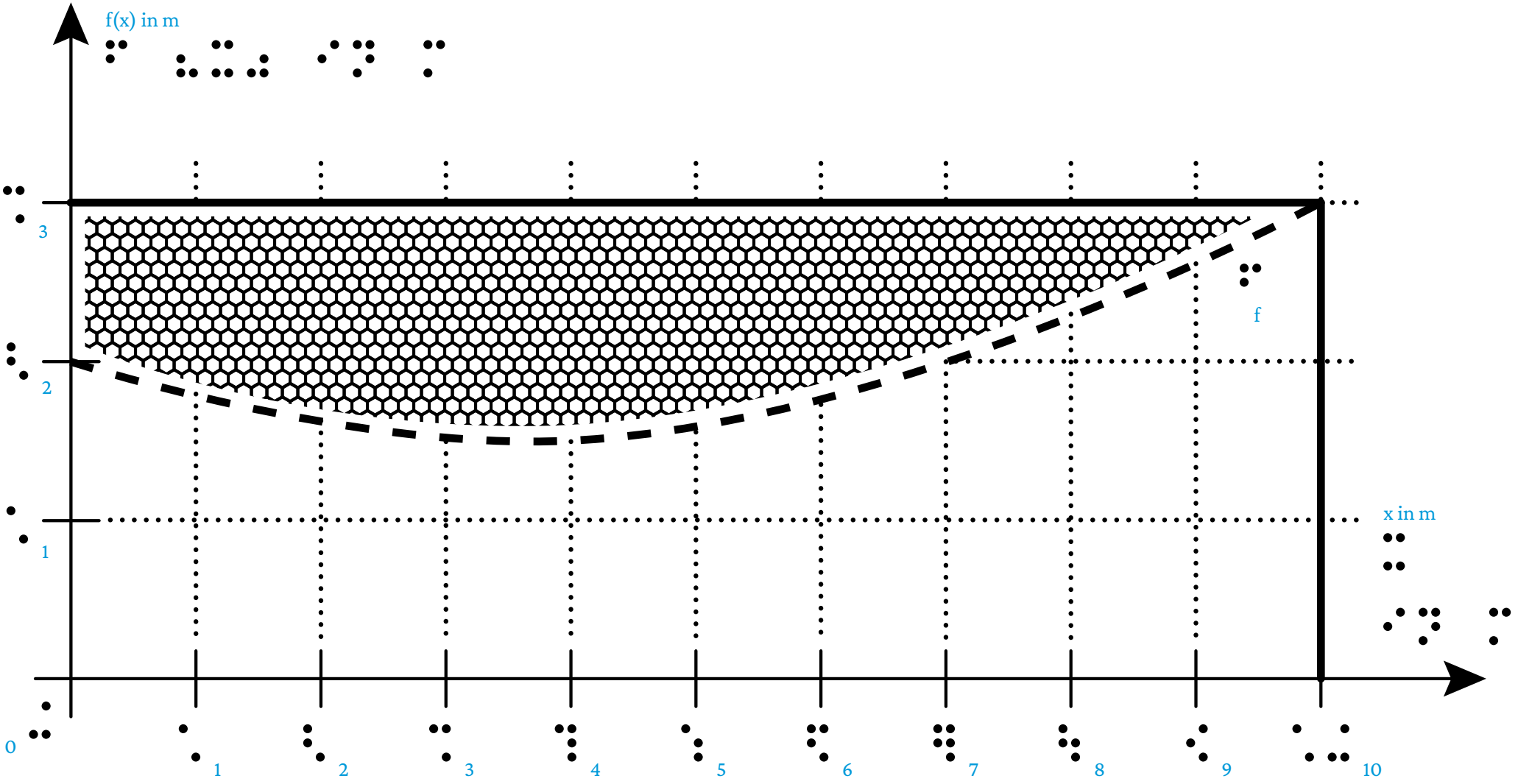


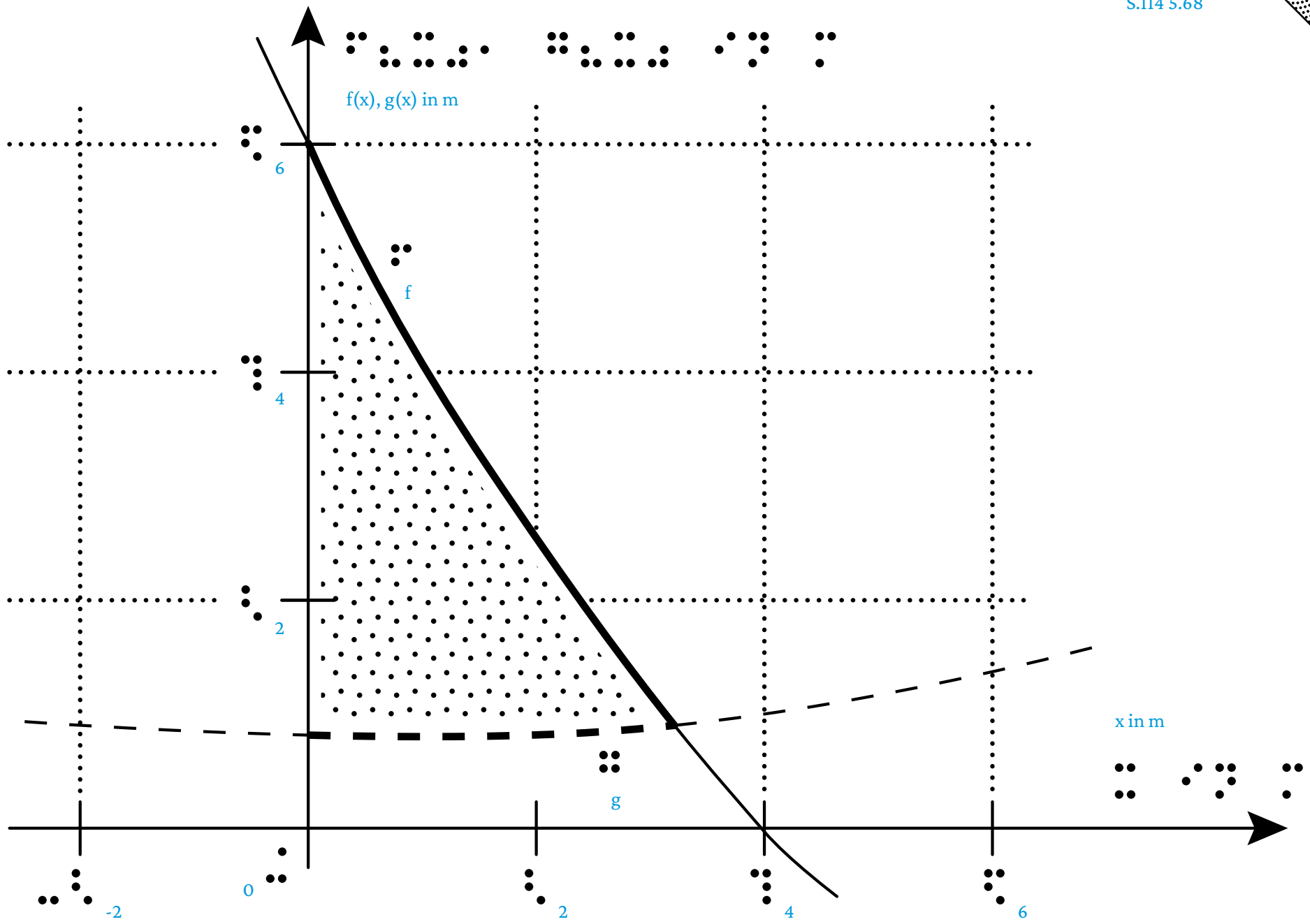
A. 3

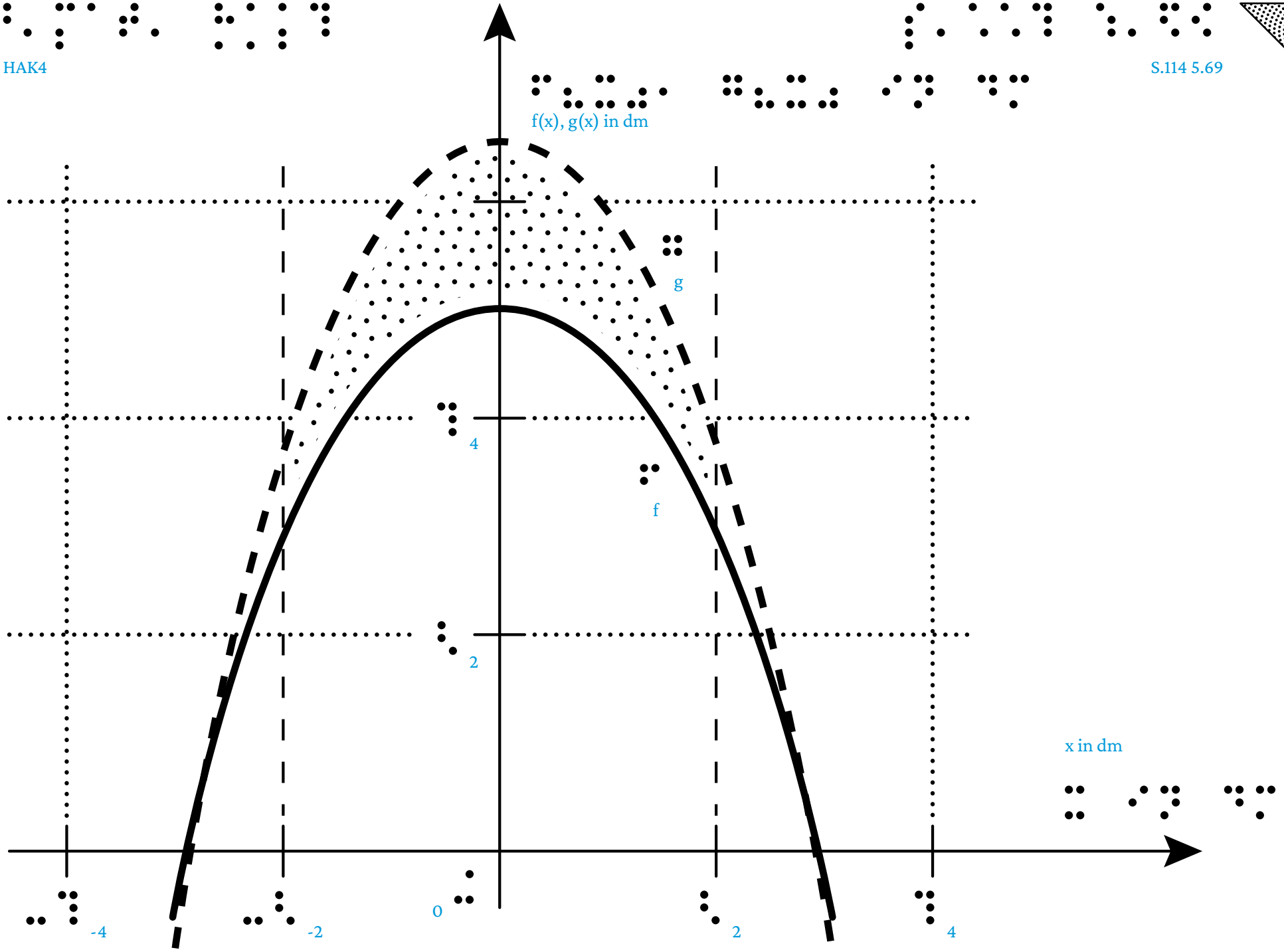


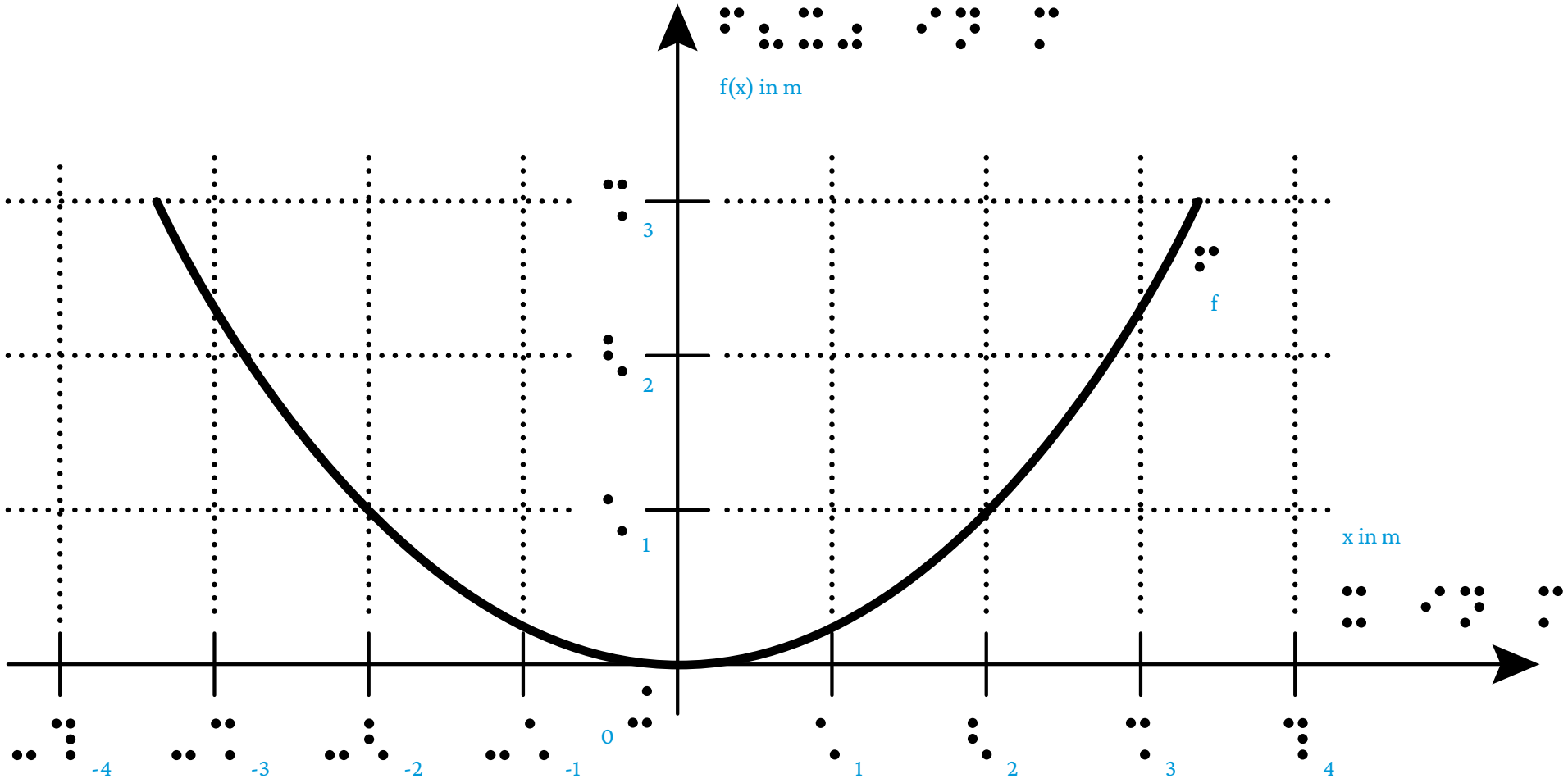

N'(t) in 100000 AbonnentInnen pro Jahr
t in Jahren
3
2
1
0
-1
1
2
3
4
5

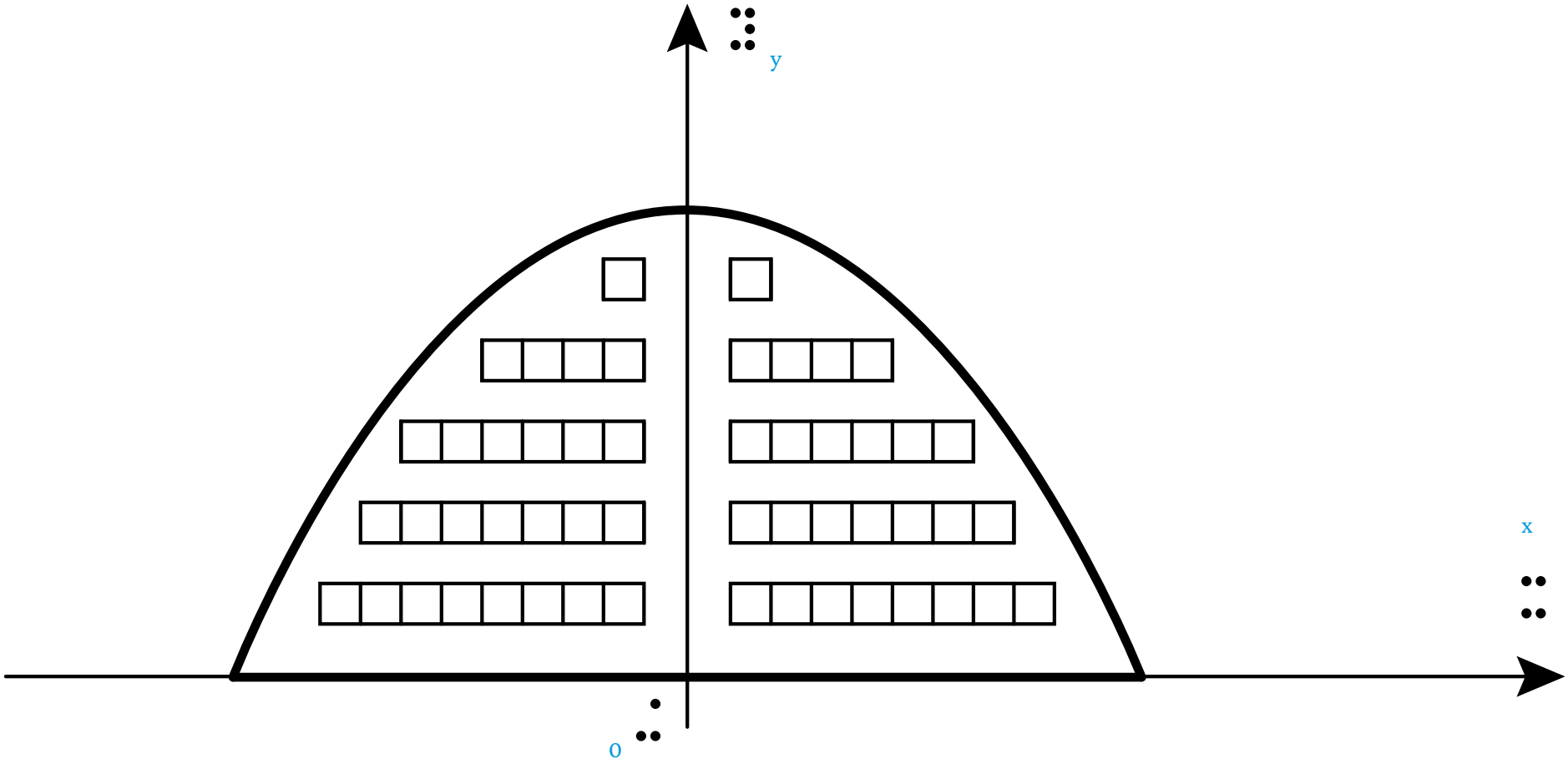

Änderungsrate Q(t) in m^3 pro Tag
Q_ab
Q_zu
Zeit in Tagen
0
58
91
182
274
365
Jänner
April
Juli
Oktober
Dezember

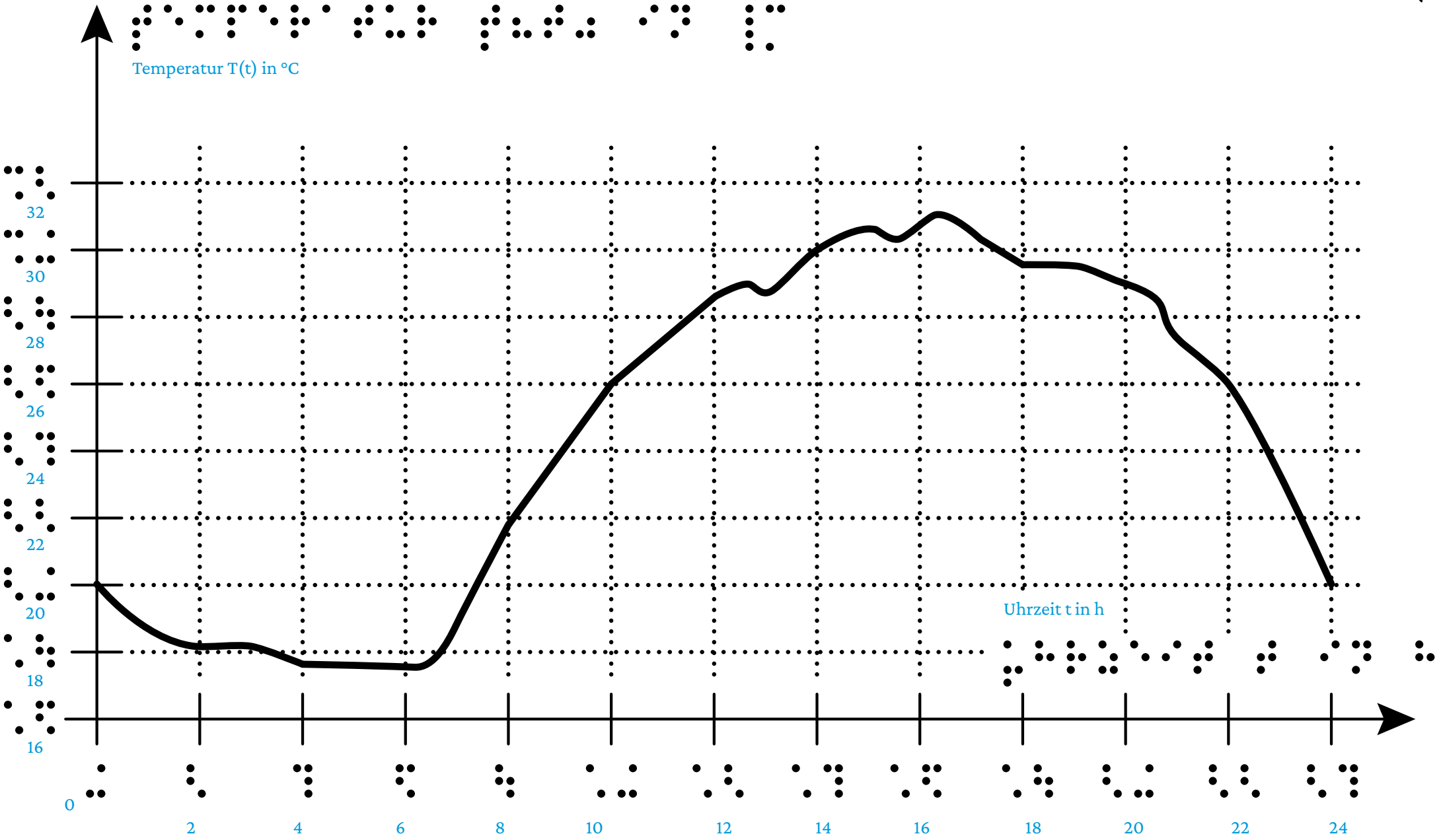

f(x) in m
3
2
1
0
f
x in m
1
2
3
4
5
6
7
8
9
10


f(x), g(x) in m
6
4
2
f
g
x in m
-2
0
2
4
6

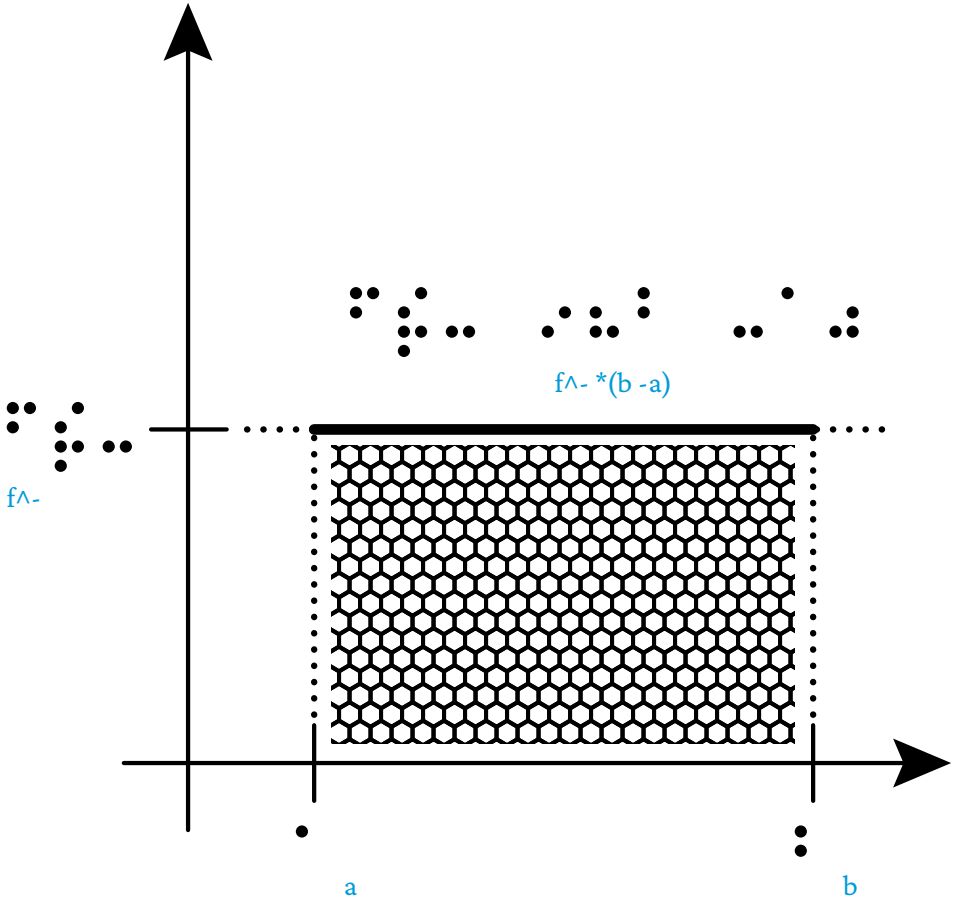

f(x), g(x) in dm
g
f
4
2
0
-4
-2
2
4
x in dm


f(x) in m
3
2
1
f
x in m
0
-4
-3
-2
-1
1
2
3
4

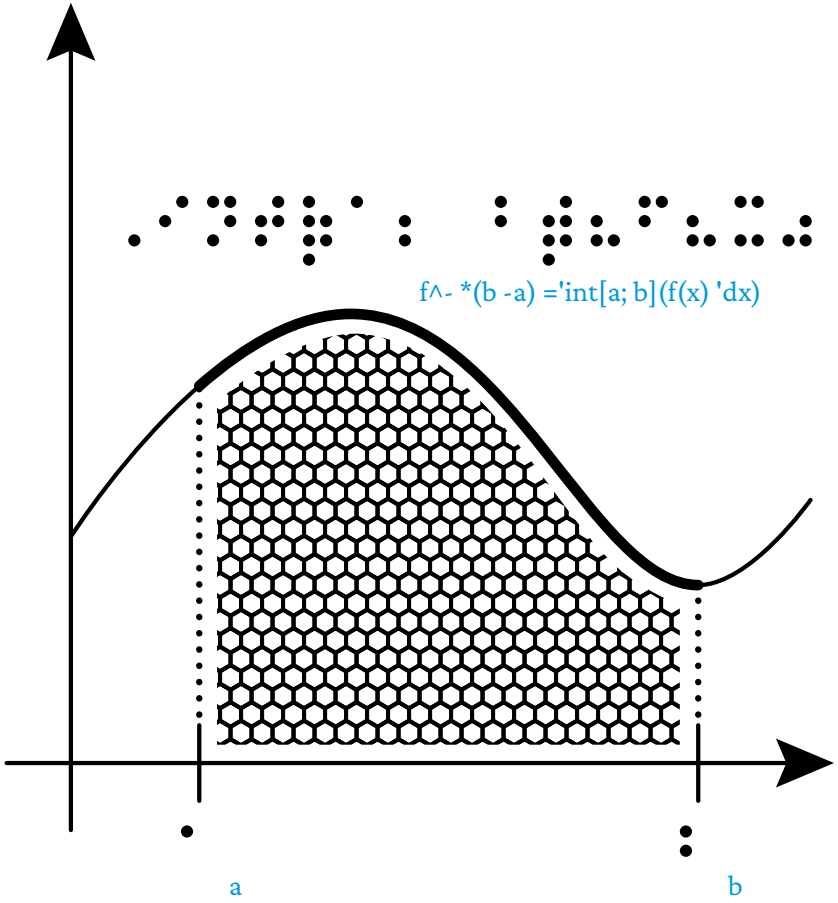

y
x
0

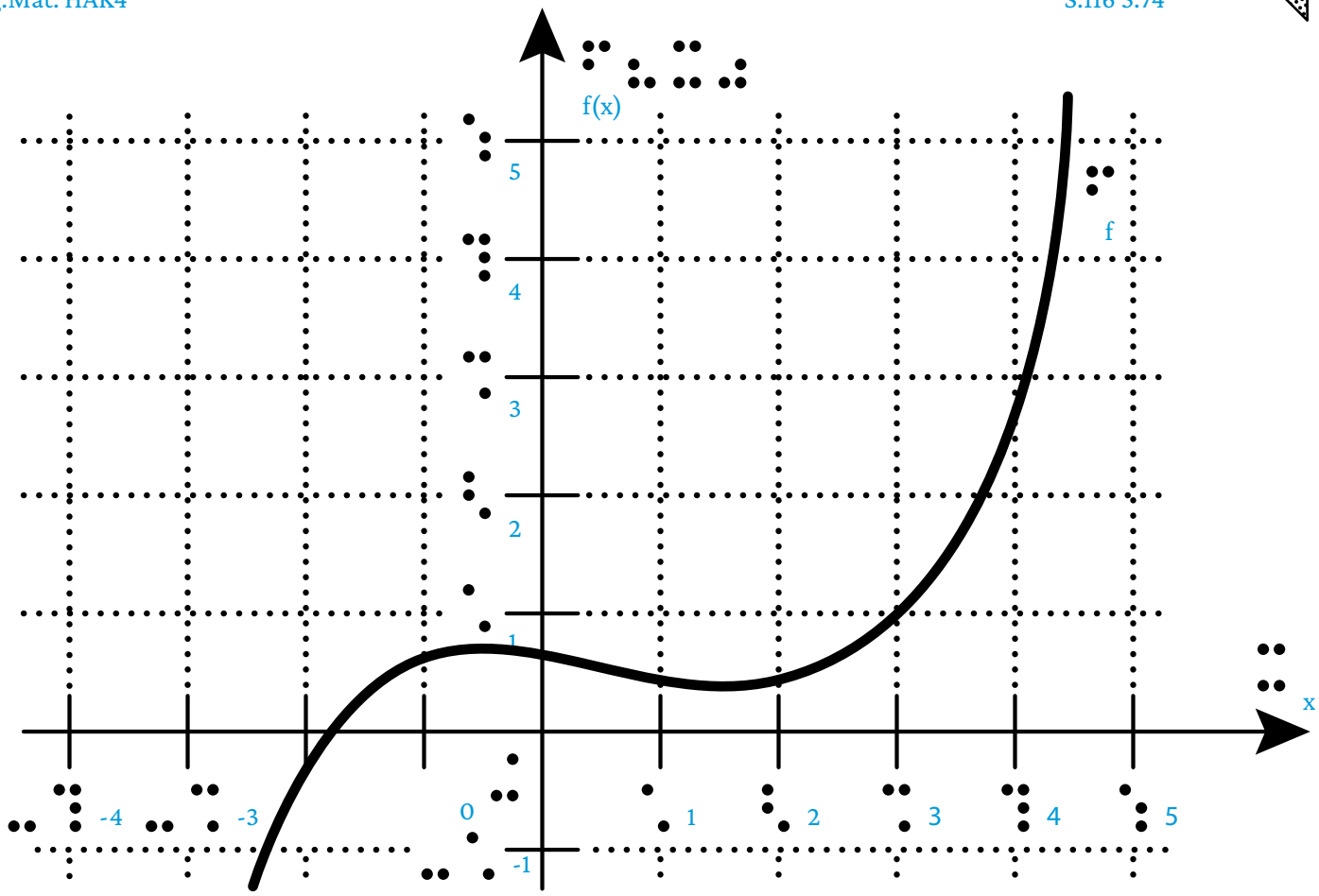

Temperatur T(t) in °C
32
30
28
26
24
22
20
18
16
0
2
4
6
8
10
12
14
16
18
20
22
24
Uhrzeit t in h

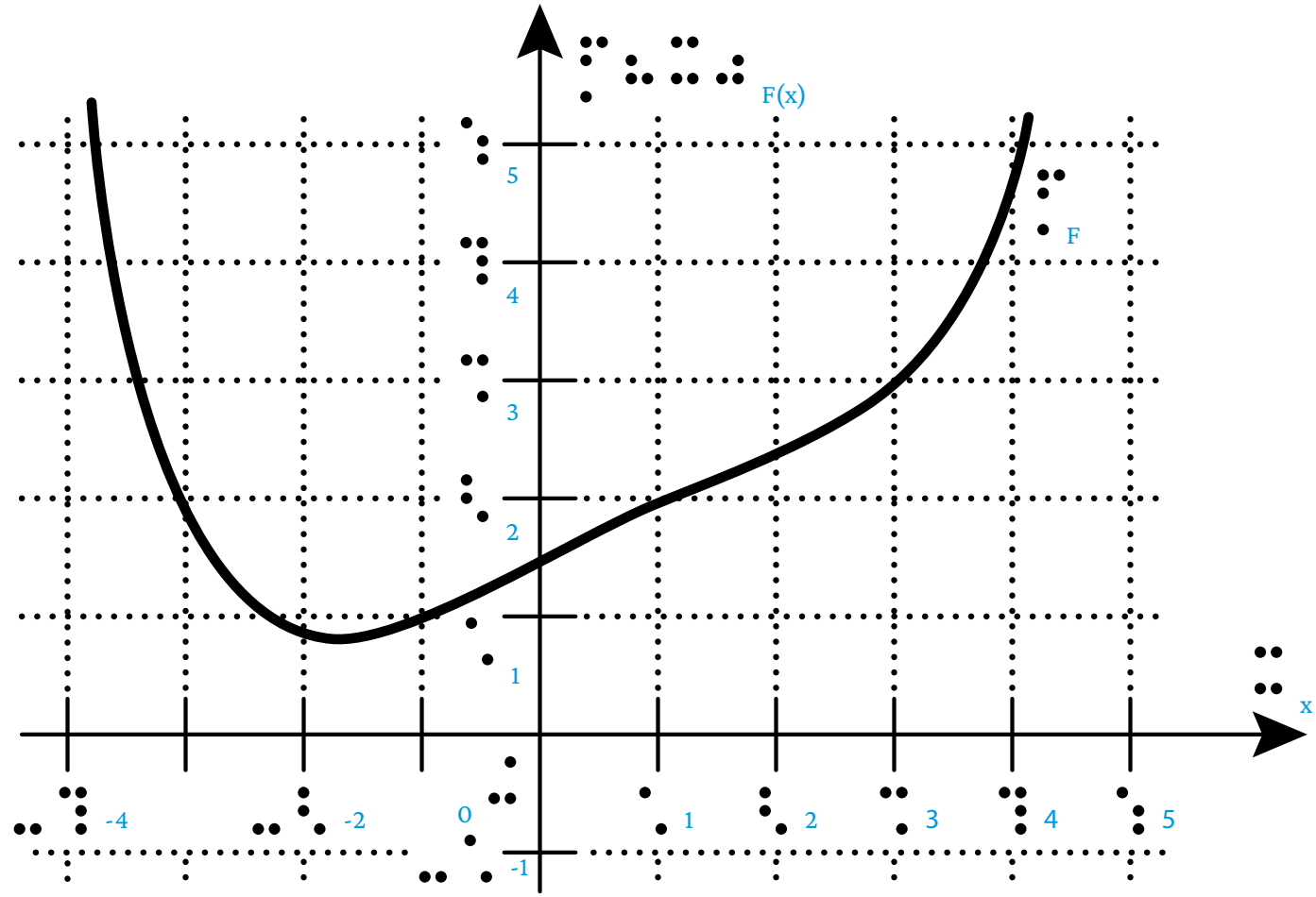
f^- *(b -a) ='int[a; b](f(x) 'dx)

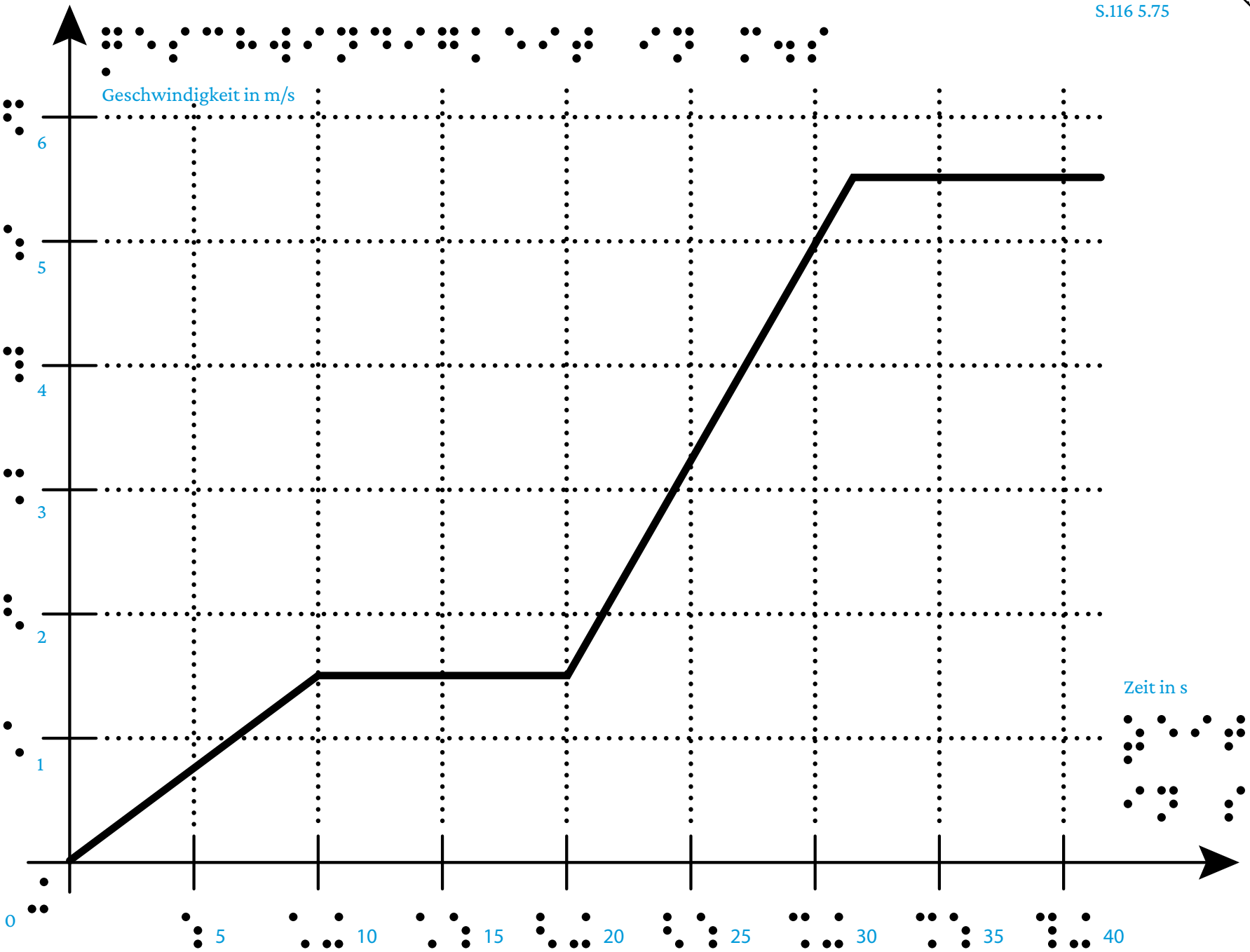


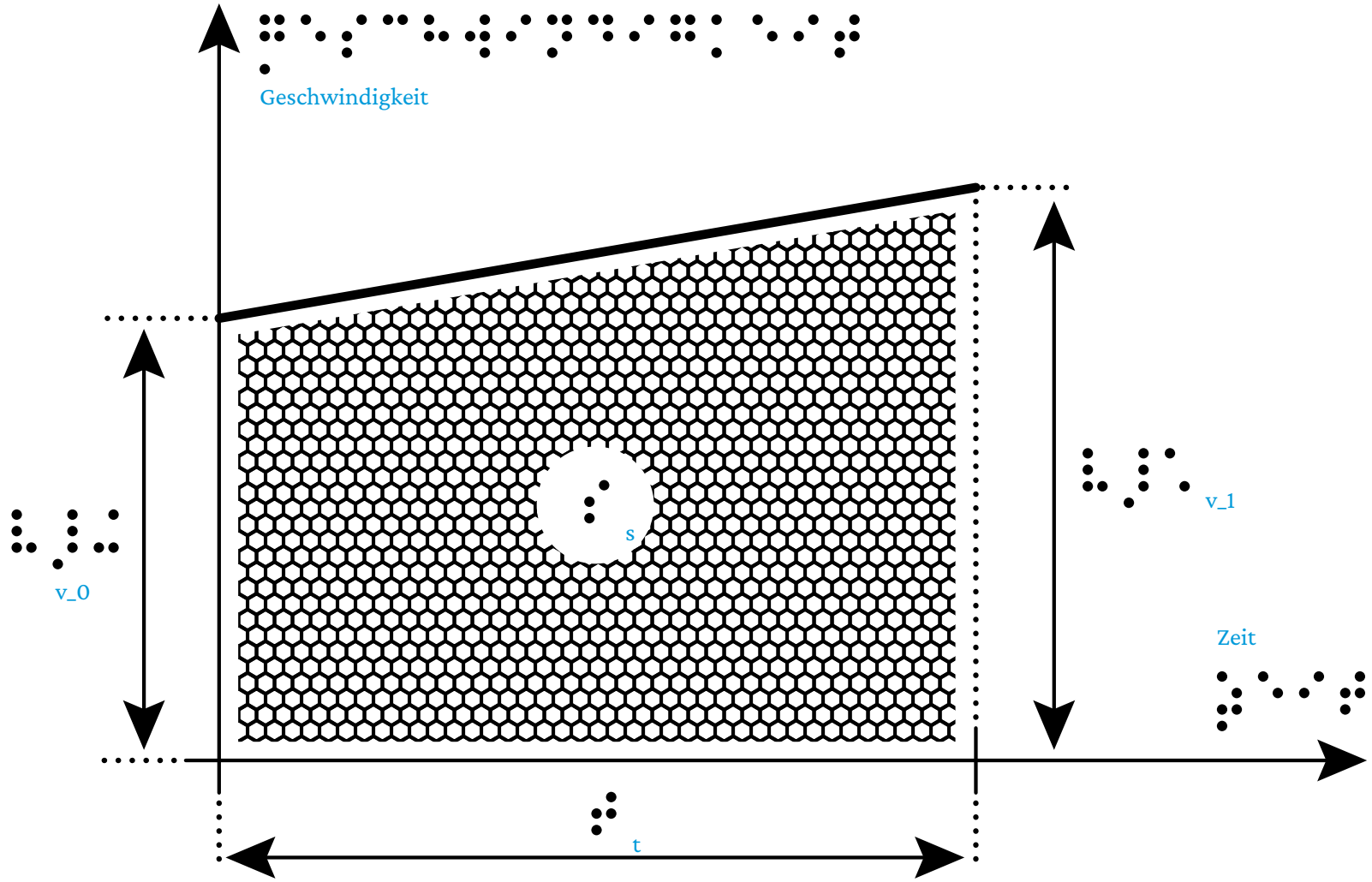
f^- *(b -a) ='int[a; b](f(x) 'dx)

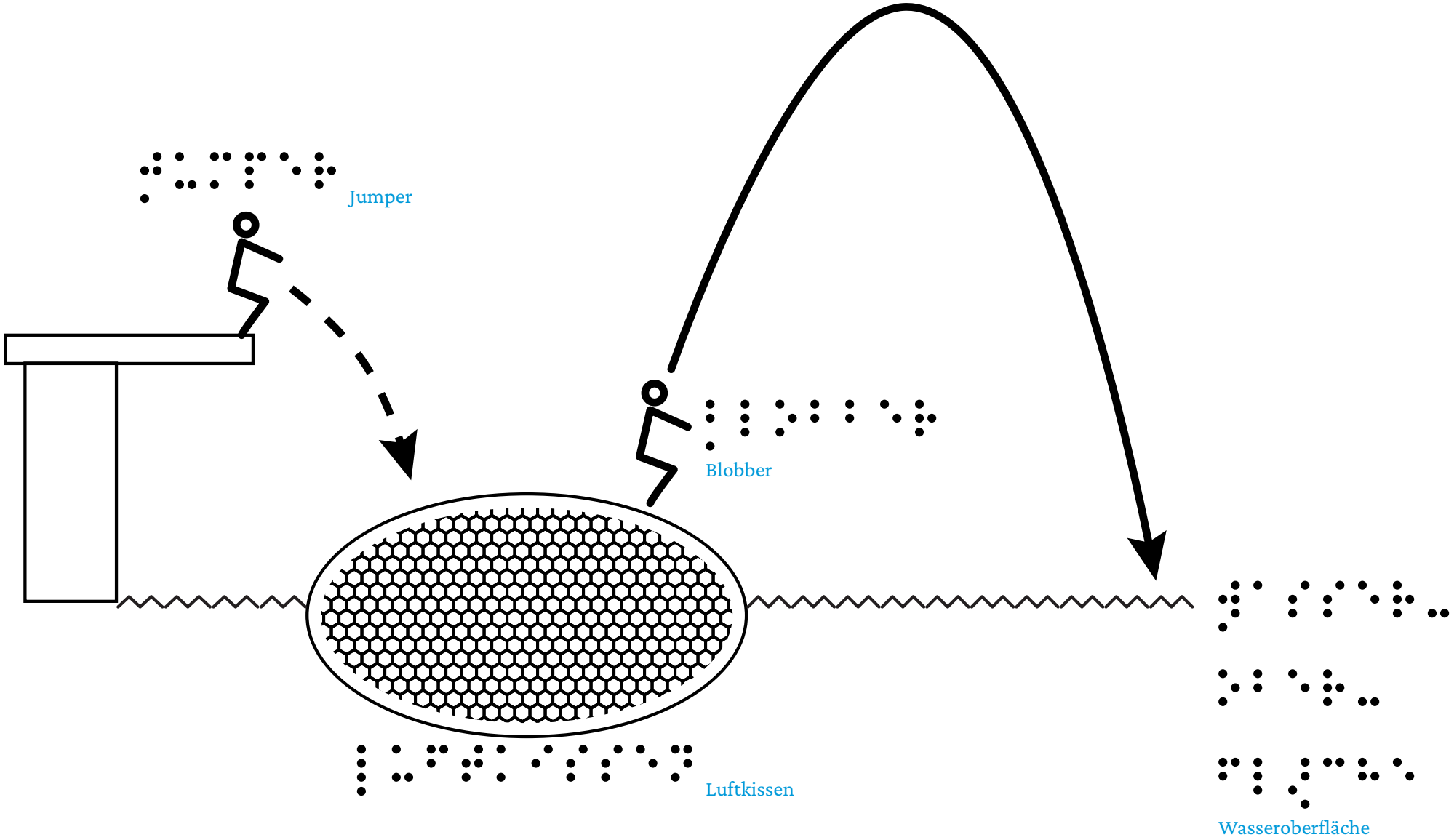


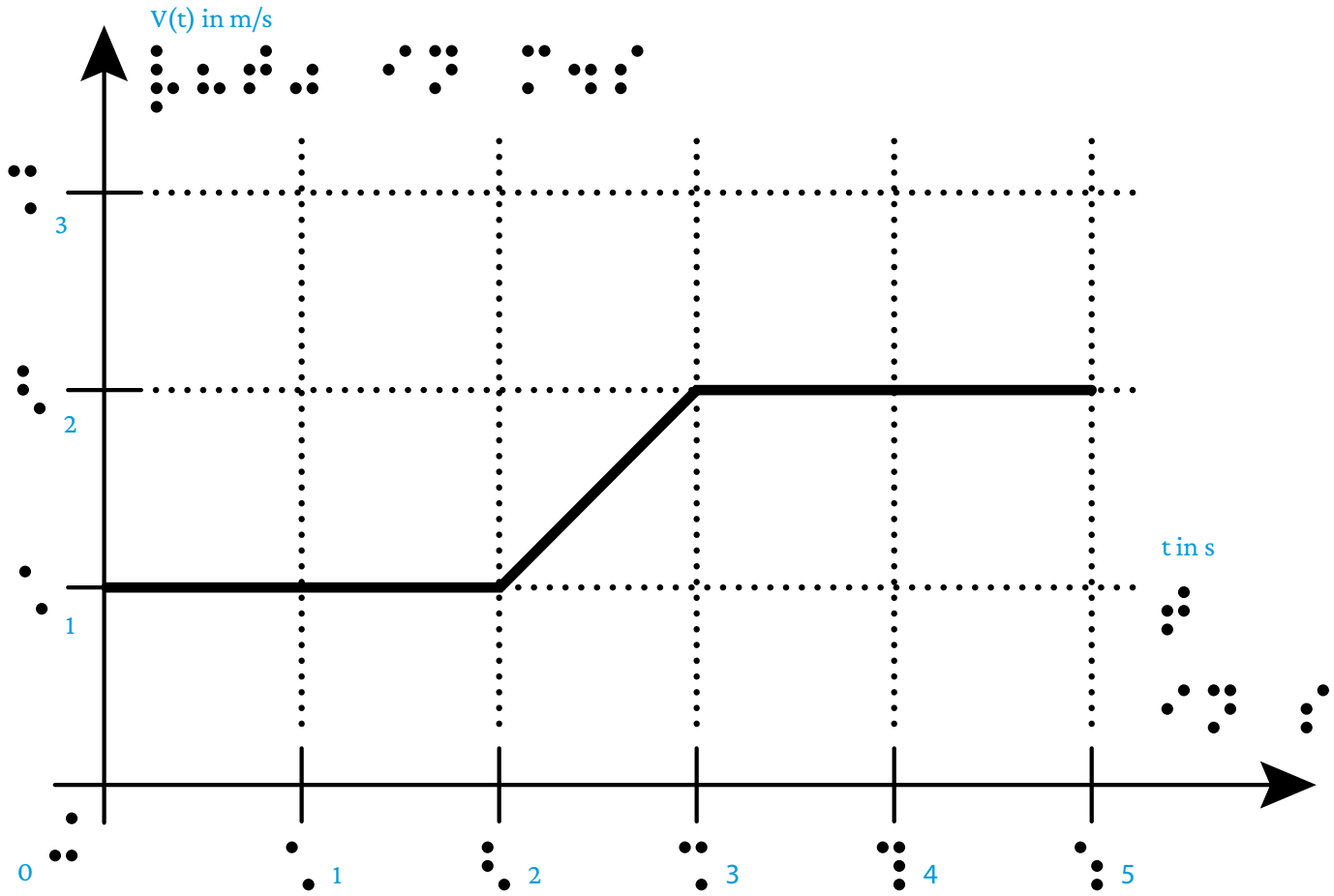

f(x)
f
x
5
4
3
2
1
-1
-4
-3
0
1
2
3
4
5

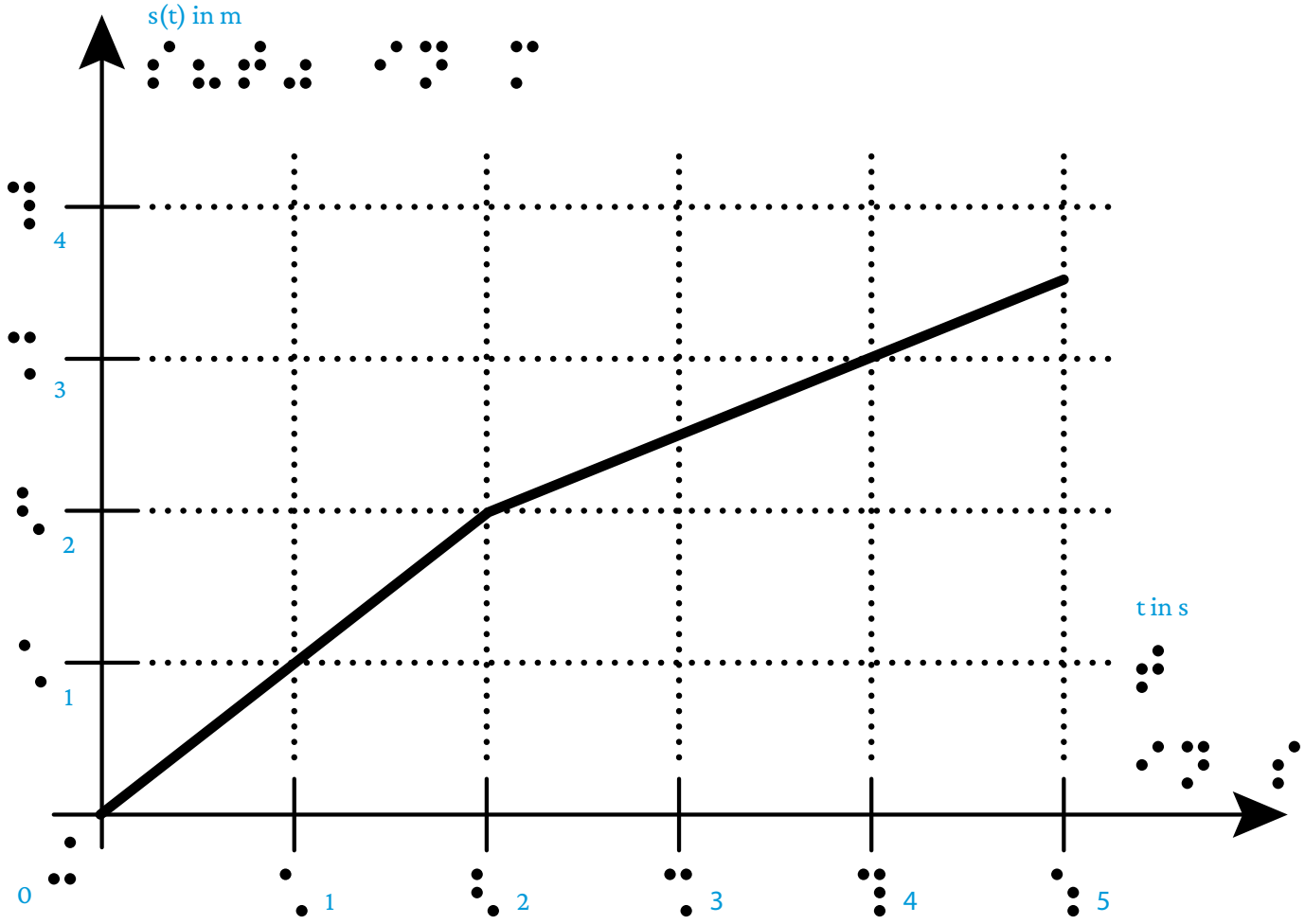

F(x)
F
x
5
4
3
2
1
-1
-4
-2
0
1
2
3
4
5

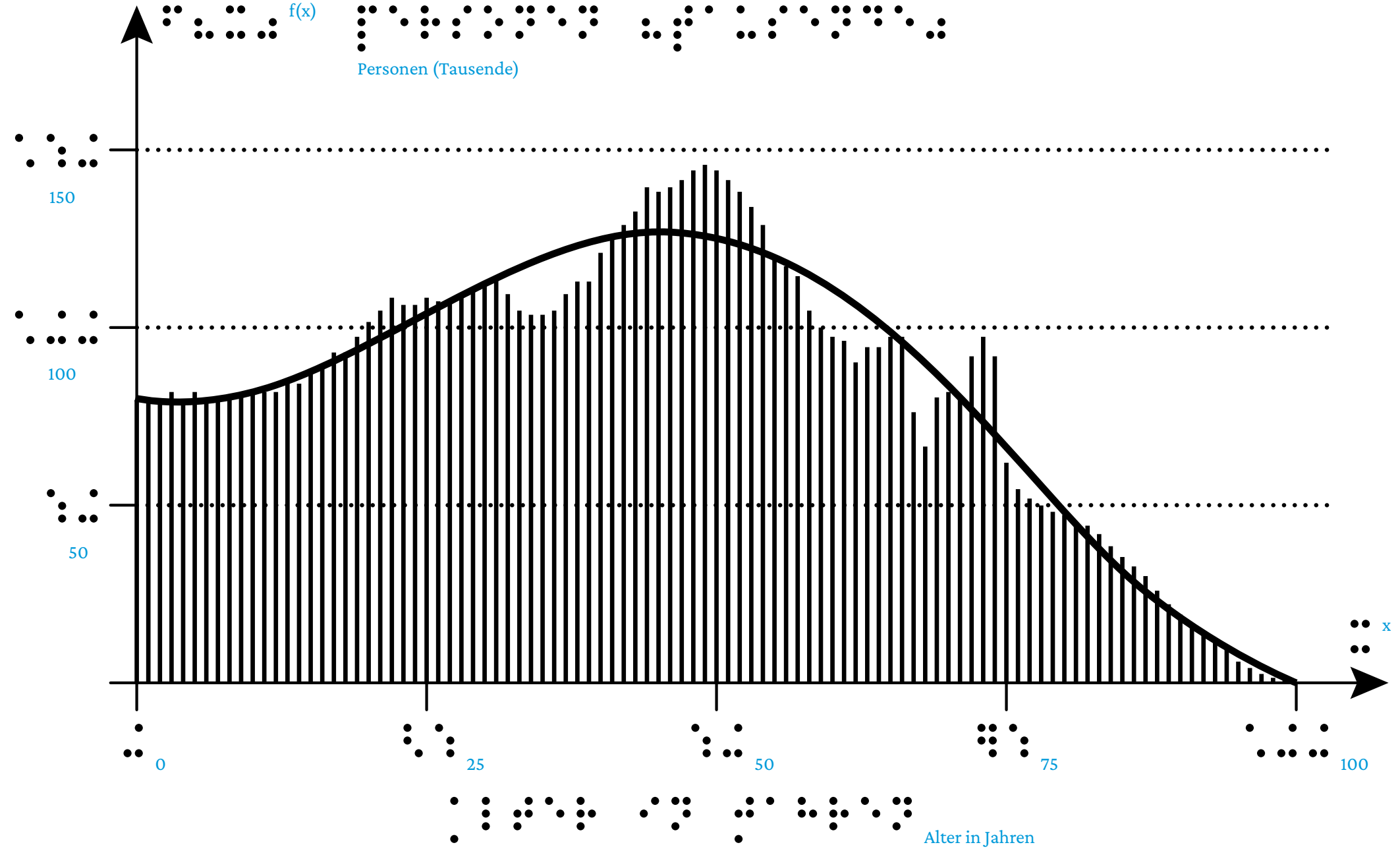

Geschwindigkeit in m/s
6
5
4
3
2
1
0
Zeit in s
5
10
15
20
25
30
35
40

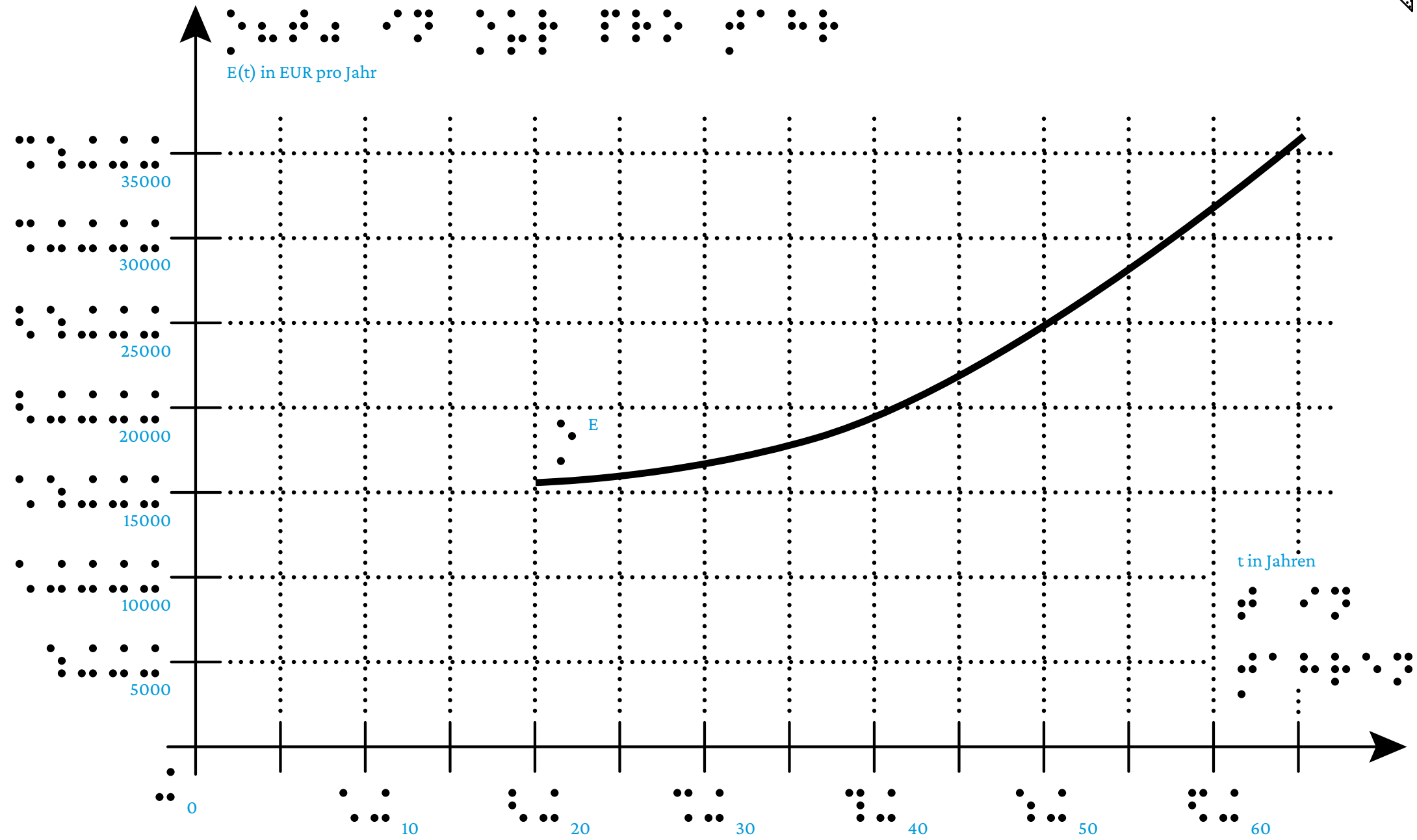

Geschwindigkeit
v_0
s
v_1
Zeit
t

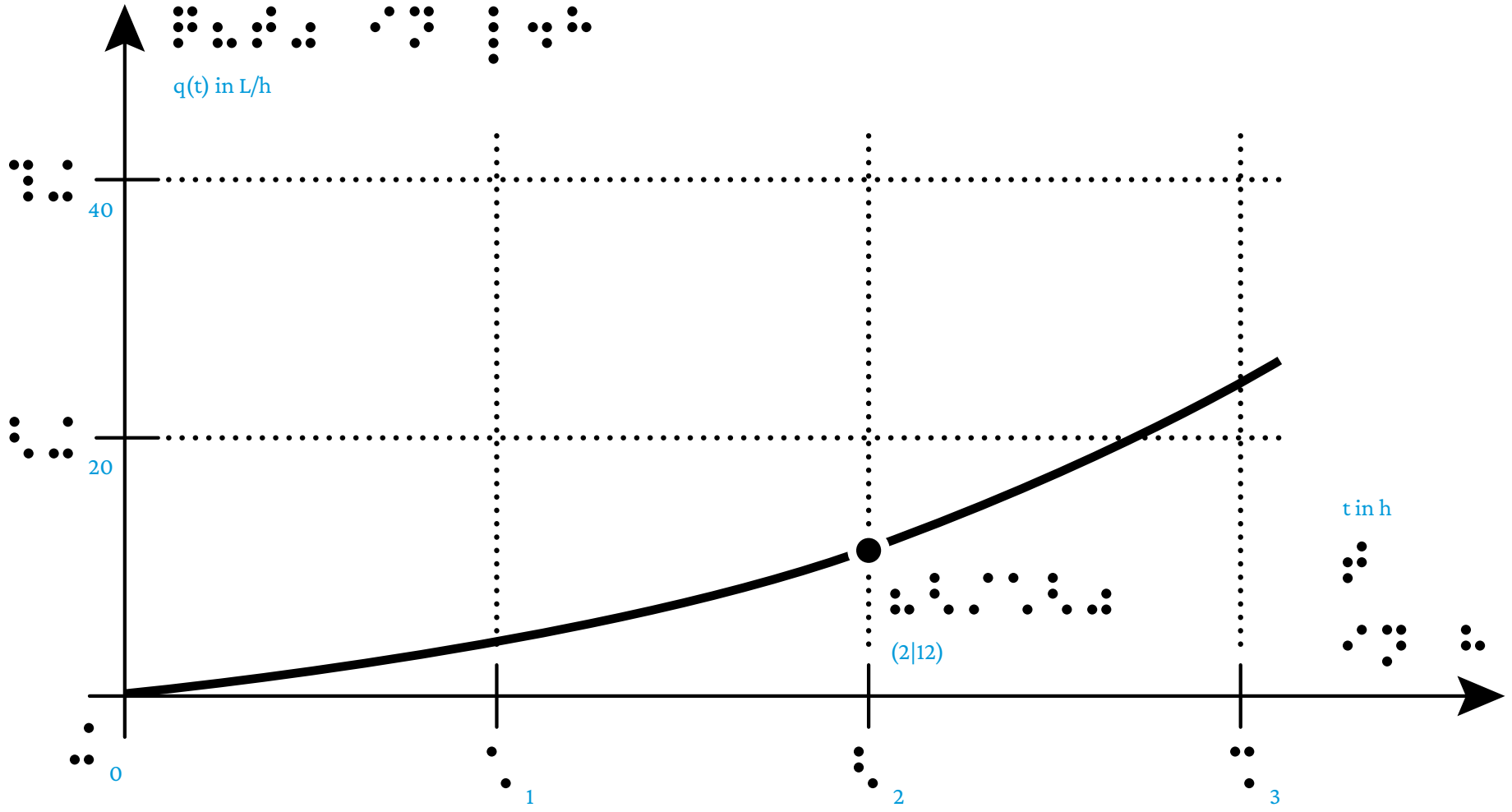

Jumper
Blobber
Luftkissen
Wasseroberfläche

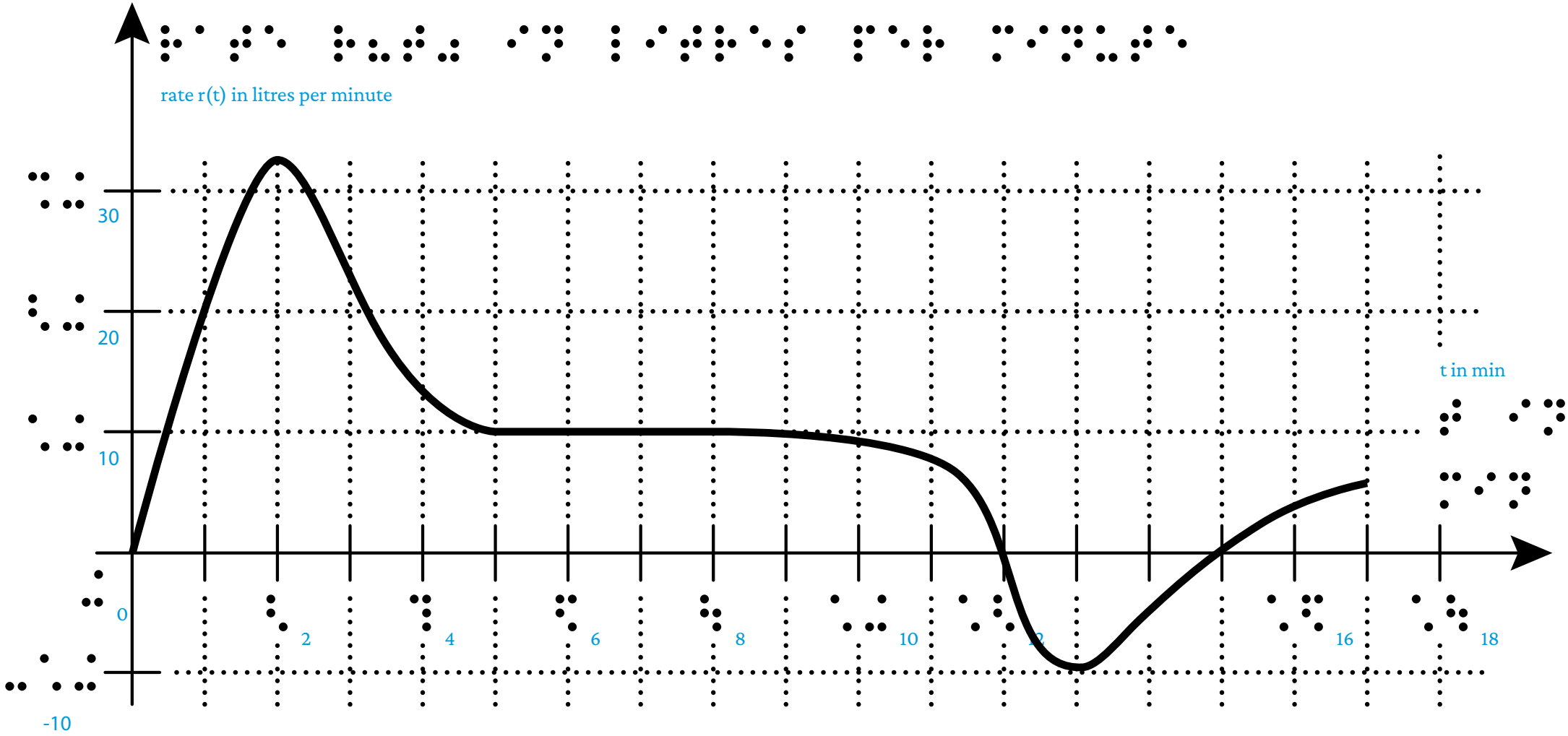

V(t) in m/s
3
2
1
t in s
0
1
2
3
4
5

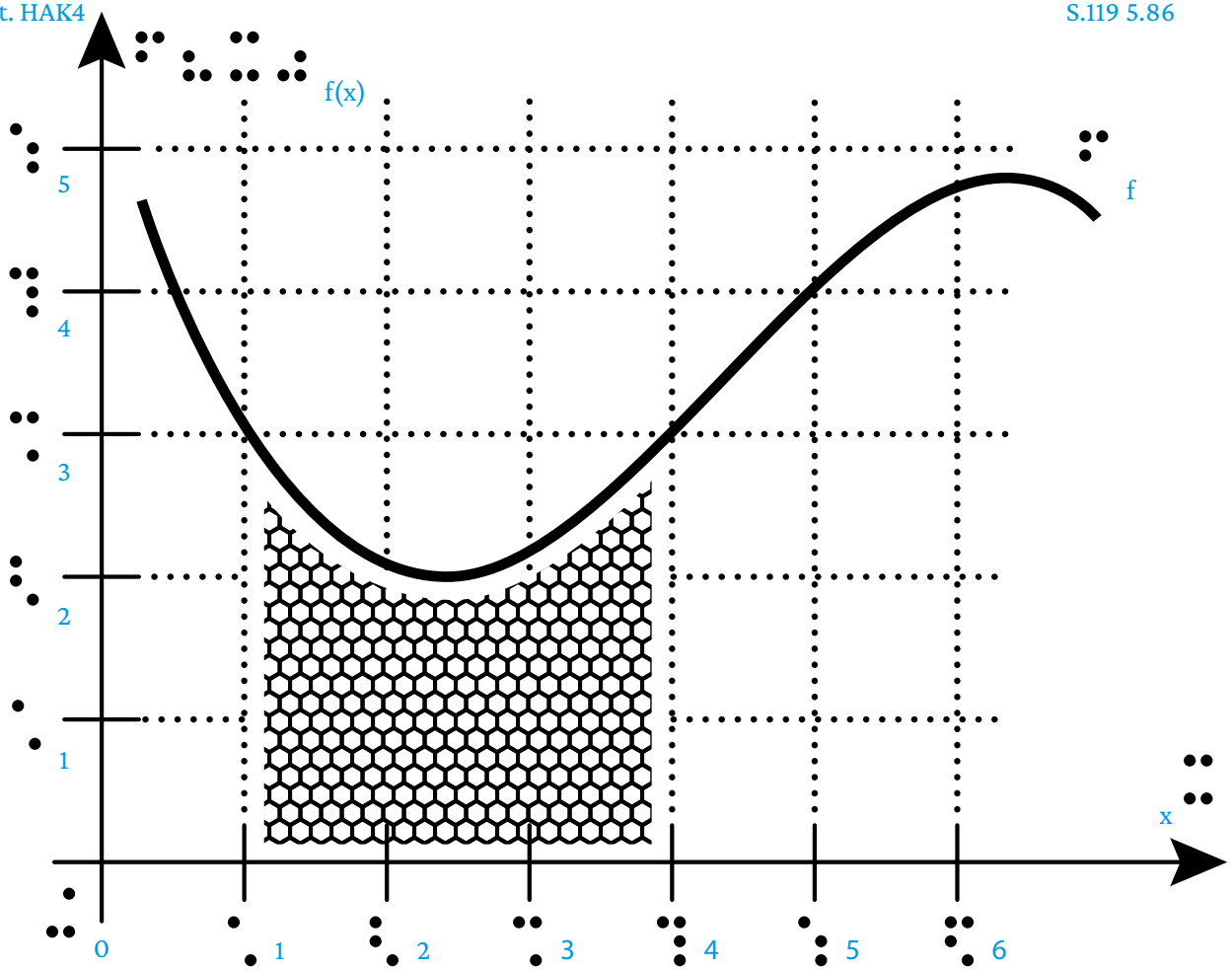

s(t) in m
4
3
2
1
t in s
0
1
2
3
4
5

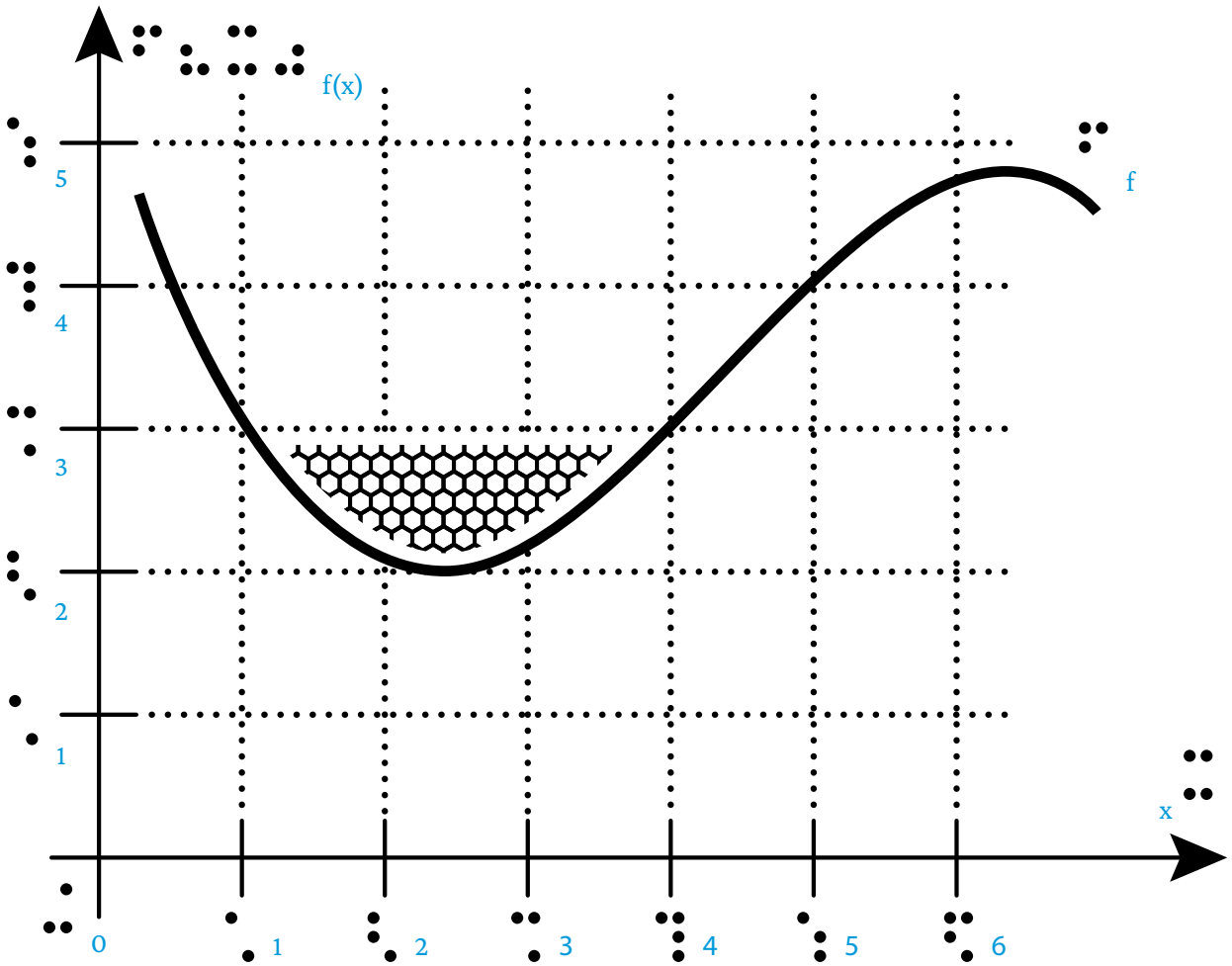

f(x)
Personen (Tausende)
150
100
50
x
0
25
50
75
100
Alter in Jahren

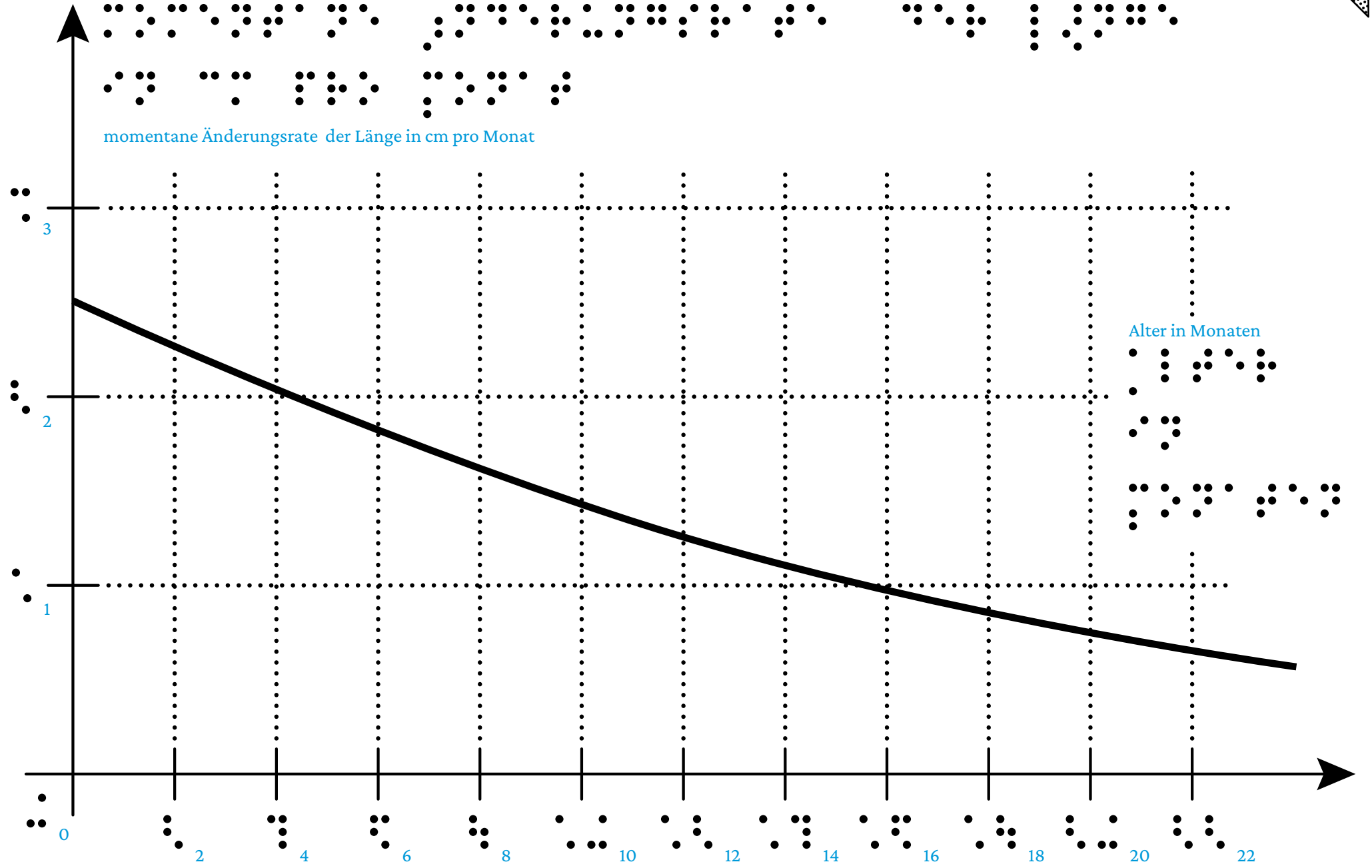

E(t) in EUR pro Jahr
35000
30000
25000
20000
15000
10000
5000
0
E
t in Jahren
10
20
30
40
50
60

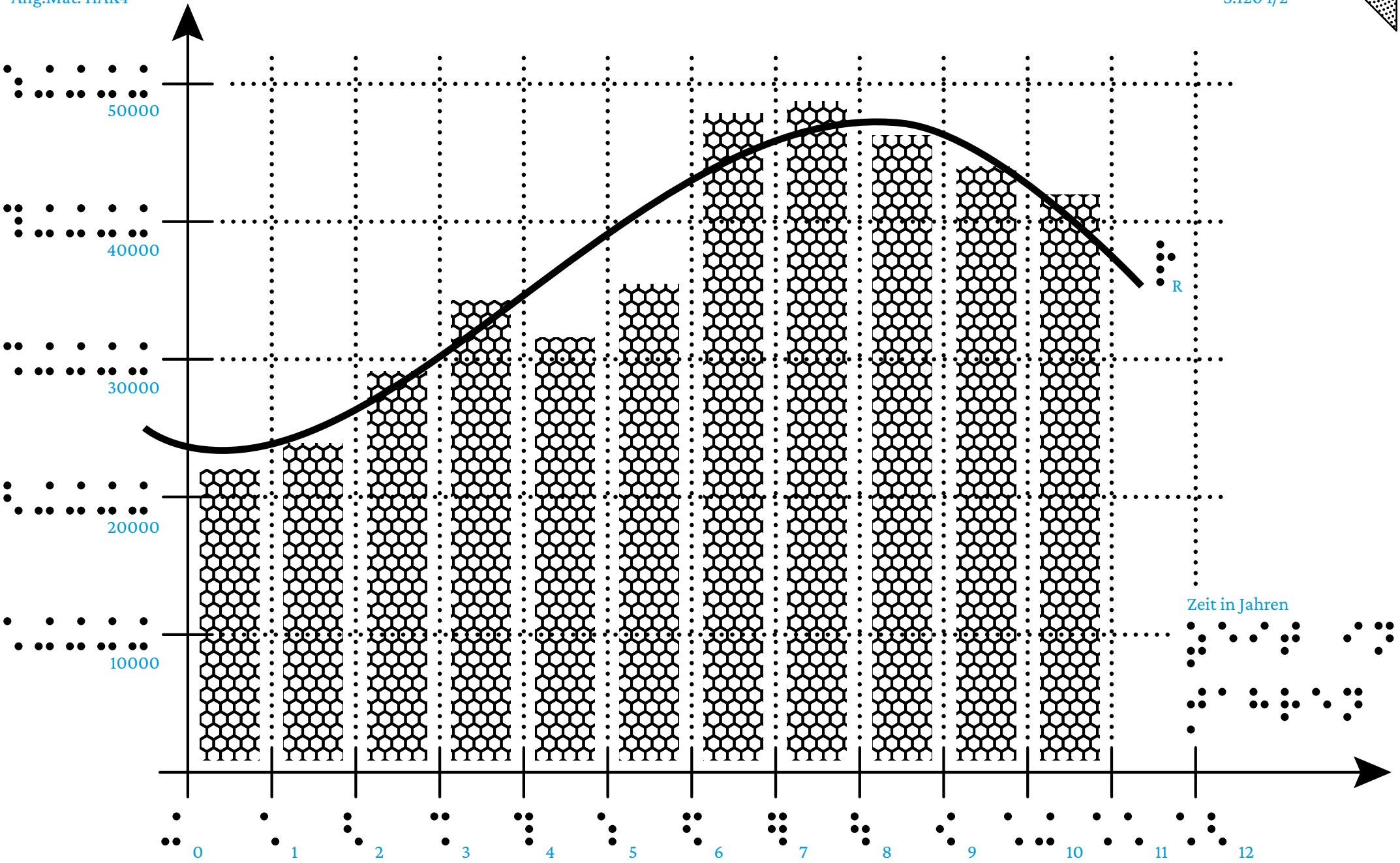

q(t) in L/h
40
20
t in h
(2|12)
0
1
2
3

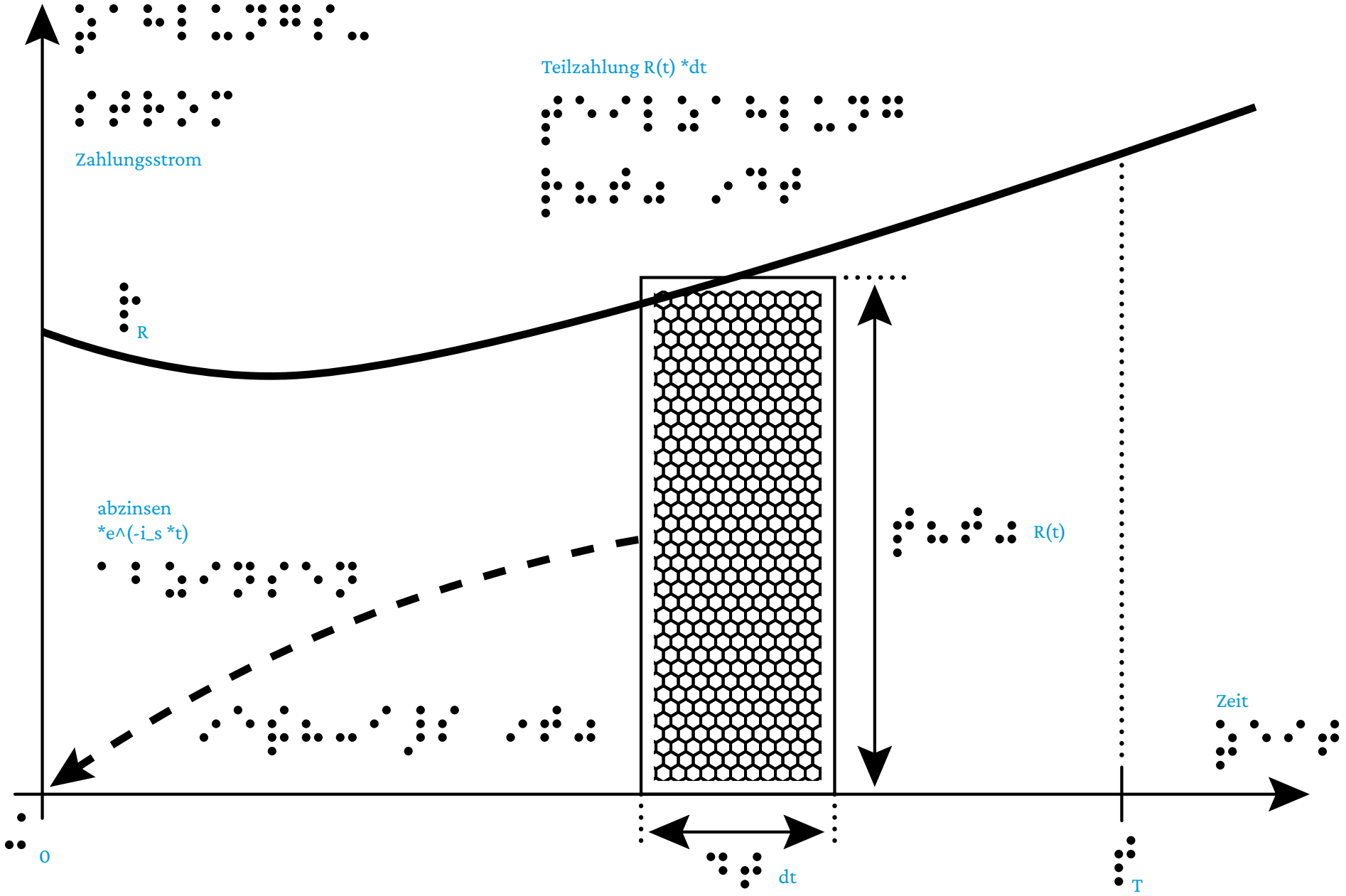

rate r(t) in litres per minute
30
20
10
0
-10
2
4
6
8
10
16
18
t in min

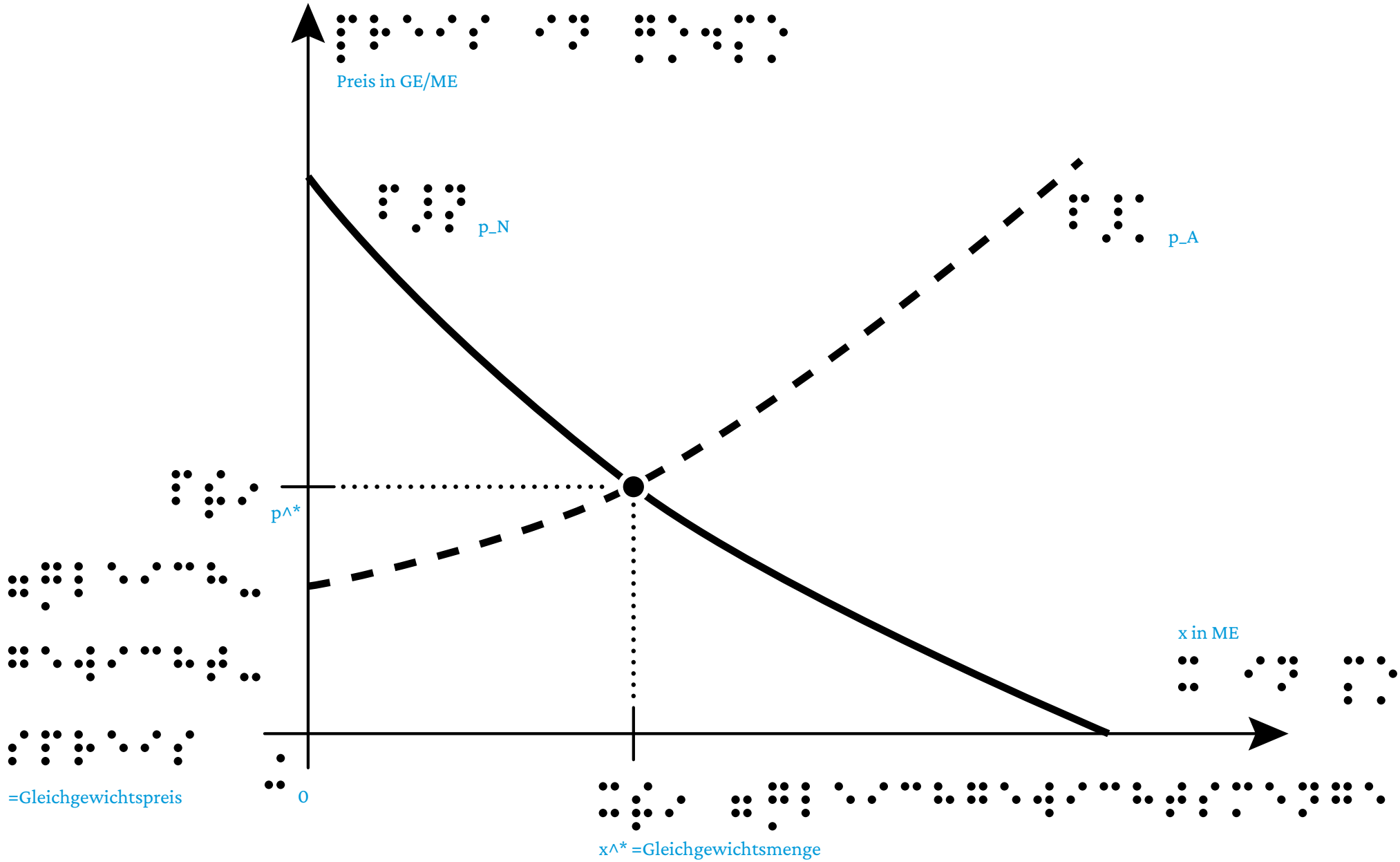

f(x)
f
x
0
1
2
3
4
5
6

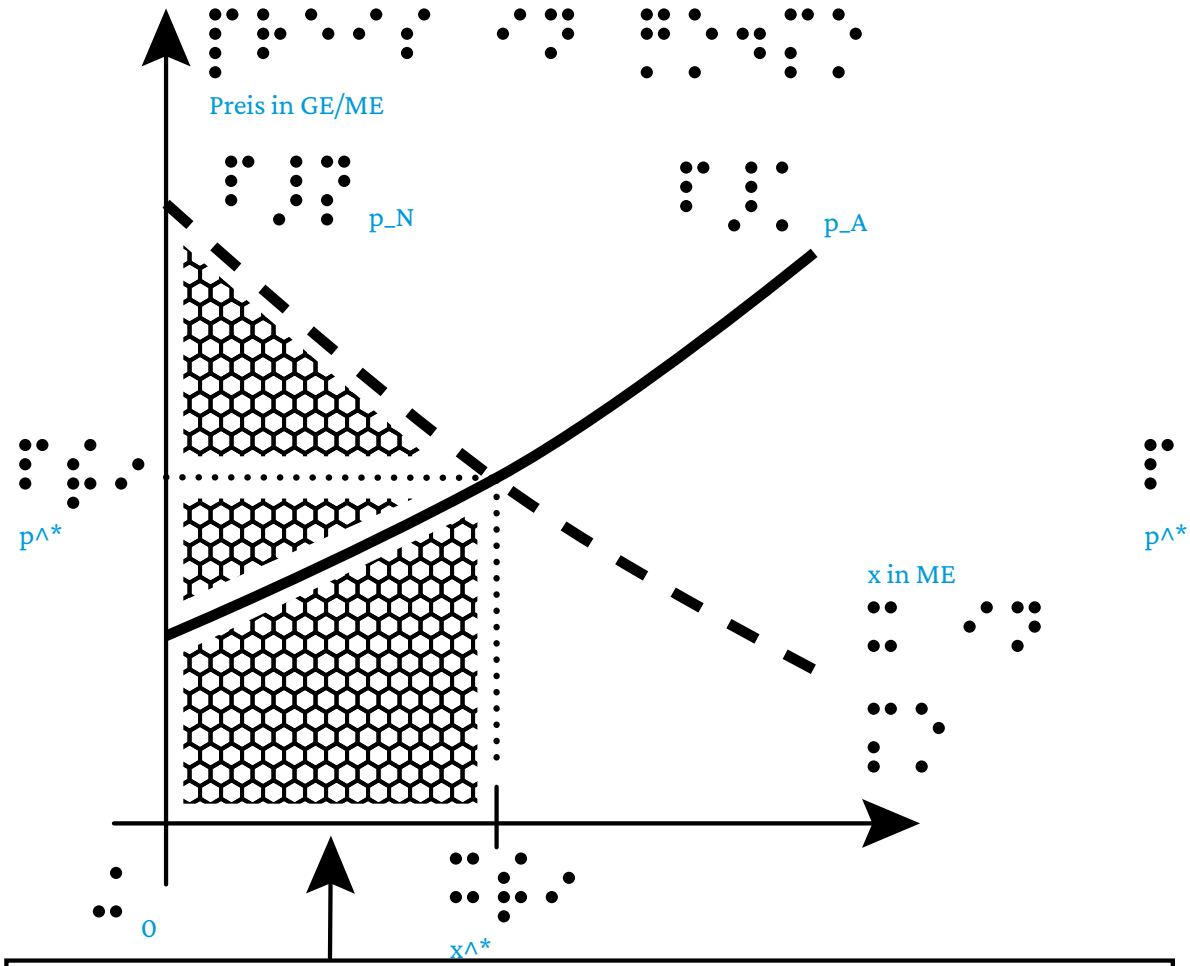

f(x)
f
x
0
1
2
3
4
5
6


momentane Änderungsrate der Länge in cm pro Monat
3
2
1
0
Alter in Monaten
2
4
6
8
10
12
14
16
18
20
22

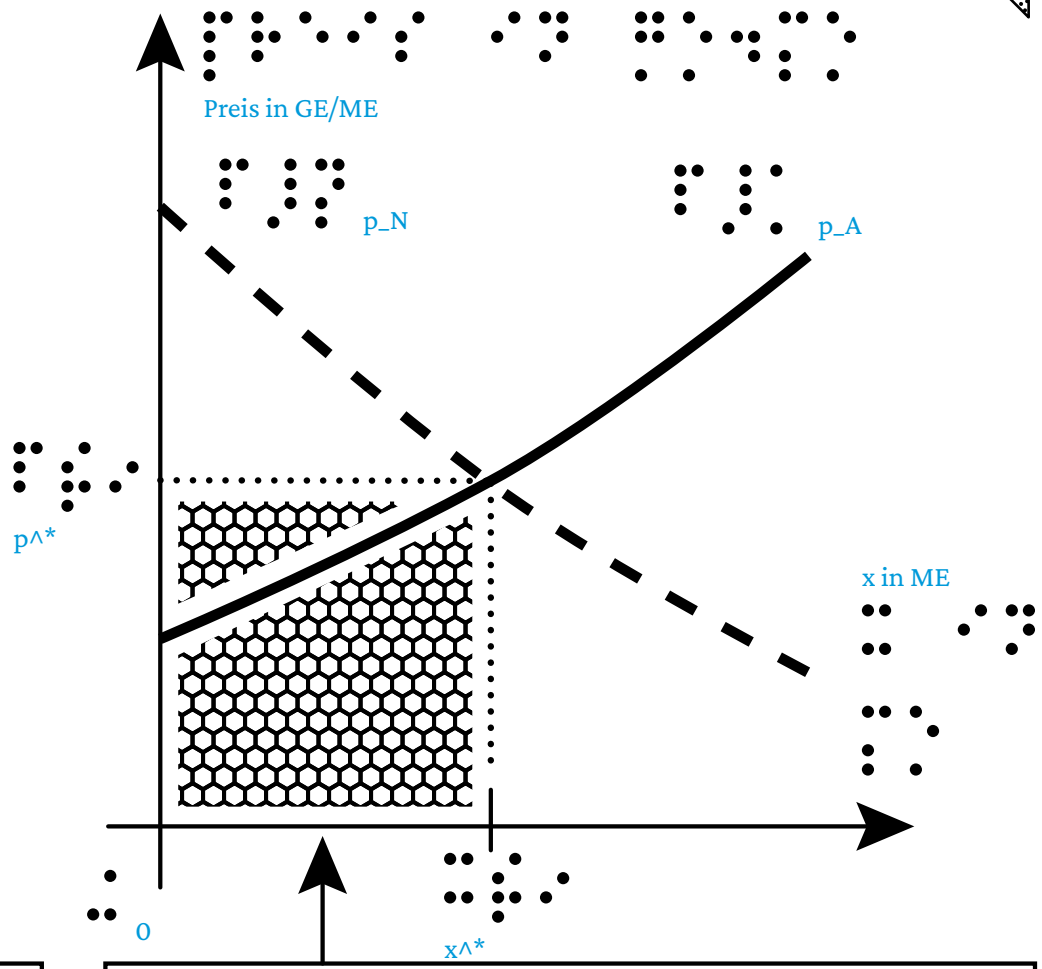

50000
40000
30000
20000
10000
R
Zeit in Jahren
0
1
2
3
4
5
6
7
8
9
10
11
12


Zahlungsstrom
Teilzahlung R(t) *dt
R
abzinsen
*e^(-i_s *t)
R(t)
Zeit
0
dt
T

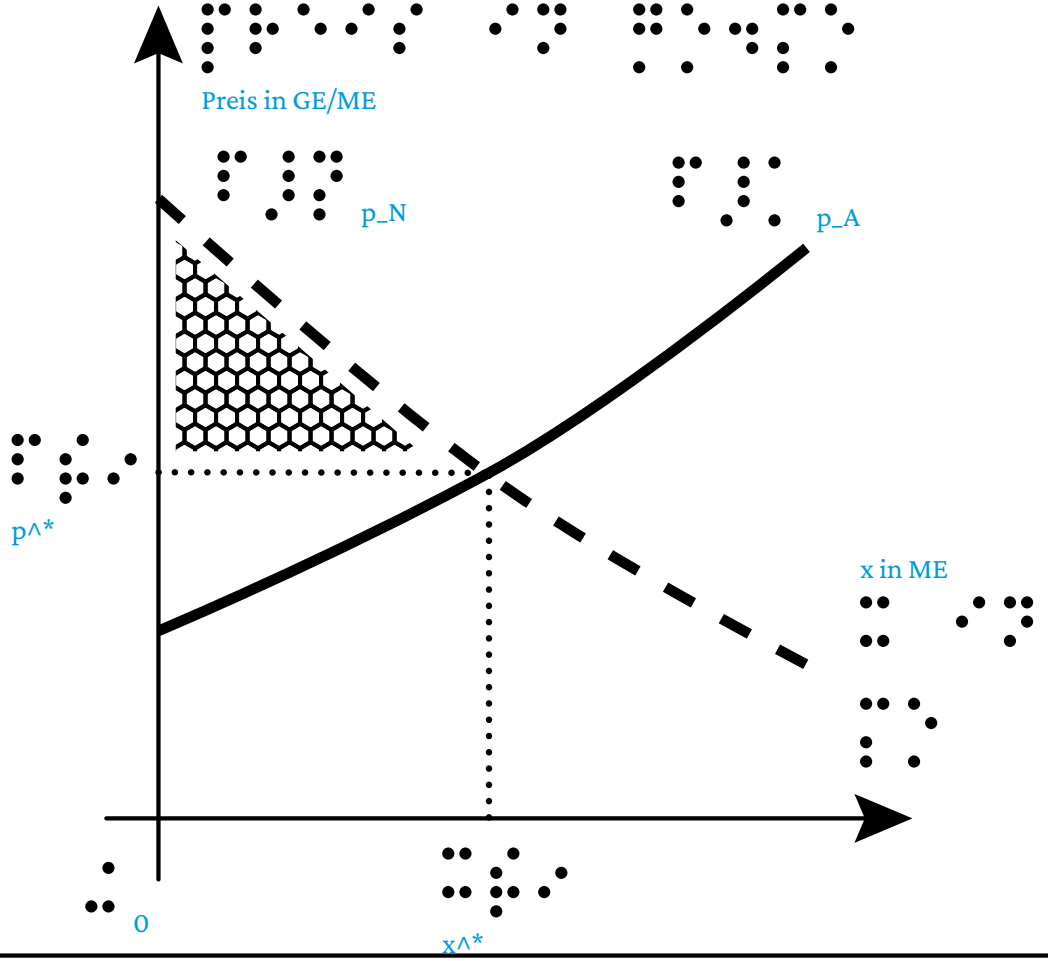

Preis in GE/ME
p_N
p_A
p^*
x in ME
=Gleichgewichtspreis
0
x^* =Gleichgewichtsmenge



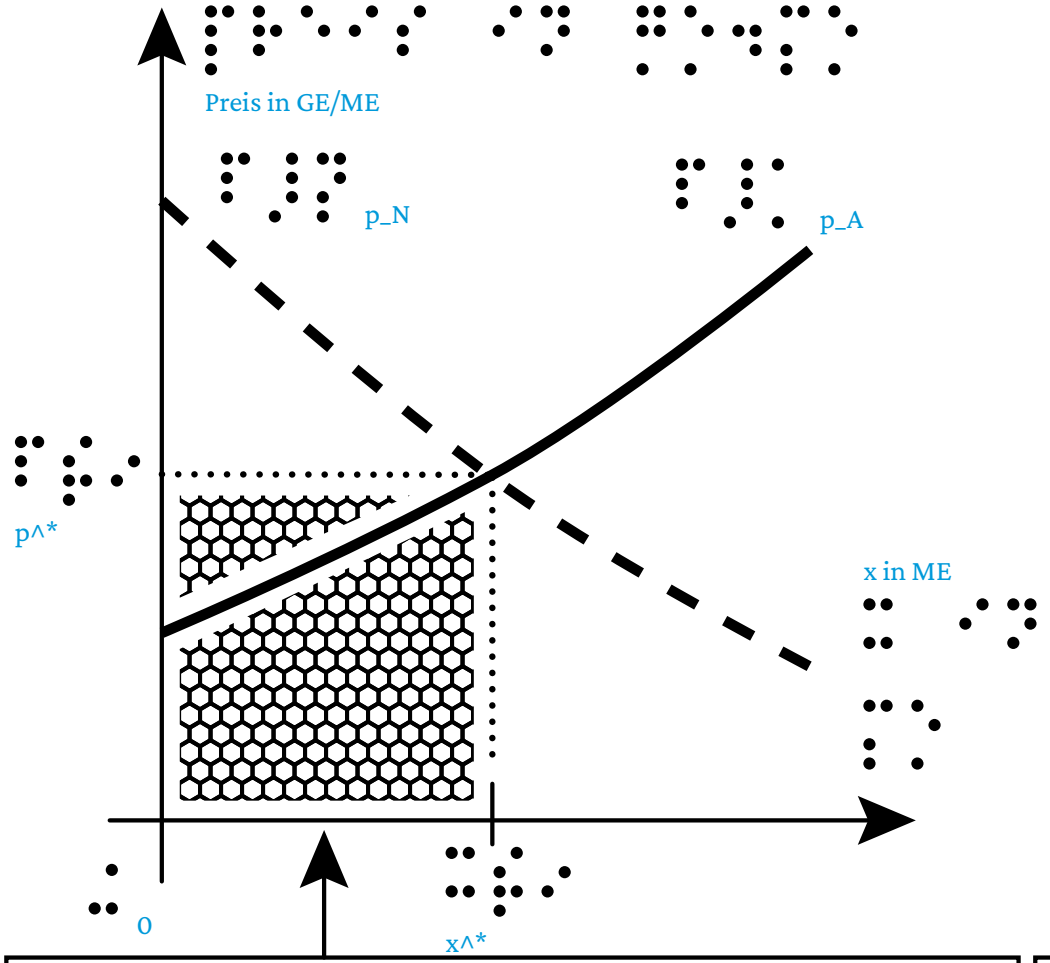
Konsumenten sind bereit 'int[0, x^*](p_N(x) 'dx) zu zahlen



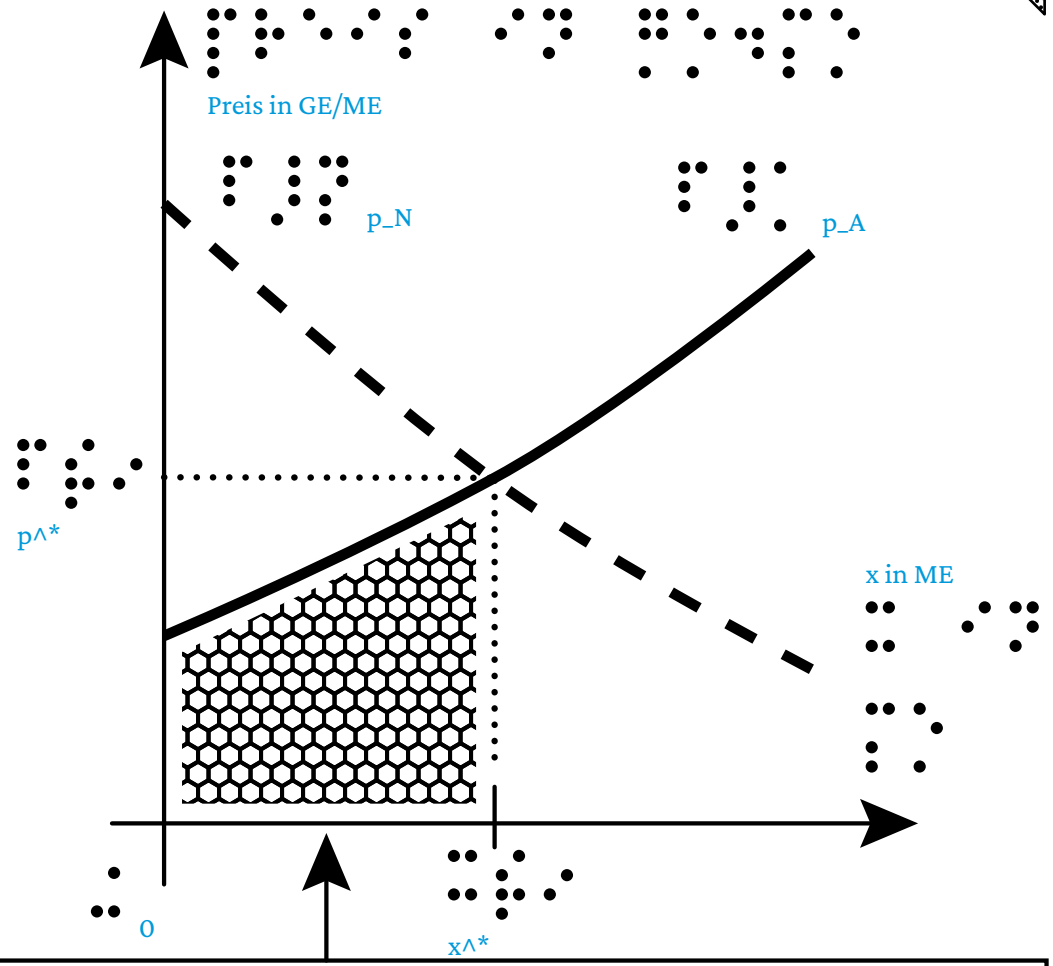
Tatsächlich zahlen sie aber nur p^* *x^*



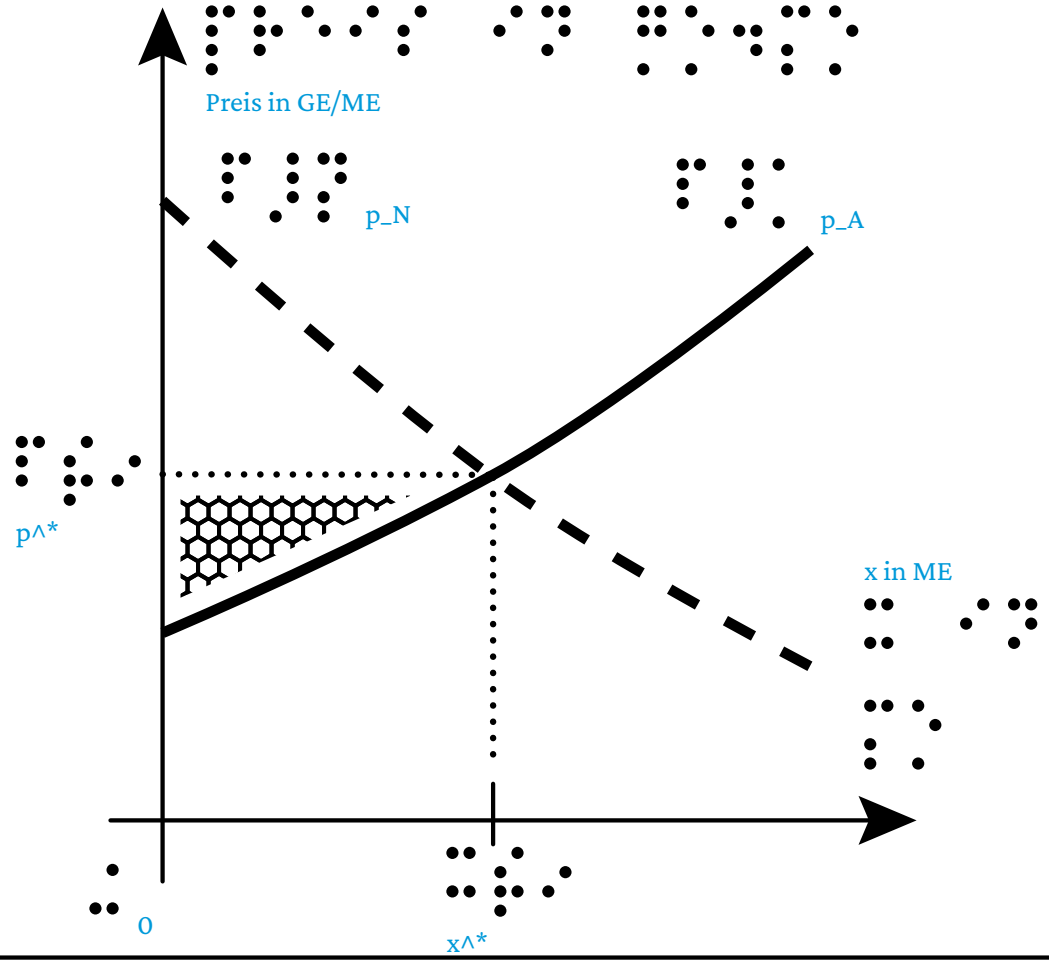
Konsumenten sparen also: 'int[0, x^*](p_N(x) -p^* *x^*. Das nennt man Konsumentenrente



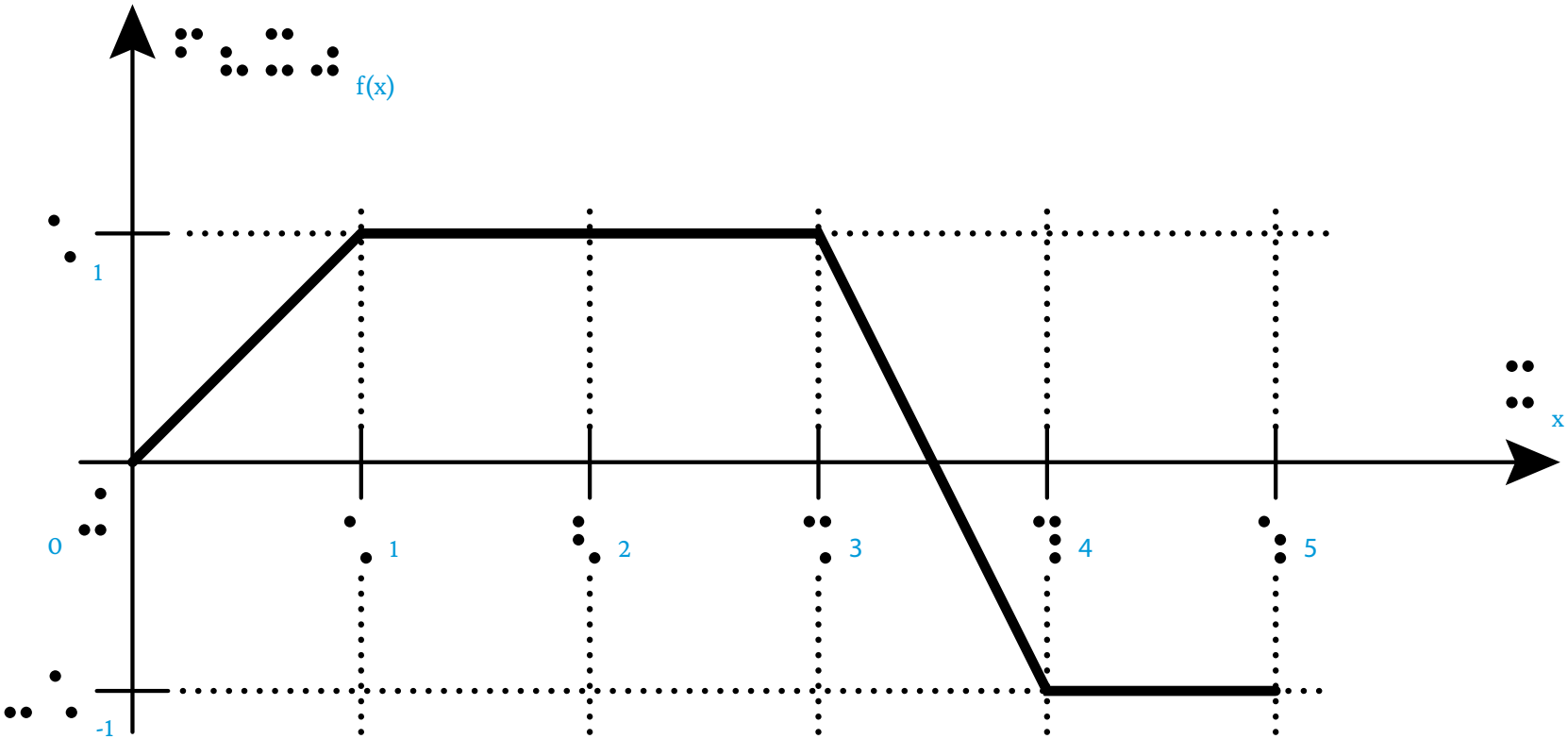
Produzenten erzielen einen Erlös von p^* *x^*

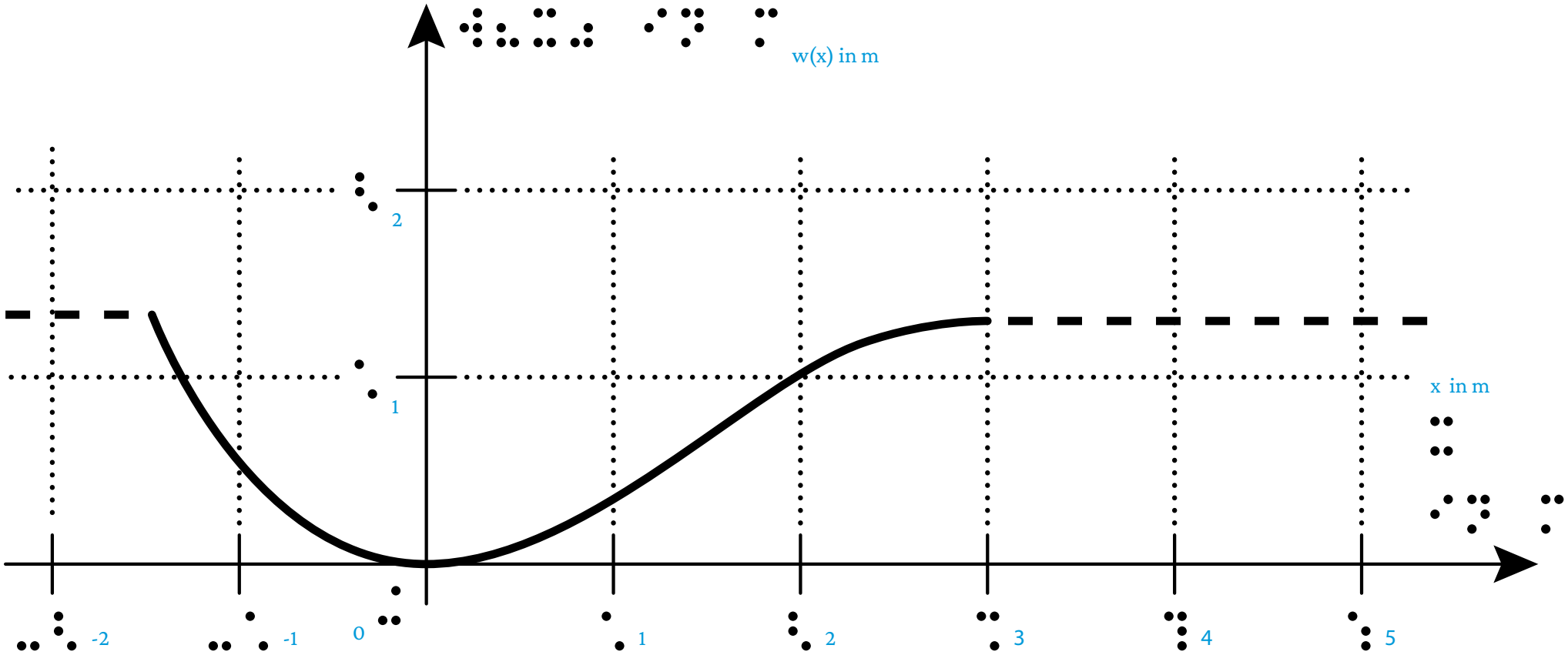


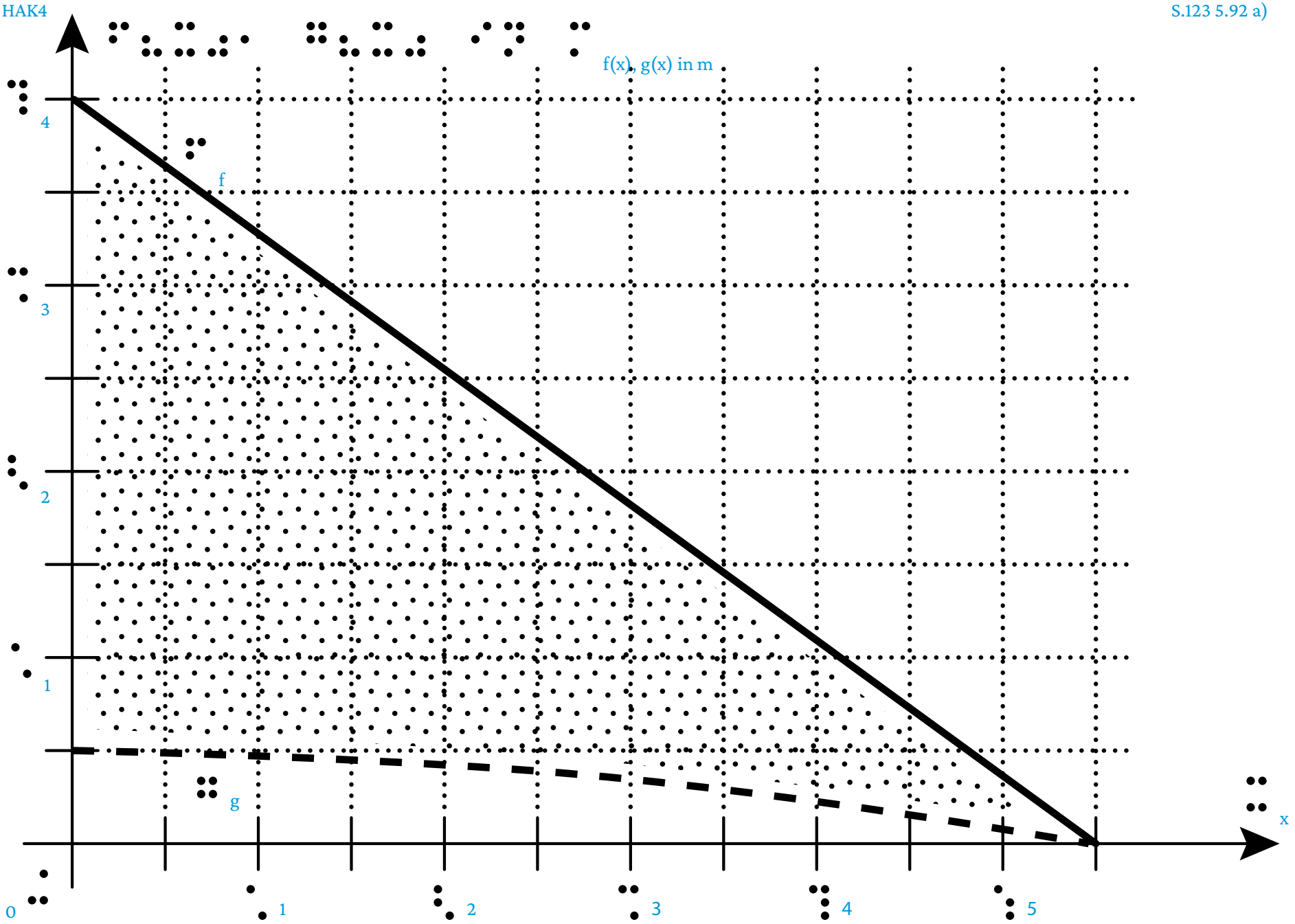
Würden aber Erlös von 'int [0, x^*](p_N(x) 'dx) akzeptieren



Produzenten haben somit p^* *x^* -'int[0, x^*](p_N(x) gespart. Das nennt man Produzentenrente.

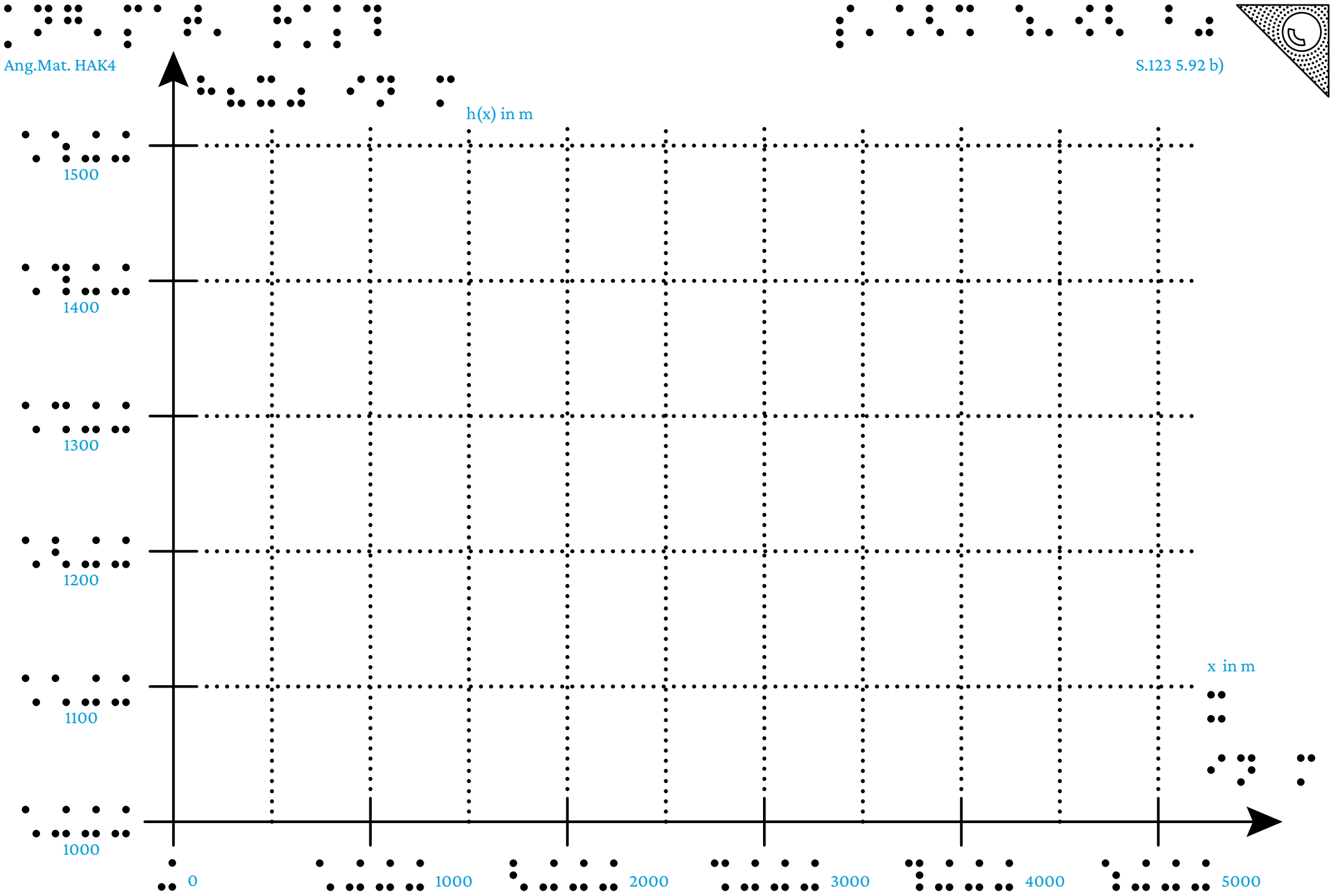

f(x)
1
0
-1
1
2
3
4
5
x

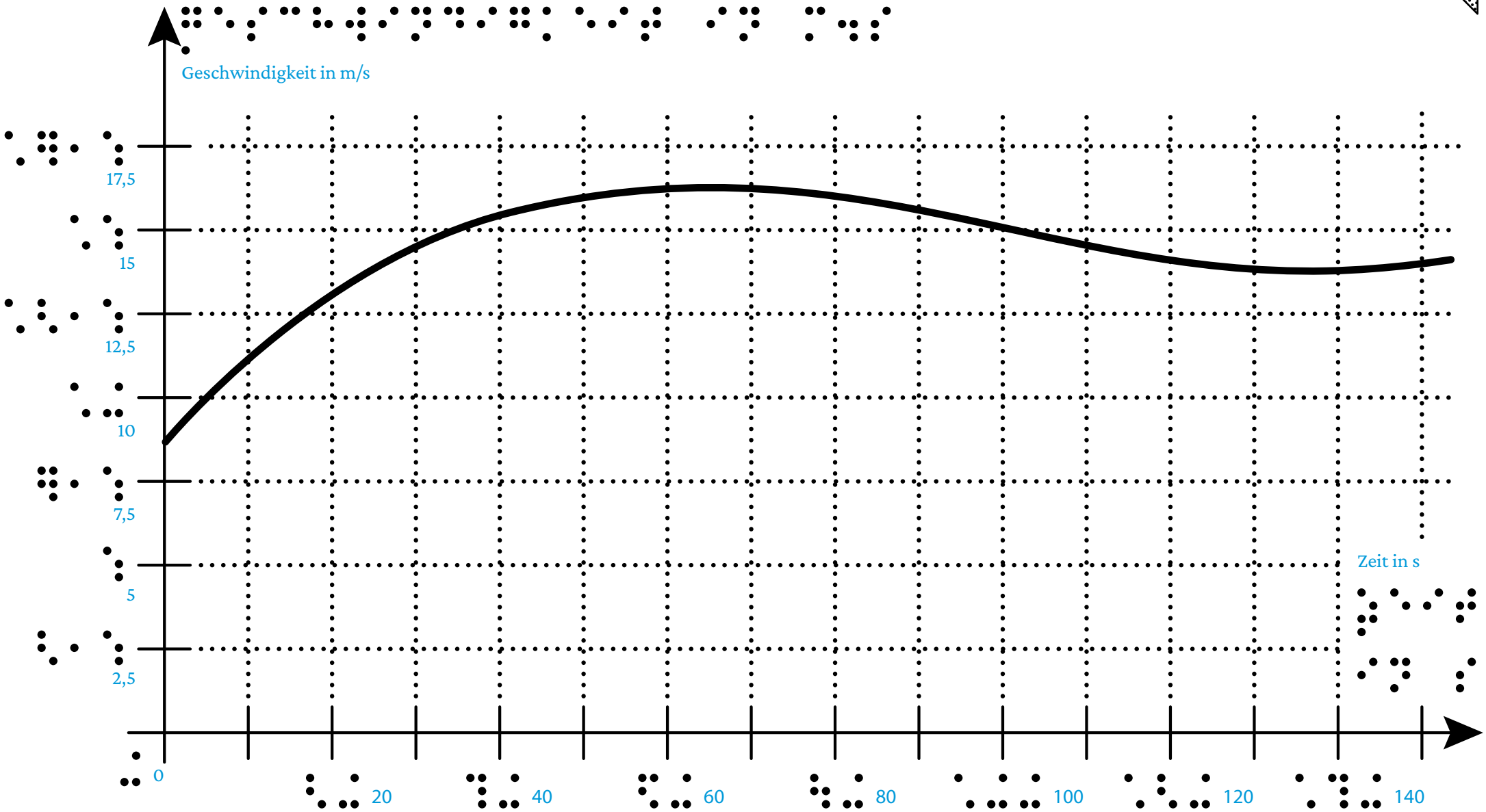

w(x) in m
2
1
x in m
-2
-1
0
1
2
3
4
5

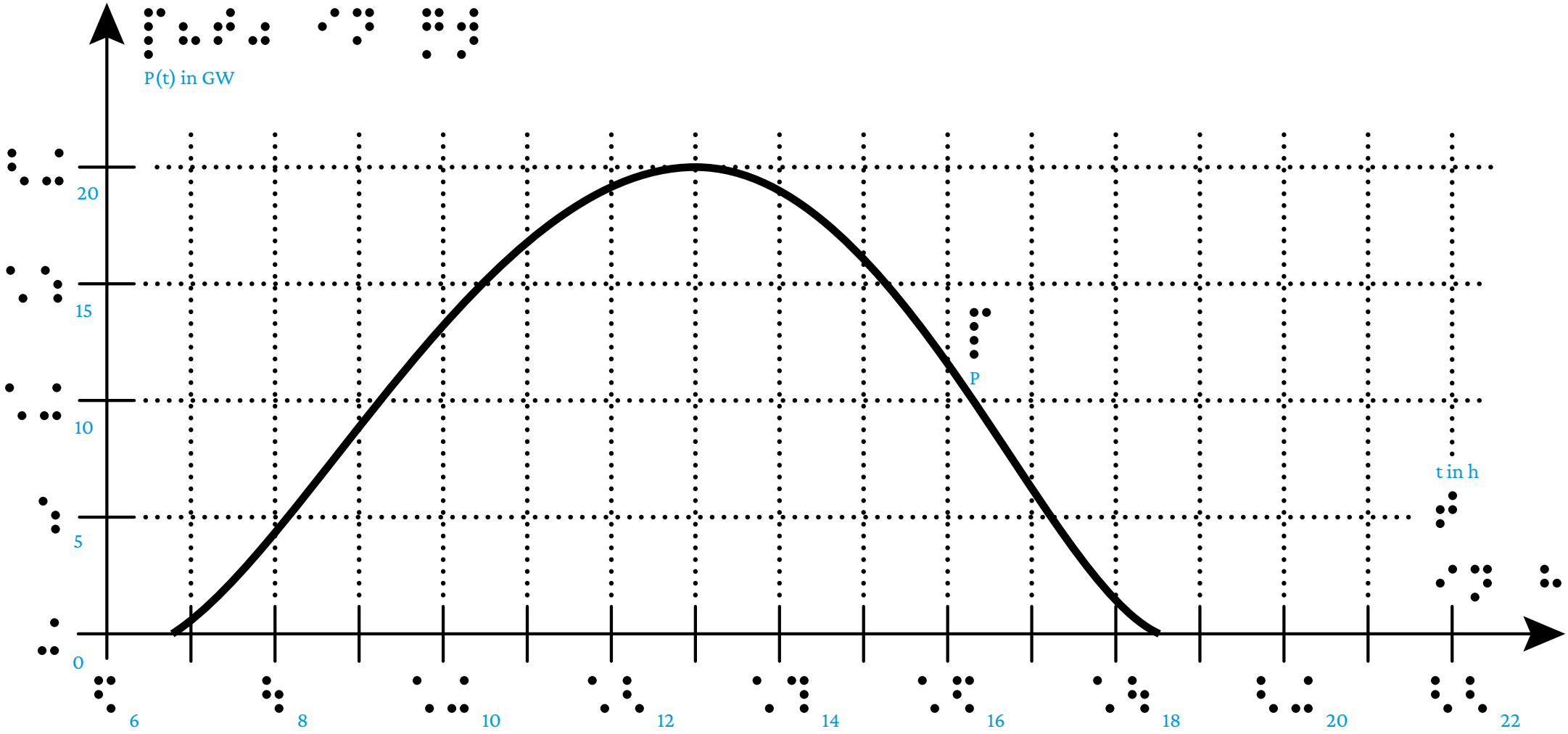

f(x), g(x) in m
4
3
2
1
f
g
x
0
1
2
3
4
5

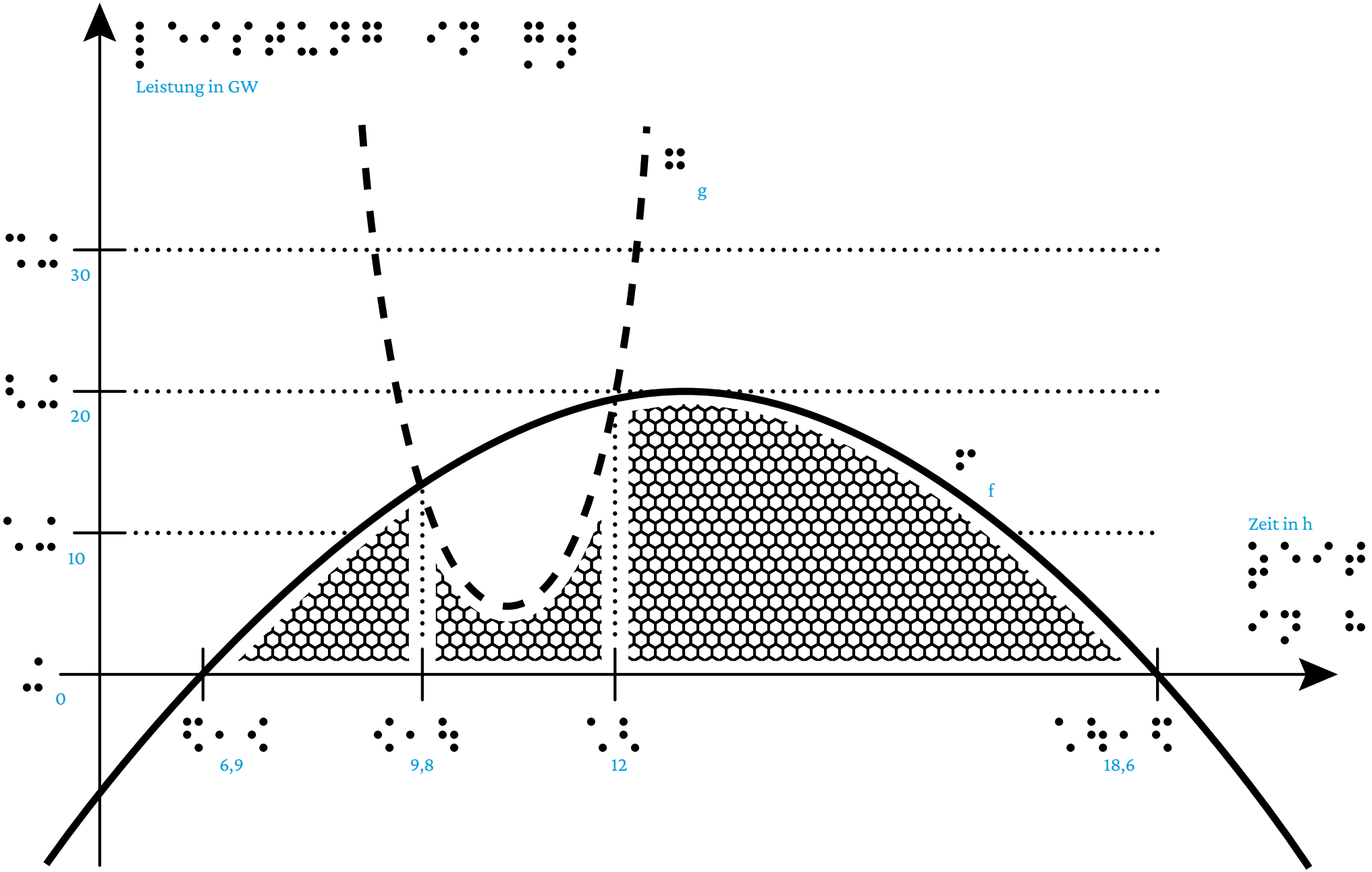
Ang.Mat. HAK4

S.123 5.92 b)




Geschwindigkeit in m/s
17,5
15
12,5
10
7,5
5
2,5
0
20
40
60
80
100
120
140
Zeit in s


P(t) in GW
20
15
10
5
0
P
t in h
6
8
10
12
14
16
18
20
22


Leistung in GW
g
30
20
f
10
Zeit in h
0
6,9
9,8
12
18,6